Stability of Inertial Neural Network with Time-Varying Delays Via Sampled-Data Control

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Abstract

In this paper, the stability problem is studied for inertial neural network with time-varying delays. The sampled-data control method is employed for the system design. First, by choosing a proper variable substitution, the original system is transformed into first-order differential equations. Then, an input delay approach is applied to deal with the stability of sampling system. Based on the Lyapunov function method, several sufficient conditions are derived to guarantee the global stability of the equilibrium. Furthermore, when employing an error-feedback control term to the slave neural network, parallel criteria regarding to the synchronization of the master neural network are also generated. Finally, some examples are given to illustrate the theoretical results.

Keywords Inertial neural network · Sampled-data control · Time-varying delays · Stability · Synchronization

1 Introduction

Recurrent neural networks (RNNS), especially Hopfield neural networks, cellular neural networks (CNNs), Cohen–Grossberg neural networks (CGNNs), bidirectional associative memory (BAM) neural networks have been extensively investigated during the past years, since their potential application in the fields of pattern recognition, associative memories, signal processing, fixed-point computations, and so on. In the design of neural networks, the dynamical properties of networks, such as the stability of the networks, play important roles. And there have been many literatures on the stability of neural networks [1–14].

It has been confirmed that the time delay, which is an inherent feature of signal transmission between neurons, is one of the main sources for causing oscillation, divergence or instability of neural networks. Therefore, stability analysis for neural networks with time

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delays has been an attractive subject of research in the past few years [1–21]. Noticing that lots of previous studies mainly focused on neural networks with only first derivative of the states, whereas it is also of significant importance to introduce an inertial term, or equivalently, the influence of inductance, into the artificial neural networks, because the inertial term is considered to be a critical tool to generate complicated bifurcation behavior and chaos [22,23]. Babcock and Westervelt pointed out that dynamics may become complexity when the neuron couplings contain an inertial nature [24]. Therefore, the dynamical characteristics of the inertial neural network have been investigated extensively [15–20,25–27]. For instance, resorting to homeomorphism and inequality technique, the stability of inertial Cohen–Grossberg neural networks with time delays was studied [16,17]. In [18], the matrix measure strategies were used to analyze the stability and synchronization of inertial BAM neural network with time delays. Stability of inertial BAM neural network with time-varying delay was studied via impulsive control [19].

As we all know, external forcing or stimuli plays a very important role in nonlinear systems. By adding certain types of forcing, rich phenomena can be observed in physical, ecological systems. Dynamic features including stability types can be greatly affected by external forcing. For master–slave neural networks, some kind of control term can be added to the slave system in order to guarantee the synchronization between the master system and the slave system. There are many control strategies such as impulsive control [7–9,19,25], feedback control [10,28], intermittent control [11,12,15], sampled-data control [13,14] and so on to stabilize the RNNs. To be noted that the sampled-data control drastically reduces the amount of transmitted information and increases the efficiency of bandwidth usage, because its control signals are kept constant during the sampling period and are allowed to change only at the sampling instant. On the other hand, due to the rapid growth of the digital hardware technologies, the sampled-data control method is more efficient, secure and useful. Due to these advantages, there are many results on the sampled-data control of finite-dimensional systems over the past decades [13,14,29–37]. For example, the stabilization of BAM neural networks with time-varying delay in the leakage terms was studied via sampled-data control [14]. Using sampled-data control, the asymptotical synchronization problem of Chaotic Lur’e systems with time delays was investigated in [31]. Based on the fuzzy-model-based control approach and LMI technique, several conditions were derived to guarantee the stability of the T-S fuzzy delayed neural networks with Markovian jumping parameters via sampled-data control [33].

Motivated by the above discussions, this paper mainly focuses on the fundamental problem of stability discussion of a class of time-varying delayed inertial neuron network via sampled-data control. To the best of our knowledge, this extension has not been reported in the literatures at the present stage. The main contributions of this paper are as follows. (1) We make the first attempt to address the sampled-data stability control problem for inertial neuron networks with time-varying delays. (2) Because sampled-data control signals are kept constant during the sampling period and are allowed to change only at the sampling instant, the control signals have discontinuous form. For the sake of discussion, the sampling system is changed into a continuous time-delay system by using the input delay approach. (3) A sampled-data controller is designed to achieve the synchronization of the master–slaver neural networks.

The remaining paper is organized as follows. Section 2 describes some preliminaries including definition, assumption and lemmas. The main result is stated in Sect. 3. The application concerning the synchronization of master–slave systems is investigated in Sect. 4. Section 5 gives some numerical examples to testify the theoretical analysis. Finally, conclusions are drawn in Sect. 6.

2 Preliminaries

For the sake of convenience, we introduce some notations as follows. R^n denotes the n -dimensional Euclidean space. The notation $X > Y$ ($X \geq Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive definite (positive semidefinite). The superscript “ T ” represents the transpose, and $\text{diag}\{\dots\}$ stands for a block-diagonal matrix. For a symmetry matrix, “ $*$ ” denotes the symmetric elements in it. Let $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ be the maximum and minimum eigenvalue of matrix P , respectively.

In this paper, we consider the following inertial neural network:

$$\begin{aligned} \frac{d^2 u_i(t)}{dt^2} = & -a_i \frac{du_i(t)}{dt} - b_i u_i(t) + \sum_{j=1}^n c_{ij} g_j(u_j(t)) \\ & + \sum_{j=1}^n d_{ij} g_j(u_j(t - \tau(t))) + I_i, \quad i = 1, 2, \dots, n, \end{aligned} \quad (2.1)$$

where $u_i(t)$ denotes the state variable of the i th neuron at time t , the second derivative of $u_i(t)$ is called an inertial term of system (2.1), $a_i > 0$, $b_i > 0$ are constants, c_{ij} , d_{ij} are constants and denote the connection weights of the neural network, $g_j(\cdot)$ denotes the neurons activation function of j th neuron at time t , $\tau(t)$ is the time varying delays which satisfies $0 \leq \tau(t) \leq \tau$, $\dot{\tau}(t) \leq \bar{\tau} < 1$, I_i is the external input on the i th neuron from the neural field.

