

Long Period Tidal Force Variations and Regularities in Orbital Motion of the Earth-Moon Binary Planet System

Yu. N. Avsyuk · L. A. Maslov

Received: 19 September 2010 / Accepted: 29 March 2011 / Published online: 19 April 2011
© Springer Science+Business Media B.V. 2011

Abstract We have studied long period, 206 and 412 day, variations in tidal sea level corresponding to various moon phases collected from five observatories in the Northern and Southern hemispheres. Variations in sea level in the Bay of Fundy, on the eastern Canadian seaboard, with periods of variation 206 days, and 412 days, have been discovered and carefully studied by Desplanque and Mossman (Proceedings of the 4th Bay of fundy workshop Saint John, New Brunswick 2001, Atlantic Geol 40:1, 2004). The current manuscript focuses on analyzing a larger volume of observational sea level tide data as well as on rigorous mathematical analysis of tidal force variations in the Sun-Earth-Moon system. We have developed a twofold model, both conceptual and mathematical, of astronomical cycles in the Sun-Earth-Moon system to explain the observed periodicity. Based on an analytical solution of the tidal force variation in the Sun-Earth-Moon system, it is shown that the tidal force can be decomposed into two components: the Keplerian component and the Perturbed component. The Perturbed component of the tidal force variation was calculated, and it was shown that the observed periodicity, 206 and 412 days, of atmospheric and hydrosphere tides results from variations of the Perturbed component of tidal force. The amplitude of the Perturbed component of tidal force is 19×10^{-8} N/kg. It is the same order of magnitude as the amplitude of the Keplerian component of tidal force: 58×10^{-8} N/kg. It follows that the Perturbed component of the variation of a tidal force must always be taken into consideration along with the Keplerian component in geodynamical constructions involving tides.

Yu. N. Avsyuk
Institute Physics of the Earth, B. Gruzinskaya St., 10, 123995 Moscow, Russia
e-mail: avsyuk@ifz.ru

L. A. Maslov (✉)
Aims College, 20th Street, 5401, Greeley, CO 80634, USA
e-mail: lmaslov@aims.edu; ms_leo@hotmail.com

L. A. Maslov
Computing Center RAS, 65, Kim Yu Chen Str., 680000 Khabarovsk, Russia

Keywords Earth · Moon · Sea level tides · Seismicity · Tidal force · Baricenter · Binary planet system

1 Introduction

Variation of the tidal force in the Sun-Earth-Moon system causes of a number of tidal cycles in the solid Earth, atmosphere, and hydrosphere. The following are the main cycles: principal lunar semidiurnal M_2 , principal solar semidiurnal S_2 , larger lunar elliptic semidiurnal N_2 , lunar diurnal K_1 , and lunar diurnal O_1 . The longer period cycles are: lunar monthly M_m , lunisolar synodic fortnightly M_{sf} , and lunisolar fortnightly M_f . The lunar monthly M_m cycle has duration 27.55 days, the lunisolar synodic fortnightly cycle has duration 14.76 days, and the lunisolar fortnightly cycle has duration 13.36 days.

The tidal cycles mentioned above can act as trigger mechanisms in the seismicity of the Earth and Moon. For example, study of the seismicity of the Earth under the influence of lunar tides (Tamrazian 1967) revealed that the maximum number of earthquakes happen when the Moon is at perigee and the minimum number of earthquakes occur when the Moon is at apogee. A thorough study of seismicity triggered by diurnal and semidiurnal tides has been conducted (Métivier et al. 2009). It was shown here that there exists a clear correlation, with 99% confidence, between the phase of the Moon and the timing of seismic events. The study of space regularities in the seismicity of the Earth and Moon (Levin and Sasorova 2010) revealed a similarity in the distribution of quakes in the Earth and the Moon. The quakes are distributed within two latitudinal belts: approximately 20–40°, on each side of the equator. Past research (Kosygin and Maslov 1986) discussed possible geodynamic consequences of solid Earth tides such as seismicity, drift of the lithosphere, and magnetic field generation.

