More Aboutness in Imagination

Christopher Badura¹

Received: 8 October 2019 / Accepted: 13 September 2020 / Published online: 8 October 2020
© The Author(s) 2020

Abstract

In Berto's logic for aboutness in imagination, the output content of an imaginative episode must be part of the initial content of the episode (Berto, *Philos Stud* 175:1871–1886, 2018). This condition predicts expressions of perfectly legitimate imaginative episodes to be false. Thus, this condition is too strict. Relaxing the condition to correctly model these cases requires to consider a language with predicates and constants. The paper extends Berto's semantics for aboutness in imagination to a semantics for such a language. The new semantics models contents of formulas along the lines of Hawke's issue-based theory of topics (Hawke, *Australas J Philos* 96:697–723, 2017), while remaining faithful to the (in)validities discussed by Berto. Several relations between issues and topics are defined, which allow to overcome shortcomings of Hawke's initial framework. These relations are then discussed with respect to their usefulness in the truth condition for the imagination operator.

Keywords Imagination · Conditional logic · Aboutness · Predicate logic

1 Introduction

Francesco Berto has proposed a logic for aboutness in imagination [1].¹ He aims to model rational imagination realistically, which he takes to be a kind of mental simulation having an input content and an output content. Imagination thus understood is

¹On his account, the content of a sentence is what the sentence is about, i.e., aboutness is a relation between sentences and contents. Except for their mereological behaviour, Berto doesn't say much more about what kind of thing contents are. I will get back to that in a few paragraphs.

This research has been funded by the Ruhr University Research School PLUS, funded by Germany's Excellence Initiative [DFG GSC 98/3]. Thanks to an anonymous reviewer for their critical comments.

✉ Christopher Badura
christopher.badura@rub.de

¹ Ruhr University Bochum, Institute for Philosophy I, GA 3/37, Universitätsstraße 150, 44801, Bochum, Germany

rational because it satisfies some logical validities, e.g., imagining that B and imagining that C entails imagining that $B \wedge C$. It is modelled *realistically* because agents do not end up imagining all logical consequences of what they start out to imagine. In particular, imagination is hyperintensional.

The uses of the rational imagination as mental simulation Berto is concerned with are expressed by “In imagining (content expressed by) A , the agent also imagines (content expressed by) B ”.² The formal notation for this is $[A]B$, and I call formulas of this form “imagination-formulas”.³ The content expressed by A is the initial content and the content expressed by B is the output content. The truth condition Berto proposes for sentences of this form is conjunctive. The first conjunct requires that if a world w is accessed by the formula expressing the initial content, then the formula expressing the output content is true at w . This is the truth condition for variably strict conditional operators familiar from conditional logic.

The second conjunct requires that the output content is part of the initial content. This condition is familiar from containment logic(s), see, e.g., [2]. This, together with some other conditions provided below, takes reasonably good care of the content relations between complex formulas. For example, $[A](B \wedge C)$ entails $[A]B$ and $[A]C$, partly because the content of $B \wedge C$ includes the content of B and the content of C . Moreover, the condition on contents plausibly invalidates the inference from $[A]B$ to $[A](B \vee C)$ because the content of C might not be part of the content of A . Suppose, in imagining that there is cake on the table I imagine that the cake has candles. It is not the case that, in imagining that there is cake on the table I imagine that the cake has candles *or* Christiano Ronaldo scored a goal. My imaginative episode simply isn’t about Ronaldo (although he might wish that everything was about him). The condition on contents is too strict, however. It is particularly problematic when the subformulas in the imagination formula are atomic. Consider the following example, where Gwenny is a dog, and Helena is concerned with taking her to the lake: “In an act of imagining that Gwenny is at her favourite lake, Helena imagines that Gwenny swims in her (Gwenny’s) favourite lake”. This expresses a perfectly legitimate imaginative episode. Moreover, the episode is not especially creative, logically anarchic, or irrational. It is an imaginative episode from everyday life. Thus, if such an episode occurs, the formal counterpart of expressing it should come out as true.⁴ And it should also come out as true for the right reasons. Problematically, Berto’s account does not predict this episode as true since linguistic intuition suggests that the content of “Gwenny swims in her favourite lake” is not *a part of* the content of “Gwenny is at her favourite lake”. So, on Berto’s account, the prediction is that this imaginative episode will come out as false because the content of the output is not part of the content of the input. There is a way to force the episode to come out as true on Berto’s account. Since there is a function assigning (atomic) contents to atomic formulas one could simply define a model in which the content of “Gwenny swims

²Sometimes, I speak loosely and use “the content expressed by B ” and “ B ” interchangeably.

