The dynamic analysis of load motion during the interaction of wind pressure

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Abstract This work presents the analysis of the load motion during the interaction of wind pressure. The load was treated as a rigid body, and the rope system model as a non-deformed. The influence of effective area of wind pressure on load motion was considered. The theoretical model of load motion was presented, which may be an universal approach for transporting machines equipped with a rope-lifting system. To define the orientation of the movable Cartesian coordinate system related to the load, Bryant angles were used. An algorithm and computational program were developed to allow for analysis of dynamic phenomena. The initial problem was solved with the use of the ode45 calculation procedure in the Matlab software on the basis of the Runge–Kutta 4th Order Method. The obtained results were verified with the experimental ones achieved in the wind tunnel and Tracker program. Numerical calculations using commercial software SolidWorks were also presented. In the experiment, the spatial motion of the load was analysed. Experimental tests were carried out for gust

of wind and constant temperature and humidity. In addition, the paper presents the application of the proposed method for a load carried by a rotary crane. After taking into account the control functions resulting from the nature of the work of any machine, the formulated model can be a full description of the carried load motion taking into account external forces.

Keywords Wind pressure · Load · Experimental analysis · Dynamics

List of symbols

m	Mass of load (kg)
ρ	Density (kg/m ³)
Ψ	Bryant angle (X axis)
ϑ	Bryant angle (Y axis)
Φ	Bryant angle (Z axis)
a	Length of the load (m)
b	Width of the load (m)
c	Height of the load (m)
g	Gravitational acceleration (m/s ²)
S	Effective area of wind pressure (m ²)
Γ	Load suspension point (rotary crane)
$\mathbf{G}$	Gravity force (N)
$\mathbf{H}$	Rotational momentum vector of the load (N m)
r, χ	Rope length (m)
V_w	Wind speed (m/s)
C_D	Coefficient of aerodynamic resistance
$\mathbf{a}_c$	Total translational acceleration vector (m/s ²)

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$\mathbf{F}_n$	Resultant vector of all acting forces (N)
$\mathbf{M}_c$	Momentum vector of all acting forces (N m)
$\mathbf{F}_r$	Total vector of the tension in the rope (N)
$\mathbf{F}_w$	Wind force vector (N)
ω_c	Angular velocity vector (rad/s)
ϵ_c	Angular acceleration vector (rad/s ²)
$\mathbf{J}_c$	Matrix of inertia momentum (kg m ²)

1 Introduction

The impact of external forces, for example, the wind pressure, is essential during the process of load transfer and positioning. Dynamic changes of deflections caused by these forces can trigger load positioning errors or, in extreme cases, machine instability. However, the impact of external factors, including wind pressure, is a phenomenon often neglected during an analysis of the work cycles of transport equipment. The machine instability caused by too strong load fluctuations caused by wind [1, 2], can be the result of such neglecting phenomenon.

In the literature, several works on the subject of strength analysis, load dynamics or trajectory optimization can be found. Research on transport machines primarily concerns issues related to rotary cranes [3–10], tower cranes [11, 12] and gantry cranes [13–20].

Works [3–5] concern modelling of rotary crane load dynamics, taking into account various transport processes and positioning. The load positioning process under the influence of kinematic forcing [3] and the impact of external forces on load deflections during crane working cycle [4] were analysed. The dynamics of the lifted load and mobile crane with consideration of the flexibility of support system on crane stability [5] was also investigated. Truck crane stability was analysed in [6], where the CAD/CAE program and the neural network system were used to determine the values of the vertical reactions of the base. CAD/CAE programs, which example can be SolidWorks, are as well used for strength and vibration analyses [7–9]. Article [7] presents the strength analysis of the telescopic boom of the mobile crane. The presented approach can provide guidelines for the optimal design of telescopic parts and other structures with local stresses in the contact zone. In [8], CAE

tools were used to determine the frequency of the loading crane. The obtained results were verified by experimental research. Work [9] presents a free vibration analysis of a mobile crane taking into account load configuration changes. Analysis of the vibrations of the crane is also depicted in [10], where a mathematical description with the flexibility of individual structure members was presented.

