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EDITORIAL

New prospects in non-conventional modelling of solids and structures

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Editorial

This special issue aims to provide an overview of advanced approaches in mechanics, focusing on three main topics: fractional calculus, formulation of non-classical continua, modelling of random media. The interest in fractional calculus is motivated by its unique capabilities in describing fractal or nonlocal media; it is also a valuable and well-established framework for capturing complex long-memory or multiscale phenomena in materials. The formulation of non-classical continua keeps attracting a considerable attention in engineering science, especially to deal with new artificially engineered materials where microstructure effects may play a crucial role. Moreover, modelling random media is a challenging objective, of great relevance to capture the behaviour of materials in nature. The special issue contains fourteen articles, eight on fractional calculus, five on non-classical continua and one on modelling of random media. The brief description that follows is meant to provide the readers with a quick guide on the content.

The first group of articles concerns fractional models [1–4]. Patnaik et al. [1] propose a fractional-order homogenization approach to model the elastic wave propagation in a bi-material periodic beam, taken as a representative of a one-dimensional elastic metamaterial. The authors assume the homogenized beam is governed by a fractional-order equation analogue of the Euler-Bernoulli beam equation, where fractional-order kinematic relations involving a space-fractional Riesz-Caputo derivative are defined over a nonlocal horizon. In this context, the nonlocality of the Riesz-Caputo derivative is exploited to attain a spatial average of the material properties of the heterogeneous medium that, in classical homogenization approaches, is obtained by specifically designed convolution kernels. The fractional-order equation governing the homogenized beam is obtained via variational principles and, on assuming a nonlocal horizon equal to the length of the beam, the fractional order of the model is derived by imposing that the dispersion behaviour of homogenized and initial heterogeneous beams are equivalent. The resulting fractional differential model of the heterogeneous system has, in its most general form, a complex-valued and frequency-dependent order whose real and imaginary parts are related to the decay and the frequency modulation of the

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amplitude and phase of the wave propagating in the medium. The fractional-order finite element method developed by the authors in a previous study (see references in ref. [1]) is used to simulate the response of the periodic beam. The fractional-order framework is validated, at both band-pass and band-gap frequencies, by direct comparison against the response of a periodic beam solved via the traditional finite element method based on integer-order equations. Remarkably, the fractional-order homogenization approach proves capable of representing the dynamic behaviour of a bi-material periodic beam within the first few frequency band gaps, which is a strength of the proposed method over classical homogenization techniques whose range of validity is limited to the low-frequency regime. A discussion on potential improvements is also given [1]. Lazopoulos and Lazopoulos [2] propose a novel fractional approach to continuum mechanics, based on the definition of a fractional Λ-derivative and an associated Λ-space to introduce consistent Λ-fractional strain measures, as well as corresponding stress and displacement measures. The work focuses on a fractional bar under axial load as example and discusses generalizations to the bending problem of a cantilever beam. The proposed fractional approach to continuum mechanics is conceived as a nonlocal approach to address, e.g., size effects in small scale media; applications to linear and nonlinear elasticity problems are possible. Alotta et al. [3] propose a mechanically consistent nonlocal formulation based on fractional operators in conjunction with a peridynamic scheme where long-range interactions are taken into account. For unbounded domains, the stress is proportional to the Riesz fractional integral of the strain while the strain is proportional to the Riesz fractional derivative of the stress. Both operators are convolution integrals whose kernel is a power-law attenuation function governing the decay of long-range interactions in the nonlocal continuum. Notably, the analytical forms of the proposed nonlocal stress-driven and strain-driven relations mirror the typical result of fractional viscoelasticity, where the stress history is related to the strain history (strain driven) by the Caputo's fractional derivative and, in contrast, the strain history is related to the stress history (stress driven) by the inverse operator, namely the Riemann-Liouville fractional integral. Useful results from the proposed concept are discussed, taking an unbounded nonlocal rod as example. Barretta et al. [4] propose an original approach combining fractional hereditariness and integral stress-driven formulation of nonlocality, with focus on the nonlocal viscoelastic bending problem of an Euler-Bernoulli beam. Closed-form solutions are derived for parametric analyses addressing the influence of nonlocal and viscoelastic parameters on the mechanical response. The approach addresses small-scale structures with molecular interactions and long-range forces, non-elastic materials like polymeric and non-conventional structural continua. Additionally, nonlocalities in non-continuous materials with microstructures like artificial or biological tissues and the mechanical behaviour of multiphase materials in advanced smart composites are identified as potential targets of the proposed approach.

