Earth Moon Planet (2009) 105:367–371
DOI 10.1007/s11038-009-9299-6

Few Comments on the Relation Between the Initial Proto-planetary Disc Model and the Oort Cloud Formation

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Received: 12 November 2008 / Accepted: 23 February 2009 / Published online: 17 March 2009

Abstract A fraction of small bodies from the once existing proto-planetary disc was ejected, by the giant planets, to large heliocentric distances and start to build the comet Oort cloud. Considering four models of initial proto-planetary disc, we attempt to roughly map a dependence between the initial disc's structure and some properties of the Oort cloud. We find that it is difficult to construct the proto-planetary disc if (i) the amount of heavy chemical elements in Jupiter and Saturn is as high as currently accepted and (ii) the total mass of the minimum-mass solar nebula is assumed to be lower than $\approx 0.05\,M_{\odot}$. The behaviour of the Oort cloud formation does not crucially depend on the initial disc model. Some differences in its structure are obvious: since the cloud is known to be filled mainly by Uranus and Neptune, the efficiency of its formation is higher when the initial amount of particles in the Uranus-Neptune region is relatively higher. A significantly large number of Jupiter Trojans in our simulation appears, however, only in the case of the initially non-gapped disc, with the particles situated also close to the Jupiter's orbit.

Keywords Comets: general · Oort cloud · Solar system: formation

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The concept of the solar-system formation can be refined using the new, permanently enlarging databases of the reservoirs of small bodies in this system. In one of our previous papers (Dybczyński et al. 2008; Paper I, hereinafter), we dealt with the formation of the comet Oort cloud (OC). The structure of this distant reservoir has been determined by several factors. Among others, it could be influenced by the initial distribution of macroscopic solid bodies in the once existing proto-planetary disc (PPD), from which we believe the OC comets originate.

In the simulation of the OC formation described in Paper I, we assumed a smooth initial PPD, with the surface-density proportional to the heliocentric distance, r, as $\propto r^{-3/2}$ (e.g. Morbidelli and Brown 2004). At the same time, we assumed that the giant (Jovian) planets were already formed and moved around the Sun in their current orbits. Therefore, a part of the material of smooth-profile initial PPD had to be spent for the planet formation. This process resulted in a formation of gaps in the PPD. The gaps formation during the era of the gaseous solar nebula has been described in several papers (Lin and Papaloizou 1993; Bryden et al. 1999; Papaloizou et al. 1999; 2004). In this paper, we investigate the possible impact of gaps creation, inside the initial smooth PPD, on the OC formation.

The initial mass M_{sn} of the so-called minimum-mass solar nebula (MMSN) is expected to be in the range from ~ 0.01 to ~ 0.1 $\mathrm{M}_{\odot}$ (Weidenschilling 1977). Here, we assume two values of M_{sn}: that at the upper limit, i.e. 0.1 $\mathrm{M}_{\odot}$, and the half value, 0.05 $\mathrm{M}_{\odot}$.

As in Paper I also in the current paper, the disc is assumed to consists of a large number of test particles (TPs). More specifically, we utilize the resultant data of extensive computation, integration of the motion of TPs which was presented in Paper I, and, meanwhile, extended to 2 Gy (Leto et al. 2008; Paper II). Here, we consider another three, alternative, models of the initial PPD (besides that in Paper I), which are created by removing some TPs in a way to obtain the requested initial TP distribution.

1 A-Model: A Smooth $r^{-3/2}$ Surface-Density Profile, Arbitrary M_{sn}

This model of the initial PPD was considered in Paper I, where its detailed characteristics can be found. It is regarded as the reference model in the following. Since no TPs are removed, no specific value of M_{sn} is necessary to be assumed.

2 B-Model: Gaussian-like Depleted $r^{-3/2}$ Profile, $M_{sn} = 0.1\,\mathrm{M}_{\odot}$

In this model, we remove from the reference A-model those TPs, which are supposed to collide with the giant-planet embryos and, thus, be incorporated to the given planet. In the applied procedure, the summary mass of the removed TPs should equal the heavy-element-component mass inside the given planet, since the TPs representing the planetesimals practically consisted only of the heavy elements.

It can be shown that the partial mass of the MMSN between 4 and 50 AU was $M_{4-50} \doteq 0.0414\ \mathrm{M}_{\odot} \doteq 13\,800\mathrm{M}_{\oplus}$, if the total mass $M_{sn} = 0.1\mathrm{M}_{\odot}$. The mass of the heavy-chemical-element fraction available to form small bodies between 4 and 50 AU had to be $\approx$276 $\mathrm{M}_{\oplus}$. Since the corresponding TPs in our PPD model are 10,038, the average mass of one TP is $\approx$0.0275 $\mathrm{M}_{\oplus}$.

From the characteristics of well-known models of the internal structure of giant planets given by Weissman et al. (1999), we can deduce that the fractions of the heavy chemical elements in Jupiter, Saturn, Uranus, and Neptune are approximately 20, 29, 13.5, and 16 $M_\oplus$, respectively. So, we must remove 727, 1055, 491, and 582 TPs, respectively.

A detailed heliocentric-distance distribution of the planetesimal PPD, after the gas has gone, is not known. We simply assume a random depletion of the given planetary zone and, consequently, the Gaussian distribution of the removed TPs. Specifically, the Gaussian is centred at the current mean heliocentric distance of the given planet. The peak of the Gaussian profile is equal to the number of TPs at that distance, all TPs at the mean planet distance are removed, its dispersion is fitted to remove the given, above mentioned number of TPs.

