Optimal design of gears contact interface modification for an objective as minimum impact resistance of initial meshing-in time domain

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Received: 6 July 2020 / Accepted: 4 December 2020 / Published online: 3 January 2021
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Abstract In gear pair actual alternating meshing process, the comprehensive errors of the transmission system and the thermal elastic deformation of the teeth body cause the gears in the meshing state to high pair contact nodes inconsistently, especially the transient process of meshing-in and meshing-out points will deviate from the theoretical line, which will cause a sudden change in the meshing velocity and cause instantaneous impact of meshing. Off line meshing impact will generate large additional loads, which has attracted many attentions. The gear transmission system vibration and noise are increased, and the impact of gear teeth meshing-in is significantly greater than that of meshing-out. Therefore, optimal analysis

of the impact with the minimum value of gear teeth meshing-in includes determining the initial meshing points, calculating of meshing impact velocity and the impact force of teeth meshing-out. The optimized calculation model using loaded teeth contact analysis method is validated.

Keywords Gear teeth modification · Meshing-in impact · Minimum impact resistance · Optimal design · Alternating intermeshing

1 Introduction

Gears with teeth meshing are known to be subject to undergo meshing impact ranging from mild to quite severe, depending on the extent and distribution of the gear transmission system errors. An actual and scientific phenomenon was noted that meshing outside the normal path of gear teeth may lead to impact owing to sudden fluctuation of rotation velocity [1]. In addition, meshing impact may induce dynamic load, vibration and noise. Even if the time-domain meshing gear has a fully accurate involute tooth profile under the working condition, the gear teeth will change the base joints of driving and driven gears in the position of the meshing line due to the deformation of the gear tooth load and the installation error under the initial action of meshing force, which will no longer be equal,

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resulting in the gear tooth surface interference which will cause the meshing point to deviate from the normal meshing line [2–4].

Some scholars had suggested three kinds of meshing impact (such as meshing-in impact, mesh-out impact and node impact) could appear in gear transmission system [5]. The influence of node impact was minimal and the effect of mesh-out impact was less than that of meshing-in impact. Subsequently, partly gear transmission errors were taken into account to analyze the meshing impact [6–8].

The meshing impact is derived from a meshing-in impact velocity model and the results from some typical examples simulation with meshing impact gear pairs are presented to validate the model [9–11]. Loaded teeth contact analysis (LTCA) was an important numerical method used to simulate the course of teeth meshing under applied load conditions [12–14]. Combining loaded teeth contact analysis and teeth modification, and studying loaded teeth meshing under modification can explore the best teeth surface modification design [15–17].

Many studies had been proved that through modifying addendum and dedendum, teeth meshing performance could be effectively improved meshing impact being decreased, noise being reduced, and loading capacity being increased because of teeth surface load uniformly distributing [18–20].

The optimization goal was to minimize meshing impact and uniformly distribute teeth surface load, and the modified parameters could be obtained by applying the complex method [21]. Subsequently, several typical examples which verify the correctness of the modification method was done by comparison with the loaded transmission error, maximum impact force and load distribution factor both before and after modification [22].

The time-domain transmission error and gear tooth deformation in working condition cause the normal joint of the meshing gear pair to be unequal and deviate from the theoretical meshing line at the meshing-in point and the meshing-out point, resulting in the transient meshing impact from the abrupt step of rotational speed. Out-of-line meshing impact generates a large additional load, and makes the gear transmission system vibration and noise increase, which in turn affects the comprehensive performance of gear transmission.

The discussion in this article will focus on the impact of meshing-in. The calculation process of the impact includes the determination of the position of the initial meshing-in point position, the calculation of the initial meshing-in impact velocity, and the calculation of the initial meshing-in impact force.

2 Mechanism of meshing-in impact and calculation model

The contact process between the edge of one gear teeth side and the other gear teeth surface is called interface edge contact. In this article, the tangent at the contact point along the toothed edge of the driving gear must be perpendicular to the normal at the contact point of the teeth surface of the driven gear. This article takes the helical gear's edge contact (including teeth-side edge contact and teeth tip edge contact) as the carrier object during the meshing-in to meshing-out. The coordinate relation of a pair of gear meshing is shown in Fig. 1.

