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FUTURE SATELLITE GRAVIMETRY FOR GEODESY

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Abstract. After GRACE and GOCE there will still be need and room for improvement of the knowledge (1) of the static gravity field at spatial scales between 40 km and 100 km, and (2) of the time varying gravity field at scales smaller than 500 km. This is shown based on the analysis of spectral signal power of various gravity field components and on the comparison with current knowledge and expected performance of GRACE and GOCE. Both, accuracy and resolution can be improved by future dedicated gravity satellite missions. For applications in geodesy, the spectral omission error due to the limited spatial resolution of a gravity satellite mission is a limiting factor. The recommended strategy is to extend as far as possible the spatial resolution of future missions, and to improve at the same time the modelling of the very small scale components using terrestrial gravity information and topographic models. We discuss the geodetic needs in improved gravity models in the areas of precise height systems, GNSS levelling, inertial navigation and precise orbit determination. Today global height systems with a 1 cm accuracy are required for sea level and ocean circulation studies. This can be achieved by a future satellite mission with higher spatial resolution in combination with improved local and regional gravity field modelling. A similar strategy could improve the very economic method of determination of physical heights by GNSS levelling from the decimeter to the centimeter level. In inertial vehicle navigation, in particular in sub-marine, aircraft and missile guidance, any improvement of global gravity field models would help to improve reliability and the radius of operation.

Keywords: Geodesy, gravity field, heights, GNSS levelling, inertial navigation

1. Introduction

All currently available geoid or gravity models are based on satellite orbit analysis, satellite radar altimetry in ocean areas, terrestrial and shipborne gravimetry and topographic models. Long-term observation and analysis resulted in global gravity field models such as the Earth Gravity Model EGM96 (Lemoine et al., 1998). Despite of the wealth of input data and sophistication of the computational processes these models are still rather heterogeneous in terms of resolution and accuracy. At present, a major step forward in gravity field knowledge is achieved by a first generation of dedicated gravity field satellite missions, CHAMP (Reigber et al., 2003), GRACE (Tapley et al., 2004b) and GOCE (ESA, 1999).

In geodesy the requirements of geoid and gravity field are particularly high. Highly accurate and homogeneous gravity field information is needed for the establishment of unified global height systems and for the calculation of physical heights from ellipsoidal GNSS heights. Precise and homogeneous height systems are important for engineering purposes as well as for Earth sciences, e.g. for sea level studies. With GRACE and GOCE, considerable improvements will be achieved, but the requirements in terms of accuracy and spatial resolution will not be fully met. Inertial navigation and precise orbit determination are other geodetic fields requiring very accurate gravity field knowledge.

A new and very challenging field is the observation and analysis of the time variable gravity field due to mass variations in atmosphere, oceans, continental water cycle, ice covered areas and solid Earth (Committee, 1997; Ilk et al., 2005). The satellite mission GRACE enables a first view on this field, but many open questions will remain. With GRACE, and even more with future satellite gravity missions, temporal gravity field variations will become an important subject of geodetic research. For adequate analysis and modelling, a close cooperation between geodesy, geophysics, oceanography, glaciology, hydrology and other Earth sciences is mandatory.

There exist reliable mathematical "rules" about the average behaviour of the gravity field and geoid under the assumption of stationarity and isotropy. These rules are power spectra of average signal power, expressed in terms of spherical harmonic degree variances or degree rms. We use such power spectra as a starting point for a discussion of the state-of-the-art, of shortcomings and future needs in gravity field knowledge, cf. Section 2.1. At short wavelengths gravity information is insufficient or non-existent. For wavelengths smaller than 100 km, this situation will remain unchanged even after the completion of the first generation of dedicated gravity field satellite missions. Statistically, the unknown small-scale gravity signal is dealt with as omission error, cf. Section 2.2. Much more uncertain than the stationary gravity field characteristics is the signal size and behaviour of the time variable gravity field, cf. Section 2.3. The stationary and time variable gravity field signal amplitudes and length scales give the background for the subsequent analysis of future geodetic requirements and possible improvements by future satellite missions, cf. Section 3.

2. Gravity Field and Geoid: State-of-the-Art

2.1. Stationary geoid

In the following we discuss the signal amplitudes of the Earth's gravity field at various wavelengths and spatial scales, respectively. The discussion is based on Figure 1 where degree rms values from spherical harmonic geoid expansions are shown. The figure also explains where the current geoid information comes from and where there is room and need for improvement.