Assumption 1 For any $x_1 \neq x_2 \in R$, the neuron activation functions $g_i(\cdot)$ satisfy

$$l_i^- \leq \frac{g_i(x_1) - g_i(x_2)}{x_1 - x_2} \leq l_i^+, \quad i = 1, 2, \dots, n, \quad (2.2)$$

with l_i^- , l_i^+ being constant scalars.

Denote $L^- = \text{diag}\{l_1^-, l_2^-, \dots, l_n^-\}$ and $L^+ = \text{diag}\{l_1^+, l_2^+, \dots, l_n^+\}$ for convenience.

Next, by introducing the following variable transformation:

$$v_i(t) = \frac{du_i(t)}{dt} + \xi_i u_i(t), \quad i = 1, 2, \dots, n,$$

the original neural network (2.1) can be written as

$$\begin{aligned} \frac{du_i(t)}{dt} = & -\xi_i u_i(t) + v_i(t), \\ \frac{dv_i(t)}{dt} = & -[b_i - \xi_i(a_i - \xi_i)]u_i(t) - (a_i - \xi_i)v_i(t) + \sum_{j=1}^n c_{ij} g_j(u_j(t)) \\ & + \sum_{j=1}^n d_{ij} g_j(u_j(t - \tau(t))) + I_i, \end{aligned} \quad (2.3)$$

for $i = 1, 2, \dots, n$.

Denote $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T$, $v(t) = (v_1(t), v_2(t), \dots, v_n(t))^T$, the network (2.3) can be written as

$$\begin{aligned} \frac{du(t)}{dt} = & -Au(t) + v(t), \\ \frac{dv(t)}{dt} = & -Au(t) - Bv(t) + Cg(u(t)) + Dg(u(t - \tau(t))) + I, \end{aligned} \quad (2.4)$$

where $\Lambda = \text{diag}\{\xi_1, \xi_2, \dots, \xi_n\}$, $A = \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_n\}$, $B = \text{diag}\{\beta_1, \beta_2, \dots, \beta_n\}$, $C = (c_{ij})_{n \times n}$, $D = (d_{ij})_{n \times n}$, $I = \text{diag}\{I_1, I_2, \dots, I_n\}$, $\alpha_i = b_i - \xi_i(a_i - \xi_i)$, $\beta_i = a_i - \xi_i$.

Remark 1 In order to get less conservative results, we can use the transformation: $v_i(t) = \zeta_i \frac{du_i(t)}{dt} + \xi_i u_i(t)$, $i = 1, 2, \dots, n$. Here, for simplicity, we only choose one variable to express the transformation.

Definition 1 The point (u^*, v^*) with $u^* = (u_1, u_2, \dots, u_n)^T$, $v^* = (v_1, v_2, \dots, v_n)^T$ is called an equilibrium of the neural network in (2.4) if

$$\begin{aligned} -\Lambda u^* + v^* &= 0, \\ -Au^* - Bv^* + Cg(u^*) + Dg(u^*) + I &= 0. \end{aligned} \quad (2.5)$$

Let (u^*, v^*) be the equilibrium point of (2.4). For the purpose of simplicity, we can shift the equilibrium point (u^*, v^*) to the origin by letting $x = u - u^*$, $y = v - v^*$, then the network (2.4) can be transformed into

$$\begin{aligned} \frac{dx(t)}{dt} &= -\Lambda x(t) + y(t), \\ \frac{dy(t)}{dt} &= -Ax(t) - By(t) + Cf(x(t)) + Df(x(t - \tau(t))), \end{aligned} \quad (2.6)$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$, $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T$ are the state vectors of the transformed networks. It follows from (2.2) that the function $f(x) = g(x + u^*) - g(u^*)$ satisfies

$$L^- \leq \frac{f(x)}{x} \leq L^+, \quad (2.7)$$

for any $x \neq 0 \in R$.

Remark 2 From (2.7), we can obtain $-[f(x(t)) - L^-x(t)]^T U_1 [f(x(t)) - L^+x(t)] \geq 0$ and $-[f(x(t - \tau(t))) - L^-x(t - \tau(t))]^T U_2 [f(x(t - \tau(t))) - L^+x(t - \tau(t))] \geq 0$, where U_1, U_2 are positive diagonal matrices.

In the following, we will consider the sampled-data stability problem of the neural network (2.4). Let $\mu(t) = Kx(t_k)$, $v(t) = My(t_k)$, where $K, M \in R^{n \times n}$ are the sampled-data feedback controller gain matrices to be designed. t_k denotes the sample time point and satisfies $0 = t_0 < t_1 < \dots < t_k < \dots$, and $\lim_{k \rightarrow +\infty} t_k = +\infty$. Moreover, we assume that there exists a positive constant h such that $t_{k+1} - t_k \leq h$, $\forall k \in N$.

Under the sampled-data control, (2.6) can be modeled as

$$\begin{aligned} \frac{dx(t)}{dt} &= -\Lambda x(t) + y(t) + \mu(t), \\ \frac{dy(t)}{dt} &= -Ax(t) - By(t) + Cf(x(t)) + Df(x(t - \tau(t))) + v(t). \end{aligned} \quad (2.8)$$

Let $h(t) = t - t_k$, for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} \frac{dx(t)}{dt} &= -\Lambda x(t) + y(t) + Kx(t - h(t)), \\ \frac{dy(t)}{dt} &= -Ax(t) - By(t) + Cf(x(t)) + Df(x(t - \tau(t))) + My(t - h(t)). \end{aligned} \quad (2.9)$$

The initial conditions of (2.9) are given as :

$$\begin{cases} x(t) = \varphi(t), & t \in [-\bar{h}, 0], \\ y(t) = \phi(t), & t \in [-h, 0], \end{cases} \quad (2.10)$$

where $\bar{h} = \max\{h, \tau\}$.