Setting 1 denotes for the pinion teeth side edge contact, the parameter l_1 is constant. The expression of the contact equation of the teeth surface interface is given as

$$\begin{cases} M_{f1}(\varphi_1)R_{1r}(u_1, l_1) - M_{f2}(\varphi_2)M_{e2}R_2(u_2, l_2) = 0 \\ (L_{f1} \frac{\partial R_{1r}}{\partial u_1}) \cdot (L_{f2}(\varphi_2)L_{e2}n_2(u_2, l_2)) = 0 \end{cases} \quad (1)$$

Setting 2 indicates that the pinion top edge contact, then the parameter μ_1 is constant. The contact equation of the teeth surface interface can be expressed as

Fig. 1 The coordinate relationship of a pair of gears

$$\begin{cases} M_{f1}(\varphi_1)R_{1r}(u_1, l_1) - M_{f2}(\varphi_2)M_{e2}R_2(u_2, l_2) = 0 \\ (L_{f2}\frac{\partial R_{1r}}{\partial l_1}) \cdot (L_{f2}(\varphi_2)L_{e2}n_2(u_2, l_2)) = 0 \end{cases} \quad (2)$$

Here, M_f expresses the conversion matrix of each vector in the fixed coordinate system, M_e represents the rotation sub-matrix, R_r is the component of theoretical teeth surface position vector, φ is expressed as an input variable, where the subscripts 1 and 2 represent the driving gear and driven gear, respectively. The other four unknown parameters can be solved to determine the contact point. For each instantaneous contact point, the corresponding main curvature and main direction can be determined separately, and the meshing imprint can be determined.

In Fig. 2, the coordinate relationship between the normal teeth profile of the rack-shaped cutter and the rack to form a tooth surface can be described. Hence, the related expressions of theoretical teeth surface position vector and normal vector are written as

$$r_{ci} = \begin{cases} -y_i \sin \alpha + (\mu_i - dpi) \cos \alpha \\ y_i \cos \alpha \cos \beta + l_i \sin \beta + [(\mu_i - dpi) \sin \alpha + \alpha_{mi}] \cos \beta \\ -y_i \cos \alpha \cos \beta + l_i \cos \beta - [(\mu_i - dpi) \sin \alpha + \alpha_{mi}] \sin \beta \end{cases} \quad (3)$$

In the above expression, where y_i ($i = 1, 2$ represent small and large gears respectively) is the parabolic equation in the tooth profile coordinate system $o_b x_b y_b$.

μ_i is the interface parameter of the teeth surface on the section of the rack cutter, and the distance from the point on the teeth profile to the parabolic pole is described. l_i is a tooth surface interface parameter and indicates the distance of the normal section where the points on the teeth profile are located from the coordinate origin o_{ci} . dpi is the position parameter of the parabolic pole. α and β are pressure angle and spiral angle, respectively. α_{mi} is the normal half tooth thickness on the rack pitch line. Hence, the unit normal of the tooth surface interface can be determined by the expression (4)

$$\vec{n}_{ci} = \frac{(\vec{\partial r}_{ci} / \partial \mu_i) \times (\vec{\partial r}_{ci} / \partial l_i)}{|\vec{\partial r}_{ci} / \partial \mu_i \times \vec{\partial r}_{ci} / \partial l_i|} \quad (4)$$

Tooth surface is depicted in the gear coordinate system $X_i O_i Y_i$, where $[M(\theta_i)]_{i,ci}$ is the transformation matrix from the coordinate system S_{ci} to S_i in the expression (5), $[L(\theta_i)]_{i,ci}$ is the rotator matrix.

$$\begin{cases} \vec{r}_i(\mu_i, l_i, \theta_i) = [M(\theta_i)]_{i,ci} \vec{r}_{ci}(\mu_i, l_i) \\ \vec{n}_i(\mu_i, l_i, \theta_i) = [L(\theta_i)]_{i,ci} \vec{n}_{ci}(\mu_i, l_i) \end{cases} \quad (5)$$

The following meshing equation must be met in the process of the tool machining the teeth surface

enveloping the gear interface, where $\vec{v}_{ci}^{(ci,i)}$ is the relative velocity of the tool and the gear being machined

Fig. 2 Coordinate relationship between normal teeth profile and extended teeth surface

$$f(\mu_i, l_i, \theta_i) = \overrightarrow{n_{ci}} \cdot \overrightarrow{v_{ci}(ci, i)} = 0 \quad (6)$$

Considering various operating conditions, the transmission error of gear pair is defined as the value of the actual rotation angle of the driving gear deviates from the theoretical rotation angle with the change of the rotation angle of the driven gear, and its expression is written as

$$\psi(\varphi_1) = (\varphi_1 - \varphi_1^0) - N_1/N_2(\varphi_2 - \varphi_2^0) \quad (7)$$

Here, φ_1 is the initial angle of the pinion, φ_2 is the initial angle of the large gear. N_1 and N_2 represent the number of teeth of pinion and large gear, respectively.