³Context should make clear whether a formal expression is used or mentioned. For better readability I omit quotation marks sometimes.

⁴It is the (formal representation of the) expression expressing the episode which is the truth-value-bearer. For ease of exposition, I will also say that imaginative episode itself is true or false.

at her favourite lake” is included in the content of “Gweny is at her favourite lake”. Although this gets the truth value right, it pays the price of an implausible assignment of contents. Thus, Berto’s account faces a dilemma: either it posits implausible content assignments or it makes wrong predictions about the truth values of expressions of perfectly legitimate imaginative episodes.

In this paper, I solve this dilemma by combining Berto’s semantics for imagination with Peter Hawke’s issue-based theory of topics (IBTT), and thus improve Berto’s account. Berto’s account comes unequipped with a specific account of content. So far in none of his subsequent published work has he given an explicit semantic specification of his account of content. The paper shows that there is at least one specific account of content which provides a promising specification, which is compatible with Berto’s semantics and allows solving a dilemma of Berto’s initial account due to the combination of the two frameworks. It also amounts to a first step towards extending Berto’s account to the first-order case. In addition, the various relations between topics defined are *not* present in Hawke’s original account. In fact, Hawke doesn’t consider any relations between contents except overlap and set-theoretic inclusion.

I assume that solving the dilemma requires (at least) considering a language with constant and relation symbols for the following reason: the described imaginative episode seems legitimate, at least partly, because initial and output content are both about Gweny and her favourite lake. The main reason we have to assume that they both are about Gweny and her favourite lake is that “Gweny” and “her favourite lake” occur in both sentences (in a specific way), and that they refer to the same objects across sentences. The sentences differ only in what Gweny does at the lake: simply being there, or swimming in the lake. Moreover, notably, swimming in the lake entails being at/in the lake. So, what matters for the relation between initial and output content are the individuals and relations among them, and also the metaphysical or conceptual entailments between the relations. A propositional language doesn’t have expressions referring to individuals or relations and also, usually, in its semantics there are no formal entities to represent either. So, syntax and semantics don’t provide enough resources for a more adequate model.⁵

There are various reasons for choosing Hawke’s theory of content over others by, e.g., Perry, Lewis, Yablo, or Fine [3, 4, 11, 12, 14]. First, Hawke argues convincingly that Perry’s, Lewis’s, and Yablo’s accounts each violate some plausible linguistic intuitions concerning aboutness of sentences. Second, the resources Hawke uses are familiar from semantics of first-order modal logic [5]. So, in a way, his account could be considered more conservative than the truthmaker account by Fine [3, 4]. Developing a solution for the dilemma in terms of truthmaker semantics is worth investigating – but in another paper. Third, Hawke’s theory is versatile and provides several options for defining relations between topics besides strict set-theoretic inclusion or primitive mereological parthood.

As for the structure of the paper, Section 2 rehearses Berto’s semantics. From Section 3 on, I introduce a language with constants and relation symbols with a

⁵This is not meant to assume that propositions are structured. We could also give an unstructured account in terms of, e.g., truthmaker semantics. I leave this option for another paper.

varying domain possible world semantics. Secondly, topics in the sense of Hawke's IBTT are defined on this semantics in Section 3.1 and a problem for IBTT is raised in Section 3.2. I proceed to define various relations between issues, and various relations between topics, which are then investigated in more detail in Sections 3.3 and 3.4. Then, in Section 3.5, I discuss which of the various relations between topics seem most suitable for the truth condition of the imagination operator. In Section 4, I summarise the results and point to further topics of research.

2 Berto's Semantics of Imagination

To make the paper self-contained, I start out by presenting Berto's logic for aboutness in imagination. Berto is only concerned with the case of a single agent (and I will be, too). So any agent indices are omitted. The key idea of Berto's account is that imagination is a kind of mental simulation. It proceeds similarly to evaluating a variably strict conditional, while being subject to a constraint on the contents of antecedent and consequent. His account is based on a propositional language:

Definition 1 (Alphabet) The alphabet has countably many propositional variables (or atomic formulas) p_i , operators $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (strict implication), $\Box$ (necessity), and auxiliary symbols $(,), [,]$.