3 C-Model: Gaussian-like Depleted $r^{-3/2}$ Profile, $M_{sn} = 0.05\,\mathrm{M}_\odot$

In this model, we remove 1460, 2117, 985, and 1168 TPs from Jupiter, Saturn, Uranus, and Neptune region, respectively.

The efficiency of the accretion of Jupiter and Saturn had to be enormously high, if they formed in the PPD described by the C-model and actually contain the considered amount of heavy chemical elements. If the latter is the fact, it is difficult to believe that the total mass of MMSN could be even smaller than 0.05 $\mathrm{M}_\odot$. In such a case, the embryos of these two planets would have, very efficiently, accreted the material not only from an adjacent region, but from the terrestrial planet, Uranus-Neptune, and, possibly, Kuiper-belt regions as well.

The problem with the high accretion efficiency would not appear if the amount of heavy elements in Jupiter and Saturn was lower than that generally and also in this work accepted. A lower amount can be actual due to its quite large determination uncertainty. Guillot and Gladman (2000) demonstrated that the amount can be as low as 10 and 20 $M_\oplus$ in Jupiter and Saturn, respectively (cf. 20 and 29 $M_\oplus$, respectively, in this work).

4 D-Model: Gaussian-like Depleted r^{-2} Profile, $M_{sn} = 0.05\ \mathrm{M}_\odot$

Another way to avoid the too high formation efficiency of Jupiter and Saturn, required in the case of C-model, is to assume a centrally more concentrated initial PPD. Kusaka et al. (1970) demonstrated that the surface density of the MMSN may depend on the heliocentric distance as $\propto r^{-1}$ to $\propto r^{-2}$.

The D-model of the initial PPD is derived from the heliocentric-distance TP distribution corresponding with the more centrally concentrated r^{-2} surface-density profile. Since we intend to use only the data obtained within the large computation for the A-model, we create the r^{-2} surface-density profile of the PPD from the $r^{-3/2}$ profile discarding the redundant TPs in the 4–50-AU range, considered in Paper I and II. Having the model of the initial PPD with the heliocentric-distance distribution corresponding with the r^{-2} surface-density profile, we further remove the TPs assumed to be accreted by the planet embryos in the same way as in the case of B or C model.

5 A comparison of results for the considered PPD models

Let us now compare the results from Paper I and II, for the A-model of the initial PPD, with the corresponding results for the other models introduced here. It appears that the outer OC (OOC) population reaches its maximum at $\approx$200 My, while the inner OC (IOC) population at $\approx$1.2 Gy, in the case of all considered PPD models. The height of the OOC-population peak depends, as expected, on the amount of TPs in the Jupiter-Saturn region of the PPD, from where the TPs come to the OOC during the very early period. The peak is lowest for the C-model with a large amount of TPs spent for Jupiter and Saturn formation and, consequently, a poor remnant of the TPs that remained for the OOC formation.

The final population of the IOC, OOC, and whole OC at 2 Gy are [1.11; 0.14; 1.25], [1.15; 0.18; 1.33], [1.24; 0.19; 1.43], and [0.92; 0.18; 1.10] for A-, B-, C-, and D-model, respectively. The values are given in the percents of the corresponding total initial PPD population. From the presented figures, we can deduce that the efficiency of formation of the OOC is a little higher for the gapped, B-, C-, and D-models than for the smooth A-model of the PPD (cf. $0.18-0.19\%$ and 0.14%). This is likely the effect of the gaps formation that diminishes the number of TPs in orbits being very close to the planetary orbits. The absence of such TPs is obviously also a reason of a higher efficiency of IOC formation for B- and C-models in comparison with A-model.

The significant difference occurs for Jupiter's Trojans. Our simulation predicts that $0.41 \pm 0.07\%$ for the A-model and 0.01, 0.02, and 0.04% of all considered TPs for B-, C-, and D-models, respectively, orbit around the Sun in the position of Jupiter Trojans at 2 Gy. In the case of the last three models, only a single TP occurs in the Jupiter-Trojan orbit, therefore the statistical uncertainty (calculated applying the Poisson statistics) is larger than 100% in this case. Nevertheless, the above stated figures indicate that the occurrence rate of bodies in the Jupiter-Trojan orbits is at least one order smaller for these models than for the A-model. The Jupiter Trojans are believed to be captured by the growing planet during the era, when the solar nebula was not dissipated, yet. However, the capture was possible only if a large number of small bodies were present in the adjacent region as implied by the A-model. In other words, it means that not all the small bodies close to Jupiter's orbit could be spent for Jupiter formation. The models not containing such bodies are disfavoured by this condition.

Acknowledgements T.P., M.J., and L.N. thank the project "Enabling Grids for E-sciencE II" (http://www.eu-egee.org/) for the provided computational capacity and support in the development of the computer code, which was necessary for the management of tasks on the GRID. They also acknowledge the partial support of this work by VEGA—the Slovak Grant Agency for Science (grant No. 7047). P.A.D. acknowledges the partial support of this work from Polish Ministry of Science and High Education (year 2008, grant No. N N203 302335). G.L. thanks PI2S2 Project managed by the Consorzio COMETA, http://www.pi2s2.it and http://www.consorzio-cometa.it for the computational resources and technical support.

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