– The two central lines are those denoted "Tscherning-Rapp" and "Kaula". They represent models of the geoid signal size (in meters) as a function of spherical harmonic degree or average spatial scale (half-wavelength in kilometers). They are based on satellite and terrestrial data translated into a simple mathematical expression (rule of thumb). At small length scales these curves are extrapolated and pure speculation. In addition, they assume homogeneous signal characteristics all over the globe. Nevertheless, the two rules give an excellent impression of the overall (logarithmic) signal strength decrease with increasing spherical harmonic degree.
– The line for the Earth Gravity Model EGM96 shows the signal decrease of the geoid based on real data. It is derived from satellite, terrestrial and altimetric gravity information. Its maximum spherical harmonic degree is 360 (about 60 km). It fits well between the Tscherning-Rapp and Kaula lines.

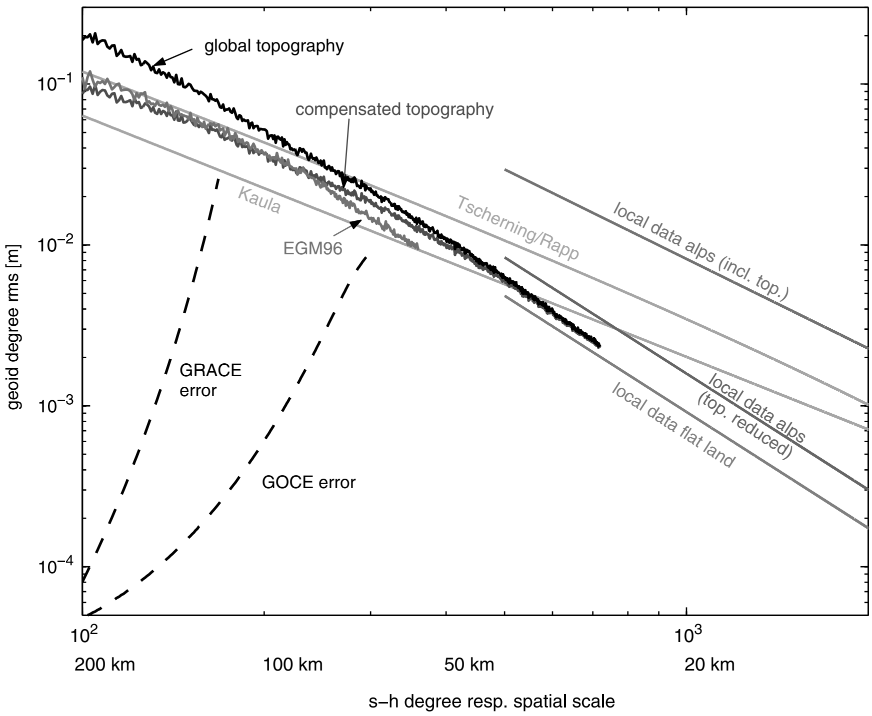

Figure 1. Single content of the static gravity field, shown between spherical harmonics degree 100 and 2000, in terms of geoid signal degree rms-values, as well as GRACE and GOCE error rms-values. See the explanation in the text.

– The "global topography" line is based on a geoid computation from the gravity potential of all visible topographic masses (and ocean depths) assuming constant density. It seriously overestimates the geoid signal strength because it neglects any isostatic compensation of topographic masses.
– Therefore, in a second computation, mass compensation has been taken into account using a simple Airy compensation model. Now the "compensated topography" line fits well to the measured EGM96 line. One can also observe that at smaller length scales, say below 50 km, topographic masses are not compensated anymore according to this model.
– From the GRACE and GOCE error curves one can deduce the state-of-art of geoid knowledge after GRACE and GOCE, respectively.
– At short wavelengths (10–40 km) two representative areas were selected with very good terrestrial gravity data coverage: a local test area in the Alps, which results in the geoid spectral line "local data Alps" and an area of similar size but flat, resulting in the spectral line "local data flat land". One observes the large difference in amplitude (one order of magnitude).
– Finally, for the Alpine test area, the geoid spectrum has been computed after subtracting the effect of the topographic masses; this is the "local data Alps (top. reduced)" line. It is much closer to the Kaula model spectrum, but more important, it demonstrates that at short wavelengths the major part of the observed geoid can be explained by the visible topographic masses of the area and its surroundings (cf. Flury, 2002, 2005).

Improving the static gravity field by means of a new gravity field satellite mission means to penetrate into spatial scales between 40 and 100 km. Scales larger than 100 km will be very well resolved by GOCE; at scales smaller than 40 km the signal amplitude is quite small, and a major part of it can be computed from topographic models. In between, neither sufficient global gravity data (of good quality) nor adequate topographic data are currently available.