Definition 2 (Formulas) The set of formulas is given by the following Backus-Naur form:

$$p \mid \neg A \mid \Box A \mid A \wedge B \mid A \vee B \mid A \rightarrow B \mid [A]B,$$

where p is an atomic formula. The set of atomic formulas is denoted by $Atom$.

The Boolean formulas receive their standard natural language interpretation. Formulas of the form $A \rightarrow B$ are interpreted as strict implications. Formulas of the form $[A]B$ are interpreted as "In an act of imagining (the content expressed by) A , the agent also imagines (the content expressed by) B ", cf. [1].

Since imagination is meant to work similar to a variably strict conditional, the frames are frames familiar from conditional logic (sometimes called Chellas-frames), where the accessibility relations are indexed by formulas from the language:

Definition 3 (Frame) A frame is a structure $\langle \mathcal{W}, \{\mathcal{R}_A \mid A \in K\} \rangle$, where $\mathcal{W}$ is a set of possible worlds, K is a (possibly improper) subset of the set of formulas, and each $\mathcal{R}_A$ is a binary relation on $\mathcal{W}$.

Instead of considering the accessibility relation $\mathcal{R}_A$ it is sometimes more convenient to consider a set-selection function that assigns to each formula-world pair the worlds that are accessible from that world via $\mathcal{R}_A$. That is, to consider a function defined by $f_A(w) = \{w' \mid \mathcal{R}_A w w'\}$. In the context of Berto's account, the accessibility

relations are meant to capture that from a given world, the agent can access other worlds via the content expressed by a formula. Models are defined as usual:

Definition 4 (Model) Let $\langle \mathcal{W}, \{\mathcal{R}_A | A \in K\} \rangle$ be a frame and v a valuation function from the set of atomic formulas to the powerset of possible worlds. Then we call a structure $\mathfrak{M} = \langle \mathcal{W}, \{\mathcal{R}_A | A \in K\}, v \rangle$ a model.

According to Berto, imagination as mental simulation also obeys a constraint on the initial and output content. He defines content models to account for this:

Definition 5 (Content model) A *content model* is a structure $\mathfrak{M}_C = \langle \mathcal{C}, \otimes, c \rangle$, where $\mathcal{C}$ is a non-empty set of contents and $\otimes$ satisfies for all elements in $\mathcal{C}$:⁶

- $x \otimes x = x$
- $x \otimes y = y \otimes x$
- $(x \otimes y) \otimes z = x \otimes (y \otimes z)$

We define a partial order $\leq$ on $\mathcal{C}$ by setting $x \leq y \Leftrightarrow x \otimes y = y$ for all $x, y \in \mathcal{C}$. We say that x is an atomic topic, or c-atom, iff there is no $y \in \mathcal{C}$, s.t. $(y < x)$. We define the notion of overlap as follows $x \oslash y := \exists z \in \mathcal{C} : z \leq x \wedge z \leq y$. Let $c : Atom \rightarrow \{x \in \mathcal{C} | x \text{ is a c-atom}\}$. We extend c to the set of all formulas as follows: $c(A) = c(B_1) \otimes \dots \otimes c(B_n)$, where $B_i \in Atom(A)$. Fusion $\otimes$ is unrestricted: $\forall xy \in \mathcal{C} \exists z \in \mathcal{C} (z = x \otimes y)$.

(Chellas-)Models and content models combine into imagination models as follows:

Definition 6 (Imagination Model/I-model) Let $\langle \mathcal{W}, \{\mathcal{R}_A | A \in K\}, v \rangle$ be a model, $\langle \mathcal{C}, \otimes, c \rangle$ a content model. We call the structure $\mathfrak{M} = \langle \mathcal{W}, \{\mathcal{R}_A | A \in K\}, \mathcal{C}, \otimes, c, v \rangle$ an imagination model, or I-model. Given an imagination model $\mathfrak{M} = \langle \mathcal{W}, \{\mathcal{R}_A | A \in K\}, \mathcal{C}, \otimes, c, v \rangle$, I write $w \in \mathfrak{M}$ if $w \in \mathcal{W}$.

