Intrinsic mechanical properties of food in relation to texture parameters

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Received: 22 June 2020 / Accepted: 3 March 2021 / Published online: 24 March 2021
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Abstract The texture profile analysis test is an imitative test to determine texture properties of food, which quantify the consumer's perception of eating food. The instrumental texture parameters obtained from this test depend on the specimen size and the nonstandardized test conditions. To overcome this problem, texture properties are here related to intrinsic mechanical properties, which are independent of the test conditions. Two types of materials are used to investigate the effect of viscoelasticity, plasticity and damage on the texture parameters for varying test conditions. Analytical relations between mechanical properties, test conditions, and the instrumental hardness, springiness, cohesiveness, and adhesiveness are determined. The hardness is obtained from the stiffness of the material, which is potentially rate-dependent, and the yield stress of a material in case of plasticity. The springiness is determined by the recoverable or irrecoverable strain in the material, which results from the mechanical properties in combination with the test conditions. Cohesiveness and springiness are found to be strongly related, unless structural damage is present in the material. Adhesiveness is only an indirect measure of the adhesion between the material and compression plate and depends on the test conditions and stiffness of the material. Finite element simulations reveal a decrease of hardness in case of a nonflat top surface, indicating the importance of geometrical effects.

Keywords Texture profile analysis · Mechanical properties · Hardness · Springiness · Cohesiveness · Adhesiveness

1 Introduction

The instrumental texture profile analysis (TPA) test is commonly used to measure food texture, since its introduction by Friedman et al. (1963). The TPA test is a double compression test that is a simplified representation of the first two bites during the complex action of oral

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processing (Pascua et al. 2013; Chen 2015). The purpose of the test procedure is to deliver an objective measurement related to perceived sensory attributes. These sensory attributes are evaluated by a trained test panel. The hardness is evaluated as the required force to compress a product by a certain amount, the springiness as the recovery of the deformation, and the cohesiveness as a measure of the deformation before rupture. The deformation is applied by the test panel participant with the teeth or fingers (Di Monaco et al. 2008; Pascua et al. 2013). Besides these three main parameters, many more texture properties are defined, such as adhesiveness, chewiness, and resilience. Many researchers have investigated relations between the sensory evaluation of the test panel and the instrumental texture parameters, obtained by a TPA test. Meullenet et al. (1998) tested a wide variety of food products and found high correlations for the hardness and springiness, but not for cohesiveness and chewiness. However, Wee et al. (2018) did a similar study and found significant correlations for springiness, cohesiveness, chewiness, and resilience but not for the hardness. Breuil and Meullenet (2001) found reasonable predictions for the texture in cheese using an extended instrumental test procedure by considering cone penetration and needle puncture tests next to the uniaxial compression test. Paula and Conti-Silva (2014) used different probes with the texture analyzer as well to study extruded snacks.

Comparing these disparate findings, suggests that the nonstandardized test conditions of the TPA test make the interpretation of results challenging, if not troublesome. The obtained texture parameters differ when using different test parameters (i.e., applied time-displacement profile) with varying types of probes on different specimen sizes and shapes (Rosenthal 2010; Alvarez et al. 2002; Peleg 2006, 2019; Hyldig and Nielsen 2001; Patel et al. 1992). Clearly, this makes it problematic to compare results. Moreover, it is questionable how meaningful these instrumental texture parameters are in terms of sensory attributes. Consequentially, there is a need for well-defined properties that are ideally independent of the test conditions (Peleg 2019).

Mechanical properties are intrinsic material characteristics that fulfill these requirements. In the TPA test the food material is mechanically loaded under the chosen test conditions. Although instrumental texture parameters are believed to be relevant to the sensory attributes, relating these attributes to mechanical properties that influence the texture parameter may provide novel insights. Di Monaco et al. (2008) related sensory attributes to the results of cyclic compression tests and stress-relaxation experiments, whereas Vincent and Saunders (2002), Kim et al. (2012), and Vliet and Primo-Martín (2011) linked the sensory evaluation of brittle food to fracture mechanics. These studies directly relate mechanical properties to sensory evaluation, without considering the TPA test in relation to mechanical properties.

The TPA test is a test method with a lot of disadvantages and the usefulness of the parameters that are normally derived from the test is limited at best. However, the results of a double compression test are clearly influenced by the mechanical properties of a material. The goal of this study is to analyze the mechanical response during the TPA test and to identify relations between the instrumental texture parameters and intrinsic mechanical properties. This knowledge is useful for multiple purposes. Firstly, it provides insight into the effect of varying test conditions. Secondly, it shows which intrinsic material properties affect specific texture parameters, which also indicates suitable material characterization methods. Moreover, such relations are useful in designing customized food as mechanical properties can be tailored by the microstructure, which is altered by the ingredient composition or processing conditions (Attenburrow et al. 1989; Sozer et al. 2011; Gao et al. 2018), for example, by 3D printing (Le Tohic et al. 2018). To achieve these relations, two different types of materials, namely a linear viscoelastic and an elasto-viscoplastic material, are represented by a suitable constitutive model. The effect of structural damage is included. The modeling

approach is, however, limited to food products that remain macroscopically intact during the TPA test. Using these models, the intrinsic material response during the TPA test is described in detail. Each material has its own characteristic mechanical properties, for which analytical relations with the resulting texture parameters are established. The used probe is a flat compression plate, because other types of probes inherently result in heterogeneous deformation that depends on the geometry of the probe. The considered texture parameters are hardness, cohesiveness, and adhesiveness, for which the original definitions (Friedman et al. 1963) are used, and the springiness (Meullenet et al. 1997). The presented methods are easily extended to additional texture parameters such as resilience and chewiness or to modifications of the original definitions (Peleg 1976; Bourne 1978; Fiszman et al. 1998). In addition, finite element simulations are performed to investigate the effect of friction and surface roughness on the resulting texture properties.

2 Texture profile analysis test

2.1 Test procedure

Figure 1 shows the result of a typical texture profile analysis test, in which the applied compressive force is measured as a function of time. First, the compression plate is moving down with constant velocity v to a maximum applied displacement u_t , inducing a compressive load on the sample. The time this takes is defined as the test time t_t according to

$$t_t = \frac{u_t}{v}. \quad (1)$$

Next, the compression plate is moving up with the same velocity, reaching its original position at time $2t_t$. The compressive force in the sample decreases to zero if there is no adhesion, or to a tensile force if there is adhesion. Subsequently, there is a waiting time t_w during which the plate remains at its starting position, after which the loading–unloading step is repeated.

During the unloading step, the sample and plate may lose contact after time t_{lc} , counted from the moment at which the unloading started. The compressive strain that is still present in the sample may be recoverable or irrecoverable. The plate starts to move down again at time $2t_t + t_w$. Contact is made again (i.e., the force increases) after time t_{mc} relative to the start of the second cycle. It therefore takes a time interval t_{rec} between losing contact with the sample and touching it again in the second cycle.

