Earth Moon Planets (2014) 113:99–107
DOI 10.1007/s11038-014-9447-5

Analytical Estimation of the Scale of Earth-Like Planetary Magnetic Fields

Mauro Bologna · Bernardo Tellini

Received: 25 June 2012 / Accepted: 16 August 2014 / Published online: 24 August 2014

Abstract In this paper we analytically estimate the magnetic field scale of planets with physical core conditions similar to that of Earth from a statistical physics point of view. We evaluate the magnetic field on the basis of the physical parameters of the center of the planet, such as density, temperature, and core size. We look at the contribution of the Seebeck effect on the magnetic field, showing that a thermally induced electrical current can exist in a rotating fluid sphere. We apply our calculations to Earth, where the currents would be driven by the temperature difference at the outer-inner core boundary, Jupiter and the Jupiter's satellite Ganymede. In each case we show that the thermal generation of currents leads to a magnetic field scale comparable to the observed fields of the considered celestial bodies.

Keywords Planetology of solid surface planets: Magnetic field and magnetism · Planetology of fluid planets: Magnetic field and magnetism · Earth's interior structure and properties

1 Introduction

The origin of the Earth's magnetic field and more in general of the celestial body's magnetic field, is a problem that has been faced by many authors (see Stevenson 1983, 2010 for a review and prospectives about planetary magnetic fields). For many years the intuitive idea that the magnetic field is generated by heavy fluid in the center of Earth subjected to the rotational motion of our planet, has been conjectured. Many numerical works have started to shed light on the possible mechanism of the generation of Earth's

M. Bologna (✉)
Instituto de Alta Investigación, Universidad de Tarapacá, Casilla 7-D, Arica, Chile
e-mail: mauroh69@gmail.com

B. Tellini
Department of Energy, Systems, Territory and Construction Engineering, University of Pisa, Largo L. Lazzarino, 56122 Pisa, Italy
e-mail: bernardo.tellini@ing.unipi.it

magnetic field (Glatzmaier and Roberts 1995; Kuang and Bloxham 1997; Roberts and Glatzmaier 2000; Busse 2000; Kono and Roberts 2002). The basic model for the generation of Earth's magnetic field or of other planets, is based upon the dynamo effect of a turbulent convection in rotating fluids. This idea has received much attention in the past few years and many numerical studies based on the dynamo model (see the above references) attempted to reproduce some of the main properties of the magnetism of celestial bodies, among them the phenomenon of magnetic field reversal. Magnetic field reversal, the phenomenon for which the positions of magnetic north and magnetic south are interchanged, is another important feature of the terrestrial magnetic field that has been studied intensively, and recently a similar phenomenon has been reproduced in the laboratory (Berhanu et al. 2007).

In a previous work (Bologna and Tellini 2010) the authors showed that a magnetic field can be generated in the laminar region of a fluid velocity under the condition $\rho = \eta\sigma\mu$, but such a condition is far from the usual condition of the celestial body and, more important, its magnitude would be of the order of $B \sim \Omega R\sqrt{\rho\mu_0}$ where Ω is the rotational velocity, R is the radius of the outer core, and ρ is the density of the fluid in the outer core of Earth (Lee 2002). Inserting Earth's parameters would imply an intensity field of $B \sim 10^5$ gauss that is very far from Earth's actual magnetic field value, e.g. 25 gauss in the central region. To authors' knowledge no magnetic field scale as a function of the physical system parameters (such as density, temperature, and core size etc.) is known for Earth. We shall consider planets with a core wherein the electrons can be considered approximately in a degenerate state (degenerate Fermi's gas) and where the currents would be driven by the temperature difference at the outer-inner core boundary. We shall show that such a characteristic magnetic field exists if we consider the thermal contribution at the basis of the generation of the magnetic field. The intensity of such a field is close to the actual value of the Earth's field and we shall apply the basic idea also to the Jupiter planet and its satellite, Ganymede.