In the works related to tower cranes, the most common topics discussed are dynamic analysis [11, 12]. The dynamic reliability induced by wind loading was investigated in [11]. The resonance frequencies of the analysed crane were determined and the influence of wind on displacements of respective nodes was discussed. Whereas in work [12], an experimental analysis of load motion in the frequency domain was presented. A method of controlling transferred load oscillations was also introduced.

Gantry and overhead cranes are one of the most common devices used for transporting cargo [13]. The problem of load dynamics during transport and positioning was analysed in the works [14–16]. In [14], the vibration control of the gantry crane with a suspended load was presented. Different trolley kinematic configurations on load oscillations were analysed in [15]. The finite element method was used to determine the forces acting on the device due to the load transportation process. A theoretical model of a load carried by overhead crane using Lagrange equation of the second kind was shown in [16]. The simulation model, which was developed using Matlab software, allowed the trolley and payload trajectory to be determined. Due to the fact that overhead cranes are used in external conditions, several works analyse the impact of external forces (e.g. wind pressure) on the machine's working cycle [17–20]. A dynamical model of a gantry crane and its control system was presented in [17]. Issues related to the load operation and its positioning under the wind disturbances were analysed. Hence, the experimental tests were conducted in the motionless state. The problem of payload deflections and positioning of the load has been widely described in the works [18–20]. These articles used neural networks [18], output-based command shapers [19] and a control scheme with command smoother [20]. In addition, the verification process was carried out using experimental tests.

In the literature of the subject, one can find works that do not strictly relate to the load transport. However, they are a valuable complement to the topic of rigid body motion, which includes the load motion. The motion of rigid bodies as a motion of the mathematical and physical pendulum is shown in works [21–27]. The deformability of the rope system was included in the paper [21]. Whereas in the work [22], the motion of rigid bodies connected by non-deformable ropes is presented. The harmonical and kinematical excite dynamical system provided by spring pendulum is analysed in [23]. In addition, the proposed model enables the determination of resonances using the multi-scale (MS) method. In work [24], MS method is used to analyse a non-linear system with two degrees of freedom. Transport of energy between modes of vibration is also presented. The problem of spring pendulum nonlinear vibrations and resonances has also been extensively described in articles [25, 26]. In both of these works, the asymptotic method was used for analytical calculations. The spring motion equations were obtained using the 2nd type Lagrange's equations.

This paper presents the analysis of the load motion during its free motion under the interaction of wind pressure. The results obtained on the basis of analytical and numerical simulations have been compared and verified with the experimental results. The application of the proposed analytical method has been also presented on the example of a rotary crane working cycle. Part of this research was presented at the 15th International Conference Dynamical Systems Theory and Applications (DSTA2019) [28].

2 Research methods

Three research methods that allow for motion analysis of the load during the interaction of the wind pressure have been examined in this work: an analytical approach using the Matlab environment, numerical calculations in the SolidWorks program and experimental research in the low-speed wind tunnel.

The model, shown in Fig. 1, was developed on the basis of the following assumptions. The load was treated as a rigid body, attached at point O on a non-deformable rope with fixed length r . The load motion has been treated as the spatial motion. The direction of wind force ($\mathbf{F}_w$) was adopted from experimental tests

Fig. 1 The dynamic model used for an analytical approach

performed in the wind tunnel. Aerodynamic drag has been omitted in numerical simulations due to the negligible impact on analysed motion in application (rotary cranes). Due to the adopted assumptions, two coordinate systems have been introduced: $OXYZ$ —inertial system, O_1xyz —movable system connected with the load centre of mass. In addition, Bryant's angles (Ψ , ϑ , Φ) were introduced in an analytical approach to determine the position of the load [29].

2.1 Analytical approach

The general motion of the rigid body can be presented as a combination of translational motion relative to the origin of the global coordinate system (1) and rotational motion relative to the load's centre of mass (2) [17, 30–35]:

$$m\mathbf{a}_c = \mathbf{F}_n, \quad (1)$$

$$\frac{d}{dt}\mathbf{H} = \mathbf{M}_c, \quad (2)$$

where m is the mass of the transferred load, $\mathbf{a}_c$ is the total translational acceleration vector of the centre of mass of load in the inertial coordinate system, $\mathbf{M}_c$ is the momentum vector of all forces acting on the system with respect to the centre of mass, $\mathbf{H}$ is

the rotational momentum vector of the load with regard to its centre of mass, $\mathbf{F}_n$ is the resultant force vector of all forces acting on the system.