From the TPA test, multiple texture parameters can be extracted. In this research, focus is put on the parameters hardness H , adhesiveness A_3 , springiness S , and cohesiveness C . The hardness is the maximum force in the first cycle. The adhesiveness A_3 is equal to the area of the force–time response, for a negative measured force, in the first cycle. The springiness and cohesiveness are defined from Fig. 1 as

$$S = \frac{s_2}{s_1} \quad \text{and} \quad C = \frac{A_2}{A_1}, \quad (2)$$

where s_1 is equal to the test time t_t , s_2 is determined by the time between making contact in the second cycle and the test time, that is, $s_2 = s_1 - t_{mc}$, and A_1 and A_2 are the areas under the loading–unloading curve (for a positive measured force) in cycles 1 and 2, respectively. The test conditions of the TPA test are determined by the maximum applied displacement u_t , the velocity v , and the waiting time between the two cycles t_w .

Fig. 1 Definitions in the texture profile analysis test. Parameters in blue denote relative times, and parameters in red are used for calculating texture parameters (Color figure online)

2.2 Finite element model

Although the aim is obtaining analytical relations between texture parameters and intrinsic mechanical properties, additional simulations of the TPA test are performed using the finite element package MSC.Marc to take the nonlinear material behavior and heterogeneous deformation into account. The considered geometry is a cylinder with a length and diameter of 1 cm. The vertical displacement of the bottom of the cylinder is suppressed, and the compression plate is displaced according to the test parameters u_t , v , and t_w .

The deformation is homogeneous in an ideal test situation, meaning that the sample is flat, the sample and compression plate are perfectly aligned, and friction is negligible. In the homogeneous case, modeling the geometry by a single 2D axisymmetric linear element is sufficient to simulate the TPA experiment, using a time step size of $5.0 \cdot 10^{-4}$ s.

In a real experiment, friction and surface roughness are potential sources that may cause the deformation to be heterogeneous. To investigate these effects, the geometry is modeled using 400 axisymmetric linear elements. The effect of surface roughness is investigated by applying a gradual increase of the sample length from 0.95 cm at the boundary to 1.00 cm at the center of the cylinder. The corresponding mesh of a sample with this arbitrary irregular surface is depicted in Fig. 2.

A smoothed Coulomb friction model is implemented by relating the tangential friction force F_t to the normal force F_n , the coefficient of friction μ , and the relative sliding velocity v_r :

$$F_t = -\mu F_n \frac{2}{\pi} \arctan\left(\frac{|v_r|}{0.5}\right) \text{sgn}(v_r). \quad (3)$$

A coefficient of friction $\mu = 0.3$, corresponding to friction between starch and steel, is chosen as a reference (Emami and Tabil 2008).

Fig. 2 Mesh considering surface roughness in the finite element model

3 Materials

3.1 Uniaxial material response

The test conditions are converted into relevant quantities for material characterization. The engineering strain ε and strain rate $\dot{\varepsilon}$ are calculated for a deformation u , velocity v , and initial specimen length L_0 using

$$\varepsilon = \frac{u}{L_0} \quad \text{and} \quad \dot{\varepsilon} = \frac{v}{L_0}. \quad (4)$$

The true strain definition, denoted by $\bar{\varepsilon}$, is accounting for the deformation at each instant of the deforming sample length and is applied for large deformations:

$$\bar{\varepsilon} = \ln \left(\frac{u}{L_0} + 1 \right) \quad \text{and} \quad \dot{\bar{\varepsilon}} = \frac{v}{L_0 + u}. \quad (5)$$

The maximum applied deformation u_t is converted into the maximum applied engineering strain ε_t and true strain $\bar{\varepsilon}_t$ by substituting $u = u_t$ in Equations (4) and (5), respectively. Because the velocity is kept constant in the TPA test, the true strain rate is not constant during the measurement. The engineering strain rate $\dot{\varepsilon}_t$ is therefore used as the relevant test parameter. Note that ε_t and $\bar{\varepsilon}_t$ are negative in the TPA test, because the material is loaded in compression. The strain rate $\dot{\varepsilon}_t$ is negative during loading and positive during unloading. Therefore $\dot{\varepsilon}_t$ is defined positive.

During the test, the compressive force F is measured. The engineering stress σ and the true stress $\bar{\sigma}$ are calculated using the initial cross-sectional area A_0 and the corresponding deformed area A :

$$\sigma = -\frac{F}{A_0}, \quad \bar{\sigma} = -\frac{F}{A}. \quad (6)$$

The difference between the engineering and true stress is important because changes in the cross-sectional area during deformation are generally hard to measure. This prevents the direct measurement of the true stress. The adhesion between the material and the compression plate is characterized by an engineering stress σ_a or true stress $\bar{\sigma}_a$, which implies that they are separated when the tensile stress in the material reaches this adhesion stress.

To investigate the relation between mechanical properties and texture properties, two types of materials are considered that exhibit a different characteristic mechanical response,

Fig. 3 Schematic force response of the materials of interest for a specific applied displacement (left picture) as a function of time

namely a linear viscoelastic and an elasto-viscoplastic material. The elasto-viscoplastic material is considered with and without damage. Figure 3 sketches the characteristic force response of these materials for a displacement that is first increasing and subsequently kept constant in time. The key characteristic of a linear viscoelastic material is its time-dependent response. The stiffness depends on the applied deformation rate, and the force increase is, in general, nonlinear, whereas the force is decreasing if the displacement is kept constant. The stiffness of an elasto-viscoplastic material does not depend on the deformation rate. An elastic regime is followed by a plastic regime in which a small increasing force is measured due to an increasing cross-sectional area. The plastic strain in the material is irrecoverable, whereas the strain in the linear viscoelastic material is always recoverable. By introducing damage the force decreases with increasing plastic deformation.

First, the constitutive models of the two materials are presented, after which the constitutive equations are solved for the different steps in the TPA test. From this, analytical relations are obtained to link the texture properties to the mechanical properties.

3.2 Tensor notations

The constitutive models are formulated in a tensorial form in the three-dimensional space. Using a Cartesian vector basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and summation over repeated indices, the second-order tensor $\mathbf{A}$ and the transpose $\mathbf{A}^T$ are denoted as

$$\mathbf{A} = A_{ij} \mathbf{e}_i \mathbf{e}_j \quad \text{and} \quad \mathbf{A}^T = A_{ji} \mathbf{e}_j \mathbf{e}_i. \quad (7)$$

The second-order tensor $\mathbf{C}$ results from the inner product of tensors $\mathbf{A}$ and $\mathbf{B}$ according to

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B} \quad \text{with} \quad C_{ik} = A_{ij} B_{jk}, \quad (8)$$

and the scalar c from the double inner product of two tensors is

$$c = \mathbf{A} : \mathbf{B} = A_{ij} B_{ji}. \quad (9)$$

The hydrostatic part $\mathbf{A}^h$ and deviatoric part $\mathbf{A}^d$ of a tensor are given by

$$\mathbf{A}^h = \frac{1}{3} \text{tr}(\mathbf{A}) \mathbf{I} \quad \text{and} \quad \mathbf{A}^d = \mathbf{A} - \mathbf{A}^h, \quad (10)$$

where $\mathbf{I} = \mathbf{e}_i \mathbf{e}_i$ is the unit tensor, and the trace of a tensor is

$$\text{tr}(\mathbf{A}) = A_{ii}. \quad (11)$$

Fig. 4 Mechanical analog of the standard solid model

Fig. 5 Stress-relaxation response of the standard solid model

The material time derivative is denoted as $\dot{A}$.