We define truth at a world in an I-model inductively as follows:

Definition 7 (Truth at a world in a model) Let $\mathfrak{M}$ be an imagination model. Then:

- $\mathfrak{M}, w \Vdash p \Leftrightarrow w \in v(p)$
- $\mathfrak{M}, w \Vdash \neg A \Leftrightarrow \mathfrak{M}, w \not\Vdash A$
- $\mathfrak{M}, w \Vdash A \wedge B \Leftrightarrow \mathfrak{M}, w \Vdash A \ \& \ \mathfrak{M}, w \Vdash B$
- $\mathfrak{M}, w \Vdash A \vee B \Leftrightarrow \mathfrak{M}, w \Vdash A \text{ or } \mathfrak{M}, w \Vdash B$
- $\mathfrak{M}, w \Vdash \Box A \Leftrightarrow \forall w' \in \mathcal{W} : \mathfrak{M}, w' \Vdash A$
- $\mathfrak{M}, w \Vdash A \rightarrow B \Leftrightarrow \forall w' \in \mathcal{W} (\mathfrak{M}, w' \Vdash A \Rightarrow \mathfrak{M}, w' \Vdash B)$
- $\mathfrak{M}, w \Vdash [A]B \Leftrightarrow \forall w' (\mathcal{R}_A w w' \Rightarrow \mathfrak{M}, w' \Vdash B) \ \& \ c(B) \leq c(A)$

⁶Berto restricts the definition to a *finite* non-empty set of contents. But there's no immediate reason for why the set should be finite.

The notions of truth in a model, validity on a frame, and validity on a class of frames are defined as usual. We define $\llbracket A \rrbracket^{\mathfrak{M}} := \{w \in \mathfrak{M} \mid w \Vdash A\}$.⁷

The definition of logical consequence is also standard:

Definition 8 (Consequence and Logical Truth) Let Γ be a set of formulas and A a formula. Then A is a logical consequence of Γ , $\Gamma \models A$, just in case the following holds: for all imagination models $\mathfrak{M}$ and all worlds $w \in \mathfrak{M}$, if for all formulas $B \in \Gamma$, $\mathfrak{M}, w \Vdash B$, then $\mathfrak{M}, w \Vdash A$. If $\Gamma = \emptyset$ and $\emptyset \models A$, then A is a logical truth.

Berto is only concerned with a class of admissible models, however, namely the class of models that satisfy the following basic constraint:

(BC) Let $\mathfrak{M}$ be an I-model. For all $w \in \mathfrak{M}$ and all formulas A , whenever $\mathcal{R}_A w w_1$, then $\mathfrak{M}, w_1 \Vdash A$

Or, in terms of the set-selection function,

(BC) Let $\mathfrak{M}$ be an I-model. For each $w \in \mathfrak{M}$ and each formula A , $f_A(w) \subseteq \llbracket A \rrbracket^{\mathfrak{M}}$

This ensures that “the content of the explicit input is always imagined” [1] because BC entails that $\models [A]A$. In what follows, I will only be concerned with the class of imagination models that satisfy BC.

Note 1 Berto proves each of these [1]:

- $[A](B \wedge C) \models [A]B$ and $[A](B \wedge C) \models [A]C$
- $[A]B \not\models [A](B \vee C)$
- $[A]B, [A]C \models [A](B \wedge C)$
- $[A](B \vee C) \not\models [A]B \vee [A]C$
- $[A]B \not\models [A \wedge C]B$
- $A \rightarrow B \not\models [A]B$
- $\not\models [A \wedge \neg A]B$
- $\not\models [A](B \rightarrow B)$

Note 2 The following holds, too [6]:

- $[A](B \rightarrow C), [A]B \models [A]C$

Not all of these are uncontroversial but discussing them in detail is beyond the scope of this paper. However, for the purposes of this paper, I assume them to be correct. I show below that the content constraints I suggest for Berto’s account do not affect them.

There is another constraint Berto considers to impose on the models, the Principle of Imaginative Equivalents. Let $\mathfrak{M}$ be an imagination model:

(PIE) If $f_A(w) \subseteq \llbracket B \rrbracket$ and $f_B(w) \subseteq \llbracket A \rrbracket$, then $f_A(w) = f_B(w)$.

⁷I sometimes omit the superscript if the model is clear from context.

This validates the following:

Note 3 If PIE holds, then Substitutivity and Special Transitivity hold [1]:

Sub $[A]B, [B]A, [A]C \models [B]C$

ST $[A]B, [A \wedge B]C \models [A]C$

Berto notes that “[i]t may be, however, that there are intuitive counterexamples to ST, forceful enough to lead us to reject PIE”. I show below that some of the available content constraints I suggest don’t validate ST but still validate PIE. Let me now extend the framework to one with a predicate language.