Remark 3 Based on the input delay approach [14,33,38], the system (2.8) is changed into a continuous-time system (2.9) because it is difficult to solve the stabilization problem of system (2.8) with the discrete term $\mu(t) = Kx(t_k)$, $v(t) = My(t_k)$.

In order to investigate the stabilization problem of the network (2.9), the following lemmas are needed.

Lemma 1 [39] For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t) : [a, b] \rightarrow R$ such that the integrals concerned are well defined, then the following inequality holds:

$$\left[\int_a^b x(s) ds \right]^T M \left[\int_a^b x(s) ds \right] \leq (b-a) \int_a^b x^T(s) M x(s) ds.$$

Lemma 2 [19] If $P \in R^{n \times n}$ is a symmetric positive definite matrix and $Q \in R^{n \times n}$ is a symmetric matrix, then

$$\lambda_{\min}(P^{-1}Q)x^T Px \leq x^T Qx \leq \lambda_{\max}(P^{-1}Q)x^T Px, \quad x \in R^n.$$

Lemma 3 [40] If $\omega(t) : [t_0, +\infty) \rightarrow R$ is uniformly continuous, and $\int_{t_0}^t \omega(s) ds$ has a finite limit as $t \rightarrow +\infty$, then $\lim_{t \rightarrow +\infty} \omega(t) = 0$.

3 Main Results

In this section, we shall establish some sufficient conditions to ensure the stability of network (2.9).

Theorem 1 Under Assumption 1, if there exist positive-definite matrices $P_i (i = 1, \dots, 9)$, $Q_i (i = 1, 2)$, positive-definite diagonal matrices $U_i (i = 1, 2)$ and any matrices X, Y such that the following LMI holds:

$$\Pi = \begin{bmatrix} \pi_{1,1} & \pi_{1,2} & \pi_{1,3} & 0 & \pi_{1,5} & 0 & \pi_{1,7} & \pi_{1,8} & 0 & 0 & \pi_{1,11} & 0 \\ * & \pi_{2,2} & 0 & 0 & \pi_{2,5} & 0 & \pi_{2,7} & 0 & 0 & 0 & 0 & 0 \\ * & * & \pi_{3,3} & \pi_{3,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \pi_{3,12} \\ * & * & * & \pi_{4,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \pi_{5,5} & \pi_{5,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \pi_{6,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \pi_{7,7} & \pi_{7,8} & \pi_{7,9} & 0 & \pi_{7,11} & \pi_{7,12} \\ * & * & * & * & * & * & * & \pi_{8,8} & \pi_{8,9} & 0 & \pi_{8,11} & \pi_{8,12} \\ * & * & * & * & * & * & * & * & \pi_{9,9} & \pi_{9,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \pi_{10,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \pi_{11,11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \pi_{12,12} \end{bmatrix} < 0 \quad (3.11)$$

where

$$\begin{aligned}
 \pi_{1,1} &= P_3 - P_4 + P_5 - P_7 - (1 - \bar{\tau})P_9 - L^-U_1L^+ - 2Q_1A, \\
 \pi_{1,2} &= P_1 - Q_1 - Q_1A, \pi_{1,3} = P_4 + (1 - \bar{\tau})P_9, \pi_{1,5} = P_7 + X, \pi_{1,7} = Q_1 - Q_2A, \\
 \pi_{1,8} &= -Q_2A, \pi_{1,11} = \frac{1}{2}U_1(L^- + L^+), \\
 \pi_{2,2} &= \tau^2P_4 + h^2P_7 + \tau^2P_9 - 2Q_1, \pi_{2,5} = X, \pi_{2,7} = Q_1, \\
 \pi_{3,3} &= -(1 - \bar{\tau})P_3 - 2P_4 - (1 - \bar{\tau})P_9 - L^-U_2L^+, \pi_{3,4} = P_4, \\
 \pi_{3,12} &= \frac{1}{2}U_2(L^- + L^+), \\
 \pi_{4,4} &= -P_4, \\
 \pi_{5,5} &= -2P_7, \pi_{5,6} = P_7, \\
 \pi_{6,6} &= -P_5 - P_7, \\
 \pi_{7,7} &= P_6 - P_8 - 2Q_2B, \pi_{7,8} = P_2 - Q_2 - Q_2B, \pi_{7,9} = P_8 + Y, \\
 \pi_{7,11} &= Q_2C, \pi_{7,12} = Q_2D, \\
 \pi_{8,8} &= h^2P_8 - 2Q_2, \pi_{8,9} = Y, \pi_{8,11} = Q_2C, \pi_{8,12} = Q_2D, \\
 \pi_{9,9} &= -2P_8, \pi_{9,10} = P_8, \\
 \pi_{10,10} &= -P_6 - P_8, \\
 \pi_{11,11} &= -U_1, \\
 \pi_{12,12} &= -U_2,
 \end{aligned}$$

then the equilibrium of network (2.9) is globally asymptotically stable. Moreover, the desired controller gain matrices are given as $K = Q_1^{-1}X$, $M = Q_2^{-1}Y$.