3.3 Linear viscoelastic material

1D constitutive model

The standard solid model is a time-dependent model to describe linear viscoelastic materials. The mechanical analog of the model consists of a Maxwell element, with Young's modulus E_1 and viscosity η , and a parallel spring with Young's modulus E_∞ ; see Fig. 4. The characteristic time of the model is the relaxation time λ given by

$$\lambda = \frac{\eta}{E_1}. \quad (12)$$

The formulation of this standard solid model is restricted to small strains. Therefore analysis is limited to a maximum compressive strain of 0.1, at which the engineering stress and strain are approximately equal to the true stress and strain. The total stress is calculated from the stress in the parallel spring σ_∞ and the stress in the Maxwell element σ_M using

$$\sigma = \sigma_\infty + \sigma_M = E_\infty \varepsilon + \eta \dot{\varepsilon}_v, \quad (13)$$

where $\dot{\varepsilon}_v$ is the strain rate in the viscous part of the Maxwell element, obtained by subtracting the strain rate $\dot{\varepsilon}_e$ in the elastic part as follows:

$$\dot{\varepsilon}_v = \dot{\varepsilon} - \dot{\varepsilon}_e = \dot{\varepsilon} - \frac{\dot{\sigma}_M}{E_1}. \quad (14)$$

By substituting Equation (14) into Equation (13) the differential equation that relates the stress and strain is given by

$$\sigma + \lambda \dot{\sigma} = E_\infty \varepsilon + \lambda (E_\infty + E_1) \dot{\varepsilon}. \quad (15)$$

By introducing the total stiffness E and the fraction of the material response that is viscoelastic ϕ ,

$$E = E_1 + E_\infty \quad \text{and} \quad \phi = E_1/E, \quad (16)$$

the stress-strain relation given by Equation (15) is rewritten as

$$\sigma + \lambda \dot{\sigma} = E((1 - \phi)\varepsilon + \lambda \dot{\varepsilon}). \quad (17)$$

The material parameters are easily determined from a stress-relaxation experiment; see Fig. 5. The stress is measured as a function of time for an applied strain ε , which is kept constant during the experiment. The time t_{step} , in which this strain is applied, is typically smaller than the relaxation time to accurately measure the mechanical properties. The material reacts initially with a total stiffness E . After loading, the stress is exponentially decreasing toward a plateau value that corresponds to the stress in the parallel spring. The relaxation time λ is obtained by linearizing the curve around $t = t_{\text{step}}$ and determining its intersection point with the plateau stress $E_\infty \varepsilon$.

3D constitutive model

The 3D integral formulation of Equation (15), which is a built-in material model in the finite element package MSC.Marc, is

$$\sigma(t) = \int_0^t 3K(t-t')\dot{\mathbf{e}}^h + 2G(t-t')\dot{\mathbf{e}}^d dt' \quad (18)$$

with $\dot{\mathbf{e}}^h$ and $\dot{\mathbf{e}}^d$ the hydrostatic and deviatoric parts of the strain-rate tensor. The time-dependence of the shear modulus G and bulk modulus K is expressed as

$$G(t) = G_\infty + G_1 \exp\left(\frac{-t}{\lambda}\right) \quad \text{and} \quad K(t) = K_\infty + K_1 \exp\left(\frac{-t}{\lambda}\right), \quad (19)$$

of which the shear and bulk moduli are related to the corresponding Young's moduli and Poisson's ratio ν by

$$G = \frac{E}{2(1+\nu)} \quad \text{and} \quad K = \frac{E}{3(1-2\nu)}. \quad (20)$$

The influence of the Poisson's ratio on the TPA test results is small. A standard value of the initial Poisson ratio's $\nu = 0.15$ is therefore chosen in the analysis. The 3D formulation is used in finite element simulations to include the effect of friction and surface roughness. Without these effects, a uniaxial stress state is recovered. This implies that exact analytical relations between the TPA parameters, test conditions, and material properties are then derived by solving Equation (15).

Analytical solution

Fig. 6 shows the stress-strain response for the TPA test of Fig. 1 by solving Equation (15) using the test parameters $\varepsilon_t = -0.1$, $\dot{\varepsilon}_t = 2.0 \text{ s}^{-1}$, and $t_w = 0.2 \text{ s}$ and the material parameters $E_1 = 40 \text{ MPa}$, $E_\infty = 0.1 \text{ MPa}$, $\lambda = 0.01 \text{ s}$, and $\sigma_a = 0.2 \text{ MPa}$. The strain and stress as functions of time are depicted in Fig. 7.

Fig. 6 Stress-strain response in the TPA test of a linear viscoelastic material

Fig. 7 (a) Strain-time and (b) stress-time response in the TPA test of a linear viscoelastic material

The loading response of the linear viscoelastic material for compression cycle i with a strain rate $\dot{\varepsilon} = -\dot{\varepsilon}_t$ is given by

$$\sigma = E \left[\dot{\varepsilon}_t \lambda \phi \left(\exp \left(\frac{-(t - t_{0,i})}{\lambda} \right) - 1 \right) + (1 - \phi)(\varepsilon_{0,i} - \dot{\varepsilon}_t(t - t_{0,i})) \right] \quad \forall t \in [t_{0,i}, t_{1,i}] \quad (21)$$

with the initial state given by time $t_{0,i}$, strain $\varepsilon_{0,i}$, and stress $\sigma_{0,i} = 0$. In the first compression cycle (i.e., $i = 1$) it holds that $t_{0,i} = 0$, $\varepsilon_{0,i} = 0$ and the loading step is ended at time $t_{1,1} = t_t$. The second loading step starts at $t_{0,2} = 2t_t + t_w + t_{mc}$ and is ended at time $t_{1,2} = 3t_t + t_w$. The time it takes to retrieve contact t_{mc} is obtained by calculating the material response throughout the whole first cycle and during the waiting time, which also provides the initial condition $\varepsilon_{0,2}$.