Considering manufacturing and manufacturing errors, the actual tooth surface of helical gear is different from its theoretical tooth surface, and a smooth surface is formed after a certain time of running-in. Theoretical teeth surfaces (pinion and large gear) of helical gear with rack spread are shown in Fig. 3. The precision requirement of less than $1.0 \mu\text{m}$ is often difficult to achieve the actual tooth surface by NURBS fitting the sampled three-coordinate data directly. The teeth surface errors are obtained by three coordinate measurements and the actual teeth surface are constructed by superposition of theoretical teeth surface and normal error surface. Due to the gear teeth errors and the elastic deformation of the bearing gear, the time domain meshing gear produces the “meshing synthetic base node error”, which makes a pair of gear teeth deviate from the theoretical meshing-in point on the meshing line when entering the initial meshing-in, and then causing the meshing-in

impact. This appearance mechanism is the focus of the research group and the next step to study the problem.

2.1 Mechanism of meshing-in impact

In the gear transmission system dynamics, the impact caused by the “meshing synthetic base node error” is called the meshing-in and meshing-out impact of the gear teeth surface meshing process. For the paper, revealing the mechanism of the initial meshing-in impact is the core of this study. Considering the equivalent error of the system and the comprehensive deformation of the gear teeth, the exact position of the initial meshing-in point D out-of-line is calculated according to the meshing principle and geometric relationship, as shown in Figs. 4 and 5.

In $\triangle O_1DO_2$ and $\triangle PDO_2$, then

$$r_{O_1D} = \sqrt{a^2 + r_{a2}^2 - 2ar_{a2}\cos(\angle PO_2D)} \quad (8)$$

$$r_{O_1D} = \sqrt{a^2 + r_{a2}^2 - 2ar_{a2}\cos(\frac{\pi}{2} - \alpha - \arcsin \frac{r_2 \sin(\frac{\pi}{2} + \alpha)}{r_{a2}} + \varphi_k + \Delta\varphi_2)} \quad (9)$$

where, α represents the center distance between two gears, r_{a2} is the radius of the tooth top circle of the driven gear, r_2 is the pitch circle radius of the driven gear, r_{b1} is the base circle radius of the driving gear, α is the pressure angle of the index circle.

A pair of gears in the meshing process, the gear has a completely accurate involute profile. Theoretically, the normal pitch is equal to each other in normal mesh condition. In practice, under the gear meshing impact

Fig. 3 Theoretical teeth surfaces of helical gear with rack spread

Fig. 4 Schematic of out-of-line initial meshing-in impact

force action, due to the teeth load deformation and installation errors, which may lead to the driving gear and driven gear base line position change, and then generate teeth surface interference which may cause the meshing point to deviate from the normal meshing line. When the gear teeth are affected by the load deformation and the system error will generate off line meshing. And then can cause the contact teeth relative velocity difference where the actual meshing point D along teeth profile normal line direction. That is, meshing-in impact instantaneous velocity v_s , as shown in Fig. 6.

The deformation of the gear tooth and the system error will produce the out-of-line meshing, which leads to the relative velocity difference of the contact tooth pair along the common normal direction of the tooth profile at the actual meshing-in point.

The circumferential velocity v_1 and v_2 of the gear pair at the D point are decomposed into two components (v_{n1}, v_{s1}) and (v_{n2}, v_{s2}) along the instantaneous

Fig. 5 Geometric position of any point during initial out-of-line meshing-in

meshing line and its perpendicular line, respectively. The velocity of the two teeth profiles along the direction $(\vec{N}_1, \vec{N}_2)$ of the instantaneous meshing line at point D can be obtained from the geometric relation. The practice meshing process of meshing-in impact mechanism is: the driven gear engages in advance from point D and the meshing-in impact occurs. Then the driven gear scratches along the flank of driving gear until the gear mesh along the line of action.

2.2 Calculation model of initial meshing-in impact

In heavy duty and high line speed gear transmission system, the main effect of factors of load distribution coefficient and dynamic impact force are loaded transmission error. According to the mechanism of meshing-in impact, the main sources of the meshing-in impact are the load distribution coefficient and

Fig. 6 Schematic diagram of initial meshing-in impact instantaneous velocity

meshing impact. Moreover, load distribution coefficient is origin from manufacturing and pitch error, and then the meshing impact is mainly due to the assembling error and normal backlash. The effect of meshing impact is larger than that of uneven load distribution, so the minimum value of meshing impact is considered in the present work.