The degree rms curves in Figure 1 are derived as follows: For global models, degree variances c_l are obtained from spherical harmonic coefficients of degree l and order m

$$c_l = \sum_m (c_{lm}^2 + s_{lm}^2),$$

their square roots give the degree rms values. For the local data sets (gravity anomalies), empirical signal autocovariance models $C(\psi)$ related to the spherical distance ψ have been determined and converted to degree variances (Wenzel and Arabelos, 1981; Forsberg, 1984):

$$c_l = \int_{\psi=0}^{\Psi} C(\psi) P_l(\cos\psi) \sin\psi \mathrm{d}\psi.$$

$P_l(\cos\psi)$ are Legendre polynomials. The integration is performed up to an appropriate maximum distance ψ. The gravity anomaly degree variances obtained from local data can be represented by simple power laws (cf. Flury, 2005) and converted to geoid heights. The GRACE and GOCE error models are obtained from mission simulations, cf. Sneeuw et al., this issue.

2.2. Geoid omission part

Any satellite gravity field mission will measure the global geoid with a certain precision and a certain spatial resolution. The size of the signal not resolved by the measurements constitutes the so-called omission part. For most geophysical science applications a certain finite spatial resolution is sufficient, corresponding e.g. to the grid spacing of a finite element model. Then the omission part does not enter into considerations as long as the geoid resolution fits to the model resolution. For some other applications, the full geoid information is needed at individual points on earth, i.e. the full spectral content from zero to infinity. In those cases the omission part enters into the error budget. The omission part can be reduced by improving the spatial resolution of a space mission or by adding local geoid information as derived e.g. from local gravity surveys and topographic data.

Figure 2 shows the geoid omission part in centimetres as a function of spherical harmonic degree in the window between degree 300, which represents the situation after GOCE, and degree 700, which a future mission could possibly resolve, corresponding to the range of 70–30 km half-wavelength, respectively. The full omission error, shown by the black line, is still rather high, decreasing from 28 cm at degree 300 to 10 cm at degree 700. It should be noted, however, that by adding local gravity information in a circle of radius 0.5°, 1°, 2° or 5° (50 km, 100 km, 200 km, 500 km) around the computation point the omission error can be reduced significantly. The error for a 0.5° radius cap is of special interest: such an area could – even if no local data were available – be filled up with about 100 terrestrial gravity measurements very fast and economically in sufficient accuracy.

2.3. Time variable gravity field

The geoid and gravity are changing with time due to the tides of sun, moon and planets, due to mass movements and mass exchange in Earth system and, as a secondary effect, due to deformation of the Earth's surface as a consequence to such mass motions. The changes are small, ranging from below a

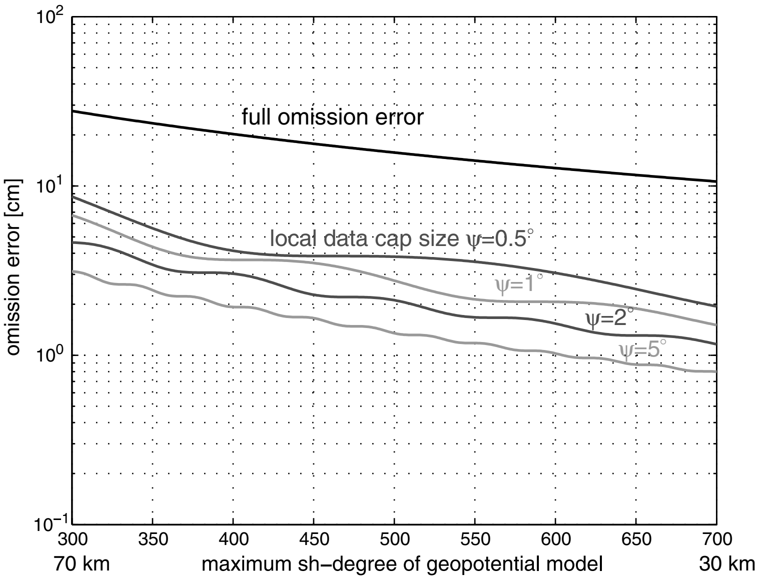

Figure 2. Omission error modeled using Tscherning/Rapp degree variance model. For each spherical harmonic degree the omission part up to infinity is shown, assuming that the signal up to this degree is covered by a geopotential model. The omission error reduces greatly when local gravity data for a relatively small cap size is added, with spherical cap radii of 0.5°, 1°, 2° or 5°.

