Natural Computing (2023) 22:409–410
https://doi.org/10.1007/s11047-023-09952-3

Preface

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Accepted: 4 July 2023 / Published online: 25 July 2023

Cellular automata as well as automata networks were introduced in the 1940s at the time of the basic developments of modern computer science. The former stem from discussions between Ulam and von Neumann, when von Neumann was particularly interested in both the capacity of the machines to highlight the phenomenon of self-reproduction of species and the principle of parallel computation. The latter were born from the collaboration between McCulloch and Pitts in their quest to model neural interactions at the heart of human brain function and to explore their computational properties.

These two models of computation are thus intrinsically linked to natural computating. Indeed:

- they were fundamentally inspired by natural phenomenology;
- they have contributed to major advances in computing and keep providing theoretical and applied problems at the interface of computer science and mathematics; and
- they constitute relevant representation models of phenomena (in physics, in biology, or even in sociology) whose analysis provides a better understanding of the world.

Roughly speaking, cellular automata and automata networks are discrete models based on the principles of interaction and locality essential to the vision of computation dear to Turing. In this special issue, for the sake of simplicity and abusing the most general definitions, we make a clear distinction between cellular automata and automata networks: cellular automata evolve on regular spaces, where cells are governed by a common rule, whereas automata networks evolve on general graphs where any node has its own rule. Despite these differences, one of their primary characteristics is that they are particularly simple to describe and define syntactically, but, despite this intrinsic simplicity, the discrete dynamical systems captured by their semantics are of great richness and complexity allowing fruitful application in a multitude of scientific domains. In this sense, they are "canonical" representatives of complex systems, defined here as systems of interacting entities where global properties emerge which cannot be explained by our knowledge of their local characteristics. The papers in this issue illustrate those principles through research of mainly theoretical nature. They provide a good overview of recent developments and new results that reflect quite well the dynamics of the field.

This special issue was initiated at the joint event which took place at the CIRM in Marseille (France) from the 12th to the 17th of July 2021:

- AUTOMATA'2021, the 27th International Workshop on Cellular Automata and Discrete Complex Systems; and
- WAN'2021, an international Workshop on Automata Networks.

This event also shared a half-day session dedicated to Professor Eric Goles on the occasion of his 70th birthday. Professor Goles is a reknown Chilean computer scientist who has made numerous major contributions to both cellular automata and automata networks, especially in the areas of discrete dynamical systems, complexity theory and computability.

Moreover, in order to provide a more comprehensive presentation of the field, this special issue is not limited to articles resulting from presentations given during the event; an international call for contributions was widely distributed through multiple channels. A rigorous refereeing process resulted in nine articles accepted for this issue.

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The first contribution is by Julio Aracena, Florian Bridoux, Luis Gómez and Lilian Salinas. It focuses on the complexity of conjunctive automata networks, namely Boolean networks in which every automaton computes its state as the conjunction of that of each of its in-neighbours, to have limit cycles of specific length. The paper considers several variants of the main problem by taking into account deterministic update modes inside the block-sequential family.

Aurélien Naldi, Adrien Richard and Elisa Tonello study a new class of automata networks, namely those admitting linear cuts, *i.e.*, components able to eliminate some regulatory conflicts that might prevent some asynchronous dynamics to take place. The interaction graph of a Boolean network admits a linear cut when a linear component occurs in each cycle and in each path from components with multiple targets to components with multiple regulators. The authors prove that under this structural condition the attractors are in one-to-one correspondence with the minimal trap spaces. Moreover, they provide a characterization of the reachability of attractors.

The paper of Pacôme Perrotin provides a further illustration of how automata networks and cellular automata are strictly related. Indeed, the paper proves that the limit dynamics of any finite automata network under the parallel update schedule corresponds exactly to the fixed points of a strictly one-way cellular automaton. This result is based on the use of *output functions*, a concept developed by the author.

The contribution of Barbara Wolnik, Adam Dzedej, Maciej Dziemiańczuk, Aleksander Wardyn and Bernard - De Baets explores one-dimensional reversible number conserving cellular automata with radius 1, with a focus on alphabet of prime size for which an exhaustive list has been obtained. The authors also use theory of bundle graphs and d-definite automata in order to compute the precise scope of inverses of such rules.

The article by Luca Mariot deals with cellular automata as a means to design pseudorandom number generators which could be used in the context of computational security and symmetric cryptography. The contribution suggests, through both empirical and theoretical approaches, that using orthogonal cellular automata could be appropriate for such a design. The author argues on the fact that discrete dynamical systems can be defined which exhibit the longest possible limit cycles, on which generators are built.

Alexandre Fernandez, Luidnel Maignan and Antoine - Spicher continue the study of cellular automata from the point of view of category theory. In particular, they clarify the meaning of the fact that the application of a cellular automaton to partial configurations is a natural extension of its local transition function through the categorical notion of Kan extension. The arguments and the ideas in the paper allow to *easily* generalize concepts pioneered by cellular automata to arbitrary kinds of possibly evolving spaces. The authors made a huge didactical effort so that almost no *a priori* knowledge of category theory is necessary to go through the paper.

The contribution by Firas Ben Ramdhane and Pierre Guillon proposes to study substitution dynamical systems (which can be seen as an extension of cellular automata) in non Cantor topologies. After having studied these objects in the Besicovitch topology, they propose to proceed to the topology induced by the Feldman-Katok pseudometric. Finally, they generalize results about cellular automata and substitutions to dill maps (a class of discrete dynamical systems which generalizes both).

The contribution by Ville Salo studies von Neumann regularity, a notion of weak inverse in semi-groups, for cellular automata on mixing subshifts of finite type. It establishes that von Neumann regularity is equivalent to the category-theoretic condition of being split epic onto the image. In particular this implies, based on a previous work of the author, that it is a decidable property. Following this approach, the author answers a question raised by Castillo-Ramirez *et al.* concerning elementary cellular automata.

This issue ends with a contribution by Jarkko Kari and Victor Lutfalla about Planar Rosa, a family of rhombus tilings with a $2n$-fold rotational symmetry that are generated by a primitive substitution and are also discrete plane tilings (*i.e.*, they are obtained as a projection of a higher dimensional discrete plane). In particular, the authors prove that the Sub Rosa substitution tilings with $2n$-fold rotational symmetry defined by Kari and Rissanen do not satisfy even the weaker discrete plane condition. They prove these results for all even $n \geq 4$ completing in this way the previously published results for odd values of n.

We would like to warmly thank all the authors for the papers that they share with us. Also, we are grateful to the referees for the rigorous and generous efforts that made this special issue possible.