2 Magnetohydrodynamic Equations

To make the paper as self-contained as possible, we review briefly the set of equations for a plasma with finite conductivity and constant density (Landau and Lifshitz 1981; Jackson 1998)

$$\rho\left[\frac{\partial}{\partial t}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla P + [\nabla\times\mathbf{H}]\times\mathbf{B} + \mathbf{f} + \boldsymbol{\sigma} \tag{1}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{2}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times[\mathbf{v}\times\mathbf{B}] + \frac{1}{\mu\sigma}\nabla^2\mathbf{B} \tag{3}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{4}$$

where $\mathbf{v}$ is the flow velocity, $\mathbf{H}$ is the magnetic field related to the magnetic induction $\mathbf{B}$ via the relation $\mathbf{B} = \mu\mathbf{H}, P$ is the pressure of the fluid, and ρ is the mass density. The gravity force density, $\mathbf{f}$, takes the form $\mathbf{f} = \rho\nabla\psi$ where ψ is the gravitational potential. The vector $\boldsymbol{\sigma}$ is defined through its components as follows (Landau and Lifshitz 1987)

$$\sigma_i = \frac{\partial \sigma'_{ik}}{\partial x_k}, \quad \sigma'_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) \tag{5}$$

where σ'_{ik} is the viscous stress tensor, and η is the coefficient of viscosity which is assumed constant. We also used the convention of dropping the symbol of sum for the repeated indexes. The current density $\mathbf{J}$ is given by the constitutive relation $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, i.e. Ohm's law, where σ is the electrical conductivity of the fluid.

The dynamic system has to be implemented using the equation of heat transfer in magnetohydrodynamics (Landau and Lifshitz 1981)

$$\rho c_p \left(\frac{\partial}{\partial t} T + \mathbf{v} \cdot \nabla T\right) = \sigma'_{ik} \frac{\partial v_i}{\partial x_k} + \kappa \nabla^2 T + \frac{J^2}{\sigma} + Q \tag{6}$$

where c_p is the specific heat at constant pressure, κ is the thermal conductivity, T is the temperature of the fluid, and Q is the quantity of heat generated by external sources of heat contained in a unit volume of the fluid per unit time.

As many authors have pointed out the analytical solution of this equation is a very hard task and only a few exact or approximate cases are known (see for example Sutton and Sherman 2006 for a review). Our main aim is to find an estimation for the scale of the magnetic field as a function of the physical parameters of the center of Earth (such as density, temperature, etc.) without necessarily solving the system (1)–(4).

3 Remarks on Field Velocity Components for a Rotating Sphere

In this section we will show that for a rotating sphere, in general the ϕ component of the velocity is not enough to describe a time dependent motion, with it being understood that $v_\phi = \Omega r \sin\theta$, where Ω is the angular velocity, is an exact solution. Note that Kerr (2005) shown that the inner core of the Earth may spin faster than the rest of the planet so that the above exact solution does not hold for the fluid motion of the terrestrial core. This conclusion implies that the temperature distribution can not be a radial distribution due to the fact that Eq. (6) is, in general, coupled with the velocity field. To enforce this statement we shall show that in general a rotating fluid sphere, as previously stated, can not be described by the ϕ component of velocity. Let us then assume that a sphere starts to rotate with $v_\phi(t)$ such that $v_\phi(0) = 0$ and $v_\theta = v_r = 0$. We also assume that for symmetry all physical variables do not depend on ϕ. Writing only the hydrodynamic part of the set of equations (1)–(4), i.e. setting $\mathbf{B} = 0$, we obtain

$$\cot\theta \frac{v_\phi^2}{r} = \frac{1}{\rho r} \frac{\partial P}{\partial \theta} \tag{7}$$

$$\frac{v_\phi^2}{r} = \frac{1}{\rho} \frac{\partial P}{\partial r} \tag{8}$$

$$\frac{\partial v_\phi}{\partial t} = \frac{\eta}{\rho} \nabla^2 v_\phi. \tag{9}$$

Combining Eqs. (7) and (8) we obtain

$$\cot\theta \frac{\partial}{\partial r} v_\phi^2 = \frac{1}{r} \frac{\partial}{\partial \theta} v_\phi^2. \tag{10}$$