Concerning fractional calculus, the second group of articles deals with fractional models of complex phenomena in materials [5–8]. Mashayekhi et al. [5] investigate the relaxation behaviour of fractal polymers and propose a generalized Scott-Blair fractional model of viscoelasticity, where the excluded volume effect and the hydrodynamic interaction are explicitly taken into account to derive the microscopic stress within the molecular theory of Rouse and Zimm. The excluded volume effect is added to consider how the finite volume of a set of polymer segments, neighbouring segments and solvent interactions may swell the polymer and influence time-dependent deformation. The model requires an appropriately modified distribution of the end-to-end polymer vectors, which is used for a pre-averaging approximation of the mobility matrix in the Zimm model. On this basis and through the application of thermodynamic laws, the generalized fractional model of viscoelasticity is obtained in a linear form based on its spectral dimension, fractal dimension and the excluded volume parameter. Specifically, continuum thermodynamic energy and entropy balance equations are used to obtain the Riemann-Liouville integral and fractional-order relations between the internal state order parameter and the deformation for rate-dependent, finite deformation of the polymer. The derivation unveils that the order of the fractional derivative in the linear fractional model of viscoelasticity is strongly correlated with fractal structure and excluded volume effects. Zhokh and Strizhak [6] deal with anomalous transport dynamics in porous fractal media due to crossover between different transport regimes. Taking as example methanol and methane transport through a zeolite/alumina porous particle, the authors investigate the mass transfer of the diffusing agents using as an analytical tool a one-dimensional time-fractional advection-diffusion equation. The results are validated against experimental data. It is shown that transport mechanisms differ at short and long times, with pure advection occurring at shorter times and convective mechanism at medium and large times. Moreover, it is seen that the longtime transport may experience either Fickian or non-Fickian kinetics, depending on the diffusing agent. Essential conclusions of the study are that the diffusing agent with higher adsorption energy on the porous material exhibits non-Fickian transport. At the same time, the origin of the mass transfer abnormality is associated with the adsorption strength of the corresponding diffusing agent on the surface of the porous material. Sun et al. [7] propose a state-dependent fractional plasticity model for over-consolidated soft soils. The model involves a new state-dependent stress-dilatancy equation, developed from a fractional differentiation of an elliptic yielding surface; in this context, the dilatancy ratio is determined by the current stress state and the distance from current to critical state, via the fractional order and the shape factor of the yielding surface. Additionally, the model involves a combined isotropic-deviatoric fractional hardening rule expressed as a function of incremental plastic volumetric and shear strains. The authors demonstrate the model can reasonably reproduce the key stress-strain features, such as the strain hardening and volumetric contraction under normally consolidated or lightly over-consolidated conditions and the strain softening and volumetric dilation under heavily over-consolidated conditions in different re-modelled and natural clays. The model parameters are seven and can be obtained from laboratory tests. Blaszczyk et al. [8] focus on computational issues of fractional calculus and, specifically, those related to the Riesz-Caputo fractional derivative of variable order with fixed memory. This operator is of interest for certain types of phenomena in physics, with promising applications to diffusion processes in inhomogeneous and heterogeneous media, as detailed in ref. [8]. Scope of the study is to propose three different approximations of the operator for computational purposes, based on piecewise constant, piecewise linear and piecewise quadratic interpolations. The errors generated by the three methods and the experimental rate of convergence are discussed. Finally, an application of the Riesz-Caputo fractional derivative of space-dependent order is proposed for a one-dimensional continuum body.

