



Counterpart Theory and Actuality

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Abstract

Lewis (*The Journal of Philosophy*, 65(5), 113–126, 1968) attempts to provide an account of modal talk in terms of the resources of *counterpart theory*, a first-order theory that eschews transworld identity. First, a regimentation of natural language modal claims into sentences of a formal first-order modal language L is assumed. Second, a translation scheme from L -sentences to sentences of the language of the theory is provided. According to Hazen (*The Journal of Philosophy*, 76(6), 319–338, 1979) and Fara & Williamson (*Mind*, 114(453), 1–30, 2005), the account cannot handle certain natural language modal claims involving a notion of *actuality*. The challenge has two parts. First, in order to handle such claims, the initial formal modal language that natural language modal claims are regimented into must extend L with something like an actuality operator. Second, certain ways that Lewis' translation scheme for L might be extended to accommodate an actuality operator are unacceptable. Meyer (*Mind*, 122(485), 27–42, 2013) attempts to defend Lewis' approach. First, Meyer holds that in order to handle such claims, the formal modal language L^* that we initially regiment our natural language claims into need not contain an actuality operator. Instead, we can make do with other resources. Next, Meyer provides an alternative translation scheme from L^* -sentences to sentences of an enriched language of counterpart theory. Unfortunately, Meyer's approach fails to provide an appropriate counterpart theoretic account of natural language modal claims. In this paper, I demonstrate that failure.

Keywords Modal logic · Counterpart theory · Translation · Actuality

1 Counterpart Theory

According to David Lewis, we should understand our modal talk in terms of the resources of his *counterpart theory*, a first-order theory that eschews transworld identity.

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The language of counterpart theory (CT) is a standard first-order language L^{CT} (with identity) whose signature includes exactly: (i) for every $n \geq 1$, denumerably-many n -ary predicate constants F_1^n, F_2^n, \dots ; (ii) two unary predicate constants A and W ; (iii) two binary predicate constants I and C ; and (iv) the individual constant $@$.¹

The additional four predicate constants are to be read as follows:

- Wx x is a world
 Ixy x is in y
 Ax x is actual
 Cxy x is a counterpart of y

The individual constant $@$ is employed to denote the actual world.

The theory CT contains exactly the following L^{CT} -sentences, which impose conditions on objects, worlds, and the counterpart relation:

- (P1) $\forall v_1 \forall v_2 (Iv_1 v_2 \rightarrow Wv_2)$
 Objects are only in worlds.
- (P2) $\forall v_1 \forall v_2 \forall v_3 ((Iv_1 v_2 \wedge Iv_1 v_3) \rightarrow v_2 = v_3)$
 No object is in more than one world.
- (P3) $\forall v_1 \forall v_2 (Cv_1 v_2 \rightarrow \exists v_3 Iv_1 v_3)$
 Any object that is a counterpart is in a world.
- (P4) $\forall v_1 \forall v_2 (Cv_1 v_2 \rightarrow \exists v_3 Iv_2 v_3)$
 Any object that has a counterpart is in a world.
- (P5) $\forall v_1 \forall v_2 \forall v_3 (((Iv_1 v_2 \wedge Iv_3 v_2) \wedge Cv_1 v_3) \rightarrow v_1 = v_3)$
 An object is a counterpart of no object in its world other than itself.
- (P6) $\forall v_1 \forall v_2 (Iv_1 v_2 \rightarrow Cv_1 v_1)$
 Any object in a world is a counterpart of itself.
- (P7) $\exists v_1 (Wv_1 \wedge \forall v_2 (Iv_2 v_1 \leftrightarrow Av_2))$
 There is a world containing exactly all actual objects.
- (P8) $\exists v_1 Av_1$
 Some object is actual.
- (A) $\forall v_1 (Wv_1 \vee \exists v_2 Iv_1 v_2)$
 Each object is a world or is in some world.
- (B) $\forall v_1 (Wv_1 \rightarrow Iv_1 v_1)$
 Each world is in itself.
- (C) $\forall v_1 (\forall v_2 (Iv_2 v_1 \leftrightarrow Av_2) \leftrightarrow v_1 = @)$
 $@$ is the actual world.²

¹ Lewis [12, 113–116] presents counterpart theory. The original language of the theory includes no individual constants. However, the addition of $@$ here is an innocuous simplification.

² (P1)–(P8) are given in [12]. (A) is included because Lewis notes that “the domain of quantification is to contain every possible world and everything in every world” [12, 114]. Note that Lewis’ ontology includes more objects than every world and everything in every world. Namely, his ontology also includes (at least) sets and transworld sums. However, in a later postscript, Lewis clarifies that the language of counterpart theory is best understood as quantifying only over worlds and objects in worlds. (B) is also included to account for Lewis’ clarification in a later postscript. Note that from (A) and (B), every object in the domain of quantification is in a world. In the context of discussing counterpart theory, my use of *individual* concerns exactly all those objects in worlds. Finally, (C) is included to characterise the primitive $@$.

Lewis analyses his counterpart relation in terms of similarity. Schematically: individual b_1 in world w_1 is a counterpart of individual b_2 in world w_2 iff b_1 is (i) *appropriately similar* to b_2 , and (ii) at least as similar to b_2 as any other individual in w_1 is. As a result, Lewis holds that: (a) the counterpart relation need not be symmetric; (b) the counterpart relation need not be transitive; and (c) an entity need not have at most one counterpart per world.

The central idea in using the resources of CT to understand our modal talk is that *de re* modal claims are to be understood in terms of the counterpart relation (rather than transworld identity).