The above equation is satisfied by a function $v_\phi(r,\theta)$ of the form $v_\phi(r,\theta) = v_\phi(r\sin\theta)$. Using this result we may rewrite Eq. (9) as

$$\frac{\partial}{\partial t} v_\phi(t,x) = \frac{\eta}{\rho}\left(\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} - \frac{1}{x^2}\right) v_\phi(t,x), \quad x \equiv r\sin\theta. \tag{11}$$

The general solution of the above equation, via Laplace transform and with the condition $v_\phi(0,x) = 0$, is

$$\hat{v}_\phi(s,x) = aI_1\left(\frac{x\sqrt{s}}{\sqrt{D}}\right) + bK_1\left(-\frac{x\sqrt{s}}{\sqrt{D}}\right) \tag{12}$$

where, by definition, $\hat{v}_\phi(s,x) = \int_0^\infty \exp[-st] v_\phi(t,x)dt$, $D = \eta/\rho$ and $I_1(z)$ is the modified Bessel function of the first kind while $K_1(z)$ is the modified Bessel function of the second kind, divergent at the origin as z^{-1}. Assuming a finite solution for $r \to 0$ and consequently $x \to 0$, then $b = 0$. It is evident by the form of Eq. (12) that the boundary condition for the velocity, i.e. $\hat{v}_\phi(s, R\sin\theta) = aI_1\left(R\sin\theta\sqrt{s/D}\right)$ where R is the sphere radius, can be satisfied only by a restricted class of functions. One could consider adding another component, for example the radial component v_r, to describe matter falling in the center. But the independency of the dynamic by the angular variable ϕ implies that the θ component of the velocity v_θ also has to be considered. This fact comes directly from the continuity equation. Let us assume that there is another non vanishing component of velocity, the radial component v_r. The continuity equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) = 0. \tag{13}$$

The solution $v_r = f(\theta)/r^2$ diverges at the origin and can not vanish on the surface of the sphere $r = R$. To avoid this inconsistency we are forced to add also the third component v_θ to the flow. From this we can infer that from the early stages of Earth's formation to the present, the velocity of the fluid could not be described by only one component of the velocity vector and we conclude that the hydrodynamics of the Earth's interior is a three-D problem. This analytical conclusion is in agreement with the numerical works presented in the references (see for example Glatzmaier and Roberts 1995).

4 Thermal Generation of Magnetic Field

Looking at Eq. (3), we notice that is a diffusive-like equation. For the time evolution of **B** it is important to give an estimation of its initial value. If the magnetic field driving equation was purely diffusive, the field would then exponentially decay with a characteristic time of the order of $\Delta R^2 \mu_0 \sigma \sim 10^5$ years (Stevenson 2010). As a consequence, the term $\mathbf{v} \times \mathbf{B}$ plays the role to maintain the field since the field would disappear in few hundreds thousands of years via the diffusion process, while we know that the Earths magnetic field exists from billions of years (Tarduno et al. 2010).

As a matter of fact, it is known from the Faraday's law that a magnetic field can not be varied through a perfect conductor. Thus, it is not possible to vary the internal field distribution by external fields under the hypothesis of perfect conductor. If we consider that at the very beginning the velocity of the fluid is very low, and it is growing due to angular momentum conservation, we can make the hypothesis that at this stage of planetary formation the thermal contribution could play a crucial role while the contribution of $\mathbf{v} \times \mathbf{B}$ could be negligible. Successively, due to the increasing of the metal core density the conductivity tend to increase making difficult the insertion of an external magnetic field. Modelling at a first approximation the planet's core as a perfect conductor leads us to search for possible internal source mechanisms at the basis of the magnetic field generation. What we are going to illustrate in this section is that the magnetic field produced by a thermally induced electrical current has an intensity of the order of the magnetic field of the Earth and other celestial bodies. Note that field produced by a thermally induced electrical current would be produced by internal mechanism instead by an external source as discussed for example in Stevenson (1983). The non uniform temperature can generate a contribution to the electrical current that is proportional to the gradient of the temperature, known as Seebeck effect. The idea of a Seebeck effect powered Earth dynamo has been considered in the past (see for example Stevenson 1987; Giampieri and Balogh 2002) where the authors focused their attention to the mantle-core interaction. In this paper we consider the Seebeck effect in the inner-outer core region. We can write the total current as (Landau and Lifshitz 1981)

$$\mathbf{J} = \sigma[\mathbf{E} + \mathbf{v} \times \mathbf{B} - \alpha(T)\nabla T]. \tag{14}$$