In addition, the modification of clockwise diagonal trimming and teeth profile may be considered. Generally, the load distribution coefficient and meshing impact are random variables, and follow a standard normal distribution. The effects of meshing impact value, load distribution coefficient and optimized modification on the transmission precision and load-carrying properties are associated. The contact teeth relative velocity difference where the actual meshing point D along teeth profile normal line direction, namely, meshing-in impact instantaneous velocity v_s ,

the relative velocity along common tangent direction are as follows:

$$v_s = v_{n1} - v_{n2} = \omega_1 r_{b1} (1 + 1/i) \left(1 - \frac{\cos(\lambda_1 + \gamma_1)}{\cos \alpha} \right) \quad (10)$$

$$\begin{cases} v_{n1} = \omega_1 \cos \angle DO_1 N_1 = r_{b1} \omega_1 \\ v_{n2} = \omega_2 \cos \angle DO_2 N_2 = \frac{r'_{b2} \omega_2}{i} \end{cases} \quad (11)$$

Here, v_{n1} and v_{n2} are the meshing-in impact real time velocity of driving gear (active gear) and driven gear (passive gear) respectively. ω_1 , ω_2 are the angle velocity of driving and driven gear respectively.

The meshing-in impact velocity and the impact force from the beginning to the end are always in the process of a pair of gears intermeshing alternately along the normal contact path.

The Load distribution coefficient, defined as ratio of tangential impulse and normal impulse, is obtained and approved with the presented reference. Considering the gear teeth elasticity and its own mass and inertia, the out-of-line meshing leads to the impact velocity, and the impact is inevitable at the meshing point D. The magnitude and variation law of the meshing impact force are closely related to the impact velocity, gear teeth meshing stiffness and load. The meshing-in impact transient dynamics model is shown in Fig. 7. The moment of inertia of two meshing gears is transformed into the induced mass of the unit teeth width on the instantaneous meshing line.

$$\begin{cases} m_{red1} = \frac{J_1}{r_{b1}^2} \\ m_{red2} = \frac{J_2}{r_{b2}^2} \end{cases} \quad (12)$$

$$\begin{cases} J_1 = \frac{\pi \rho b}{2} (r_{b1}^4 - r_h^4) \\ J_2 = \frac{\pi \rho b}{2} (r_{b2}^4 - r_h^4) \end{cases} \quad (13)$$

where J_1 is the moment of inertia of the driving gear, J_2 is the moment of inertia of the driven gear, m_{red1} is the induced mass of unit tooth width on the instantaneous meshing line of the driving gear, m'_{red2} is the induced mass of unit tooth width on the instantaneous meshing line of the driven gear, r_{b1} is the instantaneous base circle radius corresponding to the

Fig. 7 Dynamic model of initial meshing-in impact

instantaneous meshing line of the driving gear, r'_{b2} is the instantaneous base circle radius corresponding to the instantaneous meshing line of the driven gear, ρ is density, r_h is inner hole radius of gear hub.

The impact force is closely related to meshing-in impact velocity, impact duration, mesh stiffness and load. Then the impact kinetic energy is

$$E_k = \frac{1}{2} \frac{J_1 J_2}{b(J_1 r_{b2}^2 + J_2 r_{b1}^2)} v_s^2 \quad (14)$$

The maximum impact force is

$$F_s = v_s \sqrt{\frac{b J_1 J_2}{(J_1 r_{b2}^2 + J_2 r_{b1}^2) q_s}} \quad (15)$$

According to impact mechanics theory, the maximum deformation δ_s and maximum impact force F_s with relative to energy as

$$E_k = \frac{1}{2} \frac{J_1 J_2}{b(J_1 r_{b2}^2 + J_2 r_{b1}^2)} v_s^2 = \frac{1}{2 q_s} \delta_s^2 \quad (16)$$

Fig. 8 Heavy duty and high line speed transmission system gear pair meshing-in impact optimization

Here, E_k is the impact kinetic energy of meshing gear and δ_s is the maximum deformation, F_s is the maximum impact force. q_s is the comprehensive performance of the meshing-in impact meshing point D (not including Hertz contact); b is teeth surface width; r_i is the radius of internal cycle of gear, $i = 1, 2$; r_{b1} , r_{b2} are the base cycle radius of driving and driven gear respectively.