Variations in the tidal sea level, with periods of variation 206 days, and 412 days, have been discovered and carefully studied by Desplanque and Mossman (2001, 2004). These authors studied variations of sea tides in the Bay of Fundy, on the eastern Canadian seaboard. However, diurnal, semidiurnal, and lunar monthly influences are more commonly discussed in the literature. The study of the seismicity of the Moon, conducted by the Apollo mission, showed that the seismicity of the Moon has a period of 27 days and also a period of 206 days (Lammlein 1974). Yu. Barkin and coauthors (Barkin et al. 2005) reported that the seismicity of the Moon has periods between 200–240 days and 360–440 days. Study of atmospheric air temperature variations in the Moscow region, (absolute values relative to the average) for the years 1960–2003, revealed periods of 27, 87, 206 and 355 days (Sidorenkov 2009).

In this work we aim to:

- (a) Represent variations in tidal sea level corresponding to various moon phases collected from five different observatories worldwide in order to study the periodicity of these tides.
- (b) Develop a model of astronomical cycles in the Sun-Earth-Moon system to explain the observed periodicity in tidal sea level variations.
- (c) Apply an analytical solution for the tidal force variation that occurs in the Sun-Earth-Moon system to show that the tidal force can be decomposed into two components: the Keplerian component and the Perturbed component.
- (d) Calculate the Perturbed component and show that the observed periodicity, 206 and 412 days, of atmospheric and hydrosphere tides results from variations of the Perturbed component of the tidal force.

2 Sea Tides and Their Interpretation

Figure 1 shows graphs of syzygial sea tides in Murmansk, Russia; Puerto Williams, Chili; Suva, Fiji; Lerwick, Scotland; and Magadan, Russia from the years 1977 to 1979.

As can be seen from these graphs, tides happen synchronously and independently at different observatories in the Northern and Southern hemispheres. It can also be seen that there exists a clear periodicity in tidal variations. The time interval between two closest full moon and new moon maximums is equal to 7 months (≈ 206 days), and the period of each of these variations is equal to 14 months (≈ 412 days). Figure 2 shows syzygial and quadrature sea level tidal variations for the years 1977–1983 in Murmansk, and the years 1977–1994 in Magadan. Here, as in Fig. 1, syzygial and quadrature sea level tidal variations with periods 206 and 412 days can be clearly seen throughout the record. The quadrature tides are shifted relative to syzygial tides by 3.5 months (≈ 103 days).

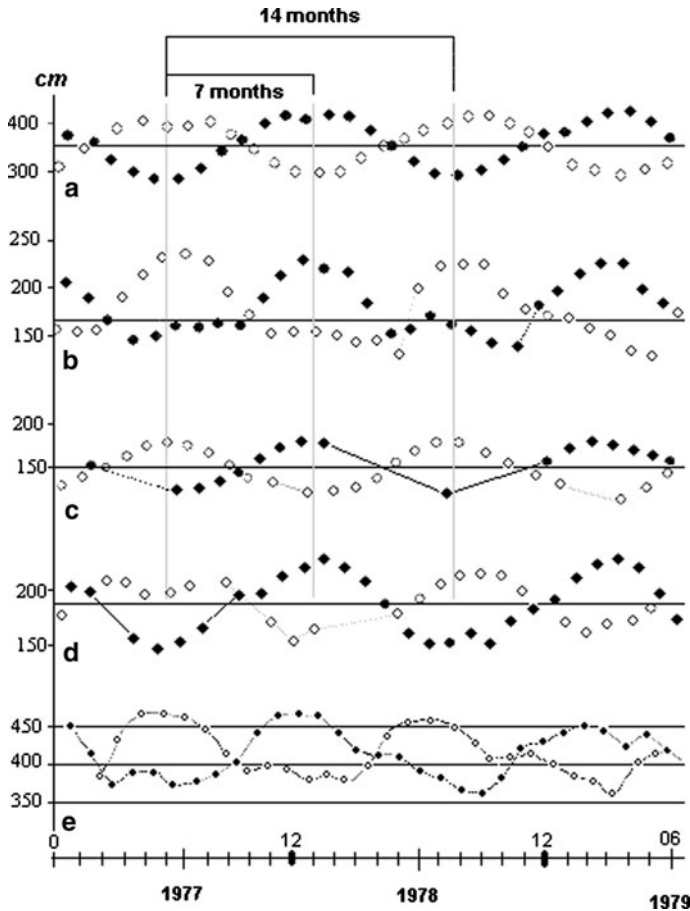


Fig. 1 Heights of syzygial sea tides at full and new moons. **a**—in Murmansk (Russia), 68.97 N, 33.08 E. **b**—in Puerto Williams (Chili), 54.93 S, 67.62 W. **c**—in Suva (Fiji), 18.13 S, 178.43 E. **d**—in Lerwick (Scotland), 60.15 N, 1.15 W. **e**—in Magadan (Russia), 59.57 N, 150.8 E. *Opened circle, bullet*—syzygial full moon and new moon tides