If $\sigma \to \infty$, we can generate the term $\mathbf{E} + \mathbf{v} \times \mathbf{B}$ trough the thermal contribution $-\alpha(T)\nabla T$. Indeed for $\sigma \to \infty$ then to have a finite current density it must be that $\mathbf{E} + \mathbf{v} \times \mathbf{B} - \alpha(T)\nabla T \to 0$, i.e. $\mathbf{E} + \mathbf{v} \times \mathbf{B} \approx \alpha(T)\nabla T$.

It is accepted that the Earth's core is made mainly of iron with a solid inner core the size 10^3 km and a liquid outer core of about 2×10^3 km thick (Lee 2002). The temperature distribution of the core is not uniform and it ranges from approximatively 4.5×10^3–8×10^3 °K at the very center to 3×10^3–4×10^3 °K at the surface of the outer core.

Note that Eq. (3) does not change if $\alpha(T)\nabla T$ is written as a gradient of a function. To evaluate the coefficient α, we have to consider the fact that the density of either the solid inner core, or the fluid outer core, is such that the electrons can be considered a degenerate Fermi's gas. Indeed, according to de Wijs Gilles et al. (1988), the core density is of the order of 10^4 Kg m^{-3}. This implies that the Fermi energy of the electrons of the Earth's core is

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3} \approx 2 \times 10^{-18}\ \mathrm{J} \tag{15}$$

corresponding to a Fermi temperature $T_F \approx 1.4 \times 10^5$ °K. This is at least one order of magnitude higher than the Earth's core temperature therefore justifying the degenerate Fermi's gas approximation. Quantum calculations show that (Landau and Lifshitz 1981)

$$|\ \alpha(T)\ | \sim k_B\frac{k_B T}{e\varepsilon_F} \tag{16}$$

where k_B is the Boltzmann constant and e is the electron charge. Let us consider a very simplified model of the early stage's of Earth's formation. Models suggest that Earth's core was completely molten (Buffett 2003). Since the temperature change of the core has a time scale on the order of Earth's age (Jacobs 1953) we infer that whatever is the contribution from the thermal term in Eq. (14) this contribution still holds today with the same order of magnitude. A widely accepted estimation of the core temperature is approximatively 8×10^3 °K for the inner core and 4×10^3 °K for the outer core (see for example Poirier

1991; de Wijs Gilles et al. 1988; Alfè et al. 2002). Let us consider the contribution to the magnetic field due to thermal term. From Maxwell's equation we obtain

$$\nabla \times \mathbf{B}_T = \mu_0 \sigma \mathbf{J}_T = -\mu_0 \sigma \alpha(T) \nabla T \tag{17}$$

where μ_0 is the vacuum permeability and the subscript T stands for thermal. In general, the temperature distribution in time and space is coupled with the velocity field, Eq. (6), and as shown in the previous section, all components of the velocity are present so that the temperature distribution can not be only radial. Note also that we use μ_0 as the value of the magnetic permeability since at such temperature we assume that there is no magnetization. We can deduce the field scale via the relation

$$\frac{B_T}{R} \approx \mu_0 \sigma \alpha(T) \frac{\Delta T}{R} \tag{18}$$

where ΔT is the difference in temperature, and R is the length scale of the system. We obtain the scale strength of the thermal magnetic field

$$B_T = \mu_0 \sigma \frac{k_B T_c}{e \varepsilon_F} k_B \Delta T. \tag{19}$$

The range of values of the central temperature is $T_c \approx 4.5 \times 10^3 - 8 \times 10^3\,^\circ$K, while the range of values of the temperature difference is $\Delta T \approx 5 \times 10^2 - 4 \times 10^3\,^\circ$K. The conductivity value ranges from a theoretical estimation $\sigma \approx 10^5$ S m^{-1} (Stacey 1967), to a more recent computational evaluation $\sigma \approx 1.4 \times 10^6$ S m^{-1} (Pozzo et al. 2012). Plugging these values in Eq. (19), we obtain the numerical range of values for the strength of the core of Earth's magnetic field