The uniaxial unloading response of the material with $\dot{\varepsilon} = \dot{\varepsilon}_t$ is given by

$$\begin{aligned} \sigma = E \left[\left(\frac{\sigma_{1,i}}{E} - (1 - \phi)\varepsilon_t - \dot{\varepsilon}_t \lambda \phi \right) \exp \left(\frac{-(t - t_{1,i})}{\lambda} \right) \right. \\ \left. + \dot{\varepsilon}_t \lambda \phi + (1 - \phi)(\varepsilon_t + \dot{\varepsilon}_t(t - t_{1,i})) \right] \quad \forall t \in [t_{1,i}, t_{1,i} + t_{c,i}], \end{aligned} \quad (22)$$

in which the initial conditions for the first and second cycle are $t_{1,1} = t_t$ and $\sigma_{1,1} = \sigma(t_t)$ and $t_{1,2} = 3t_t + t_w$ and $\sigma_{1,2} = \sigma(3t_t + t_w)$, respectively. Equation (22) is only valid when the material is in contact with the compression plate, which is not the case during the entire

Fig. 8 Schematic representation of the different regimes during unloading. The loading step is finished at time t_t (a). In the adhesion-free case the material is in contact with the plate until $\sigma = 0$ is reached upon unloading (b). In case of adhesion, contact is lost when the adhesion tensile stress σ_a is reached (c), followed by an instantaneous decrease of the sample height (d), after which the strain recovers toward zero (e)

unloading step. Figure 8 shows the different regimes. At t_t the plate starts moving upward: the sample is initially in contact with the plate, that is, $\dot{\varepsilon} = \dot{\varepsilon}_t$. Upon unloading, the material reaches a stress equal to zero after a time interval $t_{lc,1}^0$. The sample is still in contact with the plate until the tensile stress in the material reaches the adhesion stress σ_a at $t_t + t_{lc,1}$.

Contact with the compression plate is lost when exceeding σ_a in tension, and at this moment the stress drops instantaneously to zero. The time at which contact is lost depends on the material properties, the adhesion stress, the applied strain, and applied strain rate; $t_{lc,i}$ is the time interval it takes until the stress in Equation (22) reaches σ_a , which results in

$$t_{lc,i} = \lambda \Omega \left(\frac{-(\sigma_{1,i}/E - (1-\phi)\varepsilon_t - \dot{\varepsilon}_t \lambda \phi) \exp\left(\frac{\dot{\varepsilon}_t \lambda \phi + \varepsilon_t(1-\phi) - \sigma_a/E}{\dot{\varepsilon}_t \lambda (1-\phi)}\right)}{\dot{\varepsilon}_t \lambda (1-\phi)} \right) - \frac{\dot{\varepsilon}_t \lambda \phi + \varepsilon_t(1-\phi) - \sigma_a/E}{\dot{\varepsilon}_t (1-\phi)}, \quad (23)$$

in which $\Omega(x)$ denotes the principle branch of the Lambert-W function of the argument x (Corless et al. 1996). Note that the moment of losing contact in the adhesion-free case equals $t_{lc,i}^0$, which is found by substituting $\sigma_a = 0$ into Equation (23), and t_a is the additional time it takes to reach the adhesion stress σ_a starting from $\sigma(t_t + t_{lc,1}^0) = 0$:

$$t_a = t_{lc,1} - t_{lc,1}^0. \quad (24)$$

Figure 7 reveals the strain and stress at the moment of losing contact. The strain in the material at an infinitesimally small time before the moment of losing contact is

$$\varepsilon(t_{1,i} + t_{lc,i}^-) = \varepsilon_t + t_{lc,i} \dot{\varepsilon}_t, \quad (25)$$

because the strain in the material is prescribed by the compression plate. In the case of adhesion, this is followed by an instantaneous change in stress and strain, exactly at the moment of losing contact. The stress becomes equal to zero because there is no external force on the material. The change in compressive strain is characterized by the linear elastic response of the material due to the infinitely fast stress rate. The strain in the material at an infinitesimally small time after the moment of losing contact is

$$\varepsilon(t_{1,i} + t_{lc,i}^+) = \varepsilon_t + t_{lc,i} \dot{\varepsilon}_t - \frac{\sigma_a}{E}. \quad (26)$$

From the moment contact is lost the material is unconstrained, and the strain will recover toward zero. This material relaxation between cycles 1 and 2 results from solving Equation (17) with boundary conditions $\sigma = 0$ and $\dot{\sigma} = 0$:

$$\varepsilon = \varepsilon(t_t + t_{lc,1}^+) \exp\left(-\frac{1-\phi}{\lambda}(t - (t_t + t_{lc,1}))\right) \quad \forall t \in [t_t + t_{lc,1}, 2t_t + t_w + t_{mc}]. \quad (27)$$

The springiness and cohesiveness depend on the deformed sample height at the time contact is re-established. The time between the moments of losing contact and making contact is the time the strain recovers toward zero and is defined as t_{rec} in Fig. 1. To determine the time t_{rec} , the total recovery time is split into two parts, the time between losing contact in the first cycle and the time the plate starts to move to perform the second cycle results in $\varepsilon(2t_t + t_w)$ using Equation (27). From this moment the plate starts to move while the material is still relaxing according to Equation (27). To determine the time t_{mc} , the equation

$$\varepsilon(2t_t + t_w) \exp\left(-\frac{1-\phi}{\lambda}t_{mc}\right) + \dot{\varepsilon}_t t_{mc} = 0, \quad (28)$$

is solved, obtaining

$$t_{mc} = \frac{\lambda}{1-\phi} \Omega\left(\frac{\varepsilon(2t_t + t_w)(1-\phi)}{-\dot{\varepsilon}_t \lambda}\right). \quad (29)$$

The second cycle is completed by repeating the steps taken to derive the first cycle, starting with Equation (21). The initial conditions are given by $\varepsilon_{0,2} = -t_{mc}\dot{\varepsilon}_t$ and $t_{0,2} = 2t_t + t_w + t_{mc}$. The analytical derivations allow to establish an expression of the TPA parameters in terms of the mechanical properties and test parameters.

Texture parameters

The stress in the material after the loading step in the first compression cycle $\sigma(t_t)$ is given by Equation (21), from which the hardness results, making use of the fact that the test time $t_t = -\varepsilon_t/\dot{\varepsilon}_t$:

$$H = -\sigma(t_t)A_0 = A_0 E \left(\dot{\varepsilon}_t \lambda \phi \left(1 - \exp\left(\frac{\varepsilon_t}{\dot{\varepsilon}_t \lambda}\right) \right) - (1-\phi)\varepsilon_t \right). \quad (30)$$

To calculate the springiness, the times s_1 and s_2 indicated in Fig. 1 are required. The resulting springiness equals

$$S = \frac{t_t - t_{mc}}{t_t} = 1 + \frac{\dot{\varepsilon}_t t_{mc}}{\varepsilon_t}, \quad (31)$$

in which the time to make contact t_{mc} is given by Equation (29). The areas A'_1 and A'_2 in Fig. 7 are equivalent to A_1 and A_2 in Fig. 1 and are calculated by integrating the stress as a function of time, whereby the stress is negative. The area A'_i of cycle i is split into a loading part $A'_{l,i}$ and an unloading part $A'_{u,i}$, which are found by integrating Equations (21) and (22):

$$A'_{l,i} = E \left[-\dot{\varepsilon}_t \lambda^2 \phi \left(\exp\left(-\frac{t_{1,i} - t_{0,i}}{\lambda}\right) - 1 \right) - \dot{\varepsilon}_t \lambda \phi(t_{1,i} - t_{0,i}) \right]$$

$$+ (1 - \phi) \left(-\frac{1}{2} \dot{\varepsilon}_t (t_{1,i} - t_{0,i})^2 + (t_{1,i} - t_{0,i}) \varepsilon_{0,i} \right) \Big], \quad (32)$$

$$A'_{u,i} = E \left[\lambda (\sigma_{1,i} / E - (1 - \phi) \varepsilon_t - \dot{\varepsilon}_t \lambda \phi) \left(1 - \exp \left(-\frac{t_{lc,i}^0}{\lambda} \right) \right) + \dot{\varepsilon}_t \lambda \phi t_{lc,i}^0 \right. \\ \left. + (1 - \phi) \left(\frac{1}{2} \dot{\varepsilon}_t (t_{lc,i}^0)^2 + t_{lc,i}^0 \varepsilon_t \right) \right]. \quad (33)$$