3 Combining Berto’s Sermantics and IBTT

As I have pointed out in the introduction, accounting for content connections between atomic sentences requires a language involving individual symbols and predicates. So, accordingly, I define such a language:

Definition 9 (Alphabet) The language $\mathcal{L}$ has a set of individual symbols $\{c_i\}_{i \in \mathbb{N}}$, a set of relation symbols $\{R_i^j\}_{j > 0, i, j \in \mathbb{N}}$ of arity j and index i , a special binary relation symbol = (equality).⁸ The language $\mathcal{L}$ has the operators $\neg$ (negation), $\wedge$ (conjunction), and $\Box$ (necessity). Additionally, $\mathcal{L}$ has auxiliary symbols brackets $[,]$ and parantheses $(,)$.

For relation symbols R_i , I use R as a metavariable, too.

Speaking of *relation* symbols comprises unary relation symbols, i.e. predicate symbols, too. As can be seen from the definition, there are no nullary relation symbols.⁹

Definition 10 (Terms) The set of terms of $\mathcal{L}$, $\text{TERM}_{\mathcal{L}}$, contains all constant symbols c_i . Metavariables t_i range over terms of $\mathcal{L}$.

Definition 11 (Formulas) Let t_i be terms. The set of formulas of $\mathcal{L}$, $\text{FORM}_{\mathcal{L}}$, is given by the following Backus-Naur form:

$$t_j = t_i \mid R_i^n t_1 \dots t_n \mid \neg A \mid A \wedge B \mid \Box A \mid [A]B$$

Other familiar connectives are defined in the usual way: disjunction $A \vee B := \neg(\neg A \wedge \neg B)$, the material conditional $A \supset B := \neg A \vee B$ and a strict conditional $A \rightarrow B := \Box(A \supset B)$. (Meta)variables A, B, C range over formulas of the formal language $\mathcal{L}$ or sentences of natural language.

⁸I often omit arities and indices. I assume that all index sets are countable, so I omit the membership constraint on indices below.

⁹Sometimes nullary relation symbols are added and then taken as propositional variables. I do not see the need, given we define atomic formulas below.

Definition 12 (Subformulas) The set of subformulas of a formula A , $SF(A)$ is inductively defined as follows:

- $SF(Rt_1...t_n) = \{Rt_1...t_n\}$
- $SF(\circ A) = SF(A) \cup \{\circ A\}$, where $\circ \in \{\neg, \Box\}$
- $SF(A \wedge B) = SF(A) \cup SF(B) \cup \{A \wedge B\}$
- $SF([A]B) = SF(A) \cup SF(B) \cup \{[A]B\}$

The set of atomic subformulas of A is denoted by “ $Atom(A)$ ”.

The semantics has to be adjusted for this language. Following [7], I use a semantics familiar from first-order modal logic [5].

Definition 13 (Frames) A frame is a structure $\mathfrak{F} = \langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A | A \in \mathcal{L}\} \rangle$ where $\mathcal{W} \neq \emptyset$ is a set of (possible) worlds, $\mathcal{D}$ is a function that assigns to each world in $\mathcal{W}$ a domain of objects, each $\mathcal{R}_A \subseteq \mathcal{W} \times \mathcal{W}$ is an accessibility relation between worlds. The domain of the frame, $\mathcal{D}(\mathfrak{F})$, is $\bigcup_{w \in \mathcal{W}} \mathcal{D}(w)$. If $\mathcal{D}$ is a constant function, we say the frame is a *constant domain* frame. If it's not constant, we say the frame is a *varying domain* frame.¹⁰ Variables o_i range over objects from the domain.

Definition 14 (Interpretation) Let $\mathfrak{F}$ be a frame. A *non-rigid interpretation* $\mathcal{I}$ in $\mathfrak{F}$ is a function such that:

1. $\mathcal{I}(w, c_i) \in \mathcal{D}(\mathfrak{F})$ for every c_i
2. $\mathcal{I}(w, R_i^n) \subseteq \mathcal{D}(\mathfrak{F})^n$ for every R_i^n
3. $\mathcal{I}(w, =) = \{ \langle d, d \rangle | d \in \mathcal{D}(\mathfrak{F}) \}$

If there is $o \in \mathcal{D}(\mathfrak{F})$ such that for all $w \in W$ we have that $\mathcal{I}(w, c_i) = o$, then c_i is a *rigid designator*.