$$22 \lesssim B_T \lesssim 300 \,\text{gauss}. \tag{20}$$

The estimated strength of the core of Earth 's magnetic field is approximatively $B_{est} \sim 25$ gauss (Buffett 2010) which is close to the minimum value given by Eq. (19). We note that selecting different values for the temperature, according to the different models present in the literature, the value of the field would of course change consequently, but the scale magnitude remains of the order of tens of gauss. The thermal current gives a strong (if not total) contribution to the magnetic field. It is worthy to stress that in principle the only phenomenological parameter, i.e. the conductivity σ, could be evaluated using quantum mechanics (Ashok and Evans 1972) so that we can conclude that the field given by the expression (19) may be written in terms of fundamental constant and physical parameters of the system such as density N/V and temperature T. Note that dependence on the radius R of the fluid region is implicit in the dependence of the temperature on R. This is the main reason why we kept explicit the temperature difference ΔT in Eq. (19). We conclude that Eq. (19) represents the scale of the strength of the magnetic field of celestial bodies with an Earth-like physical condition for the core, from a statistical point of view.

5 Jupiter and Ganymede's Magnetic Field Estimation

In principle we can apply the ideas of the previous section to other celestial bodies, particularly in our solar system. The main difficulty with this is the scarcity of information about the physical internal condition of other planets. With respect to our solar system we can make some general considerations. For example, Mercury and Mars are quite smaller than Earth. This fact surely contributes to a faster cooling of their interiors so we can expect that the cores of these planets are no longer in the fluid state (Orgzall and Franck 1988). In fact the two planets have a very weak magnetic field.

Venus does not have an appreciable magnetic field (Ness 1974; Nellis et al. 1999) and there are several possible explanations for this. Venus is a planet very similar to Earth in dimension but it does not exhibit volcanic activity. According to the models in Stevenson (1983) Venus has no inner core. It is widely accepted that the core is a fully liquid core. Thus the core would become conductive over geological time-scales and in a non-convective core, a magnetic dynamos does rapidly decay. Also it has a very slow rotational motion compared to Earth, and a rotational motion is considered to play a crucial role for the terrestrial magnetic field. Finally we should consider the possibility that Venus could be in a reversal phase.

Jupiter is a good candidate to test our model. Even though little is known about the planet, its internal structure has been modelled by several authors (see for example Hubbard 1969; Manghnani and Yagi 1998; Fortney and Nettelmann 2010) and the physical information is enough to allow a rough estimation of the scale of its magnetic field using Eq. (19). The estimate electrical conductivity ranges from $\sigma \approx 10^5$ S m^{-1}(Manghnani and Yagi 1998) up to a more recent (average) value $\sigma \approx 10^6$ S m^{-1} (French et al. 2012), its temperature ranges from $T \approx 2 \times 10^4\,^\circ$K for the core boundary to $T \approx 10^4\,^\circ$K for the metallic hydrogen boundary, and the estimated density of the metallic hydrogen is $\rho \approx 4 \times 10^3$ Kg m^{-3} (Hubbard 1969; Fortney and Nettelmann 2010). Consequently the Fermi's energy takes the value $\varepsilon_F \approx 10^{-17}$ Joule corresponding to a Fermi temperature $T_F \approx 7 \times 10^5\,^\circ$K so that we can apply the Fermi statistic for the electrons in the metallic region. Plugging these values into Eq. (19) we obtain for the magnetic field of Jupiter an estimation of its strength in the metallic hydrogen region $30 \lesssim B_J \lesssim 300$ gauss. Taking into account that this region extends for a fraction, ranging from 0.7 to 0.9 times the Jupiter radius and that the surface field, B_{JS}, is related to the core field via the relation $B_{JS} \sim B_J (R_C/R_S)^3$, we obtain for the surface value of the magnetic field a range of