mm to a few dm for the geoid, they occur at all time scales from secular to sudden and at all spatial scales from global to very local. From terrestrial gravity measurements the global pattern of Earth tides is known rather well; also temporal changes due to postglacial rebound or secular crustal movements are nowadays well measurable. All other effects, such as changes in groundwater level or ocean or atmospheric loading are so far difficult to identify and quantify by *in-situ* measurements. From satellite orbit analysis time variations in the very low degree zonal spherical harmonic coefficients can be determined rather well. Their physical interpretation proves difficult, however (Cazenave and Nerem, 2002; Cox and Chao, 2002). The situation is expected to improve significantly with the monthly sets of spherical harmonic coefficients from GRACE, from which changes in atmospheric pressure, ocean bottom pressure or in the hydrological cycle will be derived, compare Wahr et al. (1998) and Tapley et al. (2004a, b).

Like it is done for the static gravity field, it is common practice to represent the signal strength of the various contributions of temporal geoid variations in terms of signal degree variances. On the one hand, this allows to compare the spectral signal characteristics of the individual geophysical signals; on the other hand, one can compare the various signals with the expected spectral characteristics of the measurement noise of space missions and deduce thereof indications about their observability. There are two difficulties with this approach. First, for some of the geophysical signals knowledge about their temporal and spatial behaviour is rather poor; consequently the degree variance lines may be unrepresentative. Second, some of the considered phenomena are confined to land or ocean or certain geographical regions, which makes the use of spherical harmonic degree variances somewhat problematic. Thus, the spectra are to be seen with a certain ‘‘grain of salt’’.

In essence, the temporal variations can be divided into two classes: those that need to be studied by future gravity field satellite missions and those that are to be considered as ‘‘disturbances’’. A special class of disturbances arises from periodic, high frequency geophysical phenomena that map as ‘‘alias’’ into the spectral range of interest due to the peculiar space-time sampling of a satellite. In particular, semi-diurnal and diurnal phenomena such as the tides of the solid Earth, oceans and atmosphere belong to this category.

Figures 3 and 4 give an impression of the spectral geoid signal of some geophysical effects. Comparison with the geoid degree rms-values of the

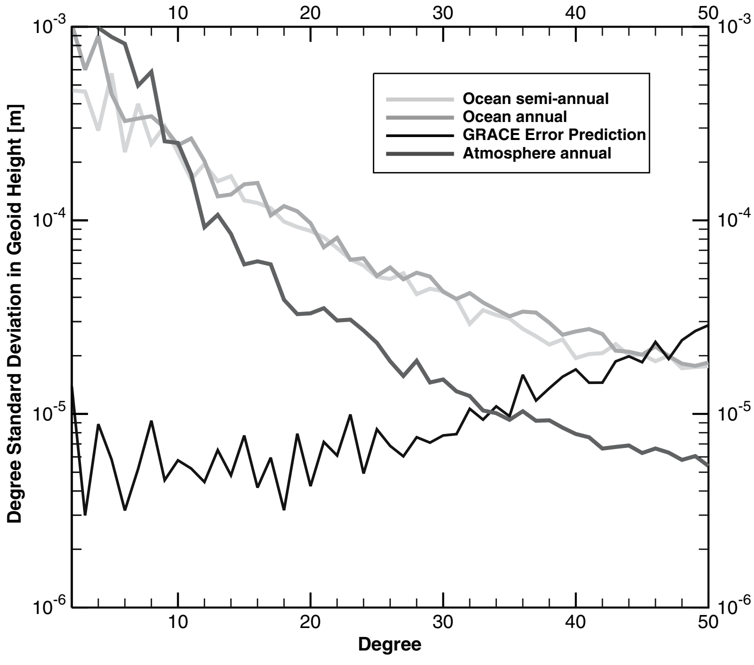

Figure 3. Geoid degree rms-values of the annual variations in atmospheric density of the annual and semi-annual ocean mass changes. For comparison, the expected GRACE noise spectrum is added.