2 A Translation Scheme

To capture the counterpart theoretic analysis of our modal talk, Lewis provides a method for translating natural language modal claims into L^{CT} -sentences [12, 116–118].

First, we assume a regimentation of natural language modal claims into sentences of a standard first-order modal language L , whose modal operators are exactly \diamond and \square , and whose signature contains exactly: for every $n \geq 1$, denumerably-many n -ary predicate constants F_1^n, F_2^n, \dots

Next, we define a *translation scheme* that maps an L -sentence to an L^{CT} -sentence. The translation of L -sentence ϕ is ϕ^α , to be read as: ϕ holds at the actual world.

To define this, we simultaneously define by recursion, for every $\alpha \in [\text{VAR} \cup \{\alpha\}]$, a unary function $(\cdot)^\alpha: \text{Form}(L) \rightarrow \text{Form}(L^{CT})$. We read ϕ^α as: ϕ holds at world α .³

- (i) $(Px_1 \dots x_n)^\alpha$ is $Px_1 \dots x_n$.
- (ii) $(x_1 = x_2)^\alpha$ is $x_1 = x_2$.
- (iii) $(\sim \phi)^\alpha$ is $\sim(\phi^\alpha)$.
- (iv) $(\phi \rightarrow \psi)^\alpha$ is $(\phi^\alpha \rightarrow \psi^\alpha)$.
- (v) $(\exists x \phi)^\alpha$ is $\exists x(Ix\alpha \wedge \phi^\alpha)$.
- (vi) $(\forall x \phi)^\alpha$ is $\forall x(Ix\alpha \rightarrow \phi^\alpha)$.
- (vii) $(\diamond \phi(x_1, \dots, x_n))^\alpha$ is $\exists w \exists y_1 \dots \exists y_n (Ww \wedge Iy_1 w \wedge Cy_1 x_1 \wedge \dots \wedge Iy_n w \wedge Cy_n x_n \wedge (\phi(y_1, \dots, y_n))^w)$.

³ VAR is the set of individual variables. For simplicity, when presenting formulas in this paper, I may omit some brackets. In which case, assume the written string is an abbreviation of a formula that is bracketed to the left. Sometimes, to stress scope, I shall include additional brackets that are not part of the object language formulas. Notation-wise, for an L -formula ϕ with n -many free variables, for distinct (from each other) individual variables x_1, \dots, x_n , the L -formula $\phi(x_1, \dots, x_n)$ is the same as ϕ , except that every free occurrence of its first free variable is replaced with x_1, \dots , and every free occurrence of its n^{th} free variable is replaced with x_n . Of course, the following is not yet a recursive definition of the function $(\cdot)^\alpha$ as for clauses (vii) and (viii), we have given a string containing the meta-variables w, y_1, \dots, y_n (which range over the individual variables of our object language), but no way to determine exactly which individual variables are contained in the output. For simplicity, I shall leave this specification implicit and talk in terms of strings involving meta-variables ranging over object language variables. The only desideratum in determining the object language variables is that the resultant translation scheme does not deliver formulas with incorrectly bound occurrences of individual variables.

For some world w , w -counterpart y_1 of x_1, \dots , and w -counterpart y_n of x_n , ϕ holds of y_1, \dots, y_n at w .

- (viii) $(\Box\phi(x_1, \dots, x_n))^\alpha$ is $\forall w\forall y_1\dots\forall y_n((Ww \wedge Iy_1w \wedge Cy_1x_1 \wedge \dots \wedge Iy_nw \wedge Cy_nx_n) \rightarrow (\phi(y_1, \dots, y_n))^w)$.

For every world w , w -counterpart y_1 of x_1, \dots , and w -counterpart y_n of x_n , ϕ holds of y_1, \dots, y_n at w .

For examples of the translation scheme:

- $(\forall xFx)^\textcircled{\alpha}$ is $\forall x(Ix@ \rightarrow Fx)$
- $(\Box\forall xFx)^\textcircled{\alpha}$ is $\forall y(Wy \rightarrow \forall x(Ixy \rightarrow Fx))$
- $(\forall x\Box Fx)^\textcircled{\alpha}$ is $\forall x(Ix@ \rightarrow \forall y\forall z((Wy \wedge Izy \wedge Cz x) \rightarrow Fz))$

For an example of the overall translation process:

- (1) Some individual is rich but could have been poor.
- (1*) $\exists x(Rx \wedge \Diamond Px)$
- (1**) $\exists x(Ix@ \wedge Rx \wedge \exists w\exists y(Ww \wedge Iyw \wedge Cyx \wedge Py))$

3 Actuality

One of the central challenges made against Lewis’ overall account is that it cannot handle certain natural language modal claims involving a notion of *actuality* [3, 10].

The challenge has two parts. First, in order to handle certain natural language modal claims involving a notion of actuality, the initial formal modal language that natural language modal claims are regimented into must extend L with something like an actuality operator **A**.

Second, certain ways that Lewis’ translation scheme for L might be extended to accommodate an actuality operator **A** are unacceptable. That is, certain candidate clauses for **A** in the recursive definition of our function $(.)^\alpha$ are unacceptable.

Let L_A be the extension of L with exactly **A**. Hazen argues that we cannot extend $(.)^\alpha$ with either of the following clauses for **A** [10, 330]:

- (\exists -**A**) $(\mathbf{A}\phi(x_1, \dots, x_n))^\alpha$ is $\exists y_1\dots\exists y_n(Iy_1@ \wedge Cy_1x_1 \wedge \dots \wedge Iy_n@ \wedge Cy_nx_n \wedge (\phi(y_1, \dots, y_n))^\textcircled{\alpha})$.