$$10 \lesssim B_{JS} \lesssim 220\,\text{gauss}. \tag{21}$$

While the upper limit is a couple of order of magnitude larger than the actual strength field, the lower value is near to the observed values (Smith et al. 1975). As for the Earth, the values of the parameters are affected by a certain degree of error but the strength of the proposed field scale still is not very far from the order of the observed field. Finally we applied our analysis to Ganymede, satellite of Jupiter. The Galileo spacecraft measured at the surface of the satellite a magnetic field $B_{GS} = 7.5 \times 10^{-3}$ Gauss. The available data are affected by a certain degree of uncertainty and some values of the relevant parameters are extrapolated by the mathematical models. Nevertheless we can apply Eq. (19) to this case and the result is in good agreement with the observation. Using Showman and Malhotra (1999), Bland et al. (2008) as source of data and estimation of the parameters we have that the central density is the iron density, $\rho_G \sim 7 \times 10^3$ Kg m^{-3}, the central temperature is 1,500–1,700 °K with $\Delta T \sim 200\,^\circ$K (Bland et al. 2008). Using as value of the core's conductivity the conductivity of Earth's core, $\sigma \approx 10^5 - 1.4 \times 10^6$ S m^{-1} (Stacey 1967; Pozzo et al. 2012), we obtain for the central strength of the field in the central region the value $B_G \approx 3.6 \times 10^{-1} - 5.1$ gauss corresponding to the surface value of the magnetic field

$$2.6 \times 10^{-3} \lesssim B_{GS} \lesssim 3.6 \times 10^{-2} \text{ gauss.} \tag{22}$$

The range of values is of the order of the observed field.

6 Conclusions

We provided an analytical estimation of the magnetic field scale of planets with physical core conditions similar to that of Earth from a statistical point of view. The magnetic field strength was evaluated directly from the physical parameters of the center of the planet, considering density, temperature and core's size. We showed that an electrical current generated by a thermal gradient, i.e. the Seebeck effect, can exist in a rotating fluid sphere and can give an important contribution to the magnetic field. Our conjecture was supported by estimating the magnetic field strengths of Earth, Jupiter and Ganymede. The range of values of the estimated field is in agreement with the observed magnetic field intensity of the celestial bodies.

Acknowledgments M.B. acknowledges financial support from FONDECYT Project No 1110231. The authors thank Catherine Beeker for her editorial contribution.

References

D. Alfè, M. Gillan, G.D. Price, Composition and temperature of the Earth's core constrained by combining ab initio calculations and seismic data. EPSL **195**, 91 (2002)
J. Ashok, R. Evans, Calculation of the electrical resistivity of liquid iron in the Earth's core. Nature **235**, 165 (1972)
M. Berhanu, R. Monchaux, S. Fauve, N. Mordant, F. Pétrélis, A. Chiffaudel, F. Daviaud, B. Dubrulle, L. Marié, F. Ravelet, M. Bourgoin, Ph Odier, J.-F. Pinton, R. Volk, Magnetic field reversals in an experimental turbulent dynamo. Europhys. Lett. **77**, 59001 (2007)
M.T. Bland, A.P. Showman, G. Tobie, The production of Ganymedes magnetic field. Icarus **198**, 384 (2008)
M. Bologna, B. Tellini, Generating a magnetic field by a rotating plasma. Europhys. Lett. **91**, 45002 (2010)
B.A. Buffett, The thermal state of Earth's core. Science **299**, 1675 (2003)
B.A. Buffett, Tidal dissipation and the strength of the Earths internal magnetic field. Nature **468**, 952 (2010)
F.H. Busse, Homogeneous dynamos in planetary cores and in the laboratory. Annu. Rev. Fluid Mech. **32**, 383 (2000)
A. de Wijs Gilles, G. Kresse, L. Vočadlo, D. Dobson, D. Alfè, J. Gillan Michael, D. Price Geoffrey, The viscosity of liquid iron at the physical conditions of the Earths core. Nature **392**, 805 (1988)
J.J. Fortney, N. Nettelmann, The interior structure, composition, and evolution of giant planets. Space Sci. Rev. **152**, 423 (2010)
M. French, A. Becker, W. Lorenzen, N. Nettelmann, M. Bethkenhagen, J. Wicht, R. Redmer, Ab initio simulations for material properties along the Jupiter adiabat. ApJSS **202**, 5 (2012)
G. Giampieri, A. Balogh, Mercurys thermoelectric dynamo model revisited. Planet. Space Sci. **50**, 757 (2002)
G. Glatzmaier, P. Roberts, A three-dimensional self-consistent computer simulation of a geomagnetic field reversal. Nature **377**, 203 (1995)
W.B. Hubbard, Thermal models of Jupiter and Saturn. Astrophys. J. **155**, 333 (1969)
J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1998)
J.A. Jacobs, The earth's inner core. Nature **172**, 297 (1953)
R.A. Kerr, Earth's inner core is running a tad faster than the rest of the planet. Science **309**, 1313 (2005)
M. Kono, P. Roberts, Recent geodynamo simulations and observations of the geomagnetic field. Rev. Geophys. **40**, 1013 (2002)
W. Kuang, J. Bloxham, An Earth-like numerical dynamo model. Nature **389**, 371 (1997)
L. Landau, E. Lifshitz, *Physical Kinetics* (Pergamon Press, Oxford, 1981), pp. 329–332
L. Landau, E. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1981)