I use a non-rigid interpretation function because constants are placeholders for all singular terms, i.e. also non-rigid definite descriptions, and not only proper names [5].¹¹

Definition 15 (Non-rigid Model) Let $\langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\} \rangle = \mathfrak{F}$ be a frame and $\mathcal{I}$ be a non-rigid interpretation. A non-rigid model is a structure $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\}, \mathcal{I} \rangle$. We say that $\mathfrak{M}$ is constant (varying) domain if $\mathfrak{F}$ is constant (varying) domain. $\mathcal{D}(\mathfrak{M}) = \mathcal{D}(\mathfrak{F})$ is the domain of the model.

Definition 16 (Extensions) Let $\mathfrak{M} = \langle \mathcal{D}, \mathcal{W}, \{\mathcal{R}\}, \mathcal{I} \rangle$ be a model and $w \in \mathcal{W}$. The extension of a term t , and a relation R in $\mathfrak{M}$, at a world w , are defined as follows:

1. $\llbracket c \rrbracket_w^{\mathfrak{M}} = \mathcal{I}(w, c)$
2. $\llbracket R^n \rrbracket_w^{\mathfrak{M}} = \mathcal{I}(R, w)$
3. $\llbracket = \rrbracket_w^{\mathfrak{M}} = \mathcal{I}(w, =)$

¹⁰In what follows, I use varying domains, to maintain generality. Formally, nothing depends on this assumption. If one prefers a certain metaphysics of imagined objects, it is possible to change the semantics accordingly by adding an existence predicate [5].

¹¹Adding ι - and ϵ -operators to model definite and indefinite descriptions more fine-grainedly, I leave for further research.

Definition 17 (Intensions) Let $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\}, \mathcal{I} \rangle$ be a model. Then intensions are defined as follows.

1. $\llbracket c \rrbracket^{\mathfrak{M}} = \{ \langle w, o \rangle \in \mathcal{W} \times \mathcal{D}(\mathfrak{F}) \mid o = \llbracket c \rrbracket_w^{\mathfrak{M}} \} = \{ \langle w, o \rangle \in \mathcal{W} \times \mathcal{D}(\mathfrak{F}) \mid o = \mathcal{I}(w, c) \}$
2. $\llbracket R \rrbracket^{\mathfrak{M}} = \{ \langle w, \langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_w^{\mathfrak{M}} \rangle \in \mathcal{W} \times \mathcal{D}(\mathfrak{F})^n \mid \langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_w^{\mathfrak{M}} \rangle \in \llbracket R \rrbracket_w^{\mathfrak{M}} \}$
3. $\llbracket = \rrbracket^{\mathfrak{M}} = \{ \langle w, \langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \llbracket t_2 \rrbracket_w^{\mathfrak{M}} \rangle \in \mathcal{W} \times \mathcal{D}(\mathfrak{F})^2 \mid \langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \llbracket t_2 \rrbracket_w^{\mathfrak{M}} \rangle \in \llbracket = \rrbracket_w^{\mathfrak{M}} \}$

Variables F and G range over intensions of relation symbols, variables i_i and i_j range over intensions of terms.

In a Carnapian spirit, and following Hawke, I call intensions of relation symbols, including the identity symbol, *general concepts*, and intensions of terms *individual concepts*.