Then the cohesiveness C is calculated as

$$C = \frac{A'_{1,2} + A'_{u,2}}{A'_{1,1} + A'_{u,1}}. \quad (34)$$

The adhesiveness A_3 is found by integrating the unloading curve, given by Equation (22), in the interval $[t + t_{lc}^0, t + t_{lc}]$ and multiplying it with the initial cross-sectional area A_0 :

$$A_3 = A_0 E \left[(\lambda (1 - \phi) \varepsilon (t + t_{lc,1}^0) + \dot{\varepsilon}_t \lambda^2 \phi) \left(\exp \left(-\frac{t_a}{\lambda} \right) - 1 \right) + \dot{\varepsilon}_t \lambda \phi t_a \right. \\ \left. + (1 - \phi) \left(\frac{1}{2} \dot{\varepsilon}_t t_a^2 + t_a \varepsilon (t + t_{lc,1}^0) \right) \right]. \quad (35)$$

3.4 Elasto-viscoplastic material

Constitutive model

The constitutive model that is used to describe an elasto-viscoplastic material is a specific case of the model presented by Jonkers et al. (2020) and is implemented in MSC.Marc using the user subroutine HYPELA2. To separate elastic and plastic deformation, the total deformation gradient tensor is multiplicatively decomposed (Lee 1969) in an elastic part $\mathbf{F}_e$ and a plastic part $\mathbf{F}_p$:

$$\mathbf{F} = \mathbf{F}_e \cdot \mathbf{F}_p. \quad (36)$$

The Cauchy stress tensor that results from this deformation is calculated as

$$\bar{\boldsymbol{\sigma}} = \frac{G}{J} \tilde{\mathbf{B}}_e^d + \frac{K}{2} (J^{\frac{1}{3}} - J^{-\frac{5}{3}}) \mathbf{I} \quad (37)$$

with the shear modulus G and the bulk modulus K . The volume change ratio J and the isochoric elastic finger tensor $\tilde{\mathbf{B}}_e$ are defined as

$$J = \det(\mathbf{F}_e) \quad \text{and} \quad \tilde{\mathbf{B}}_e = J^{-\frac{2}{3}} \mathbf{F}_e \cdot \mathbf{F}_e^T. \quad (38)$$

The plastic rate of deformation tensor $\hat{\mathbf{D}}_p$ is calculated from the equivalent plastic shear strain rate $\dot{\gamma}_p^{\text{eq}}$, the equivalent shear stress τ^{eq} , and the deviatoric part of the elastic second Piola–Kirchhoff-like stress tensor $\hat{\mathbf{S}}^d$ using

$$\hat{\mathbf{D}}_p = \frac{\dot{\gamma}_p^{\text{eq}}}{2\tau^{\text{eq}}} \hat{\mathbf{S}}^d \quad (39)$$

Fig. 9 Intrinsic material response of the elasto-viscoplastic material (Color figure online)

The deviatoric part of the elastic second Piola–Kirchhoff-like stress tensor is defined as

$$\hat{\mathbf{S}}^d = (J \mathbf{F}_e^{-1} \cdot \bar{\boldsymbol{\sigma}} \cdot \mathbf{F}_e^{-T})^d, \quad (40)$$

and the equivalent shear stress is given by

$$\tau^{\text{eq}} = \sqrt{\frac{1}{2} \hat{\mathbf{S}}^d : \hat{\mathbf{S}}^d}. \quad (41)$$

The equivalent plastic strain rate is obtained from the equivalent shear stress as

$$\dot{\gamma}_p^{\text{eq}} = \dot{\gamma}_0 \left(\frac{\tau^{\text{eq}}}{\tau_c} \right)^m \quad (42)$$

with the reference shear rate $\dot{\gamma}_0$, the characteristic stress τ_c , and the exponent m . The value $m = 50$ is chosen to ensure that the yield stress is approximately independent of the applied strain rate, and $\dot{\gamma}_0 = 10^{-3} \text{ s}^{-1}$ is chosen arbitrarily as the reference shear rate for τ_c . The resulting uniaxial intrinsic material response is shown in Fig. 9. The material parameters that influence this response are the elastic moduli G and K and the characteristic stress τ_c , which defines the maximum stress, that is, the yield stress $\bar{\sigma}_y$ at the corresponding yield strain $\bar{\varepsilon}_y$. Note that the material parameters $\bar{\sigma}_y$ and $\bar{\varepsilon}_y$ are equivalent measures (i.e., defined positive), whereas in compression it holds that $\bar{\sigma} = -\bar{\sigma}_y$ and $\bar{\varepsilon} = -\bar{\varepsilon}_y$ at the yield point. Figure 3 reveals a small force increase when reaching the yield stress, whereas the true stress remains constant in the plastic regime. This is due to the increase of the cross-sectional area during plastic deformation.

By introducing damage in the material the stress does not remain equal but is decreasing after reaching the yield stress. This effect is also known as strain softening. The softening regime is characterized by a damage parameter ξ , which decreases the initial, undamaged shear modulus G_0 and characteristic stress τ_0 of the material:

$$\tau_c = \tau_0(1 - \xi) \quad \text{and} \quad G = G_0(1 - \xi). \quad (43)$$

In the undamaged case the parameters retain their initial values, that is, $\tau_c = \tau_0$ and $G = G_0$. The damage parameter, which is initially zero, evolves with the equivalent plastic strain rate according to

$$\dot{\xi} = d \left(1 - \frac{\xi}{\xi_\infty} \right) \dot{\gamma}_p^{\text{eq}}. \quad (44)$$

The damage parameter d determines the rate at which the damage develops, and ξ_∞ defines the limit value of ξ . The influence of these parameters on the intrinsic material response is depicted in Fig. 9. The addition of damage in the material may have mesh-dependent results as a consequence if no precautions are taken. Here this effect is regularized by the viscoplastic response of the material, in particular, by the model parameter m in Equation (42).

Analytical approximation

The material is nonlinear, and a numerical method is required to obtain the exact response of the material to the prescribed deformation in the TPA test. However, by considering a uniaxial stress state and by making some approximations on the effective material response, analytical relations between the texture parameters and intrinsic mechanical properties can be recovered, which closely describe the actual material response.