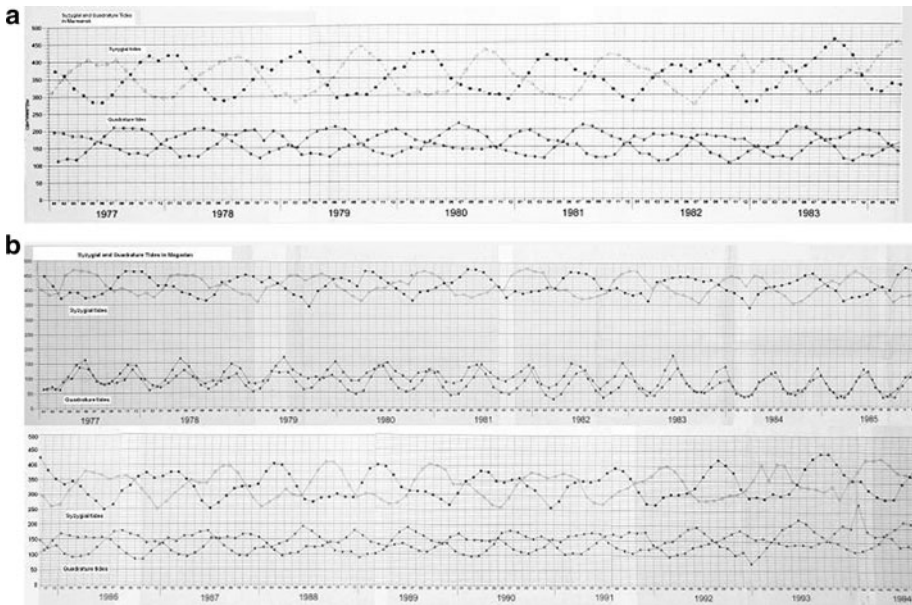


Fig. 2 Heights of syzygal and quadrature tides in Murmansk (a), and in Magadan (b) for the time intervals 1977–1983 and 1977–1994, correspondingly

The data presented above have been kindly given to us by the following institutions: N.N. Zubov State Oceanographic Institute, www.oceanography.ru (Murmansk observatory); British Oceanographic Data Centre, UK, www.bodc.ac.uk, (Puerto Williams, Suva, and Lerwick observatories); Institute of Marine Geology and Geophysics FEBRAS, www.imgg.ru/rus/index.php, (Magadan observatory). To the data above can be added also long-period tidal sea level variations in the Bay of Fundy, eastern Canadian seaboard, from Desplanque and Mossman (2001, Figs. 7 and 8).

The difference

$$H(t)_{fm} - H(t)_{nm} = H(t) \quad (1)$$

(tide at full moon minus tide at new moon) calculated from observed data, Fig. 1a, was approximated by the function:

$$D(t) = 120 \cdot \sin\left(\frac{2\pi \cdot t}{14}\right)$$

with period $T \approx 14$ months, or ≈ 412 days, Fig. 3.

A similar approximation was made for the other data in Fig. 1. In all cases the best fit was shown to be for the period $T \approx 14$ months.

To show more clearly the correspondence between the tidal and astronomical periodicities, zeros of the tidal function $H(t)$ and the minimal anomalistic months were plotted on the same graph, Fig. 4. As seen from this Figure, the zeros of the function $H(t)$ correspond well to minimal anomalistic months of the motion of the Moon, and the Earth, with period 7 months (≈ 206) days.

It can be seen from all the material presented, that the periodicity of the Earth's long period hydrosphere tides is the same as the periodicity of the variations of atmospheric air