Before going on showing that IBTT allows us to solve the initial dilemma, one might wonder if we could already solve the problem as follows.¹² Consider content models in which the elements of $\mathcal{C}$ are sets of relation symbols and terms. Then we define $c(Rt_1 \dots t_n) = \{R, t_1, \dots, t_n\}$ and $c(A) = \bigcup_{B \in \text{Atom}(A)} c(B)$. Relations Q^s and Q^o are be define as follows: $Q^s(c(A), c(B))$ iff $c(A) \subseteq c(B)$ and $Q^o(c(A), c(B))$ iff $c(A) \cap c(B) \neq \emptyset$. Then, however, $c(Rab) = c(Rba)$, which is not correct because “my dog bites my pillow” has a different content than “my pillow bites my dog”. Rather than taking *sets* of terms and relation symbols, one could consider ordered tuples. So, $c(Rt_1 \dots t_n) = \langle R, t_1, \dots, t_n \rangle$. But then $c(\text{Tom is a bachelor}) = c(Bt) = \langle B, t \rangle$ and $c(\text{Tom is an unmarried man}) = c(Ut) = \langle U, t \rangle$ come out as different because $U \neq B$. Of course, *semantically*, being an unmarried man and being a bachelor are identical. So one might argue that they should be represented by the same (unary) relation symbol. This puts the cart before the horse, though, because the semantics of these should be reflected, well, in the semantics. This runs the natural language sentence through a semantic filter that gives us identical formulas. Instead, it should be the different formulas, which receive the same interpretation in the semantics.¹³ So, instead of considering syntax only, could we consider the content of a formula to be world-relative and defined in terms of extensions, e.g., $c(w, Rt_1 \dots t_n) = \langle \mathcal{I}(w, R), \mathcal{I}(w, t_1), \dots, \mathcal{I}(w, t_n) \rangle$? Then we’d need a function that assigns to each world-formula pair a world-relative content, namely a function $f : (w, Rt_1 \dots t_n) \mapsto \langle \mathcal{I}(w, R), \mathcal{I}(w, t_1), \dots, \mathcal{I}(w, t_n) \rangle$ (and then extend it to complex formulas). This is the idea behind the issue-based theory of topics. The issue of an atomic formula is going to be a function that assigns to each world-formula pair the tuple consisting of the extension of the relation symbol at the world and the extensions of the terms at the world.

3.1 Issue-Based Theory of Topics

Berto leaves it unspecified what exactly counts as an atomic content, and so what exactly the contents of formulas *are*. The role contents play in Berto’s logic can be

¹²Thanks to an anonymous reviewer for raising this issue and providing the initial example that is to follow.

¹³We can run a similar example with proper names with the same extension like “Samuel Clemens” and “Mark Twain”.

taken over by Hawke's topics in the case for $\mathcal{L}$. First, I introduce the notion of an *issue*. In what follows, an issue is informally thought of as the question whether certain objects stand in a certain relation, or whether a single object has a certain property. Objects are designated by terms and relations are designated by relation symbols. Unary relation symbols designate properties. An issue divides logical space into answers to the question. Given the formal tools, the idea of an issue being a question can be made formally precise:

Definition 18 (Issue) [7, p.16, notation adjusted] Let $\mathfrak{M}$ be a model. An *issue* is a tuple $\langle \llbracket R \rrbracket^{\mathfrak{M}}, \llbracket t_1 \rrbracket^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket^{\mathfrak{M}} \rangle$, where $\llbracket R \rrbracket^{\mathfrak{M}}$ is an n -ary general concept and each $\llbracket t_k \rrbracket^{\mathfrak{M}}$ is an individual concept. For a formula, $Rt_1 \dots t_n$, we say that $\langle \llbracket R \rrbracket^{\mathfrak{M}}, \llbracket t_1 \rrbracket^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket^{\mathfrak{M}} \rangle$ is the issue associated with the formula under $\mathfrak{M}$. Note that since our language is infinite, the set of issues could also be infinite (unless we assume that after an arbitrarily large finite number of relation symbols and constants, the further symbols keep associating to all previously associated concepts). I use $\langle F, i_1, \dots, i_n \rangle$ and $\langle G, j_1, \dots, j_m \rangle$ as placeholders for issues without committing to particular intensions of particular relation symbols or individual constants. Thus, F (G) is a placeholder for a general concept, and each i_k (j_l) is a placeholder for an individual concept. Variables I and J are variables ranging over issues.

Each issue $\langle \llbracket R \rrbracket^{\mathfrak{M}}, \llbracket t_1 \rrbracket^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket^{\mathfrak{M}} \rangle$ generates a partition of logical space because, given world w , it is either the case that

$$\langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_w^{\mathfrak{M}} \rangle \in \llbracket R \rrbracket_w^{\mathfrak{M}}$$

or that

$$\langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_w^{\mathfrak{M}} \rangle \notin \llbracket R \rrbracket_w^{\mathfrak{M}}.$$

Consequently, given an issue $\langle \llbracket R \rrbracket^{\mathfrak{M}}, \llbracket t_1 \rrbracket^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket^{\mathfrak{M}} \rangle$, two worlds u and v are equivalent with respect to the issue if it is the case that

$$\langle \llbracket t_1 \rrbracket_u^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_u^{\mathfrak{M}} \rangle \in \llbracket R \rrbracket_u^{\mathfrak{M}}$$

iff

$$\langle \llbracket t_1 \rrbracket_v^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_v^{\mathfrak{M}} \rangle \in \llbracket R \rrbracket_v^{\mathfrak{M}}$$