The specific form of Equation (42) implies that the plastic deformation is continuously increasing. Therefore the identified yield strain $\bar{\epsilon}_y$ is not purely elastic, but has a small plastic contribution. By neglecting this plastic contribution and, in addition, the nonlinearity of the elastic regime, the yield strain $\bar{\epsilon}_y$ can be approximated by

$$\bar{\epsilon}_y = \bar{\sigma}_y / E, \quad (45)$$

and the irrecoverable plastic strain $\bar{\epsilon}_p$ by

$$\bar{\epsilon}_p = \bar{\epsilon}_t + \bar{\epsilon}_y. \quad (46)$$

Moreover, all analytical relations are expressed in true stress and strain definitions because the purpose is to relate the TPA parameters to the intrinsic material parameters. If damage is included, then the plastic deformation induced by the applied strain affects the mechanical properties. The equivalent plastic shear strain $\gamma_{p,t}^{\text{eq}}$, resulting from ϵ_t , is calculated as

$$\gamma_{p,t}^{\text{eq}} = \sqrt{3} |\bar{\epsilon}_p|, \quad (47)$$

where the factor $\sqrt{3}$ converts the uniaxial strain to an equivalent shear strain. The damage parameter ξ is obtained by integrating Equation (44), which results in

$$\xi = \xi_\infty \left(1 - \exp \left(-\frac{d}{\xi_\infty} \gamma_p^{\text{eq}} \right) \right). \quad (48)$$

The damaged Young's modulus is calculated by combining Equations (20) and (43), resulting in

$$E(\xi_t) = 2G_0(1 - \xi_t)(1 + \nu), \quad (49)$$

in which ξ_t is the induced damage, found by substituting $\gamma_p^{\text{eq}} = \gamma_{p,t}^{\text{eq}}$ into Equation (48).

Since here the time-dependence of the material behavior is neglected, which is a valid assumption for a high power-law exponent m in Equation (42), the only relevant test parameter is the applied strain $\bar{\epsilon}_t$. The standard values of $t_w = 1$ s and $\dot{\epsilon}_t = 2$ s⁻¹, which correspond to a velocity of 20 mm/s (Vliet and Primo-Martín 2011) for a specimen length $L_0 = 10$ mm, are chosen.

Fig. 10 TPA test of an elasto-viscoplastic material without damage: **(a)** force-time response and **(b)** intrinsic material behavior

Texture parameters

Figure 10a shows the result of a typical TPA experiment, obtained by a finite element simulation of an elasto-viscoplastic material without damage, and Fig. 10b shows the corresponding stress-strain response. The applied strain exceeds the yield point, resulting in plastic deformation. The hardness is found from the deformed cross-sectional area and the stress in the material as

$$H = -A\bar{\sigma} = A \min(-E\bar{\varepsilon}_t, \bar{\sigma}_y). \quad (50)$$

Changes in cross-sectional area in the elastic regime result from the Poisson's ratio ν . The deformed area A is, neglecting nonlinearities in the elastic regime, given by

$$A = (1 - \nu\varepsilon)^2 A_0. \quad (51)$$

In the plastic regime, the change in cross-sectional area is based on the incompressibility of plastic deformation, that is, the volume remains constant. The actual hardness H is calculated using the deformed cross-sectional area. If the area changes are small, then the hardness can be approximated from the initial cross-sectional area:

$$H = A_0 \min(-E\bar{\varepsilon}_t, \bar{\sigma}_y). \quad (52)$$

If the deformation is purely elastic, then the springiness and cohesiveness are equal to one. In the case of plastic deformation, at times s_1 and s_2 in Fig. 1, the corresponding strain values at these time instances can be extracted from Fig. 10b. The strain corresponding to s_1 is the applied strain $\bar{\varepsilon}_t$, and that corresponding to s_2 is the recovered elastic strain $\bar{\varepsilon}_y$, resulting in the springiness

$$S = \frac{\bar{\varepsilon}_y}{-\bar{\varepsilon}_t}. \quad (53)$$

Similarly, the areas A_1 and A_2 of Fig. 1 have equivalent energies per unit volume, which are found by extracting W_1 and W_2 in Fig. 10b. The required energy per unit volume for the loading of the first compression cycle is given by $W_1 = \frac{1}{2}\bar{\varepsilon}_y\bar{\sigma}_y - (\bar{\varepsilon}_t + \bar{\varepsilon}_y)\bar{\sigma}_y$. The recovered energy during unloading, which is also equal to the loading and unloading energy in the

Fig. 11 TPA test of a damaging elasto-viscoplastic material ($d = 5$, $\xi_{\infty} = 0.4$) for different applied strains $\bar{\varepsilon}_t$: **(a)** force-time response and **(b)** intrinsic material behavior (Color figure online)

second compression cycle, is given by $W_2 = \frac{1}{2} \bar{\varepsilon}_y \bar{\sigma}_y$. From this the cohesiveness is calculated as

$$C = \frac{2W_2}{W_1 + W_2} = \frac{\bar{\varepsilon}_y}{-\bar{\varepsilon}_t}. \quad (54)$$

Clearly, the springiness and cohesiveness are both measures of plastic deformation in the material and provide exactly the same information for the considered material.

Figure 11a shows the stress-strain and force-time responses of a damaging elasto-viscoplastic material for different values of $\bar{\varepsilon}_t$. Figure 11a reveals the interplay between the damage, causing the force to decrease and the increase in cross-sectional area, which result in a force increase. Once $\xi = \xi_{\infty}$ is reached, only the drive for an increasing force prevails. Depending on $\bar{\varepsilon}_t$, this increasing force may result in a second peak that is larger than the first peak. In that case the force of the first peak is known as the fracturability F_r , and the force of the second peak corresponds to the hardness H (Fizman et al. 1998).

The approximations of the hardness (52) and springiness (53) still hold. The calculation of cohesiveness requires the energy per unit volume in the softening regime W_d , which is approximated by

$$\begin{aligned} W_d &= \int_0^{\gamma_{p,t}^{\text{eq}}} \frac{\bar{\sigma}_y}{\sqrt{3}} (1 - \xi) d\gamma_p^{\text{eq}} \\ &= \frac{\bar{\sigma}_y}{\sqrt{3}} \left(\gamma_{p,t}^{\text{eq}} (1 - \xi_{\infty}) + \frac{\xi_{\infty}^2}{d} \left(1 - \exp \left(-\frac{d}{\xi_{\infty}} \gamma_{p,t}^{\text{eq}} \right) \right) \right). \end{aligned} \quad (55)$$

Analogously to the calculation of cohesiveness for the undamaged case, the cohesiveness of the damaging material is approximated by

$$C = \frac{\bar{\varepsilon}_y \bar{\sigma}_y (1 - \xi_t)}{\bar{\varepsilon}_y \bar{\sigma}_y (1 - \frac{\xi_t}{2}) + W_d}. \quad (56)$$

Due to the nearly time-independent material behavior, adhesion is not affecting any TPA parameter except for the adhesiveness. The time t_a to reach the adhesion stress $\bar{\sigma}_a$ is calcu-

Table 1 Overview of different parameters

Material	Mechanical properties	Test parameters
Linear viscoelastic	E, ϕ, λ	$\varepsilon_t, \dot{\varepsilon}_t, t_w$
Elasto-viscoplastic	$E, \bar{\sigma}_y, d, \xi_\infty$	$\bar{\varepsilon}_t$
Contact	σ_a or $\bar{\sigma}_a$	

lated from the damaged Young's modulus $E(\xi_t)$ as

$$t_a = \frac{\bar{\sigma}_a}{E(\xi_t)\dot{\varepsilon}_t}, \quad (57)$$

from which the adhesiveness is approximated by

$$A_3 = \frac{1}{2}\bar{\sigma}_a A_0 t_a = \frac{\bar{\sigma}_a^2 A_0}{2E(\xi_t)\dot{\varepsilon}_t}. \quad (58)$$

4 Results

4.1 Homogeneous deformation

The analytical derivations enable investigating the relation between texture parameters, mechanical properties, and test conditions. Analytical relations for the linear viscoelastic material are exact and show the effect of time-dependent properties. We compare the analytical results of the elasto-viscoplastic material with the numerical solution to investigate the accuracy of the approximations made. An overview of the two materials, including the analyzed mechanical properties and test parameters, is presented in Table 1.