For some actual counterpart y_1 of x_1, \dots , and some actual counterpart y_n of x_n , ϕ holds of y_1, \dots, y_n at $@$.

- (\forall -**A**) $(\mathbf{A}\phi(x_1, \dots, x_n))^\alpha$ is $\forall y_1\dots\forall y_n((Iy_1@ \wedge Cy_1x_1 \wedge \dots \wedge Iy_n@ \wedge Cy_nx_n) \rightarrow (\phi(y_1, \dots, y_n))^\textcircled{\alpha})$.

For every actual counterpart y_1 of x_1, \dots , and every actual counterpart y_n of x_n , ϕ holds of y_1, \dots, y_n at $@$.

For example, if we adopt $(\exists\text{-}\mathbf{A})$, then the supposedly intuitively unsatisfiable $L_{\mathbf{A}}$ -sentence (2) is translated to an L^{CT} -sentence that is satisfied by some L^{CT} -structures that model CT.

$$(2) \diamond \exists x (\mathbf{A} \exists y x = y \wedge \mathbf{A} Fx \wedge \mathbf{A} \sim Fx)$$

Proof (2)[@] is:

$$\exists u (Wu \wedge \exists x (Ix u \wedge$$

$$\exists z (Iz @ \wedge Cz x \wedge \exists y (Iy @ \wedge z = y)) \wedge$$

$$\exists z (Iz @ \wedge Cz x \wedge Fz) \wedge$$

$$\exists z (Iz @ \wedge Cz x \wedge \sim Fz)).$$

Let M be an L^{CT} -structure that models CT such that some non-actual individual d has an actual counterpart d_1 that is F and an actual counterpart d_2 that is not F . Then M satisfies (2)[@]. \square

The problem is that a non-actual individual may have multiple actual counterparts, some of which are F , and some of which are not F .

Fara & Williamson [3] go further. They show that in virtue of each of (a) the potential of a non-actual individual having multiple actual counterparts, and (b) the potential of a non-actual individual having no actual counterparts, Lewis' translation scheme for L cannot be extended to accommodate an actuality operator by unpacking the actuality operator as a kind of logical quantifier over actual counterparts.

We can block this actuality challenge against Lewis' translation account by adopting additional axioms concerning counterparts. For example, we might hold that each individual has exactly one actual counterpart. However, given Lewis' analysis of the counterpart relation in terms of similarity, such a condition seems extremely implausible.

4 Meyer [16]

In response to this actuality challenge, Meyer [16] presents a counterpart theoretic analysis of natural language modal talk that retains Lewis' overall translation approach, but whose formal modal language L^* that we initially regiment our natural language claims into does not contain an actuality operator.

4.1 The Eliminability of \mathbf{A}

The motivating idea behind Meyer's approach is that \mathbf{A} is not required for an account of natural language modal claims. We can work with a formal modal language without \mathbf{A} (given we have certain other expressive resources). He writes [16, 33]:

“The actuality operator allows us to make claims about the actual world *inside* the scope of other modal operators. That is a nice feature, but describing the actual world is something we could do already, by using unmodalized sentences

outside the scope of other operators. Offhand, one would therefore expect the actuality operator to make no difference to the expressive capacity of modal logic. Whenever a claim about the actual world is made within the scope of another modal operator, we should be able to move the claim outside that operator's scope, where we no longer need the actuality operator to talk about the actual world.

However, this assumes that the rest of our logic is strong enough to permit this kind of transformation, and that is not always the case. The actuality operator does make a difference in expressive capacity when it is combined with a quantified modal logic that is too weak to permit its elimination. But there is no reason why counterpart theorists should measure the expressive capacity of their theory of modality against such a weak system, rather than against a stronger one from which **A** can be eliminated."

As [9] shows, **A** is eliminable in propositional modal logic. We can capture this in model theoretic terms. Let L^+ be the standard propositional modal language with **A** and assume it is interpreted with its usual Kripkean semantics. L^+ -formulas ϕ and ψ are *equivalent* iff for every L^+ -structure M , M satisfies ϕ (at the actual world of the structure) iff M satisfies ψ (at the actual world of the structure). The eliminability of **A** for this logic amounts to: for every L^+ -formula ϕ with an occurrence of **A**, there is an equivalent L^+ -formula ψ with no occurrence of **A**. Meyer himself [15, 234] demonstrates this for the temporal analogue of modal propositional logic.⁴

As [8, 9] note, **A** is not eliminable for L_A interpreted with the usual varying quantifier domain Kripkean semantics. Here, let L_A -formulas ϕ and ψ be equivalent iff for every varying domain L_A -structure M , and variable assignment s for M , M satisfies ϕ relative to s (at the actual world of the structure) iff M satisfies ψ relative to s (at the actual world of the structure). Then the ineliminability of **A** for this logic amounts to: for some L_A -formula ϕ with an occurrence of **A**, there is no equivalent L_A -formula ψ with no occurrence of **A**. For example, consider $\diamond\exists x\sim A\exists yx = y$ [9, 622]. Meyer [15] notes this for an analogous varying quantifier domain temporal semantics.

However, Meyer [16] makes an eliminability claim concerning **A** for a certain enriched constant quantifier domain modal semantics. This is close to the eliminability claim made in [15] concerning the now operator **N** for a certain enriched constant quantifier domain temporal semantics.