Proof Consider the following Lyapunov-Krasovskii functional :

$$V(t) = \sum_{i=1}^6 V_i(t) \quad (3.12)$$

where

$$\begin{aligned}
 V_1(t) &= x^T(t)P_1x(t) + y^T(t)P_2y(t), \\
 V_2(t) &= \int_{t-\tau(t)}^t x^T(s)P_3x(s)ds, \\
 V_3(t) &= \tau \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)P_4\dot{x}(s)dsd\theta, \\
 V_4(t) &= \int_{t-h}^t x^T(s)P_5x(s)ds + \int_{t-h}^t y^T(s)P_6y(s)ds, \\
 V_5(t) &= h \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s)P_7\dot{x}(s)dsd\theta + h \int_{-h}^0 \int_{t+\theta}^t \dot{y}^T(s)P_8\dot{y}(s)dsd\theta, \\
 V_6(t) &= \tau \int_{-\tau(t)}^0 \int_{t+\theta}^t \dot{x}^T(s)P_9\dot{x}(s)dsd\theta.
 \end{aligned}$$

Then take the derivative of $V(t)$ along the system (2.9),

$$\dot{V}_1(t) = 2x^T(t)P_1\dot{x}(t) + 2y^T(t)P_2\dot{y}(t), \quad (3.13)$$

$$\dot{V}_2(t) \leq x^T(t)P_3x(t) - (1 - \bar{\tau})x^T(t - \tau(t))P_3x(t - \tau(t)), \quad (3.14)$$

$$\begin{aligned}
 \dot{V}_3(t) &= \tau^2\dot{x}^T(t)P_4\dot{x}(t) - \tau \int_{t-\tau}^t \dot{x}^T(s)P_4\dot{x}(s)ds, \\
 &\leq \tau^2\dot{x}^T(t)P_4\dot{x}(t) - 2x^T(t - \tau(t))P_4x(t - \tau(t)) + 2x^T(t - \tau(t))P_4x(t - \tau) \\
 &\quad - x^T(t - \tau)P_4x(t - \tau) - x^T(t)P_4x(t) + 2x^T(t)P_4x(t - \tau(t))
 \end{aligned} \quad (3.15)$$

$$\dot{V}_4(t) = x^T(t)P_5x(t) - x^T(t-h)P_5x(t-h) + y^T(t)P_6y(t) - y^T(t-h)P_6y(t-h), \quad (3.16)$$

$$\begin{aligned} \dot{V}_5(t) &= h^2 \dot{x}^T(t)P_7\dot{x}(t) - h \int_{t-h}^t \dot{x}^T(s)P_7\dot{x}(s)ds + h^2 \dot{y}^T(t)P_8\dot{x}(t) \\ &\quad - h \int_{t-h}^t \dot{x}^T(s)P_8\dot{x}(s)ds \\ &\leq h^2 \dot{x}^T(t)P_7\dot{x}(t) - 2x^T(t-h(t))P_7x(t-h(t)) + 2x^T(t-h(t))P_7x(t-h) \\ &\quad - x^T(t-h)P_7x(t-h) - x^T(t)P_7x(t) + 2x^T(t)P_7x(t-h(t)) \\ &\quad + h^2 \dot{y}^T(t)P_8\dot{y}(t) - 2y^T(t-h(t))P_8y(t-h(t)) + 2y^T(t-h(t))P_8y(t-h) \\ &\quad - y^T(t-h)P_8y(t-h) - y^T(t)P_8y(t) + 2y^T(t)P_8y(t-h(t)), \end{aligned} \quad (3.17)$$

$$\begin{aligned} \dot{V}_6(t) &\leq \tau^2 \dot{x}^T(t)P_9\dot{x}(t) - \tau(1-\bar{\tau}) \int_{t-\tau(t)}^t \dot{x}^T(s)P_9\dot{x}(s)ds \\ &\leq \tau^2 \dot{x}^T(t)P_9\dot{x}(t) - (1-\bar{\tau})x^T(t)P_9x(t) + 2(1-\bar{\tau})x^T(t)P_9x(t-\tau(t)) \\ &\quad - (1-\bar{\tau})x^T(t-\tau(t))P_9x(t-\tau(t)). \end{aligned} \quad (3.18)$$

In addition,

$$\begin{aligned} 0 &= 2[x^T(t) + \dot{x}^T(t)]Q_1[-\dot{x}(t) + \dot{x}(t)] \\ &= 2[x^T(t) + \dot{x}^T(t)]Q_1[-\dot{x}(t) - \Lambda x(t) + y(t) + Kx(t-h(t))] \\ &= -2x^T(t)Q_1\dot{x}(t) - 2x^T(t)Q_1\Lambda x(t) + 2x^T(t)Q_1y(t) + 2x^T(t)Q_1Kx(t-h(t)) \\ &\quad - 2\dot{x}^T(t)Q_1\dot{x}(t) - 2\dot{x}^T(t)Q_1\Lambda x(t) + 2\dot{x}^T(t)Q_1y(t) + 2\dot{x}^T(t)Q_1Kx(t-h(t)). \end{aligned} \quad (3.19)$$

$$\begin{aligned} 0 &= 2[y^T(t) + \dot{y}^T(t)]Q_2[-\dot{y}(t) + \dot{y}(t)] \\ &= 2[y^T(t) + \dot{y}^T(t)]Q_2[-\dot{y}(t) - \Lambda x(t) - By(t) + Cf(x(t)) + Df(x-\tau(t)) \\ &\quad + My(t-h(t))] \\ &= -2y^T(t)Q_2\dot{y}(t) - 2y^T(t)Q_2\Lambda x(t) - 2y^T(t)Q_2By(t) + 2y^T(t)Q_2Cf(x(t)) \\ &\quad + 2y^T(t)Q_2Df(x(t-\tau(t))) + 2y^T(t)Q_2My(t-h(t)) \\ &\quad - 2\dot{y}^T(t)Q_2\dot{y}(t) - 2\dot{y}^T(t)Q_2\Lambda x(t) - 2\dot{y}^T(t)Q_2By(t) + 2\dot{y}^T(t)Q_2Cf(x(t)) \\ &\quad + 2\dot{y}^T(t)Q_2Df(x(t-\tau(t))) + 2\dot{y}^T(t)Q_2My(t-h(t)). \end{aligned} \quad (3.20)$$

$$\begin{aligned} 0 &\leq -[f(x(t)) - L^-x(t)]^T U_1[f(x(t)) - L^+x(t)] \\ &= -f^T(x(t))U_1f(x(t)) + x^T(t)U_1(L^- + L^+)f(x(t)) - x^T(t)L^-U_1L^+x(t). \end{aligned} \quad (3.21)$$

$$\begin{aligned} 0 &\leq -[f(x(t-\tau(t))) - L^-x(t-\tau(t))]^T U_2[f(x(t-\tau(t))) - L^+x(t-\tau(t))] \\ &= -f^T(x(t-\tau(t)))U_2f(x(t-\tau(t))) + x^T(t-\tau(t))U_2(L^- + L^+)f(x(t-\tau(t))) \\ &\quad - x^T(t-\tau(t))L^-U_2L^+x(t-\tau(t)). \end{aligned} \quad (3.22)$$