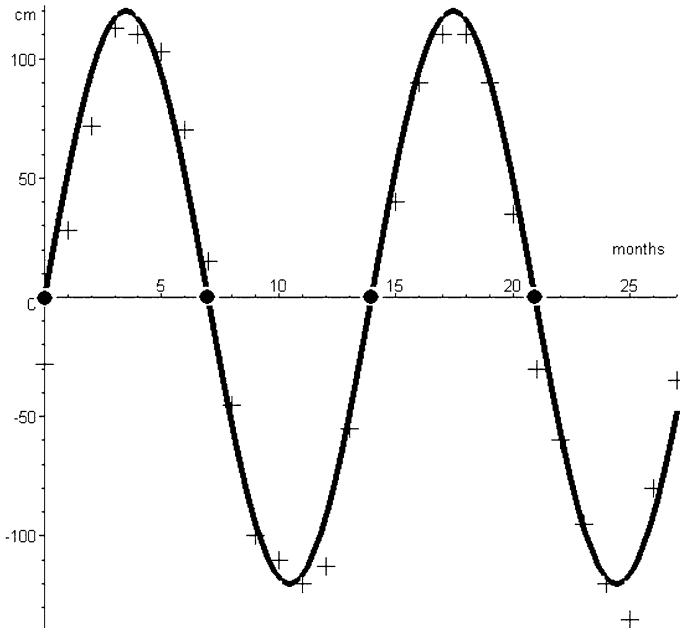


Fig. 3 Difference in heights for full and new moon tides at Murmansk, Fig. 1a, formula (1)

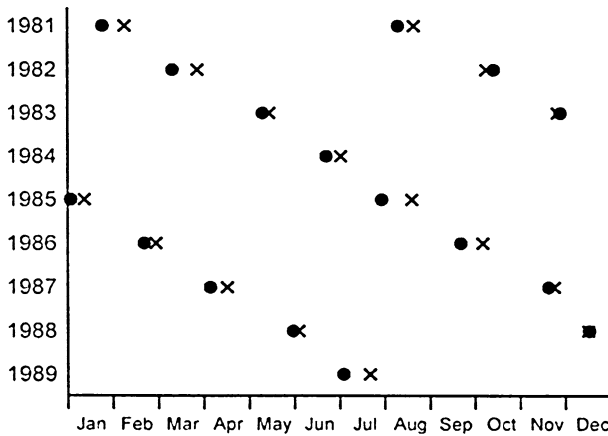


Fig. 4 Zeros of the function $H(t)$ and of minimal anomalistic month distribution. *Bullet*—zeroes of the function $H(t)$, *times*—minimal anomalistic month

temperature, as well as the periodicity of lunar seismicity, 206 days as registered by the seismic stations of the Apollo mission.

3 Cycles in Orbital Motion of the Earth and Moon Binary Planet System

Study of the orbital motion of the Earth-Moon barycenter around the Sun, and the orbital motion of the Earth and Moon centers around their common barycenter, (Avsyuk 1996;

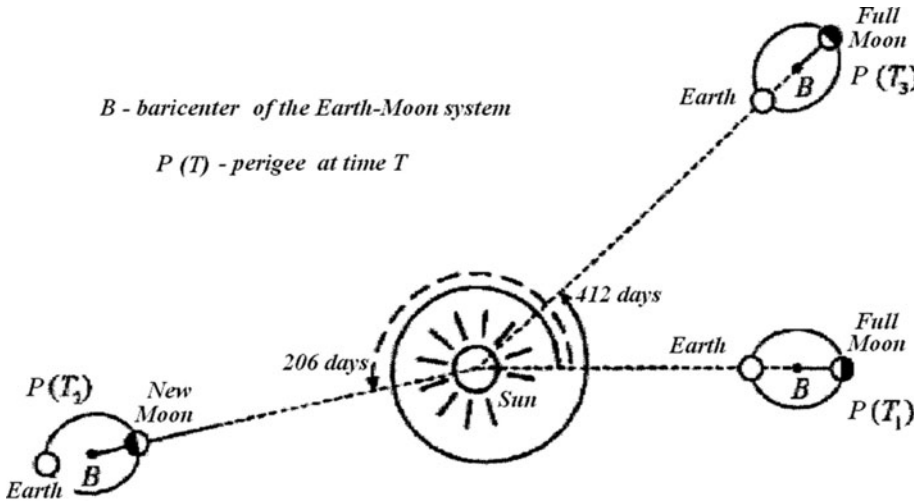


Fig. 5 Illustration of the time recurrence of full moon coinciding with the passage through the orbital perigee. Since the major axis of the Moon's (Earth's) orbit is not fixed in space (the period of perigee revolution is 3230.25 days), the full moon phase and the passage through perigee of the full moon coincide at the time moment $T_3 = T_1 + 412$ days. The time moment T_2 fixes the new moon that coincides with the passage through the perigee)

Avsyuk and Suvorova (2006) showed that the time from one full moon in perigee until the next full moon in perigee is the same as the time from one high tide to another, and is 412 days. Let T_1 be the time that we observe a full Moon with the entire system in perigee, as shown in Fig. 5.