The rotational momentum vector of the load $\mathbf{H}$ with respect to the continuous mass density is given by [32]:

$$\mathbf{H} = \int \mathbf{r} \times \mathbf{v} dm. \quad (3)$$

where $\mathbf{r}$ is a position vector from reference point with respect to dm element and $\mathbf{v}$ is a velocity vector.

Omitting the air resistance, the resultant force vector $\mathbf{F}_n$ may be shown as [36]:

$$\mathbf{F}_n = \mathbf{F}_r + \mathbf{G} + \mathbf{F}_w, \quad (4)$$

where $\mathbf{F}_r$ is the total vector of the tension in the rope, $\mathbf{G}$ is the load's gravity force, and $\mathbf{F}_w$ is the wind power.

Taking into account the angular velocity vector:

$$\omega_c = \omega_1 \mathbf{e}_1 + \omega_2 \mathbf{e}_2 + \omega_3 \mathbf{e}_3, \quad (5)$$

matrix of inertia momentum $\mathbf{J}_c$ and the derivative of the absolute angular momentum of the load in the inertial system:

$$\frac{d}{dt} \mathbf{H} = \frac{d}{dt} \mathbf{H} + \omega_c \times \mathbf{H}, \quad (6)$$

the rotational motion (2) can be represented as [32]:

$$M_1 = J_1 \epsilon_1 + (J_3 - J_2) \omega_2 \omega_3, \quad (7)$$

$$M_2 = J_2 \epsilon_2 + (J_1 - J_3) \omega_3 \omega_1, \quad (8)$$

$$M_3 = J_3 \epsilon_3 + (J_2 - J_1) \omega_1 \omega_2, \quad (9)$$

where the O_{1xyz} is the coordinate system with the principal axes of inertia and $\frac{d}{dt}$ is a derivative in a movable (relative) coordinate system.

Based on works [3, 5], vector equation (1) may be presented as:

$$\ddot{X} = -a_{O_x} + \frac{F_{w_x}}{m} + \frac{X}{r^2} \Upsilon, \quad (10)$$

$$\ddot{Y} = -a_{O_y} + \frac{F_{w_y}}{m} + \frac{Y}{r^2} \Upsilon, \quad (11)$$

$$\ddot{Z} = a_{O_z} + g + \frac{F_{w_z}}{m} + \frac{Z}{r^2} \Upsilon, \quad (12)$$

where

$$\begin{aligned} \Upsilon = r\ddot{r} + \dot{r}^2 + Xa_{O_x} + Ya_{O_y} - Z(a_{O_z} + g) + \\ - (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2). \end{aligned} \quad (13)$$

The impact of wind pressure acting on the load can be represented using a formula (14) for aerodynamic resistance [11, 37]:

$$|\mathbf{F}_w| = \frac{1}{2} \rho S V_w^2 C_D, \quad (14)$$

where ρ is the air density, S is the effective area of wind pressure, V_w is the wind speed, C_d is the dimensionless coefficient of aerodynamic resistance.

The parameter of effective area wind loading has been extensively described in the works [36, 38]. The final formula for calculating the effective area parameter S can be represented in the form:

$$S = ab \cos(\gamma_1) + ac \cos(\gamma_2) + bc \cos(\gamma_3), \quad (15)$$

where a, b, c are the dimensions of the load and $\gamma_1, \gamma_2, \gamma_3$ are angles between normal vectors and surface on which wind pressure has influence.

Considering the above relationships, a system of six ordinary differential equations of the second order was obtained [5, 34]:

$$\mathbf{D} \ddot{\mathbf{Q}} = \mathbf{E}, \quad (16)$$

where $\mathbf{E}$ is a vector of generalized angular velocities of the rigid body and accelerations of the rope suspension point, $\mathbf{D}$ is a matrix dependent on Bryant angles, geometric parameters of the body and its density and $\ddot{\mathbf{Q}}$ is a vector of unknown Bryant angles accelerations and inertial system coordinates which can be presented in the form:

$$\ddot{\mathbf{Q}} = \begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \\ \ddot{\Psi} \\ \ddot{\Theta} \\ \ddot{\Phi} \end{bmatrix} \quad (17)$$

To solve the initial problem of load motion, the system of equations (16) was reduced to the system of twelve differential equations of the first order [33, 39]. All symbolic operations necessary to formulate the problem of initial load motion were performed using the

Wolfram Mathematica package. The initial problem has been solved using ode45 procedure available in Matlab/Simulink program [3, 4, 36, 39].