Throughout the course of the study, this article focuses on the minimum impact resistance value of teeth surface of initial intermeshing alternating. The calculation process of meshing-in impact includes determining initial meshing points, calculating of meshing-in velocity and impact force. The meshing-in impact optimization process is shown as Fig. 8.

Fig. 9 No-load teeth surface imprinting and geometric transmission errors

Fig. 10 The bar graph of meshing-in impact load of herringbone gear

A pair of normally engaged gear teeth, the driving gear modification is to eliminate the driving gear teeth flank and driven gear addendum advance into meshing arising from the meshing impact, when the modified gear teeth in the meshing point is no longer bear load, namely, this explains the meshing-in impact that caused by the early meshing interference has been eliminated.

Fig. 11 Load distribution curve of herringbone gear

3 Example verification and analysis

3.1 Optimization analysis of herringbone gear

The modification of gear is a principal method for improving performance meshing. The modification is not considered and the gear and pinion are installed right. Moreover, the deformations of shaft, bearing and gearbox are not included. In this section, No-load teeth surface imprinting and geometric transmission errors is shown in Fig. 9. The bar graph of meshing-in impact load of herringbone gear is shown in Fig. 10.

Fig. 12 Optimized clockwise diagonal modification curve and surface **a** Modification curve; **b** Modification surface

Table 1 Parameters of herringbone gear pairs

Gear geometric parameter	Pinion gear	Large gear
Normal modulus $m_n(mm)$	8	8
Tooth number	17	44
Tooth width (mm)	168	168
Shaft length (mm)	300	300
Shaft radius (mm)	45	55
pressure angle ($^\circ$)	20	20
Helix angle ($^\circ$)	24.45	24.45
Weight (kg)	17.8	101.2
Moment of inertia ($kg \cdot m^2$)	0.0157	0.586
Density (kg/m^3)	7.85×10^3	
Damping ratio coefficient (ξ)	0.10	

Table 2 Parameters of helical gear pairs

Gear geometric parameter	Pinion gear	Large gear
Normal modulus $m_n(mm)$	8	8
Tooth number	19	47
Tooth width (mm)	75	75
Shaft length (mm)	200	200
Shaft radius (mm)	45	55
pressure angle ($^\circ$)	20	20
Helix angle ($^\circ$)	9.92	9.92
Weight (kg)	7.8	40.2
Moment of inertia ($kg \cdot m^2$)	0.018	0.387
Density (kg/m^3)	7.85×10^3	
Damping ratio coefficient (ξ)	0.10	

Load distribution curve of herringbone gear is shown in Fig. 11. Optimized clockwise diagonal modification curve and surface is shown in Fig. 12. The herringbone gear parameters and material parameters used in the present paper are listed in Table 1.

It can be seen from the optimized results of clockwise diagonal modification method, as shown in Figs. 9, 10, 11 and 12 and Table 1.

- (1) The gear meshing impact combined meshing-in impact velocity model is proposed to analyze the teeth surface optimized modification. The gear meshing impact is synthesized by no-load motion error, load distribution coefficient and maximum impact force in the actual meshing

point D along the teeth profile normal line of action;

- (2) By optimizing modification, the desired results are obtained. That is, the minimum value of meshing-in impact is reduced to 97% of initial value without modification;
- (3) In the process of modification optimization analysis, no edge contact with engagement by gear teeth clockwise diagonal modification. It appears as the most interesting fraction because the minimum value of meshing-in impact can be significantly decreased without increasing the mean value.

Fig. 13 No-load teeth surface imprinting and geometric transmission errors

Fig. 14 Bar diagram of meshing-in impact load of helical gear

3.2 Optimization analysis of helical gear

In this section for the study of this paper, the clockwise diagonal method is also proposed to optimize teeth profile modification. The helical gear parameters and material parameters used in the present paper are listed in Table 2.

No-load teeth surface imprinting and geometric transmission errors are shown in Fig. 13. The bar diagram of meshing-in impact load of helical gear is shown in Fig. 14. Load distribution curve of helical gear is shown in Fig. 15. Optimized clockwise diagonal modification curve and surface is shown in Fig. 16.

Fig. 15 Load distribution curve of helical gear

The following contributions can be obtained in this section, as shown in Figs. 13, 14, 15 and 16 and Table 2.