L. Landau, E. Lifshitz, *Fluid Mechanics* (Pergamon Press, Oxford, 1987)

H.K. Lee, *International Handbook of Earthquake and Engineering Seismology* (Academic Press, Waltham, 2002)

M.H. Manghnani, T. Yagi, *Properties of Earth and Planetary Materials at High Pressure and Temperature* (AGU books board, USA, 1998)

W.J. Nellis, S.T. Weir, A.C. Mitchell, Minimum metallic conductivity of fluid hydrogen at 140 GPa (1.4 Mbar). Phys. Rev. B **59**, 3434 (1999)

N.F. Ness, Magnetic fields of mars and venus: Solar wind interactions. Earth, Moon, and Planets **9**, 43 (1974)

I. Orgzall, S. Franck, Mars and its magnetic field Some new aspects. Earth, Moon, and Planets **43**, 201 (1988)

J.P. Poirier, *Introduction to the Physics of the Earths Interior* (Cambridge University Press, Cambridge, 1991)

M. Pozzo, C. Davies, D. Gubbins, D. Alfè, Thermal and electrical conductivity of iron at Earths core conditions. Nature **485**, 355 (2012)

P. Roberts, G.A. Glatzmaier, Geodynamo theory and simulations. Rev. Modern Phys. **72**, 1081 (2000)

A.P. Showman, R. Malhotra, The Galilean satellites. Science **286**, 77 (1999)

E.J. Smith, L. Jr. Davis, D.E. Jones, P.J. Jr. Coleman, D.S. Colburn, P. Dyal, C.P. Sonett, Jupiter's magnetic field. Magnetosphere, and interaction with the solar wind: Pioneer 11. Science **188**, 451 (1975)

F.D. Stacey, Electrical resistivity of the earth's core. EPSL **3**, 204 (1967)

D.J. Stevenson, Planetary magnetic fields. Rep. Prog. Phys. **46**, 555 (1983)

D.J. Stevenson, Mercurys magnetic field: a thermoelectric dynamo? Earth Planet. Sci. Lett. **82**, 114 (1987)

D.J. Stevenson, Planetary magnetic fields: achievements and prospects. Space Sci. Rev. **152**, 651 (2010)

G.W. Sutton, A. Sherman, *Engineering magnetohydrodynamics* (Dover Publications, New York, 2006)

J.A. Tarduno, R.D. Cottrell, M.K. Watkeys, A. Hofmann, P.V. Doubrovine, E.E. Mamajek, D. Liu, D.G. Sibeck, L.P. Neukirch, Y. Usui, Geodynamo, solar wind, and magnetopause 3.4 to 3.45 billion years ago. Science **327**, 1238 (2010)