Correction to: Fermionic Formulas for Eigenfunctions of the Difference Toda Hamiltonian

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In page 63, line 3 should read:

$$(u \theta_{\beta}^{\lambda}, \bar{\theta}_{\beta}^{\lambda}) = \sum_{n_1, \dots, n_l} \prod_{i=1}^l \left(\frac{(-v_i)^{-n_i} (1 - v_i^2)^{2n_i}}{(v_i^2; v_i^2)_{n_i}} \right) v^{(\gamma, \gamma)/2 + (\gamma, \lambda + \rho - \beta)} \times$$

The original article can be found online at <https://doi.org/10.1007/s11005-009-0300-0>.

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In page 63, the first paragraph in Section 3.6 “As shown in . . . ” should read as follows.

“In [7,19], quantum difference Toda Hamiltonians have been derived from Whittaker functions on quantum groups. For example, when $\mathfrak{g} = \mathfrak{sl}_{l+1}$, the simplest Hamiltonian is given by (see [7], (6.5))

$$H_{Toda} = \sum_{i=1}^{l+1} D_i - (q - q^{-1})^2 \sum_{i=1}^l y_i^{-1} y_{i+1} D_i,$$

where D_i denotes the shift operator $(D_i f)(y_1, \dots, y_{l+1}) = f(y_1, \dots, qy_i, \dots, y_{l+1})$. In this subsection, we show that the generating function

$$F(q, z, y) = \sum_{\beta} (q(q - q^{-1})^2)^{|\beta|} \prod_{i=1}^{l+1} y_i^{(\lambda - \beta, \epsilon_i) - i} J_{\beta}^{\lambda}$$

is an eigenfunction of H_{Toda} with eigenvalue $\varepsilon = \sum_{i=1}^{l+1} q^{(\lambda, \epsilon_i) - i}$. Here $\alpha_i = \epsilon_i - \epsilon_{i+1}$ and $|\beta| = \sum_{i=1}^l n_i$ if $\beta = \sum_{i=1}^l n_i \alpha_i$. More specifically, we show this in the form of an equivalent set of recurrence relations for J_{β}^{λ} ; see formulas below Proposition 3.8.

The argument to derive these relations is similar to the one for Theorem 3.1. In place of the Drinfeld Casimir element, we use other central elements including the quadratic Casimir element.”

In page 64, lines 2–3, “In terms of orthonormal vectors ϵ_i ,” should read “In terms of orthogonal vectors ϵ_i ,”

In page 64, line 7, insert “We have $(\epsilon_i, \epsilon_i) = 2$ for B_l and $(\epsilon_i, \epsilon_i) = 1$ in the other cases.”

In page 64, Proposition 3.8 should read:

For algebras of type A_l and C_l , we have the recursion relation

$$\left(\operatorname{tr}_V q^{\lambda + \rho - \beta} - \operatorname{tr}_V q^{\lambda + \rho} \right) J_{\beta}^{\lambda} = \sum_{i=1}^l v^{-d_i} \operatorname{tr}_V \left(q^{\lambda + \rho - \beta} v^{\alpha_i} E_i F_i \right) J_{\beta - \alpha_i}^{\lambda}.$$

For algebras of type B_l and D_l , the same recursion relation holds wherein the right hand side has an additional term

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$$\begin{aligned}
& -\frac{1-v^2}{1+v^2} \operatorname{tr}_V \left(q^{\lambda+\rho-\beta} v^{2\alpha_l} E_l^2 F_l^2 \right) J_{\beta-2\alpha_l}^\lambda \quad \text{for } B_l, \\
& -v^{-2} \operatorname{tr}_V \left(q^{\lambda+\rho-\beta} v^{\alpha_{l-1}+\alpha_l} E_{l-1} E_l F_{l-1} F_l \right) J_{\beta-\alpha_{l-1}-\alpha_l}^\lambda \quad \text{for } D_l,
\end{aligned}$$

respectively.

In page 64, line 4 from below should read:

$$(C_V^{(-1)})_{\theta_\beta^\lambda, \bar{\theta}_\beta^\lambda} = \sum_{n_1, \dots, n_l \geq 0} \prod_{i=1}^l \frac{v_i^{2n_i^2-4n_i} (1-v_i^2)^{4n_i}}{(v_i^2; v_i^2)_{n_i}^2} v^{-(\gamma, \lambda-\beta+\gamma)}$$

In page 65, lines 1–3, in the proof of Proposition 3.8, the last paragraph should read

When π_V is the vector representation, a simple check shows that the trace can be non-zero only when $\sum_{i=1}^l n_i \leq 1$; the only exceptions occur for types B_l and D_l , where terms with $n_l = 2$ and $n_{l-1} = n_l = 1$ also contribute, respectively.

In page 65, lines 6–8, formula for B_l should read

$$\begin{aligned}
& \sum_{i=1}^l \left(q^{(\lambda, \epsilon_i)+2l-2i+1} (q^{-(\beta, \epsilon_i)} - 1) + q^{-(\lambda, \epsilon_i)-2l+2i-1} (q^{(\beta, \epsilon_i)} - 1) \right) J_\beta^\lambda \\
& = \sum_{i=1}^{l-1} \left(q^{(\lambda-\beta, \epsilon_i)+2l-2i+1} + q^{-(\lambda-\beta, \epsilon_{i+1})-2l+2i+1} \right) J_{\beta-\alpha_i}^\lambda \\
& \quad + (1+q) \left(q^{(\lambda-\beta, \epsilon_l)+1} + q^{-1} \right) J_{\beta-\alpha_l}^\lambda - q^{(\lambda-\beta, \epsilon_l)+2} J_{\beta-2\alpha_l}^\lambda \quad \text{for } B_l,
\end{aligned}$$

In page 76, in Acknowledgements, the following sentence is added at the end:

“The authors thank A. Tsybaliuk for pointing out that a term was missing in the Toda Hamiltonian of type B_l . ”