Based on (3.13)–(3.22), let

$$\begin{aligned} \xi(t) &= \left[x^T(t), \dot{x}^T(t), x^T(t-\tau(t)), x^T(t-\tau), x^T(t-h(t)), x^T(t-h), \right. \\ &\quad \left. y^T(t), \dot{y}^T(t), y^T(t-h(t)), y^T(t-h), f^T(x(t)), f^T(x(t-\tau(t))) \right]^T \end{aligned}$$

we have

$$\dot{V}(t) \leq \xi^T(t)\Pi\xi(t). \quad (3.23)$$

From (3.23), it can be seen that

$$V(t) - \int_0^t \xi^T(s) \Pi \xi(s) ds \leq V(0), \quad t \geq 0. \quad (3.24)$$

Moreover,

$$\begin{aligned} V(0) &\leq [\lambda_{\max}(P_1) + \tau \lambda_{\max}(P_3) + \frac{1}{2} \tau^3 \lambda_{\max}(P_4) + h \lambda_{\max}(P_5) + \frac{1}{2} h^3 \lambda_{\max}(P_7) \\ &\quad + \frac{1}{2} \tau^3 \lambda_{\max}(P_9)] \Phi^2 + [\lambda_{\max}(P_2) + h \lambda_{\max}(P_6) + \frac{1}{2} h^3 \lambda_{\max}(P_8)] \Psi^2 \\ &< +\infty, \end{aligned} \quad (3.25)$$

where

$$\begin{aligned} \Phi &= \max \{ \sup_{t \in [-h, 0]} \|\varphi(t)\|, \sup_{t \in [-h, 0]} \|\dot{\varphi}(t)\| \}, \\ \Psi &= \max \{ \sup_{t \in [-h, 0]} \|\phi(t)\|, \sup_{t \in [-h, 0]} \|\dot{\phi}(t)\| \}. \end{aligned}$$

On the other hand, by the definition of $V(t)$, we get

$$\begin{aligned} V(t) &\geq x^T(t) P_1 x(t) + y^T(t) P_2 y(t) \\ &\geq \lambda_{\min}(P_1) x^T(t) x(t) + \lambda_{\min}(P_2) y^T(t) y(t) \\ &= \lambda_{\min}(P_1) \|x(t)\|^2 + \lambda_{\min}(P_2) \|y(t)\|^2 \\ &\geq \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\} [\|x(t)\|^2 + \|y(t)\|^2]. \end{aligned} \quad (3.26)$$

Then, combining (3.24), (3.25) and (3.26), one has

$$\|x(t)\|^2 + \|y(t)\|^2 \leq \frac{V(0)}{\min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}} < +\infty \quad (3.27)$$

which demonstrates that the solution of (2.9) is uniformly bound on $[0, +\infty)$. Next, we shall prove that $(\|x(t)\|, \|y(t)\|) \rightarrow (0, 0)$ as $t \rightarrow +\infty$. On the one hand, the boundedness of $\|\dot{x}(t)\|$ and $\|\dot{y}(t)\|$ can be deduced from (2.9) and (3.27). On the other hand, from (3.23), we have

$$\dot{V}(t) \leq \xi^T(t) \Pi \xi(t) \leq \lambda_{\min}(\Pi) (x^T(t) x(t) + y^T(t) y(t)) \quad (3.28)$$

So, $\int_0^t x^T(s) x(s) ds < +\infty$ and $\int_0^t y^T(s) y(s) ds < +\infty$. In view of Lemma 3, we have $(\|x(t)\|, \|y(t)\|) \rightarrow (0, 0)$. That is, the equilibrium point of the system (2.9) is globally asymptotically stable. $\square$

Remark 4 Let $-L^- = L^+ > 0$ in Assumption 1, we can obtain the corresponding result for inertial neural network with Lipschitz-type activation function which was discussed in [18, 19]. Thus, our results possess better application.

Remark 5 In Theorem 1, the matrix Λ determines whether the condition (3.11) can hold, in which different $\xi_i (i = 1, 2, \dots, n)$ represents different variable transformation. Comparing with a certain variable substitution, Theorem 1 is less conservative by introducing a free-weight matrix Λ .

Remark 6 The construction of appropriate Lyapunov function is one of difficulties of the Lyapunov theory. The Lyapunov function we made is dependent on the time delays, which results in our conclusions less conservative.

Remark 7 Synchronization, which means that the dynamical behaviors of coupled systems obtain the same time spatial state. In real world, stability and synchronization of chaotic systems are very important in consideration of its potential applications in different kinds of areas including secure communication, biology system, optics, information processing and so on [37]. Therefore, based on the proposed results, the synchronization problem of the master–slave neural network is discussed below.

4 Application

In this section, the precious results will be applied to the master–slave synchronization problem.