2.2 Experimental research

The load was suspended using a cable, which properties correspond to a non-deformable rope [4]. The test stand consisted of a wind tunnel with dimensions $0.6 \text{ m} \times 0.6 \text{ m} \times 3.0 \text{ m}$, fan with radius 0.35 m , and anemometer UT362, which made it possible to determine the value of wind force as a function of time. The devices that allowed to record the experimental research included: Nikon D610 camera with Nikkor 24–70 lens and Nikon D90 with Nikkor 17–55 lens with a minimum aperture value both lens of 2.8° (Figs. 2, 3).

During the experiment, wind pressure acts only along X axis. The spatial motion of the load was considered. Tests were carried out for various inflow velocities (from 0 to 4.5 m/s) and constant and controlled temperature and humidity [4, 40]. The video analysis was performed in the Tracker program (Fig. 4), which allows tracking the coordinates of vertices and the mass centre of the load.

Fig. 2 Experimental test stand (top view)

Fig. 3 Experimental test stand (rear view)

Fig. 4 Video analysis using the tracker program

2.3 Numerical simulations using the SolidWorks package

The last part of this work concerns the study of the load motion using the Motion module of SolidWorks package. Motion simulation provides complete, quantitative information about the kinematics (including position, velocity, and acceleration) and the dynamics (including joint reactions, inertial forces, and power requirements) of all components of a moving mechanism. Often of great additional importance, the results of motion simulation can be obtained virtually at no additional time expense, because everything needed to perform motion simulation has been defined in the CAD assembly model already, and just needs to be transferred to the Motion module [41].

The analysed model is an assembly consisting of three parts: a rigid body (Fig. 5:1), a non-deformable cable (Fig. 5:2) and the base on which the rope was suspended. Due to limitations in the SolidWorks

Fig. 5 Analysed model in SolidWorks motion (1-rigid body, 2-cable, 3-wind acting plane, 4-wind power)

program, a constant surface area (Fig. 5:3) on which wind power (Fig. 5) has effect was adopted.

3 Sample results of performed investigations

In this work, the impact of gusts on the load behaviour in stationary state was investigated. The obtained simulation results were compared with experimental results, which were conducted in a low-speed wind tunnel. The results are presented in the form of load's centre of mass coordinates changes as a function of time (Figs. 6, 7, 8, 9). The parameters of the calculation model are presented in Table 1. Load model used during experimental tests was made using 3D printing method. The coefficient of aerodynamic drag was chosen according to the literature data [42–44].

Fig. 6 Time course of the X coordinate of load's centre of mass

Fig. 7 Time course of the Y coordinate of load's centre of mass

Fig. 8 Time course of the X coordinate of one selected vertex of load

Fig. 9 Time course of the Y coordinate of one selected vertex of load

Table 1 Parameters of the calculation model

Load mass (kg)	Rope length (m)	Load dimensions (m)	Aerodynamic resistance coefficient
0.133	0.28	$0.1 \times 0.03 \times 0.07$	1.05

The analysed load had a cuboid shape and a mass equal to 0.133 kg. The wind speed increased over time from 0 m/s until it reached 5.3 m/s (in 12 s). The swing angle relative to the X axis was three degrees.

Coordinate changes caused by wind force of load's centre of mass (Figs. 6, 7) and one selected vertex of load (Figs. 8, 9) are presented. The percentage of variation of X load's mass centre coordinate as a function of time is also shown in Fig. 10. The differences between the numerical calculations and the results of the experiment are caused among others by the omission of the air damping in the numerical studies.