The motivating idea with Meyer's enriched constant quantifier domain modal semantics is that we can enrich a first-order modal language with resources that allow us to talk about (i) "existence", and (ii) sets of n -tuples of members of our overall domain of individuals.⁵

Meyer's semantics can be captured as follows [16, 35–38; 15, 238–245].⁶ First, the language $L_{A,M}$ of the semantics is the extension of L_A with exactly: E , \in , $<$, $>$, and

⁴ See Theorem 2 of [15]. Also see [11].

⁵ There are echoes of the third strategy of [2] (section VI) in the approach.

⁶ I have made an adjustment in this presentation of Meyer's semantics. In his presentations, Meyer talks just of "set variables X, Y, Z, \dots ". And he talks of a variable assignment of a structure mapping each set variable to a set in the hierarchy of sets built up from D (the hierarchy that takes the members of the overall domain D as urelements). Set variables range over sets in the hierarchy. However, the motivation of the enrichment of the language is to be able to talk about (i) "existence", and (ii) sets of n -tuples of members of

for every $n \geq 0$, denumerably-many n -ary set variables V_1^n, V_2^n, \dots ⁷ We include the following clauses in our inductive definition of the $L_{A,M}$ -formulas:

- For every individual variable x , $[Ex$ is an $L_{A,M}$ -formula].⁸
- For every $n \geq 0$, all individual variables x_1, \dots, x_n , and every n -ary set variable X^n , $[<x_1, \dots, x_n > \in X^n$ is an $L_{A,M}$ -formula].
- For every $\ast \in \{\exists, \forall\}$, set variable X , and $L_{A,M}$ -formula ϕ , $[\ast X\phi$ is an $L_{A,M}$ -formula].

An $L_{A,M}$ -structure M is a tuple $\langle W, R, D, \text{dom}, w_{@}, I \rangle$ such that:

- W and D are non-empty sets.
- $R \subseteq W^2$.
- $\text{dom}: W \rightarrow \mathcal{P}(D)$.
- $w_{@} \in W$.
- I is a function from the set of extralogical predicate constants that, for every $n \geq 1$ and n -ary predicate constant P , assigns to P a function that assigns to each $w \in W$ a subset of $\text{dom}(w)^n$.

These elements represent, respectively: a set of worlds, the accessibility relation between worlds, the overall domain of individuals, the “existing” individuals of worlds, the actual world, and an interpretation of the extralogical vocabulary.⁹

A variable assignment s for an $L_{A,M}$ -structure M assigns to each individual variable a member of D , and for every $n \geq 0$, to each n -ary set variable a member of $\mathcal{P}(D^n)$. For a variable assignment s and variables $\alpha_1, \dots, \alpha_n$, let $s_{[o_1, \dots, o_n / \alpha_1, \dots, \alpha_n]}$ be the assignment that agrees with s , except that o_1 is assigned to α_1, \dots , and o_n is assigned to α_n .

For an $L_{A,M}$ -structure M , we simultaneously define by recursion, for each $w \in W$, and variable assignment s of M , a unary satisfaction function $v_{M,w,s} : \text{Form}(L_{A,M}) \rightarrow \{1, 0\}$ as follows:

- (i) $v_{M,w,s}(Px_1 \dots x_n) = 1$ iff $\langle s(x_1), \dots, s(x_n) \rangle \in I(P)(w)$.
- (ii) $v_{M,w,s}(x_1 = x_2) = 1$ iff $s(x_1) = s(x_2)$.
- (iii) $v_{M,w,s}(\sim\phi) = 1$ iff $v_{M,w,s}(\phi) = 0$.
- (iv) $v_{M,w,s}(\phi \rightarrow \psi) = 1$ iff $v_{M,w,s}(\phi) = 0$ or $v_{M,w,s}(\psi) = 1$.
- (v) $v_{M,w,s}(\exists x\phi) = 1$ iff for some $d \in D$, $v_{M,w,s_{[d/x]}}(\phi) = 1$.
- (vi) $v_{M,w,s}(\forall x\phi) = 1$ iff for every $d \in D$, $v_{M,w,s_{[d/x]}}(\phi) = 1$.
- (vii) $v_{M,w,s}(\diamond\phi) = 1$ iff for some $w^+ \in W$ such that Rww^+ , $v_{M,w^+,s}(\phi) = 1$.
- (viii) $v_{M,w,s}(\Box\phi) = 1$ iff for every $w^+ \in W$ such that Rww^+ , $v_{M,w^+,s}(\phi) = 1$.
- (ix) $v_{M,w,s}(A\phi) = 1$ iff $v_{M,w_{@},s}(\phi) = 1$.

our overall domain D . So, in my presentation, as we shall see, I have assigned an arity to each set variable and restricted the range of an n -ary set variable to $\mathcal{P}(D^n)$. This is all we need for the eliminability thesis.

⁷ M for Meyer’s resources.

⁸ Meyer [16, 38] states that (i) one could allow formulas EX , where X is a set variable, and (ii) there are multiple ways in which the semantics might handle such a formula. He there proposes to ignore the issue, so I have excluded formulas of this kind here.

⁹ Note that in addition to an overall set of individuals, our structures specify world domains (which will be used to interpret the primitive E). Further, the extension of an n -ary predicate constant at a world w is a subset of $\text{dom}(w)^n$. This follows the formal presentation in [15].