The master neural network is given as

$$\begin{aligned} \frac{d^2 u_i(t)}{dt^2} = & -a_i \frac{du_i(t)}{dt} - b_i u_i(t) + \sum_{j=1}^n c_{ij} g_j(u_j(t)) \\ & + \sum_{j=1}^n d_{ij} g_j(u_j(t - \tau(t))) + I_i \end{aligned} \quad (4.29)$$

and the slave neural network

$$\begin{aligned} \frac{d^2 \omega_i(t)}{dt^2} = & -a_i \frac{d\omega_i(t)}{dt} - b_i \omega_i(t) + \sum_{j=1}^n c_{ij} g_j(\omega_j(t)) \\ & + \sum_{j=1}^n d_{ij} g_j(\omega_j(t - \tau(t))) + I_i + J_i(t) \end{aligned} \quad (4.30)$$

where $J_i(t)$ is the coupling control term to be designed, the two neural networks may have different initial values. The notation of $a_i, b_i, c_{ij}, d_{ij}, g_j(\cdot)$ and I_i are the same as ones in Sect. 2. Let

$$\begin{aligned} \tilde{u}_i(t) &= \frac{du_i(t)}{dt} + \xi_i u_i(t), \quad i = 1, 2, \dots, n, \\ \tilde{\omega}_i(t) &= \frac{d\omega_i(t)}{dt} + \xi_i \omega_i(t), \quad i = 1, 2, \dots, n, \end{aligned}$$

and denote $u(t) = (u_1(t), u_2(t), \dots, u_n(t))^T$, $\tilde{u}(t) = (\tilde{u}_1(t), \tilde{u}_2(t), \dots, \tilde{u}_n(t))^T$, $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_n(t))^T$, $\tilde{\omega}(t) = (\tilde{\omega}_1(t), \tilde{\omega}_2(t), \dots, \tilde{\omega}_n(t))^T$, the above two systems can be written as

$$\begin{aligned} \frac{du(t)}{dt} &= -\Lambda u(t) + \tilde{u}(t) \\ \frac{d\tilde{u}(t)}{dt} &= -\Lambda u(t) - B\tilde{u}(t) + Cg(u(t)) + Dg(u(t - \tau(t))) + I \end{aligned} \quad (4.31)$$

and

$$\frac{d\omega(t)}{dt} = -\Lambda \omega(t) + \tilde{\omega}(t)$$

$$\frac{d\tilde{\omega}(t)}{dt} = -A\omega(t) - B\tilde{\omega}(t) + Cg(\omega(t)) + Dg(\omega(t - \tau(t))) + I + J(t) \quad (4.32)$$

where $\Lambda = \text{diag}\{\xi_1, \xi_2, \dots, \xi_n\}$, $A = \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_n\}$, $B = \text{diag}\{\beta_1, \beta_2, \dots, \beta_n\}$, $\alpha_i = b_i - \xi_i(a_i - \xi)$, $\beta_i = a_i - \xi_i$, $C = (c_{ij})_{n \times n}$, $D = (d_{ij})_{n \times n}$, $I = \text{diag}\{I_1, I_2, \dots, I_n\}$, $J(t) = \text{diag}\{J_1(t), J_2(t), \dots, J_n(t)\}$.

Here $J(t)$ is considered to be an error feedback control term with the form $J(t) = Kx(t_k) + My(t_k)$, where t_k satisfies $0 = t_0 < t_1 < \dots < t_k < \dots$, $\lim_{k \rightarrow +\infty} t_k = +\infty$ and $t_{k+1} - t_k \leq h, \forall k \in N$, where K, M are the sampled-data feedback controller gain matrices to be designed. Let $h(t) = t - t_k$ for $t \in [t_k, t_{k+1})$ and define the synchronization error signal as $x(t) = \omega(t) - u(t)$, $y(t) = \tilde{\omega}(t) - \tilde{u}(t)$. Subtracting (4.31) from (4.32), the error system can be obtained as follows

$$\begin{aligned} \frac{dx(t)}{dt} &= -\Lambda x(t) + y(t) \\ \frac{dy(t)}{dt} &= -Ax(t) - By(t) + Cf(x(t)) + Df(x(t - \tau(t))) + Kx(t - h(t)) \\ &\quad + My(t - h(t)) \end{aligned} \quad (4.33)$$

Theorem 2 Under Assumption 1, if there exist positive-definite symmetric matrices $P_i (i = 1, \dots, 9)$, $Q_i (i = 1, 2)$, positive-definite diagonal matrices $U_i (i = 1, 2)$ and any matrices X, Y such that the following LMI holds:

$$\Sigma = \begin{bmatrix} \Sigma_{1,1} & \Sigma_{1,2} & \Sigma_{1,3} & 0 & \Sigma_{1,5} & 0 & \Sigma_{1,7} & \Sigma_{1,8} & 0 & 0 & \Sigma_{1,11} & 0 \\ * & \Sigma_{2,2} & 0 & 0 & 0 & 0 & \Sigma_{2,7} & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{3,3} & \Sigma_{3,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Sigma_{3,12} \\ * & * & * & \Sigma_{4,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{5,5} & \Sigma_{5,6} & \Sigma_{5,7} & \Sigma_{5,8} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{6,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{7,7} & \Sigma_{7,8} & \Sigma_{7,9} & 0 & \Sigma_{7,11} & \Sigma_{7,12} \\ * & * & * & * & * & * & * & \Sigma_{8,8} & \Sigma_{8,9} & 0 & \Sigma_{8,11} & \Sigma_{8,12} \\ * & * & * & * & * & * & * & * & \Sigma_{9,9} & \Sigma_{9,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Sigma_{10,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Sigma_{11,11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Sigma_{12,12} \end{bmatrix} < 0 \quad (4.34)$$