Based on the obtained results, a similar trend of X coordinate variations can be seen for all research methods. The larger differences were observed for Y coordinate, when wind speed exceeds 4 m/s. At this moment, the rotational motion of the load was noticed. The phenomenon of spatial motion better reflected the analytical approach than the numerical simulation performed in SolidWorks software. It is caused by taking into account the variable surface area induced by wind in the analytical approach. The percentage differences of variation (Fig. 10) do not give an explicit determination as to which method gives results closer to the experiment. However, comparing displacements of the load, one can assume that the analytical approach better presents the three-dimensional motion of the load.

4 Application of the analytical method for load motion carried by rotary crane

As an application of the developed analytical method, the load motion during the working cycle of the mobile crane was presented. Taking into account kinematic excitations resulting from the nature of work of this type of lifting equipment (Fig. 11), formulas (10, 11 and 12) can be presented as follows:

Fig. 10 Percentage of variation of the X coordinate of load's centre of mass

$$\ddot{\kappa} = -a_{\Gamma_x} + \frac{F_{w_x}}{m} + \frac{\kappa}{\chi^2} \Xi, \quad (18)$$

$$\ddot{\tau} = -a_{\Gamma_y} + \frac{F_{w_y}}{m} + \frac{\tau}{\chi^2} \Xi, \quad (19)$$

$$\ddot{v} = a_{\Gamma_z} + g + \frac{F_{w_z}}{m} + \frac{v}{\chi^2} \Xi, \quad (20)$$

where $a_{\Gamma_x}, a_{\Gamma_y}, a_{\Gamma_z}$ are components of the crane's boom tip acceleration κ, τ, v are generalized boom tip coordinates, χ is a length of the rope and Ξ takes the form:

$$\Xi = \chi \ddot{\chi} + \dot{\chi}^2 + \kappa a_{\Gamma_x} + \tau a_{\Gamma_y} - v(a_{\Gamma_z} + g) - (\dot{\kappa}^2 + \dot{\tau}^2 + \dot{v}^2). \quad (21)$$

The kinematic part of the load analysis carried by rotary crane concerned the determination of motion parameters of the rotary crane's boom end (point Γ) which was further described in [5, 45]. The main system of the mobile crane consists of a two-member telescopic jib and a pulley system. For the purposes of this work, the following assumptions were made [5, 36]:

- Crane main elements were treated as a non-deformable bodies;
- The rotary crane was placed on a non-deformable base;
- Load was treated as a rigid body attached at point Γ on a stable, static rope with variable length χ ;
- Wind pressure was related only to the dynamic part (the wind impact on crane main system was omitted);
- Wind pressure direction was adopted as constant (acting along the X direction of the global coordinate system);
- Wind speed simulating wind gusts (Fig. 12) was determined on the basis of real data [46].

The following rectangular coordinate systems have been introduced in relation to the above assumptions [36]:

- $O_1x_1y_1z_1$ —inertial system related to the rotary crane chassis;
- $O_2x_2y_2z_2$ —movable system related to the jib support;
- $\Gamma\kappa\tau v$ —movable system related to the jib end (Γ);
- $OXYZ$ —movable system related to the load centre of mass.

Fig. 11 The model of the analysed system (rotary crane and transferred load)

Fig. 12 Time course of wind speed

In addition, the following coordinates have been identified: l —jib length, β —angle of jib inclination, χ —rope length, α —crane rotation angle, Z_0 and R_0 —coordinates of jib rotation point (O_2). Motion of the jib end (kinematic part) is caused due to changes of generalized coordinates (control function): l , α , β and χ (Fig. 13).

The presented numerical simulation concerned the initial problem where the load was hanging freely. The control functions (Fig. 14) were introduced in the form of trapezoidal impulses and resulted from:

- Platform rotational motion with a telescopic boom (α)—the motion starts at 0 s and takes 10 s, the maximum velocity equals 0.2 rad/s (Fig. 14a),

Fig. 13 Coupled kinematic-dynamic model of load motion

- Working stroke control of the system of two hydraulic actuators responsible for changing the overall length of the jib (l)—start of the motion in 10 s and lasts for 10 s, the maximum velocity equals 0.1 m/s (Fig. 14b),
- Working stroke control of the hydraulic actuator responsible for changing the boom inclination angle (β)—the forcing takes 10 s and ends at 30 s, the maximum velocity equals 0.025 rad/s (Fig. 14c),
- Winch operation control (χ)—rope extension starts at 30 s and lasts 10 s, the maximum velocity equals 0.15 m/s (Fig. 14d).