Linear viscoelastic material

The hardness given by Equation (30) is determined by the product of the relaxation time and the strain rate $\lambda\dot{\varepsilon}_t$ and the viscoelastic fraction ϕ of the material as presented in Fig. 12. If $\lambda\dot{\varepsilon}_t$ is high or if the fraction ϕ is low, then the material responds linear elastically and reaches the hardness value H_e given by

$$H_e = -A_0 E \varepsilon_t. \quad (59)$$

For very small values of $\lambda\dot{\varepsilon}_t$, the hardness takes the value $(1 - \phi)H_e$ corresponding to the elastic response of the parallel spring with Young's modulus E_∞ . A combination of material parameters and test conditions that gives a normalized hardness H/H_e close to one also results in springiness and cohesiveness values close to one because it is dominated by the elastic component.

Figure 13a shows the effect of the material parameters on the springiness (Equation (31)) and cohesiveness (Equation (34)) for the adhesion-free case and constant test conditions $\varepsilon_t = -0.1$, $\dot{\varepsilon}_t = -2.0 \text{ s}^{-1}$, and $t_w = 1.0 \text{ s}$. Two characteristic regimes are visible. A small relaxation time means that the viscous component dominates the elastic response. Contact is lost in a short time period (i.e., t_{lc} is small), but the material relaxes fast after losing contact. A large relaxation time implies that within the considered timescale, the elasticity is more important. The compressive strain decreases with the test rate for a longer time (i.e.,

Fig. 12 Normalized hardness of a viscoelastic material for different values of the viscoelastic fraction ϕ (Color figure online)

Fig. 13 Springiness and cohesiveness of a viscoelastic material: **(a)** effect of relaxation time λ and viscoelastic fraction ϕ for $\varepsilon_t = -0.1$, $\dot{\varepsilon}_t = 2.0 \text{ s}^{-1}$, and $t_w = 1.0 \text{ s}$, and **(b)** different test parameters for $\lambda = 0.05 \text{ s}$ and $\phi = 0.995$ (Color figure online)

t_{lc} is closer to t_t). In comparison to a smaller relaxation time, the unconstrained material relaxation given by Equation (27) is slower, but this is compensated by the fast constrained unloading.

The interplay between the two regimes results in a minimum for the springiness and cohesiveness (as a function of relaxation time). These minimum values decrease for an increasing fraction ϕ , meaning that the viscoelastic response becomes more important, as depicted in Fig. 13a. The value of the total stiffness E does not affect the results, only the fraction ϕ matters. For a large relaxation time, ϕ has only a small influence on the TPA parameters, which corresponds to the elastic regime, also clearly visible in the hardness; see Fig. 12.

To show the effect of the test parameters in Fig. 13b, a material with relaxation time $\lambda = 0.05 \text{ s}$ and fraction $\phi = 0.995$ is taken. The springiness and cohesiveness are functions of the test time $t_t = -\varepsilon_t/\dot{\varepsilon}_t$: a small test time corresponds to an elastic response and thus the texture parameter close to one. Larger values of t_t imply a viscous response of the material. By increasing the test time the time to relax, while the plate is moving, becomes larger, which results in higher springiness and cohesiveness values. The same effect is obtained by increasing the time between the two compression cycles t_w , as presented in Fig. 13b.

Figure 14a shows the increasing adhesiveness A_3 given by (35) for an increasing adhesion stress σ_a or a decreasing Young's modulus E . A determining factor in the adhesiveness is the time t_a it takes to reach the adhesion stress. This time depends on the combination of σ_a and E , as derived in Equation (23). Moreover, the adhesiveness depends on the re-

Fig. 14 Effect of the adhesion stress σ_a and Young's modulus E on (a) the adhesiveness and (b) the springiness and cohesiveness of a viscoelastic material for $\epsilon_t = -0.1$, $t_w = 1.0 \text{ s}$, $\phi = 0.995$, and $\lambda = 0.05 \text{ s}$ (Color figure online)

laxation time λ , the fraction ϕ , and the strain rate $\dot{\epsilon}_t$. Consequently, for this material it is questionable whether the adhesiveness is a meaningful parameter, which is also discussed by Fizman and Damasio (2000). Figure 14b shows the effect of adhesion on the springiness and cohesiveness. The effect is negligible when the adhesion stress is much smaller than the compressive stress at the applied strain ϵ_t . This is illustrated by decreasing the deformation rate, which results in a lower maximum compressive stress. Subsequently, the springiness and cohesiveness are more affected by adhesion.

Elasto-viscoplastic material

The analytical solutions for the elasto-viscoplastic material without damage are compared to the nonlinear numerical results in Fig. 15 for different combinations of Young's modulus E , yield stress $\bar{\sigma}_y$, and applied strain $\bar{\epsilon}_t$. The validity of neglecting changes in cross-sectional area to calculate the hardness (52) depends on $\bar{\epsilon}_t$. The numerical solutions for a large compressive strain ($\bar{\epsilon}_t < -0.5$) deviate significantly from the analytical solution, as depicted in Fig. 15a. A large compressive strain results in a large increase of the cross-sectional area, which is not taken into account in the analytical approximation. Next to this, neglecting the nonlinearity of the elastic regime causes a deviation between the analytical and numerical results if the yield strain ϵ_y is large.

The analytical approximations for the springiness (53) and cohesiveness (54) are in good agreement with the numerical results. These texture parameters are only sensitive to the nonlinearities in the constitutive model and insensitive to variations in cross-sectional area. In the undamaged case, the springiness and cohesiveness provide the same information on the material response, in particular, a measure of the plastic, irrecoverable deformation in the material.

