

MIKLÓS FARKAS OBITUARY



Miklós Farkas was born in Budapest in 1932.

He graduated with distinction from the Budapest University of Science (Eötvös Loránd Tudományegyetem, ELTE) as a member of the first, non-teacher mathematician class.

He earned his PhD in differential geometry. He defended his DSc thesis *On Periodic Perturbations of Autonomous Systems* in differential equations in 1973.

Farkas taught at the Budapest University of Technology (Budapesti Műszaki Egyetem, BME) for 50 years. He was the head of the Department of Mathematics at the Faculty of Mechanical Engineering for almost 20 years and then he served as emeritus professor at the Department of Differential Equations of the Mathematical Institute—the successor of the former department—until his death.

In the early 1960s a paradigm shift took place in the field of differential equations and geometric-qualitative theory began to develop more intensively. With accurate timing Miklós Farkas turned his research efforts in this direction, and became a pioneer in Hungary of this subject. He was the first mathematician in Hungary to teach structural stability, bifurcations and catastrophe theory and also the first to publish these type of results in well-known international journals. He also wrote papers on applications of differential equations in economics and biology. In his later years he spent most of his time on mathematical biology and wrote the book *Dynamical Models in Biology* (Academic Press, New York, 2001). His last course at the university, in the autumn semester of 2006, was on the same topic.

As Department Head Farkas educated generations in mathematics and pursued that professional mathematics be more recognized at the Budapest Univer-

sity of Technology. At university and on national committees as well as in educational journals he took every possible opportunity to make applied mathematics acknowledged. Because of this he sometimes had even personal conflicts with mathematicians in their ivory towers. He expressed the opinion that the best of pure mathematics is not superior to the best of applied mathematics; all that matters is quality—albeit the definition of this concept is slightly different in pure and in applied mathematics. In 1974–75, he created a new engineer-mathematician class at the Faculty of Mechanical Engineering, which greatly improved the collaboration between engineers and mathematicians. This initiative took the form of a regular five-year-long class at the university and became very popular among the best students.

Farkas served as editor of four conference proceedings as well as of the Hungarian Mathematical Encyclopedia; the Encyclopedia had an important role in maintaining and developing Hungarian mathematical culture.

He published 76 research articles and 15 books and university lecture notes, including his nearly 600-page-long opus magnum *Periodic Motions* (Applied Mathematical Sciences No. 104, Springer, Berlin, 1994) in the most prestigious series of research monographs of applied mathematics.

We will keep Miklós Farkas in our memories in esteem and honor.

His colleagues and students

The editors of this issue believe that the most appropriate way here to commemorate Miklós Farkas is to give a brief summary of two distinguished discoveries representing his various research activities.

Controllably periodic perturbations [C, Section 6.2]

In his DSc Thesis and the accompanying papers [8]–[12], Farkas considers what he calls controllable periodic perturbations

$$\dot{x} = f(x) + \mu g(t/T, x, \mu, T) \quad (1)$$

of the autonomous equation

$$\dot{x} = f(x). \quad (2)$$

The functions $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $g: \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}^n$ are of class C^1 , so that equation (1) has a unique solution $x = \Phi(\cdot, t_0, x_0, \mu, T)$ satisfying the initial condition $x(t_0) = x_0$ for all $t_0 \in \mathbb{R}$, $x_0 \in \mathbb{R}^n$, $\mu \in \mathbb{R}$ and $T > 0$. It is also assumed that (2) has a nontrivial periodic solution p of period T_0 , 1 is a simple characteristic multiplier of the variational equation $\dot{y} = f'_x(p(t))y$, and that

$$g(s+1, x, \mu, T) \equiv g(s, x, \mu, T), \quad \text{whenever } s \in \mathbb{R}, x \in \mathbb{R}^n, \mu \in \mathbb{R}, T > 0.$$