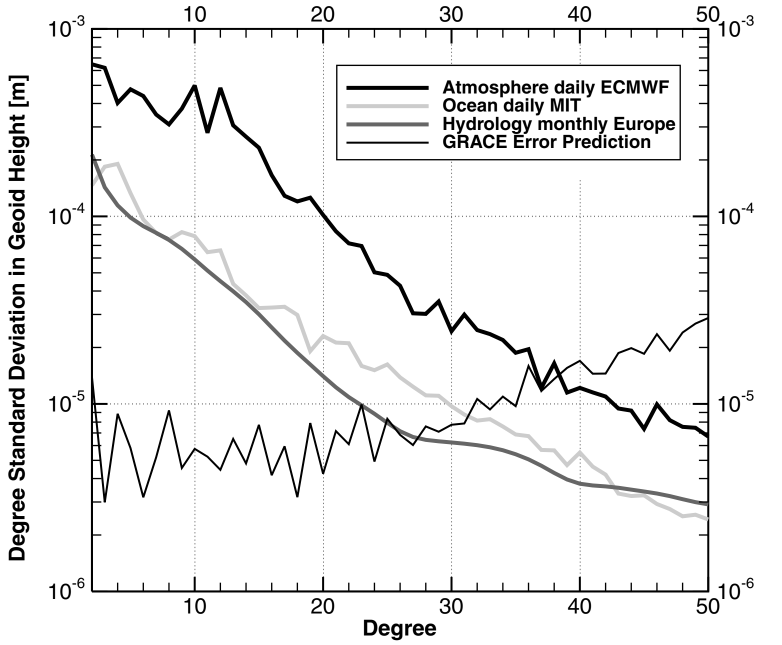

Figure 4. Geoid degree rms-values of the daily variations in atmospheric density, the daily mass changes of the oceans monthly changes in the hydrological cycle. For comparison, the expected GRACE noise spectrum is added.

static field, Figure 1, explains the smallness of the time variable signals. Figure 3 shows the geoid degree rms-values of the annual variations of the atmosphere and of the annual and semi-annual variations of the oceans. All three spectra are derived from corresponding time series of daily data. Their uncertainty is rather large. In order to get an impression of their observability the expected noise spectrum of GRACE is included as well (see Wahr et al., 1998; Wünsch et al., 2001). It can be seen that time variable signals can be derived by GRACE up to about degree and order 35, or at length scales of about 500–600 km. Figure 4 shows some daily and monthly geoid variations (daily atmosphere from ECMWF, daily ocean from MIT ocean circulation model, and – with a high uncertainty – monthly changes in the hydrological water cycle). These three time series may cause aliasing problems, depending on the sampling strategy of future gravity field satellite missions (see Gruber, 2001; Wünsch et al., 2001). The GRACE expected noise spectrum is added for completeness.

Based on this analysis of the signal behaviour of stationary and time-variable gravity and geoid and the current state-of-the-art of commission and omission error one can now turn to gravity and geoid applications in geodesy and (in the following articles) in Earth sciences.

3. Geodetic Requirements

It is common to all applications of gravity and geoid information in geodesy, mapping, geomatics and engineering that point values or differences of point values are needed. The only exception is orbit determination. This implies that the band limited geoid and gravity information deduced from satellite measurements is only part of the required total geoid and gravity point values. The missing complementary, small scale part has to come from local airborne, shipborne or terrestrial gravimetric surveys and from topographic models. After GOCE this is the signal part above spherical harmonic degree 250, see Rummel (2004, this issue). If the small scale geoid and gravity contribution is neglected, a rather high omission error has to be accepted, as was discussed in Section 2. While after GOCE the geoid error at degree and order 250 will amount to a few cm only, the omission part is about 30 cm 1 σ error.

Thus, what would be the proper strategy in these fields for the future?

(1) An improvement of GOCE precision without any improvement in resolution would reduce the geoid commission error from a few cm after GOCE to the sub-cm level. However, this will not result in major breakthroughs in view of the large omission error.
(2) An improvement in spatial resolution from a maximum degree of 250 to, say, degree 700 (or 30 km) would decrease the remaining omission error from 30 to 10 cm. This would help considerably, but it may prove very difficult to get such high resolution from space techniques.
(3) Reduction of the omission error by means of local gravimetric surveys and topographic models: In a concerted effort – based on new, well controlled measurement campaigns, re-analysis of existing data sets and new high resolution terrain models – the local omission error can be reduced to the cm-level.

As a consequence, the strategy must be: (a) Extension of the spatial resolution of future dedicated satellite missions, beyond degree 250 without loss of accuracy, and (b) reduction of the omission error for selected regions and applications to the cm-level.

3.1. Heights

In most countries, well defined and official "heights above sea level" are made available and maintained for use in engineering, mapping, cadastre, geo-referencing as well as for use in science. These are so-called physical heights which means they carry information about the direction of flow of water. National height systems usually refer to sea level and are tied to a tide gauge at an arbitrarily selected point. This results in offsets between height systems at borders and across oceans or ocean straits. For Europe, the offsets are in the order of some decimetres and are rather well known, with an accuracy of 1–10 cm (Figure 5, from Sacher et al., 1999). For some continents they are unknown and may be much larger. Physical heights are derived by spirit levelling plus gravimetry. This approach has the tendency to accumulate systematic errors that may amount to several cm or even dm on a continental scale (Haines et al., 2003), in addition to the tide gauge offsets.