- (x) $v_{M,w,s}(Ex) = 1$ iff $s(x) \in \text{dom}(w)$.
- (xi) $v_{M,w,s}(\langle x_1, \dots, x_n \rangle \in X^n) = 1$ iff $\langle s(x_1), \dots, s(x_n) \rangle \in s(X^n)$.
- (xii) $v_{M,w,s}(\exists X^n \phi) = 1$ iff for some $S \in \mathcal{P}(D^n)$, $v_{M,w,s[S/X^n]}(\phi) = 1$.
- (xiii) $v_{M,w,s}(\forall X^n \phi) = 1$ iff for every $S \in \mathcal{P}(D^n)$, $v_{M,w,s[S/X^n]}(\phi) = 1$.

Finally, for an $L_{A,M}$ -structure M , and variable assignment s of M , we define the unary function $v_{M,s} : \text{Form}(L_{A,M}) \rightarrow \{1, 0\}$ as follows:

$$v_{M,s}(\phi) = v_{M,w@s,s}(\phi).$$

This is thought of as describing satisfaction *simpliciter* of an $L_{A,M}$ -formula at an $L_{A,M}$ -structure relative to a variable assignment.

So much for the semantics. Say that two $L_{A,M}$ -formulas ϕ and ψ are equivalent iff for every $L_{A,M}$ -structure M , and variable assignment s for M , $v_{M,s}(\phi) = v_{M,s}(\psi)$. The eliminability claim concerning **A** that [16] makes is that for every L_A -formula ϕ , there is an equivalent L_M -formula ψ .¹⁰

Meyer [15] covers the temporal analogue of this thesis [15, 243, Theorem 4]. Meyer [16] informally discusses why this result holds for the modal version. For our purposes, it is sufficient to consider two examples. First, consider:

- (3) There could have been something that does not actually exist.
- (4) It might have been that everyone who is in fact rich was poor.

For (4), following the approach of [2], we restrict our attention to the plural *de re* reading. Given Meyer’s semantics, two formulas which supposedly capture the conditions of (3) and (4), respectively, are the following:

- (3*) $\diamond \exists x(Ex \wedge \sim \mathbf{A}Ex)$
- (4*) $\diamond \forall x(\mathbf{A}Rx \rightarrow Px)$

Meyer states that the following are, respectively, equivalent:

- (3**) $\exists x(\diamond Ex \wedge \sim Ex)$
- (4**) $\exists X(\forall x(\langle x \rangle \in X \leftrightarrow Rx) \wedge \diamond \forall x(\langle x \rangle \in X \rightarrow Px))$

The idea with converting an L_A -formula ϕ to an L_M -formula ψ is that in ψ , we try to capture the relevant features about $w@$ (that ϕ imposes) outside the scope of any modal operators which shift our evaluation world away from $w@$.

For the simple case of (3*) and (3**), to capture (3*), we present a formula that, outside of any modal operators, captures the relevant feature about $w@$, namely that some $d \in D$ in the domain of some world is not in $\text{dom}(w@)$.

For the case of (4*) and (4**), in (4**) we antecedently introduce a set $S = \{d \in D : d \in I(R)(w@)\}$. That is, the set containing exactly all individuals that are R at the actual world. Then, within the scope of the possibility operator occurrence, we can use individual membership of S to capture the property of an individual being R at the actual world.¹¹

¹⁰ L_M is the language that results from removing **A** from $L_{A,M}$. Note that every L_A -formula and L_M -formula is an $L_{A,M}$ -formula.

¹¹ Notice that if we didn’t restrict the range of set variables as I have suggested, (4**) would be handled slightly differently. In (4**) we would be antecedently introducing a set S whose intersection with D is

4.2 A Counterpart Theoretic Account

Meyer states that we can employ this eliminability result when presenting a counterpart theoretic account. He calls his approach the *antecedent elimination strategy*, and writes [16, 33–34]:

“Instead of uniformly translating subformulae of type $[A\phi]$ into counterpart theory, I want to propose an antecedent elimination strategy that makes do without this operator altogether. We first eliminate all occurrences of $[A]$ by translating the modal claims in question into a quantified modal logic in which this operator is redundant. After that, we apply a slightly modified version of Lewis’s translation scheme to the resulting $[A\text{-free}]$ sentences to generate substitutes in counterpart theory. This provides a systematic counterpart treatment of sentences involving $[A]$ ”

And [16, 40]:

“We begin by regimenting all modal claims about physical objects in $[L_M]$ interpreted with Meyer’s semantics]. By doing so, we eliminate all occurrences of the actuality operator $[A]$. After that, we apply our revised translation scheme to these $[A]$ -free sentences to produce substitutes in counterpart theory.”

The idea, it seems, is that for a given natural language modal claim ϕ , we first regiment ϕ as an L_M -sentence ψ that (when interpreted with Meyer’s semantics) supposedly captures ϕ . Next, we apply Meyer’s translation scheme that maps an L_M -sentence ψ to a sentence of the language of Meyer’s counterpart theory.

The language L_M^{CT} of Meyer’s counterpart theory is the extension of L^{CT} with exactly: \in , $<$, $>$, and for every $n \geq 0$, denumerably-many n -ary set variables V_1^n, V_2^n, \dots .¹² We include the following clauses in our inductive definition of the L_M^{CT} -formulas:

- For every $n \geq 0$, all individual variables x_1, \dots, x_n , and every n -ary set variable X^n , $[< x_1, \dots, x_n > \in X^n$ is an L_M^{CT} -formula].
- For every $* \in \{\exists, \forall\}$, set variable X , and L_M^{CT} -formula ϕ , $[*X\phi$ is an L_M^{CT} -formula].