This similarity does not hold if damage, resulting in the characteristic strain softening regime, is added. Figure 16a compares the analytical and numerical results of the springiness for different values of ξ_∞ . The numerical results confirm that the damage parameters do not affect the springiness value. In contrast to the springiness, the cohesiveness depends on the damage parameters. Because the effect of changing d turns out to be negligible, Fig. 16b only shows the results of the cohesiveness for different values of the damage limit value ξ_∞ . All cohesiveness values are below the prediction of the undamaged material as a result

Fig. 15 Comparison of analytical with numerical results for the elasto-viscoplastic material: **(a)** hardness and **(b)** springiness and cohesiveness. Each data point corresponds to a different combination of E , $\bar{\sigma}_y$ and $\bar{\epsilon}_t$ (Color figure online)

Fig. 16 Comparison of analytical and numerical results of the **(a)** springiness and **(b)** cohesiveness of a damaging material for different values of ξ_∞ . Each data point corresponds to a different combination of E , $\bar{\sigma}_y$, and $\bar{\epsilon}_t$ (Color figure online)

of the softening, which becomes more pronounced for larger values of ξ_∞ . Whereas the springiness only provides information on the deformation in the sample, the cohesiveness is a measure of structural damage in the material due to this deformation.

The adhesiveness A_3 of an elasto-viscoplastic material is depicted in Fig. 17. The analytical relation (58) is in adequate agreement with the actual, numerically obtained adhesiveness. If there is no damage, then the adhesiveness is directly obtained from the adhesion stress $\bar{\sigma}_a$, strain rate $\dot{\bar{\epsilon}}_t$, cross-sectional area A_0 , and Young's modulus E and does not depend on the applied strain $\bar{\epsilon}_t$ or yield stress $\bar{\sigma}_y$. This is not the case if damage is added to the material. The decrease of the stiffness induced by damage results in a higher adhesiveness. In Fig. 17, this is shown by increasing the limit value ξ_∞ , which reveals the same trends for a material with a lower yield strain $\epsilon_y = \bar{\sigma}_y/E$ or for a higher applied compressive strain $\bar{\epsilon}_t$.

4.2 Heterogeneous deformation

The effect of friction and surface roughness are analyzed for the linear viscoelastic material as the effect on the elasto-viscoplastic material is similar. The effect of friction turns out

Fig. 17 Adhesiveness of an elasto-viscoplastic material as a function of the adhesion stress $\bar{\sigma}_a$ and strain rate $\dot{\varepsilon}_t$ with $E = 40$ MPa, $\nu = 0.15$, $\bar{\sigma}_y = 2.7$ MPa, and $\bar{\varepsilon}_t = -0.22$ (Color figure online)

Fig. 18 Effect of roughness for a viscoelastic material: **(a)** TPA test for relaxation time $\lambda = 0.03$ s and **(b)** relative deviation of hardness, springiness, and cohesiveness (Color figure online)

to be negligible. The hardness increases 1% due to friction, whereas the springiness and cohesiveness remain unaffected.

We investigate the effect of surface roughness for a varying relaxation time λ . The other material parameters ($E = 20$ MPa and $\phi = 0.995$) and test parameters ($\varepsilon_t = -0.1$, $\dot{\varepsilon}_t = 2.0$ s⁻¹, and $t_w = 0.2$ s) are constant. The effect of roughness on the TPA tests with $\lambda = 0.03$ s is presented in Fig. 18a. The stiffness in the first loading cycle is smaller because the initial contact area is small. This effect is less pronounced in the second loading cycle as the contact surface is flattened in the first cycle. The relative deviation Δ of a parameter x is given by

$$\Delta = \frac{x_{\text{rough}} - x_{\text{flat}}}{x_{\text{flat}}} \quad (60)$$

and depicted for the hardness, springiness, and cohesiveness in Fig. 18b. The hardness is most affected, especially if λ is large, resulting in a more elastic response. The relative deviation of cohesiveness exceeds 15% for small relaxation times. However, the absolute deviation is below 0.06 in the whole interval and becomes significant only in the region where the cohesiveness is small, similar to the minimum cohesiveness value in Fig. 13a. The relative deviation of the springiness is below 3% and therefore negligible.

5 Conclusion

In this research, constitutive models are used to predict the effective mechanical response of the materials. However, an appropriate constitutive model is not necessary to establish relations between texture parameters and intrinsic mechanical properties. Careful experimental characterization of the mechanical behavior of a food material provides all the information that is essential to predict its texture properties. A double compression test, the TPA test, can be used to determine the mechanical characteristics of a particular food product, but the analysis should involve the mechanical properties instead of the currently used instrumental texture parameters.

Analytical relations between instrumental texture parameters, test conditions, and mechanical properties have been derived, and analyzed by numerical modeling. Two types of representative materials were chosen and characterized by a suitable constitutive model that is used to simulate the texture profile analysis test.

For the time-dependent linear viscoelastic material, exact relations are obtained by solving the differential equation for all steps of the TPA test. The characteristic relaxation time of the material in combination with the applied deformation rate is the governing factor in the interplay between the elastic and viscous components of the material. Combining these two characteristic time measures with the time-dependent stiffness of the material provides the instrumental texture parameters of a viscoelastic material. Although the values of springiness and cohesiveness are not equal for a specific set of material and test parameters, the same dependencies were identified. Consequently, the springiness and cohesiveness qualitatively provide the same structural information on the material when plasticity is not taken into account.

The large-deformation response of an elasto-viscoplastic material is studied to include the effect of plastic deformation. By neglecting some nonlinearities in the constitutive model, simple analytical relations are found between texture and material properties. Clearly, the springiness and cohesiveness are a direct result of plasticity in the material and provide exactly the same qualitative information. The plastic deformation, resulting from the applied strain, is easily found from the stiffness and yield stress of the material. By adding damage to the material, that is, a reduction of mechanical properties, it is found that the springiness is independent of the damage parameters for the considered damage model. The springiness only provides information on the current strain in the material, which may be recoverable (viscoelastic) or irrecoverable (plastic). However, the cohesiveness decreases when the damage in the material is more severe and contains therefore relevant structural information.

Two nonideal contact situations that cause heterogeneous deformation are investigated. The effect of friction on the results of the TPA test is negligible. Surface roughness has a major influence on the hardness but a minor effect on the other texture parameters as it mainly involves the first compression cycle.

Using the maximum compressive stress instead of the maximum force as the definition of hardness, which is currently affected by the specimen size, is suggested. An accurate calculation of the compressive stress requires the deformed cross-sectional area, which however can be approximated by the initial cross-sectional area if volumetric changes in the material are large, which may be the case, for instance, for porous materials. The springiness and cohesiveness provide important structural information, which is something that is definitely relevant for sensory attributes. However, the relevance of the instrumental adhesiveness is questionable for the used materials, because it does not only depend on the adhesion between the material and the probe. Clearly, the instrumental texture parameters are a result

of the mechanical characteristics of the food product. A better option is therefore to link the intrinsic mechanical properties directly to sensory attributes, as the dependence on test conditions is inherently taken into account.

Authors' contributions N. Jonkers: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing, original draft. J.A.W. van Dommelen: Conceptualization, Funding acquisition, Methodology, Project administration, Supervision, Writing, review and editing. M.G.D. Geers: Conceptualization, Funding acquisition, Methodology, Resources, Supervision, Writing, review and editing.

Funding The authors acknowledge TNO (Netherlands Organisation for Applied Scientific Research) for the financial support.

Data availability The data generated or analyzed in this study are available upon reasonable request.

Code availability Numerical simulations have been conducted using MSC.Marc 2013.

Conflicts of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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