Ideally, "heights above sea level" should be "heights above the geoid", i.e. in an ideal case all height systems should refer to one and the same level surface. Only then physical heights all over the earth would become comparable, which would lead to the establishment of a world height reference system. One would be able to decide whether an arbitrary coastal

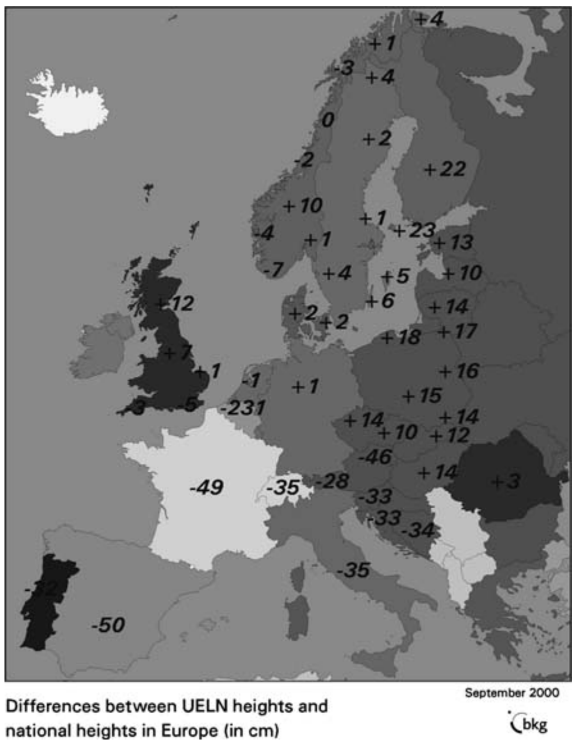

Figure 5. Offsets between national height systems in Europe (in cm), from Sacher et al. (1999). Countries in the same colour are connected to the same datum point (tide gauge). By courtesy of Bundesamt für Kartographie und Geodäsie, Frankfurt.

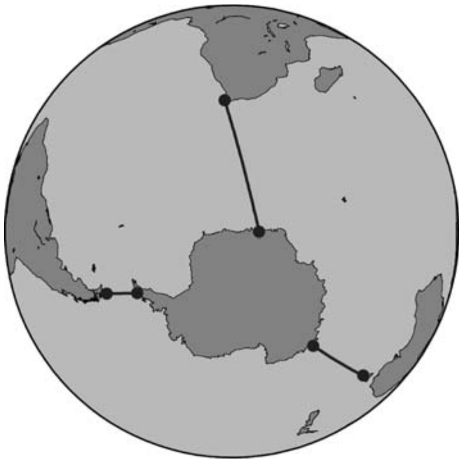

Figure 6. Examples for links between tide gauges (crossing the Antarctic Circumpolar Current), for which high precision geoid differences are required for ocean circulation modelling.

point in Australia would be higher or lower than a second point somewhere in Europe. This is of some importance for regional and global mapping, geo-referencing and geo-information, and it is of great importance for global sea level and ocean circulation studies. Tidal records would become comparable globally, too (see Figure 6). The required geoid accuracy is below 10 cm (commission plus omission part) or on the long term below 1 cm. Official institutions like the US National Geospatial-Intelligence Agency NGA strive for a continental or global accuracy of 2 cm. After GOCE, a height system accuracy of approximately 5–10 cm – commission and omission error – will be achievable for all areas where a sufficient coverage of local gravity data is available to deal with the omission part (see Section 2; Arabelos and Tscherning, 2001; Rummel, 2002). For a 1 cm accuracy, however, one has to take into account local data up to much larger distances, and a satellite mission with higher resolution is needed.

For engineering purposes (construction of bridges, long tunnels, canals) precise physical height differences and deflections of the vertical are needed. The local character of these applications emphasizes the need for very precise local gravimetric and topographic information and relaxes the requirement for very precise global (satellite derived) geoid and gravity models. However, for these applications an increased resolution of the geopotential models would lead to cost reduction in the sense that less effort would be required to check and refine the geoid and height reference locally, e.g. that the amount of required local gravity observations would decrease substantially.