The L_M^{CT} -structures are the L^{CT} -structures. The definition of satisfaction is extended in the obvious way. Next, Meyer’s counterpart theory itself seems to be the counterpart theory of Lewis. That is, CT: the set containing exactly (P1)–(P8), (A), (B), and (C).

Finally, the translation of L_M -sentence ϕ is $\phi^@$. To define this, we simultaneously define by recursion, for every $\alpha \in [VAR \cup \{@\}]$, the function $\phi^\alpha: Form(L_M) \rightarrow Form(L_M^{CT})$ [16, 37–40]:

$$(i) (Px_1 \dots x_n)^\alpha \text{ is } (Ix_1 \alpha \wedge \dots \wedge Ix_n \alpha \wedge Px_1 \dots x_n).$$

$\{d \in D : d \in I(R)(w@)\}$. That is, the intersection contains exactly all individuals that are R at the actual world. Then, within the scope of the possibility operator occurrence, we can use individual membership of S to capture the property of an individual being R at the actual world.

¹² The language of Meyer’s counterpart theory includes no individual constants. However, the addition of $@$ is an innocuous simplification again.

- (ii) $(x_1 = x_2)^\alpha$ is $(Ix_1\alpha \wedge Ix_2\alpha \wedge x_1 = x_2)$.
- (iii) $(Ex)^\alpha$ is $Ix\alpha$.
- (iv) $(\langle x_1, \dots, x_n \rangle \in X^n)^\alpha$ is $(Ix_1\alpha \wedge \dots \wedge Ix_n\alpha \wedge \langle x_1, \dots, x_n \rangle \in X^n)$.
- (v) $(\sim\phi)^\alpha$ is $\sim(\phi^\alpha)$.
- (vi) $(\phi \rightarrow \psi)^\alpha$ is $(\phi^\alpha \rightarrow \psi^\alpha)$.
- (vii) $(\exists x\phi)^\alpha$ is $\exists x(\phi^\alpha)$.
- (viii) $(\forall x\phi)^\alpha$ is $\forall x(\phi^\alpha)$.
- (ix) $(\exists X^n\phi)^\alpha$ is $\exists X^n(\phi^\alpha)$.
- (x) $(\forall X^n\phi)^\alpha$ is $\forall X^n(\phi^\alpha)$.
- (xi) $(\diamond\phi(x_1, \dots, x_n))^\alpha$ is $\exists w\exists y_1\dots\exists y_n(Ww \wedge Iy_1w \wedge Cy_1x_1 \wedge \dots \wedge Iy_nw \wedge Cy_nx_n \wedge (\phi(y_1, \dots, y_n))^w)$.
- (xii) $(\Box\phi(x_1, \dots, x_n))^\alpha$ is $\forall w\forall y_1\dots\forall y_n((Ww \wedge Iy_1w \wedge Cy_1x_1 \wedge \dots \wedge Iy_nw \wedge Cy_nx_n) \rightarrow (\phi(y_1, \dots, y_n))^w)$.

Notice that the clauses for the truth-functional connectives and modal operators remain the same as Lewis'. The differences are that (a) quantifiers are not translated as world-relative, and (b) the atomic clauses add world membership conditions. Meyer's idea with the atomic clauses is that he is supposedly building back in world-relativisation of sentences removed from the first-order quantifier clauses.

For example, then, (3**) and (4**) are translated, respectively, as follows:

- (3**) $\exists x(\diamond Ex \wedge \sim Ex)$
- (3***) $\exists x(\exists w\exists y(Ww \wedge Iyw \wedge Cyx \wedge Iyw) \wedge \sim Ix@)$
- (4**) $\exists X(\forall x(\langle x \rangle \in X \leftrightarrow Rx) \wedge \diamond\forall x(\langle x \rangle \in X \rightarrow Px))$
- (4***) $\exists X(\forall x[(Ix@ \wedge \langle x \rangle \in X) \leftrightarrow (Ix@ \wedge Rx)] \wedge \exists w[Ww \wedge \forall x((Iwx \wedge \langle x \rangle \in X) \rightarrow (Iwx \wedge Px))])$

5 Against Meyer's Account

A central desideratum for Meyer's account is that a natural language modal claim ϕ is translated to an L_M^{CT} -sentence that presents an acceptable counterpart theoretic analysis of ϕ . Unfortunately, Meyer's account does not meet this desideratum. In this section, I present two examples of this failure.

For the first example, consider Meyer's toy case: the translation of (4) to (4***). Notice that (4***) just says that there is a set S such that (a) of the actual individuals $d@$, [$d@$ is in S iff $d@$ is R], and (b) there is a world w such that of the w -individuals d_w , if d_w is in S, then d_w is P.

The first conjunct $\forall x[(Ix@ \wedge \langle x \rangle \in X) \leftrightarrow (Ix@ \wedge Rx)]$ that is supposed to define S in fact does not define S, but merely gives membership conditions for actual individuals. Given the first conjunct, it is open whether S contains non-actual individuals. The second conjunct $\exists w[Ww \wedge \forall x((Iwx \wedge \langle x \rangle \in X) \rightarrow (Iwx \wedge Px))]$

then gives us a conditional for w -individuals. Namely, that if they're in S , then they are P . In no part of the statement is a counterpart relation mentioned.