In 206 days, one half of the total 412 day period, we will again have orbital perigee of the system, but with a new moon phase instead of a full moon phase; let us call this time T_2 . The original configuration will occur at the time T_3 , 412 days after T_1 , Fig. 5. This is caused by the fact that the Earth-Moon orbit is not fixed in space and the period of its perigee revolution is 3230.25 days.

This reasoning can help us to understand the nature of the 206 and 412 day tidal cycles on a qualitative level.

4 Variation of a Tidal Force in the Earth-Moon-Sun System. Mathematical Solution

The motion of the Earth-Moon binary planet system around the Sun can be represented as the sum of two components: the orbital motion of the Earth-Moon barycenter around the Sun (Keplerian motion), and the motion of the Earth and Moon around their common barycenter (Perturbed motion). The Perturbed motion describes the periodic motion of the center of mass of the Earth in its elliptical orbit around the Earth-Moon barycenter. In astronomy, this phenomena is well known as the lunar inequality, and has angular variation $6.44''$.

The Perturbed motion of the Earth in the gravity field of the Moon and the Sun produces an additional component of tidal force, which we call the Perturbed component. In this approach, the total tidal force, acting on the Earth, is a sum of two components: Keplerian and Perturbed. The theory of the tidal force variations in cases of unperturbed (Keplerian) and Perturbed motion of a body was first developed in the work by Yu. N. Avsyuk (Avsyuk 1996, 2001). The total tidal force as a function of the positions of the Moon, Sun, and Earth

relative to each other was obtained as a solution of the system of equations of motion in Lagrange form. The tidal force was represented as a sum of Keplerian, \bar{K} , and Perturbed, \bar{P} , components:

$$\bar{F} = \bar{K} + \bar{P} \tag{2}$$

The Keplerian component is described by the formula:

$$\begin{aligned} \bar{K} = G \cdot \delta m \cdot & \left(\frac{M_2}{\rho_1^3} \cdot \bar{\rho}_1 - \frac{M_2}{R_1^3} \cdot \bar{R}_1 \right) \\ & + G \cdot \delta m \cdot \left(\frac{M_3}{\rho_2^3} \cdot \bar{\rho}_2 - \frac{M_3}{R_0^3} \cdot \bar{R}_0 \right) \end{aligned} \tag{3}$$

The Perturbed component is written in the form:

$$\begin{aligned} \bar{P} = C \cdot & \left(\frac{3}{2} \cos(2D + 0.5) \cdot \bar{r} + \sin(2D) \cdot \bar{\tau} \right) \\ C = G\delta m & \frac{R_1 M_2 M_3}{(M_1 + M_2) \cdot R_0^3} \end{aligned} \tag{4}$$

M_1, M_2, M_3 are masses of the Earth, Moon and Sun, ρ_1 is the distance from the center of the Moon to an arbitrary point in the Earth, ρ_2 is the distance from the center of the Sun to an arbitrary point in the Earth, R_0 is the distance from the Earth-Moon barycenter to the Sun, R_1 is the distance between the gravity centers of the Earth and Moon. Terms of order $(R_1/R_0)^2$ are omitted in the derivation of (3) and (4). In this formula \bar{r} and $\bar{\tau}$ are unit vectors in the radial and tangential directions respectively, $D = \lambda_S - \lambda_M$, λ_S and λ_M are longitudes of the Sun and Moon. A computer program was written to study the variation of tidal force for different input parameters. A graph of the function $|\bar{P}|/C$, calculated with two week intervals, is shown in Fig. 6. The long period variations of a tidal force with periods ≈ 206 and ≈ 412 days are well displayed by the envelope function clearly visible in Fig. 6.

It is worthwhile to note, that the amplitude of the 206 day cycle is approximately 1.803 times less than the amplitude of the 412 day cycle. The amplitude of the Perturbed component of tidal force is 19×10^{-8} N/kg. It is the same order of magnitude as the amplitude of the Keplerian component of tidal force: 58×10^{-8} N/kg. It follows that the Perturbed component of tidal force variation must always be taken into consideration along with the Keplerian component in geodynamical constructions involving tides.