In the control functions, the start-up and braking motion lasted half a second. The total time of the simulations included 40-s forced motion of the analysed system. The parameters of the rotary crane calculation model are presented in Table 2. Initial values of generalized coordinates were: $\alpha = 90^\circ$, $\beta = 20^\circ$, $l = 8\text{ m}$, $\chi = 3\text{ m}$, $Z_0 = 1.55$ and $R_0 = 1.25\text{ m}$.

The response of the system to the assumed control functions (Fig. 14) was obtained in the form of trajectories projections of the load's mass centre in the rotation (XY) and lifting (XZ and YZ) planes (Figs. 15, 16, 17). The performed investigations allowed to determine the impact of wind pressure on the trajectory of the load carried by a rotary crane.

Fig. 14 Control functions: **a** rotation of platform, **b** boom length, **c** boom inclination angle, **d** winch operation

Table 2 Parameters of the rotary crane calculation model

Load mass (kg)	Rope length (m)	Load dimensions (m)	Aerodynamic resistance coefficient
1200	3	2.21×0.85	1.05

On the basis of the obtained results of the load motion, it can be seen that the wind has an impact on the trajectory of the carried load and should not be neglected when analysing transport equipment exposed to such interactions. The highest deflections of the load caused by wind were noticed in the rotation plane (XY) along the axis of wind direction. For the first control function (rotation of crane platform) there were no differences in the load trajectory. In this time interval, the wind speed did not exceed 5 m/s. These

Fig. 15 Trajectory projections of the load's centre of mass in the rotation plane (XY)

Fig. 16 Trajectory projections of the load's centre of mass in one of the lifting planes (XZ)

results confirm the Polish standards of cranes safety and operation. When wind speed does not exceed 10 m/s, work can be done normally. When wind speed passes 17 m/s, the crane should be anchored and a load should be reduced by half. The limit value of wind speed is 20 m/s—work above this value is disallowed. The largest load deflections due to the

Fig. 17 Trajectory projections of the load's centre of mass in one of the lifting planes (YZ)

wind pressure were noticed for two control functions: boom length and inclination angles changes. During these functions, the wind was blowing at the highest speed, where gusts of wind reached 18 m/s. Deflection of the payload induced by such a high wind speed might even lead to stability loss of a rotary crane.

5 Conclusions

The paper presents an analysis of load motion with the interaction of wind pressure. Load motion was analysed through three approaches: analytical using Matlab software, numerical with the use of SolidWorks packages and experimental test. The mathematical model developed in analytical approach, after taking into account kinematic forcing resulting from the nature of machine work, can be a full description of the motion of the load carried by any transporting device. The load was treated as a rigid body suspended on a non-deformable rope. Combination of translational and rotational motion was used to present the load motion. Bryant angles were used to determine the load trajectories and deflections. Wind pressure acting on the load was included as an aerodynamic drag formula. Initial problem of load motion was solved using ode45 procedure in Matlab software. The experimental research was conducted in a low-speed

wind tunnel. Load motion has been limited in such a way as to remind the pendulum motion. The obtained experimental data made it possible, with the use of Tracker program, to present the trajectory of the mass centre and vertices of the analysed body. During the experimental test, the irregular behaviour of the load was noticed when the wind speed exceeds 4 m/s. The results obtained numerically present a similar trend of coordinates changes with the results obtained in experimental studies. The disadvantage of numerical simulations using the SolidWorks program is that the variable effective surface area is not included. Based on the obtained results, the correctness of the analytical approach can be stated. Numerical tests performed in the SolidWorks software do not show the characteristic spatial motion of the rigid body, especially the rotational motion towards Y direction. Application of a given analytical research approach was also presented. The results confirmed the recommendations regarding crane safety. When the wind speed does not exceed 10 m/s, cargo deflections are negligible. However, if the wind speed is higher, deflections also increase, which can even cause the instability of the machine. The mathematical model presented in this work will be further developed by taking into account air resistance or rope deformability.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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