1. For reducing meshing-in impact and raising working efficiency, all modifications are put in meshing gear pair. Large gear minion errors when no-load was deduced by changing the minimum value of meshing-in impact, and then obtaining the contact points trace of tooth surface of helical gear small wheel with optimized modification;
2. The tooth clockwise diagonal modification has important influence on tooth surface load distribution, which can improve the offset load caused by installation error and increase bearing the load capability of tooth;
3. The optimized modification goal is to minimize meshing-in impact value minimizing and uniformly distribute tooth surface meshing-in impact load, and the modified parameters can be obtained by applying the clockwise diagonal modification method;
4. After optimizing modification, a single tooth meshing area is generated on teeth surface, and meshing gear pair is no longer bear the load when meshing in and out, the minimum value of meshing-in impact is reduced by 93%.

3.3 Optimization analysis of spur gear

In this section, the method of gear teeth profile modification is proposed. No-load teeth surface

Fig. 16 Optimized clockwise diagonal modification curve and surface **a** Modification curve. **b** Modification surface

Fig. 17 No-load teeth surface imprinting and geometric transmission errors

Fig. 18 The bar graph of meshing-in impact load of spur gear

imprinting and geometric transmission errors are shown in Fig. 17. The bar graph of meshing-in impact load of spur gear is shown in Fig. 18. Load distribution curve of spur gear is shown in Fig. 19. Optimized clockwise diagonal modification curve and surface is shown in Fig. 20. The spur gear parameters and material parameters used in the present paper are listed in Table 3.

The following conclusions can be found, as shown in Figs. 17, 18, 19 and 20 and Table 3.

1. Meshing-in impact load after modification is obviously reduced compared with the modification before;

2. The results show that the minimum value of meshing-in impact after modification have decreased 10%, that is, the minimum value of meshing-in impact is reduced by 90%;
3. After optimizing modification, a single tooth meshing area is generated on the tooth surface, and the area has expanded more than that of non-modification.

4 Conclusions

In this study, an optimal analysis method of the impact with the minimum value of gear teeth meshing-in is proposed, the gear meshing impact combined

Fig. 19 Load distribution curve of spur gear

meshing-in impact velocity model is revealed to analyze the teeth surface optimized modification. The gear meshing impact is synthesized by no-load motion error, load distribution coefficient and maximum impact force in the actual meshing point D along the teeth profile normal line of action. The engagement gear pair with meshing-in impact load lead to modifications of the path of contact and transmission errors, which affect the marine ship power rear drive gears mesh under load.

1. Uniform gear contact surface pressure suppresses the thermoelastic deformation of teeth profile, reduces vibration and noise, and weakens initial meshing-in impact.

Table 3 Parameters of spur gear pairs

Gear Geometric Parameter	Pinion gear	Large gear
Modulus (<i>mm</i>)	6	6
Tooth number	19	48
Tooth width (<i>mm</i>)	75	75
Shaft length (<i>mm</i>)	200	200
Shaft radius (<i>mm</i>)	45	55
pressure angle (°)	20	20
Weight (<i>kg</i>)	7.5	38.6
Moment of inertia (<i>kg · m²</i>)	0.016	0.35
Density (<i>kg/m³</i>)	7.85×10^3	
Damping ratio coefficient (ξ)	0.10	

2. Optimal teeth surface modification can improve the effect of thermal elastic deformation of teeth surface on gear transmission performance, and improve teeth surface scratch resistance load capacity. Real-time lubrication conditions cannot be ignored when analyzing gear vibration and noise and describing instantaneous impact of meshing teeth surfaces.
3. Gear teeth thermal elastic deformation in initial meshing is the largest and instantaneous impact velocity reaches the peak value, and then impact force of meshing-in is the maximum.
4. After optimizing modification, the meshing-in impact load after modification is obviously reduced compared with the modification before. Moreover, a single tooth meshing area is

Fig. 20 Optimized tooth profile modification curve and surface **a** Profile modification curve; **b** Profile Modification surface

generated on the tooth surface, and the area has expanded more than that of non-modification for spur gear. The results show that the minimum values of meshing-in impact after modification have been reduced a lot.

Acknowledgement The authors gratefully acknowledge support from the Northeast Forestry University (NEFU) as well as the Heilongjiang Institute of Technology (HLJIT) and the Harbin University of Science and Technology (HUST). This work is supported by the Special Scientific Research Funds for Forest Non-profit Industry (Grant No. 201504508) and the Youth Science Fund of Heilongjiang Institute of Technology (Grant No. 2015QJ02) and the Fundamental Research Funds for the Central Universities (Grant No. 2572016CB15).

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