where

$$\begin{aligned} \Sigma_{1,1} &= P_3 - P_4 + P_5 - P_7 - (1 - \bar{\tau})P_9 - L^-U_1L^+ - 2Q_1A, \\ \Sigma_{1,2} &= P_1 - Q_1 - Q_1A, \Sigma_{1,3} = P_4 + (1 - \bar{\tau})P_9, \Sigma_{1,5} = P_7, \Sigma_{1,7} = Q_1 - Q_2A, \\ \Sigma_{1,8} &= -Q_2A, \Sigma_{1,11} = \frac{1}{2}U_1(L^- + L^+), \\ \Sigma_{2,2} &= \tau^2P_4 + h^2P_7 + \tau^2P_9 - 2Q_1, \Sigma_{2,7} = Q_1, \\ \Sigma_{3,3} &= -(1 - \bar{\tau})P_3 - 2P_4 - (1 - \bar{\tau})P_9 - L^-U_2L^+, \Sigma_{3,4} = P_4, \\ \Sigma_{3,12} &= \frac{1}{2}U_2(L^- + L^+), \\ \Sigma_{4,4} &= -P_4, \\ \Sigma_{5,5} &= -2P_7, \Sigma_{5,6} = P_7, \Sigma_{5,7} = X, \Sigma_{5,8} = X, \\ \Sigma_{6,6} &= -P_5 - P_7, \\ \Sigma_{7,7} &= P_6 - P_8 - 2Q_2B, \Sigma_{7,8} = P_2 - Q_2 - Q_2B, \Sigma_{7,9} = P_8 + Y, \Sigma_{7,11} = Q_2C, \end{aligned}$$

$$\begin{aligned}
\Sigma_{7,12} &= Q_2 D, \\
\Sigma_{8,8} &= h^2 P_8 - 2Q_2, \Sigma_{8,9} = Y, \Sigma_{8,11} = Q_2 C, \Sigma_{8,12} = Q_2 D, \\
\Sigma_{9,9} &= -2P_8, \Sigma_{9,10} = P_8, \\
\Sigma_{10,10} &= -P_6 - P_8, \\
\Sigma_{11,11} &= -U_1, \\
\Sigma_{12,12} &= -U_2,
\end{aligned}$$

then the master system (4.29) and the slave system (4.30) are globally synchronized under the desired controller gain matrix being given as $K = Q_2^{-1}X$, $M = Q_2^{-1}Y$.

Proof Noting that

$$\begin{aligned}
0 &= 2[x^T(t) + \dot{x}^T(t)]Q_1[-\dot{x}(t) + \dot{x}(t)] \\
&= 2[x^T(t) + \dot{x}^T(t)]Q_1[-\dot{x}(t) - \Lambda x(t) + y(t)] \\
&= -2x^T(t)Q_1\dot{x}(t) - 2x^T(t)Q_1\Lambda x(t) + 2x^T(t)Q_1y(t) \\
&\quad - 2\dot{x}^T(t)Q_1\dot{x}(t) - 2\dot{x}^T(t)Q_1\Lambda x(t) + 2\dot{x}^T(t)Q_1y(t).
\end{aligned} \tag{4.35}$$

$$\begin{aligned}
0 &= 2[y^T(t) + \dot{y}^T(t)]Q_2[-\dot{y}(t) + \dot{y}(t)] \\
&= 2[y^T(t) + \dot{y}^T(t)]Q_2[-\dot{y}(t) - Ax(t) - By(t) + Cf(x(t)) + Df(x - \tau(t)) \\
&\quad + Kx(t - h(t)) + My(t - h(t))] \\
&= -2y^T(t)Q_2\dot{y}(t) - 2y^T(t)Q_2Ax(t) - 2y^T(t)Q_2By(t) + 2y^T(t)Q_2Cf(x(t)) \\
&\quad + 2y^T(t)Q_2Df(x(t - \tau(t))) + 2y^T(t)Q_2Kx(t - h(t)) + 2y^T(t)Q_2My(t - h(t)) \\
&\quad - 2\dot{y}^T(t)Q_2\dot{y}(t) - 2\dot{y}^T(t)Q_2Ax(t) - 2\dot{y}^T(t)Q_2By(t) + 2\dot{y}^T(t)Q_2Cf(x(t)) \\
&\quad + 2\dot{y}^T(t)Q_2Df(x(t - \tau(t))) + 2\dot{y}^T(t)Q_2Kx(t - h(t)) \\
&\quad + 2\dot{y}^T(t)Q_2My(t - h(t)).
\end{aligned} \tag{4.36}$$

then we obtain

$$\dot{V}(t) \leq \xi^T(t)\Sigma\xi(t). \tag{4.37}$$

where $V(t)$, $\xi(t)$ are the same as Sect. 3.

Similar to the proof of Theorem 1, we get that the slave system (4.30) can be globally synchronized to the master system (4.29) under control $K = Q_2^{-1}X$, $M = Q_2^{-1}Y$. $\square$

5 Illustrative example

In this section, two examples are given to illustrate the effectiveness of our results.

Example 1 Consider the following delayed inertial neuron network

$$\begin{cases} \frac{d^2x_1(t)}{dt^2} = -\frac{dx_1(t)}{dt} - 1.5x_1(t) + 0.3g_1(x_1(t)) + 0.1g_2(x_2(t)) \\ \quad + 0.4g_1(x_1(t - \tau(t))) + 0.2g_2(x_2(t - \tau(t))) + 1 \\ \frac{d^2x_2(t)}{dt^2} = -\frac{dx_2(t)}{dt} - 1.5x_2(t) + 0.2g_1(x_1(t)) + 0.2g_2(x_2(t)) \\ \quad + 0.2g_2(x_2(t - \tau(t))) + 1 \end{cases} \tag{5.38}$$

where $g_1(x_1(t)) = \sin(x_1(t))$, $g_2(x_2(t)) = \cos(x_2(t))$, $\tau(t) = 0.5 \sin^2(t) + 0.5$.

Obviously, $g_i(\cdot)$ ($i = 1, 2$) satisfy (2.2) with $l_1^- = l_2^- = -1$, $l_1^+ = l_2^+ = 1$, and $\tau = 1$, $\bar{\tau} = 0.5$.