It has to be noted that for areas with considerable vertical land movements the time variability of the geoid has to be taken into account to maintain the well defined height reference. This applies especially to Canada, Scandinavia and Antarctica, where the geoid variation due to postglacial rebound reaches up to 2 mm/year corresponding to a 20 cm change over a century, but it also applies to low land countries (see Vermeersen, 2004, this issue). Vertical tectonic movements – typically up to some mm/year (Lambeck, 1988) – are expected to cause considerably smaller geoid changes (Marti et al., 2003). However, on the long run it may also be necessary to consider geoid variations due to tectonics for precise height systems. Therefore, geodesy would also benefit from an improved determination of secular geoid time variations.

3.2. GNSS levelling

Nowadays, by means of GPS and other space positioning methods, absolute point positions or position differences can be derived at the cm-level or sub-cm level, respectively, depending on the sophistication of the measurement setup. There is a clear trend to even higher precisions once the next generation of satellite navigation systems with more precise clocks as well as better atmospheric monitoring become available. 3-D point positions or position differences can be directly transformed into geographical coordinates or coordinate differences and ellipsoidal heights or height differences. In combination with a precise geoid model, ellipsoidal heights can be conveniently translated into physical heights (heights above the geoid). This technique circumvents the tedious, time-consuming traditional geodetic levelling and avoids the systematic errors inherent to this method. Precondition is, however, the availability of absolute geoid heights or height differences with a precision comparable to that of the ellipsoidal heights derived by means of GNSS. This is to say the required geoid precision (commission and omission part) has to be at the cm or sub-cm level.

Already now the technique of GPS-levelling is applied worldwide at a much more moderate level of accuracy. The fields of application are engineering (construction of streets, bridges, tunnels, canals), mapping and geoinformation, exploration and many more. The method is the only economic way to get a well defined height reference in unsurveyed areas, be it in developing countries or in the polar regions. It is of special interest for areas where permanent height benchmarks cannot be built, e.g. for inundation zones, where the benchmarks get destroyed by every flood.

Where no local gravity data in good quality are available, GNSS levelling is affected by the full geoid error (commission and omission) of a geopotential

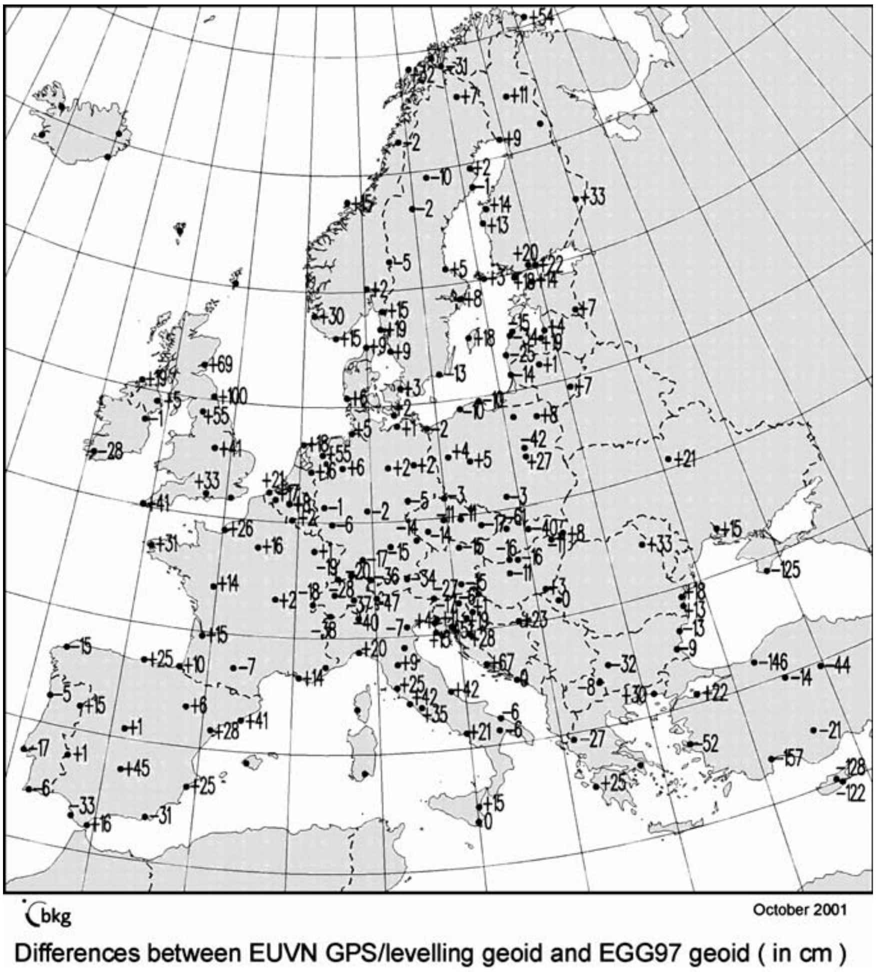

Figure 7. Errors for GNSS Levelling using the geoid model EGG97 (Denker and Torge, 1998), based on EGM96 geopotential model and a very large terrestrial gravity data base. By courtesy of Bundesamt für Kartographie und Geodäsie, Frankfurt.