Non-classical continua are the subject of the third group of articles [9–13]. Sapora et al. [9] deal with the typical borehole problem, modelled as a circular hole in an infinite elastic medium subjected to remote biaxial loading and/or internal pressure. The borehole problem is chosen for relevance in geotechnical engineering if the inner pressure is considered or, alternatively, in mechanical engineering if remote stresses are taken into account. Two nonlocal approaches are presented within the framework of gradient elasticity and finite fracture mechanics, respectively. The gradient elasticity approach is nonlocal in the elastic material behaviour but local in the failure criterion, as it considers the stress concentration factor as the governing failure parameter. On the contrary, the finite fracture mechanics approach is local in the elastic material behaviour but nonlocal in the fracture criterion since crack onset occurs when two (stress and energy) conditions are simultaneously met in front of the stress concentration point. Although the two approaches have a completely different origin, the authors observe that they present some similarities as they involve both a characteristic length. Remarkably, they lead to almost identical critical load predictions as far as the two internal lengths are properly related. The study is substantiated by comparison with experimental data related to two different rock materials. Colatosti et al. [10] analyze particle composite materials with different microstructures, modelled as an ensemble of rigid particles of arbitrary hexagonal shape and elastic interfaces. Considering a 2D panel with three different microstructure textures, static analyses are performed comparing the solutions of discrete, full unconstrained micropolar (Cosserat) and classical Cauchy models. The discrete model is implemented in ABAQUS and taken as a benchmark model, while equivalent micropolar and classical continua are formulated by a homogenization technique, where a representative volume element is built by a numerical approach. A comparison of displacement fields demonstrates the effectiveness of adopting the micropolar continuum theory for describing particle composite materials, especially for the hourglass and skew configurations, where non-symmetries and size effects play a major role. Postek et al. [11] investigate the deformation of open-cell copper foams under impacts, a topic of interest in studying open-cell multifunctional structures produced by additive manufacturing, e.g., crush-resistant heat exchangers and heat capacitors. It is assumed that the skeleton material is made of oxygen-free high, conductivity copper. The dynamic process of compression and crushing during impact is simulated by a peridynamic model, using an elastic-plastic model with isotropic hardening for the skeleton material. The study focuses on the influence of the strain rate hardening coefficient on the behaviour of a copper foam sample into an ideally elastic block, assuming the skeleton structure is obtained by tomographic images. The authors conclude that an increase of the strain rate hardening coefficient causes an increase of the maximum Huber-Mises-Hencky stress and a decrease of the maximum equivalent plastic strain in the sample, with a larger extent of the plastic zone. Di Matteo et al. [12] formulate an innovative procedure for static analysis of micro- and nano-plates, of arbitrary shape and with various boundary conditions, using the Eringen nonlocal elasticity theory to model size effects. The formulation is based on a meshfree procedure, namely the Line Element-Less Method, which involves simple line integrals of harmonic polynomials with unknown coefficients along the boundary parametric equation, to be calculated by a set of linear algebraic equations. The main advantage is that the Line Element-Less Method is completely element free: it does not require any discretization in either the domain or the boundary and it differs from other meshfree procedures since the expansion coefficients are not determined by collocation. The procedure yields approximate analytical solutions for general shapes and boundary conditions and exact solutions for some plate geometries. Various applications are discussed to show the simplicity and accuracy of the procedure. Yang et al. [13] propose the fractal scaling-law vector calculus as a connection between fractal geometry and vector calculus. The study presents the Gauss-Ostrogradsky-like, Stokes-like, Green-like theorems and Green-like identities, giving a perspective on potential applications in elasticity. Specific attention is given to the Mandelbrot scaling-law partial differential equations in elasticity.

Finally, in the concluding article of the special issue, Zhang et al. [14] deal with anti-plane shear Lamb-type elastodynamic problems in media featuring spatial randomness in mass density and anti-plane stiffness tensor fields. The randomness has fractal and Hurst characteristics, as is typically the case in many geological and biological patterns. Second-order, wide-sense stationary and isotropic models with Cauchy or Dagum correlation functions are assumed for the mass density field, while a rank-2 tensor random field, obtained as a dyadic product of two scalar random fields generated from Cauchy or Dagum correlation functions, is assumed to model full anisotropy for the stiffness tensor field. The cellular automata approach is used to simulate the transient wave propagation in a Monte Carlo sense, allowing the assignment of random heterogeneous material properties at the cell level. Considering a classical benchmark problem and comparing with the results for the model having the random mass density field and the random stiffness tensor field with local isotropy, the authors conclude that a rank-2 anti-plane stiffness tensor field with full anisotropy leads to the strongest fluctuation in the displacement responses.

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