The formula (4) does not contain angular distances between heavenly bodies and a celestial equator. This means that the declination of the Moon and Sun relative to the Earth’s equator is a secondary factor for the generation of tidal periodicities of 206 and

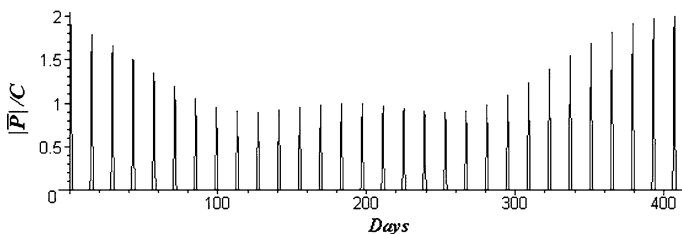


Fig. 6 Graph of the Perturbed component of tidal force $|\bar{P}|/C$, calculated with two week intervals

412 days. This fact was mentioned also by Desplanque and Mossman (2004, Fig. 68, p.106) in their study of the Bay of Fundy tides.

5 Conclusion

We have presented a large amount of new data: syzygial and quadrature tidal sea level variations for the years 1977–1985 registered at five different observatories (Murmansk, Magadan, Lerwick, Puerto Williams, and Suva) situated in the Northern and Southern hemispheres. We have shown that these tides have periods of 206 and 412 days; they are independent of the geographical position of the observatory and synchronous in time. To study this phenomena, we developed a mathematical theory of the tidal force variation in the Earth-Moon-Sun system

and from the solution evaluated periods of the Perturbed tidal force variations and its magnitude. We have shown from this solution that the long period variations of the Perturbed tidal force component have periods of ≈ 206 and ≈ 412 days, which are in the excellent correlation with periods and regularities in the orbital motion of the Earth and Moon as a binary planet system. We have also shown from this solution that the magnitude of the Perturbed component of tidal force is 19×10^{-8} N/kg. It is very important that this force is only three times smaller than the magnitude of the Keplerian component of the tidal force 58×10^{-8} N/kg. It follows that the Perturbed component of tidal force variation must always be taken into consideration along with the Keplerian component in geodynamical constructions involving tides and the other planetary processes.

We have discussed the results of other authors, that show the long period variations of a tidal force have a significant impact not only on the Earth's hydrosphere, but also on the Earth's atmosphere and the Earth and Moon interiors. The long period variations of the Earth and Moon seismicity, with periods of ≈ 206 and ≈ 412 days, are the direct manifestation of corresponding variations of the tidal force in this system.

Acknowledgments We are grateful to the Editor and reviewers of this article for their remarks, comments and patience, contributing to the considerable improvement of our work. This work was supported by RFBR grant 09-05-00426-a.

References

- Yu.N. Avsyuk, Tidal Forces and Natural Processes (UIPE RAS, Moscow, 1996) 190 p. In Russian
 Yu.N. Avsyuk, *Izvestia Phys. Solid Earth* **37**, 11 (2001)
 Yu.N. Avsyuk, I.I. Suvorova, *Izvestia Phys. Solid Earth* **42**, 7 (2006)
 Yu. Barkin, J. Ferrandiz, M. Ferrandez, Earth, Moon, Mercury and Titan seismicity: Observed and expected phenomena. (Lunar and Planetary Science XXXVI, 2005), <http://www.lpi.usra.edu/meetings/lpsc2005/pdf/1076.pdf> Accessed 10 September 2010
 C. Desplanque, D.J. Mossman, 2001. Fundamentals of fundy tides. (p.178–204), In: Chopin, T., and Wells, P.G., (eds.), 2001. Opportunities and challenges for protecting, restoring and enhancing coastal habitats in the Bay of fundy. Proceedings of the 4th Bay of fundy workshop Saint John, New Brunswick, September 19–21, 2000. Environment Canada, Dartmouth, Nova Scotia, 237p
 C. Desplanque, D.J. Mossman, *Atlantic Geol.* **40**, 1 (2004)
 Yu. Kosygin, L. Maslov, *Geotectonics* **20**, 6 (1986)
 D. Lammlein, *Rev. Geophys. Space Phys.* **16**, 224 (1974)
 B.W. Levin, E.V. Sasorova, *Doklady Earth Sci.* 434, Part 1 (2010)
 L. Métiévier, O. de Viron, C.P. Conrad, S. Renault, M. Diamant, G. Patau, *Earth Planet. Sci Lett.* **278**, 370 (2009)

N.S. Sidorenkov, *The Interaction Between Earth's Rotation and Geophysical processes* (WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim, 2009), p. 317
G.P. Tamrazian, *Ikarus*. 7, 59 (1967)