It is a natural task to look for T_μ -periodic solutions p_μ of (1) satisfying $\mu \approx 0$, $T_\mu \approx T_0$ and, for some $\tilde{t} \in \mathbb{R}$ and $t \in \mathbb{R}$ with $\tilde{t} \approx t$, $p_\mu(\tilde{t}) \approx p(t)$. The main result of Farkas is that the above class of periodic solutions can be represented as the two-parameter family of solutions to the equation

$$z(T, t_0, h, \mu) = 0$$

where the function $z: \mathbb{R} \times \mathbb{R} \times \Sigma \times \mathbb{R} \rightarrow \mathbb{R}^n$ is defined by

$$z(T, t_0, h, \mu) = \Phi(t_0 + T, t_0, p(0) + h, \mu, T) - p(0) - h.$$

Here $\Sigma = \{h \in \mathbb{R}^n \mid \langle h, \dot{p}(0) \rangle = 0\}$ is a linear subspace of \mathbb{R}^n of codimension one and thus $\{p(0) + h \mid h \in \Sigma\} \subset \mathbb{R}^n$ is a Poincaré section. Note that

$$z(T_0, 0, 0, 0) = \Phi(T_0, 0, p(0), 0, T_0) - p(0) = p(T_0) - p(0) = 0.$$

The crucial observation is that the Jacobian matrix $\frac{\partial z}{\partial(T, h)}$ evaluated at the point $(T_0, 0, 0, 0)$, as a linear map of $\mathbb{R} \times \Sigma$ to \mathbb{R}^n , is invertible. Hence, in the vicinity of $(T_0, 0, 0, 0)$, the implicit function theorem applies, $T = T(t_0, \mu)$ with $T(0, 0) = T_0$, $h = h(t_0, \mu)$ with $h(0, 0) = 0$, and $\Phi(\cdot, t_0, p(0) + h(t_0, \mu), \mu, T(t_0, \mu))$ defines the two-parameter family of periodic solutions required. Stability questions of the emerging periodic orbits, the admissible range of parameters $t_0 \in (-t_0^*, t_0^*)$, $\mu \in (-\mu^*, \mu^*)$, and various examples are also discussed. The approach of controllable periodic perturbations provides a comfortable way of investigating periodic orbits of the equation $\dot{x} = f(x) + \mu g(t/T_0, x, \mu, T_0)$, too.

The zip bifurcation [C, Section 7.4]

The second finding of Farkas [33] we report is a new type of bifurcation phenomenon he discovered when analysing the one pray, two competitive predators system

$$\begin{aligned}\dot{S} &= \gamma S \frac{K - S}{K} - S \frac{m_1 x_1}{a_1 + S} - S \frac{m_2 x_2}{a_2 + S}, \\ \dot{x}_1 &= x_1 \frac{\beta_1(S - \lambda)}{a_1 + S}, \quad \dot{x}_2 = x_2 \frac{\beta_2(S - \lambda)}{a_2 + S}.\end{aligned}\tag{3}$$

In the nonnegative octant \mathbb{R}_+^3 the equilibria of (3) are $(0, 0, 0)$, $(K, 0, 0)$, and the points on the straight line segment

$$\mathcal{L}_K = \left\{ (\lambda, \xi_1, \xi_2) \in \mathbb{R}_+^3 \mid \frac{m_1}{a_1 + \lambda} \xi_1 + \frac{m_2}{a_2 + \lambda} \xi_2 = \gamma \frac{K - \lambda}{K} \right\}.$$

Though he could not settle the most general questions of the geometry of the phase portrait in terms of the seven parameters, he gave a detailed bifurcation analysis of the equilibria above.

In order to describe what he calls zip bifurcation we take, for simplicity of the exposition,

$$\lambda = 1, \quad a_1 = 2, \quad m_1 = 3, \quad a_2 = 1, \quad m_2 = 2, \quad \gamma = 1$$

and, for $K \in (3, 4)$, set

$$P_K = \left(1, 0, 1 - \frac{1}{K} \right), \quad M_K = \left(1, 3 - \frac{9}{K}, \frac{8}{K} - 2 \right), \quad Q_K = \left(1, 1 - \frac{1}{K}, 0 \right).$$

It is readily seen that \mathcal{L}_K simplifies to the straight line segment in \mathbb{R}_+^3 connecting the points P_K and Q_K . Note that $M_K \in \mathcal{L}_K$. The crucial observation is that the equilibria on \mathcal{L}_K between P_K and M_K are unstable, between M_K and Q_K are stable. As K is increased from 3 up to 4, the point M_K moves along \mathcal{L}_K from P_K to Q_K so that the points left behind become unstable. Note that as the parameter K is varied, the line segment \mathcal{L}_K itself undergoes a parallel displacement. However, this has no bearing on the qualitative picture.

Miklós Farkas' scientific publications

Books

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- [C] *Periodic Motions*, Springer, Berlin, 1994, pp. 577.
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