Let M be an L_M^{CT} -structure that models CT such that (i) there is a single actual individual d that is R , (ii) d has no counterparts that are P , and (iii) there is a non-actual world. Then (4^{***}) is true at M . This is because there is a set S such that (a) for all actual individuals b , b is in S iff b is R , and (b) for some world w , every w -individual b in S is P (namely, $S = \{d\}$). However, it seems that if every actual rich individual has no counterpart that is poor, then our counterpart theoretic analysis should not render (4) true. So, Meyer's analysis of (4) is unacceptable.

To push the point further, if there is a single actual individual that is rich, then it seems the following natural language modal claims should have the same truth-value:

- (4) It might have been that everyone who is in fact rich was poor.
- (5) Someone in fact rich could have been poor.

On Meyer's approach, (5) is straightforwardly regimented to the following L_M -sentence:

$$(5^*) \exists x(Rx \wedge \diamond Px)$$

This is then translated to the following L_M^{CT} -sentence:

$$(5^{**}) \exists x(Ix @ \wedge Rx \wedge \exists w \exists y(Ww \wedge Iyw \wedge Cyx \wedge Iyw \wedge Py))$$

However, notice that at our L_M^{CT} -structure M that models CT such that (i) there is a single actual individual d that is R , (ii) d has no counterparts that are P , and (iii) there is a non-actual world, (5^{**}) is false even though (4^{***}) is true. That is, we have an L_M^{CT} -structure M that models CT where there is a single actual individual that is rich, and yet the counterpart theoretic translations of (4) and (5) do not have the same truth-value.

The issue here is that whilst (5) receives an acceptable counterpart theoretic translation, (4) does not.

Let us now consider the second example of the failure of Meyer's account to provide acceptable counterpart theoretic translations of natural language modal claims.

Given that Meyer's translation scheme reuses certain clauses from Lewis' original scheme, his account inherits some of the problems that Lewis' exhibits. In particular, Meyer reuses Lewis' clauses for the possibility and necessity operators. However, given the way Lewis sets these up, the possibility and necessity operators are interpreted as expressing weak notions of possibility and necessity, respectively. That is, what is possible/necessary of individuals when they all exist. As a result, a problem that Lewis' overall account faces is that it cannot render true certain natural language claims concerning contingent existence. To see this, consider:

- (6) Some individual could have failed to exist.

On Lewis' account, this would be regimented as:

$$(6^*) \exists x \diamond \sim \exists yx = y$$

Which is translated to the unsatisfiable:

$$(6^{**}) \exists x(Ix @ \wedge \exists w \exists z(Ww \wedge Iz w \wedge Cz x \wedge \sim \exists y(Iy w \wedge z = y)))$$

However, certainly a later Lewis would want his counterpart theoretic analysis of (6) to allow it to be true.¹³ Indeed, by 1986, Lewis has seemingly given up on the strategy of regimenting natural language modal claims into sentences of a formal modal language L^* , alongside presenting a counterpart theoretic interpretation of L^* , at least in part because he was unable to produce a version of the strategy that delivers an appropriate analysis of modal claims concerning existence. In [14], after discussing issues with capturing such natural language claims [14, 8–12], he writes [14, 12–13]:

“What is the correct counterpart-theoretic interpretation of the modal formulas of the standard language of quantified modal logic? Who cares? We can make them mean whatever we like. We are their master. We needn’t be faithful to the meanings we learned at mother’s knee - because we didn’t. If the language of boxes and diamonds proves to be a clumsy instrument for talking about matters of essence and potentiality, let it go hang. Use the resources of modal realism *directly* to say what it would mean for Humphrey to be essentially human, or to exist contingently”

Now, notice that Meyer inherits the problem of being unable to render true certain contingent existence claims. Presumably, on Meyer’s account, (6) would be regimented as:

$$(6^+) \exists x(Ex \wedge \diamond \sim Ex)$$

Which is translated to the unsatisfiable:

$$(6^{++}) \exists x(Ix @ \wedge \exists w \exists y(Ww \wedge Iy w \wedge Cy x \wedge \sim Iy w))$$

However, presumably Meyer does want his account to be able to render (6) true. First, holding that (6) is a contradiction would be an unusual position for the defender of counterpart theory to adopt. Second, if (6⁺) is meant to in some sense capture (6) when interpreted with Meyer’s antecedent semantics, and (6⁺) is satisfiable according to that antecedent semantics, then it would be ostensibly strange for Meyer to hold that we arrive at the correct counterpart theoretic interpretation of (6) by first capturing it in a logic which renders it satisfiable, and then translating to an unsatisfiable L_M^{CT} -sentence.

6 Methodology

We have seen that Meyer’s particular account fails to deliver acceptable counterpart theoretic analyses for certain natural language modal claims. In this section, I additionally highlight the strange nature of Meyer’s methodology.

Recall that Lewis’ overall strategy is to provide an account of a natural language modal claim ϕ by (a) regimenting ϕ into L, (b.i) presenting CT, and (b.ii) presenting a translation scheme from L-sentences to L^{CT} -sentences.

¹³ Even in [12], Lewis notes that his account is in the frying pan for rendering the translation of L-sentence $\forall x \square \exists y x = y$ valid [12, 119].

It is worth noting that (b.i) and (b.ii) together generate a semantics for L. Indeed, Lewis himself notes that his original account effectively presents an interpretation for the "standard language of modal logic" [14, 9].

To capture this, let us first define the notion of an L-sentence ϕ being satisfied by an L^{CT} -structure M that models CT: M satisfies ϕ iff M satisfies $\phi^@$. Second, we define a consequence relation: for set Γ of L-sentences and L-sentence ϕ , $\Gamma \models_{CT} \phi$ iff for every L^{CT} -structure M that models CT, if $[\forall \gamma \in \Gamma, M \text{ satisfies } \gamma]$, then M satisfies ϕ .