model. This error will be – depending on the distance between stations – at a few cm after GOCE. It could be lowered down to the centimeter level by a satellite mission resolving spatial scales of the geoid up to spherical harmonic degree 700.

Figure 7 (from Ihde et al., 2002; see also Denker and Torge, 1998) shows today's state-of-the-art, where even in a well surveyed area like Europe the geoid errors amount to several dm in general, in some cases even to more than 1 m.

3.3. Inertial navigation

The core sensors of any inertial measurement unit (IMU) are a set of three orthogonally mounted accelerometers and gyroscopes. The IMU may be rigidly fixed to the moving platform (strapped down system); then the accelerometers and gyroscopes must be able to cope with the full dynamics of the platform motion. Alternatively the IMU may be isolated from the rotational degrees of freedom of the motion by means of a gimbal system (space fixed or local level system). In any case, the accelerometers measure the sum of vehicle motion and gravitational attraction.

In order to isolate the accelerations due to vehicle motion, which are then integrated once or twice to give vehicle velocity or position differences, respectively, the gravitational accelerations have to be subtracted. As there is no way to measure them independently they have to be provided by some gravity model. Any imperfection in this model will result in a systematic error, and after integration this error will quickly accumulate to large drifts in the calculated velocities and positions. In vehicle navigation, where many zero-velocity updates (ZUPT's) can be incorporated in the survey, a rather simple ellipsoidal gravity model suffices. In sub-marine, borehole, aircraft and missile guidance no ZUPT's are possible and requirements for precise gravity information are very high. They are typically at the 0.1–1 mGal level in terms of gravity and 0.1 arcsec in terms of deflections of the vertical (DOV's). Again, such errors comprise commission and omission errors (Chatfield, 1997). Any improvement of global gravity field models towards these numbers – in particular by increasing the spatial resolution – would help (Schwarz et al., 1992). It would improve reliablility, reduce navigation drift and consequently increase the radius of operation.

3.4. Satellite orbit determination

After GRACE and GOCE a very good Earth gravity model will be available for the determination of satellite orbits. However, even then a gravity model based on only one or two missions may exhibit some specific weaknesses. This can be seen when using the current CHAMP only solutions for orbit determination of other satellites (Schrama, 2003). Complementary missions may therefore still prove to be of importance, in particular missions at higher altitude or with different orbit inclinations. For this, inexpensive missions of the type LAGEOS or high orbiting satellites equipped with continuous tracking devices such as GPS receivers could serve. For the determination of low zonal coefficients and their secular variations, long mission durations (>10 years) may remain important.

On the long term, from a perfect knowledge of the gravity field in conjunction with orbits derived from high–low SST using a GNSS (e.g. using the kinematic method), the influence of the non-gravitational forces could be studied, which could be of value for atmosphere physics. In summary, there is no immediate need from the point of view of orbit determination for a follow-on dedicated gravity mission.

4. Conclusions

Geodesy, including navigation, mapping, engineering and geo-referencing requires geoid, gravity anomaly or deflections of the vertical (DOV) values in the absolute sense. Thus, not only the (commission) error of these quantities deduced from potential future satellite gravity missions has to be taken into account. At least as important is the reduction of the omission error, i.e. the signal part that cannot be resolved from space.

A medium (and long) term accuracy goal (commission and omission part) is:

– 10 cm (1 cm) for geoid heights,
– 1 mGal (0.1 mGal) for gravity anomalies,
– 1 arcsec (0.1 arcsec) for DOV's.

The corresponding strategy must be:

– Without loss of precision extend the spatial resolution of a future gravity satellite mission from sperical harmonic degree $l_{\max} = 250$ (corresponding to 80 km half-wavelength) to $l_{\max} = 400$ (50 km) or may be even $l_{\max} = 700$ (35 km).
– Reduction of the remaining omission part by means of local gravimetric surveys (airborne, shipborne, terrestrial), re-analysis of existing gravity data sets and new high resolution digital terrain models.

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