Choosing $\xi_1 = \xi_2 = 2$, the inertial neural network (5.38) can be rewritten in the form (2.4), and the corresponding matrices can be easily obtained

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad A = \begin{pmatrix} 3.5 & 0 \\ 0 & 3.5 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$C = \begin{pmatrix} 0.3 & 0.1 \\ 0.2 & 0.2 \end{pmatrix}, \quad D = \begin{pmatrix} 0.4 & 0.2 \\ 0 & 0.2 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Taking sampling time points $t_k = 0.02k$, $k = 1, 2, \dots$, and the sampling period is $h = 0.02$. Solving LMIs (3.11), we have

$$\begin{aligned} X &= \begin{pmatrix} -0.1996 & 0.0005 \\ 0.0005 & -0.2006 \end{pmatrix}, & Y &= \begin{pmatrix} -0.4914 & 0.0123 \\ 0.0123 & -0.5132 \end{pmatrix}, \\ P_1 &= \begin{pmatrix} 1.9947 & -0.0445 \\ -0.0445 & 2.0740 \end{pmatrix}, & P_2 &= \begin{pmatrix} 0.4082 & -0.0075 \\ -0.0075 & 0.4214 \end{pmatrix}, \\ P_3 &= \begin{pmatrix} 0.9164 & -0.0007 \\ -0.0007 & 0.9176 \end{pmatrix}, & P_4 &= \begin{pmatrix} 0.2528 & -0.0018 \\ -0.0018 & 0.2559 \end{pmatrix}, \\ P_5 &= \begin{pmatrix} 0.4590 & -0.0015 \\ -0.0015 & 0.4615 \end{pmatrix}, & P_6 &= \begin{pmatrix} 0.1638 & -0.0008 \\ -0.0008 & 0.1651 \end{pmatrix}, \\ P_7 &= \begin{pmatrix} 0.7210 & 0.0009 \\ 0.0009 & 0.7195 \end{pmatrix}, & P_8 &= \begin{pmatrix} 2.3186 & -0.0027 \\ -0.0027 & 2.3234 \end{pmatrix}, \\ P_9 &= \begin{pmatrix} 0.1536 & -0.0016 \\ -0.0016 & 0.1565 \end{pmatrix}, & Q_1 &= \begin{pmatrix} 0.7600 & -0.0120 \\ -0.0120 & 0.7814 \end{pmatrix}, \\ Q_2 &= \begin{pmatrix} 0.0979 & -0.0055 \\ -0.0055 & 0.1077 \end{pmatrix}, & U_1 &= \begin{pmatrix} 0.5510 & 0 \\ 0 & 0.5510 \end{pmatrix}, \\ U_2 &= \begin{pmatrix} 0.3975 & 0 \\ 0 & 0.3975 \end{pmatrix}. \end{aligned}$$

The controller gain matrices can be obtained as follows:

$$K = Q_1^{-1}X = \begin{pmatrix} -0.2627 & -0.0033 \\ -0.0033 & -0.2567 \end{pmatrix},$$

$$M = Q_2^{-1}Y = \begin{pmatrix} -5.0258 & -0.1426 \\ -0.1426 & -4.7716 \end{pmatrix}.$$

From Figs. 1 and 2, we can see that the equilibrium is globally asymptotically stable. Thus Theorem 1 is well-verified.

Example 2 We consider the following master–slave neural networks

$$\begin{cases} \frac{d^2 x_i(t)}{dt^2} = -0.05 \frac{dx_i(t)}{dt} - 0.05 x_i(t) + 0.2 \sin(x_i(t)) - 0.6 \cos(y_i(t)) \\ \quad + 0.1 \sin(x_i(t - \tau(t))) + 0.1 \cos(y_i(t - \tau(t))) + 1 \\ \frac{d^2 y_i(t)}{dt^2} = -0.05 \frac{dy_i(t)}{dt} - 0.05 y_i(t) + 0.5 \sin(x_i(t)) + 0.3 \cos(y_i(t)) \\ \quad - 0.1 \sin(x_i(t - \tau(t))) + 0.1 \cos(y_i(t - \tau(t))) + 1 \end{cases} \quad (5.39)$$

where $i = 1, 2$, $\tau(t) = \frac{e^x}{1+e^x}$, $(x_1, y_1)^T$ is the state of the master neural network and $(x_2, y_2)^T$ is the state of the slave neural network. When no control term is added to the slave neural network, Fig. 3 indicates the error between the two neural networks always exists.

Fig. 1 The state trajectories of system (5.38) without sampled-data

Fig. 2 The state trajectories of system (5.38) with sampled-data control

Taking sampling time points $t_k = 0.1k$, $k = 1, 2, \dots$, and the sampling period $h = 0.1$, choosing

$$\Lambda = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

Fig. 3 The state trajectories of the master–slave neural networks without control

Fig. 4 The state trajectories of the master–slave neural networks with sampled-data control

and by using Theorem 2, the corresponding matrices are

$$K = \begin{pmatrix} 6.1683 & -0.1105 \\ -0.1105 & 6.0854 \end{pmatrix}, \quad M = \begin{pmatrix} -6.7376 & 0.0050 \\ 0.0050 & -6.7339 \end{pmatrix}$$

From Fig. 4, we can see the slave neural network is globally synchronized to the master neural network. Thus our criteria are well-verified.

6 Conclusion

This paper concerns with the stability of inertial neural network with time-varying delays using sampled-data control. By employing an input delay approach and the Lyapunov theory, some sufficient conditions are obtained to ensure the stability of the equilibrium. Furthermore, several criteria are given to guarantee the synchronization of the master–slave neural networks under error-feedback sampled-data control. Meanwhile, the variable transformation we introduced can assure our results are less conservative. Two examples are used to demonstrate that the theoretical results are correct and effective.

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