Given this, Lewis' overall account is really a version of a common strategy to present an account of natural language modal talk: presenting a semantics for a formal modal language L^* (leaving implicit a regimentation of natural language modal claims into L^* -sentences).

Lewis' account is an interesting version of this strategy because of the indirect nature of the specification of the semantics for L. However, this indirect approach is not essential to the semantics: one can specify a direct model theory for L that is equivalent to Lewis' semantics (in the sense that it has the same consequence relation).

Crucially, when Lewis applies this strategy, his regimentation of natural language modal claims into L ostensibly does not rely on an antecedent semantics for L.

In contrast, Meyer relies on his antecedent semantics for L_M to complete an initial regimentation of natural language modal claims into L_M -sentences. His counterpart theory and translation scheme combined then generate *another* semantics for L_M . That is, to present a counterpart theoretic analysis of a natural language claim ϕ , Meyer first requires us to consider the analysis of ϕ generated by his antecedent semantics, and then to derive the counterpart theoretic analysis (a competitor to the analysis of his antecedent semantics) from this. This is a strange approach given how ostensibly different Meyer's antecedent semantics is to counterpart theory.

At the very least for this strategy to be appropriate, it seems that the antecedent semantics for L_M and the counterpart theoretic semantics for L_M (generated by the combination of (i) the first-order theory CT, and (ii) a translation scheme from L_M -sentences to L_M^{CT} -sentences) should be equivalent (in the sense of having the same consequence relation when restricted to L_M -sentences). However, Meyer's account does not achieve this, and, more generally, Meyer's antecedent semantics arguably cannot be equivalent to an acceptable counterpart theoretic semantics. I turn to these two points now.

To see that Meyer's antecedent semantics for L_M and his counterpart theoretic semantics for L_M are not equivalent (in the given sense), consider the following L_M -sentence:

$$(6^+) \exists x (Ex \wedge \diamond \sim Ex)$$

As we saw earlier, this is a contradiction on Meyer's counterpart theoretic semantics. However, it is satisfiable on the antecedent semantics.

On the general point, a popular motivation for adopting a counterpart theoretic analysis of natural language modal claims is to be able to render true certain "contingent identity" statements. To introduce the point, consider Gibbard's material constitution case [7]:

Suppose that (i) at the exact time of its creation, a lump of clay, called 'Lumpl', is formed into a statue, called 'Goliath', and (ii) after a short while, the clay statue is

vaporised. Here, then, the lump and the statue are (a) created at exactly the same time, and (b) destroyed at exactly the same time.

The lump and the statue occupy exactly the same time period, and are spatially coincident throughout, seemingly sharing, for each instant, exactly the same physical properties. Given this, some take it to be natural to identify the lump and the statue. However, it could have been that instead of being vaporised, the statue was instead squished into a ball. Here, it seems that the lump of clay survives the squashing, but the statue does not. So, in this counterfactual scenario, it seems that we cannot identify the lump and the statue. Some thus hold the following:

(7) For some x , for some y , x and y are identical, but could have been distinct.¹⁴

Lewis [13] posits multiple counterpart relations in order to render certain “contingent identity” statements true. In a similar fashion, we might posit multiple counterpart relations and allow that (7) is rendered true by there being an actual individual d and world w such that d has exactly one lump w -counterpart l , and exactly one statue w -counterpart s (where $s \neq l$).

However, it seems that any supposed capture of (7) as an L_M -sentence ϕ interpreted with Meyer’s antecedent semantics will render ϕ unsatisfiable. Even if we extended L_M with additional resources, it is unclear how (7) would be captured by a sentence ϕ of that language (interpreted with a suitably enriched version of Meyer’s antecedent semantics) that is satisfiable. As a result, if our counterpart theoretic semantics generated by a theory and translation scheme is meant to be equivalent to the antecedent semantics, it will render ϕ unsatisfiable.

7 Final Remarks

The main result of this paper is that Meyer’s account fails to provide an acceptable counterpart theoretic analysis of certain natural language modal claims. In addition, I highlighted the strange nature of Meyer’s methodology.

However, it does not follow that there is no acceptable counterpart theoretic analysis of natural language modal claims. Recall that the challenge of [3, 10] has two parts. First, in order to handle certain natural language modal claims involving a notion of actuality, the initial formal modal language that natural language modal claims are regimented into must extend L with something like an actuality operator \mathbf{A} . Second, certain ways that Lewis’ semantics for L might be extended to accommodate an actuality operator \mathbf{A} are unacceptable. That is, certain candidate clauses for \mathbf{A} in the recursive definition of our function $(\cdot)^\alpha$ are unacceptable.

A natural response to this challenge is to accept the first part of the challenge, and instead provide an alternative counterpart theoretic semantics for the initial formal modal language (which we concede includes something like \mathbf{A}). Indeed, this overall response is the most common in the literature. One can either provide such a semantics via Lewis’ indirect approach, or via the direct specification of a model theory. For

¹⁴ The notion of distinctness I employ here requires that for x and y to be distinct, x and y must exist.

example, Forbes [4–6], Ramachandran [17–19], and Sider [21] each provide a counterpart theoretic semantics for a certain formal modal language via Lewis' indirect approach. Russell [20] and Bacon [1] each provide a counterpart theoretic semantics for a certain formal modal language via a model theory. The failure of Meyer's account, and issues with his methodology, do not count generally against this kind of response.¹⁵

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