Definition 19 (Topic) Each set of issues is a *topic*. The empty set is the smallest topic. The set of all issues is the biggest topic, *Issue*, and it is possibly infinite. Given topics $\mathbf{s}$, $\mathbf{t}$, we say that $\mathbf{s}$ is included in $\mathbf{t}$ just in case $\mathbf{s} \subseteq \mathbf{t}$. We say that $\mathbf{s}$ and $\mathbf{t}$ overlap, $\mathbf{s} \bowtie \mathbf{t}$ just in case $\mathbf{s} \cap \mathbf{t} \neq \emptyset$. Variables $\mathbf{s}$ and $\mathbf{t}$ range over topics.

Also topics partition logical space: two worlds are equivalent with respect to a topic $\mathbf{s}$ just in case they are equivalent with respect to all issues in $\mathbf{s}$. We call the partition generated by $\mathbf{s}$ the resolution of $\mathbf{s}$ [7].

We define an issue-based topic model, or IBTT-model, as follows.

Definition 20 (IBTT topic model) Let $\mathfrak{M}$ be a non-rigid model, and Topics the set of all topics, i.e., the set of all sets of issues (i.e., the powerset of the set of all issues), given $\mathfrak{M}$. The triple $\langle \text{Topics}, \cup, \mathsf{T} \rangle$ with $\mathsf{T} : \text{FORM}_{\mathcal{L}} \rightarrow \text{Topics}$, defined as follows

$T(Rt_1...t_n) = \{\langle \llbracket R \rrbracket^{\mathfrak{M}}, \llbracket t_1 \rrbracket^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket^{\mathfrak{M}} \rangle\}$, and for non-atomic formulas A , $T(A) = \bigcup_{B \in Atom(A)} T(B)$ is an IBTT topic model, or IBTT model, for short. Note that the partial order on topics, $\leq$, will correspond to set inclusion, $\subseteq$.

As mentioned, topics in the present setting will play the role of contents in Berto's logic:

Note 4 Each IBTT topic model is a content model

Note that infinite topic models are also join-semilattices because every non-empty finite subset of topics has a join in the model, namely its union.

Note 4 is not to claim that Hawke takes the content of a sentence to be its topic. Rather, it seems, Hawke would identify the content of a sentence with a pair composed of the topic of the sentence and the truth-condition for the sentence [8]. For present purposes, sentences are about their topics. Since the connection between the topics of the formula that expresses what is initially imagined and of the formula that expresses the output imagining matters for the truth condition of the imagination operator, topic models and models are combined into IBTT-imagination models:

Definition 21 (IBTT-Imagination model) Let $\langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\}, \mathcal{I} \rangle$ be a non-rigid model and $\langle \text{Topics}, \cup, T \rangle$ an IBTT model. An IBTT-imagination model is a structure $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\}, \mathcal{I}, \text{Topics}, \cup, T \rangle$.

This now allows to define truth at a world in an IBTT-imagination model, taking into account IBTT and Berto's initial definition:

Definition 22 (Truth at a world in an imagination model) The truth of a formula A at a world w in an IBTT-imagination model $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\}, \mathcal{I}, \text{Topics}, \cup, T \rangle$ can be inductively defined. Let $\ll$ be a binary relation on topics in an IBTT model $\langle \text{Topics}, \cup, T \rangle$.

- $\mathfrak{M}, w \Vdash Rt_1...t_n \iff \langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \dots, \llbracket t_n \rrbracket_w^{\mathfrak{M}} \rangle \in \llbracket R \rrbracket_w^{\mathfrak{M}}$
- $\mathfrak{M}, w \Vdash t_1 = t_2 \iff \langle \llbracket t_1 \rrbracket_w^{\mathfrak{M}}, \llbracket t_2 \rrbracket_w^{\mathfrak{M}} \rangle \in \llbracket = \rrbracket_w^{\mathfrak{M}}$
- $\mathfrak{M}, w \Vdash \neg A \iff \mathfrak{M}, w \not\Vdash A$
- $\mathfrak{M}, w \Vdash A \wedge B \iff \mathfrak{M}, w \Vdash A \ \& \ \mathfrak{M}, w \Vdash B$
- $\mathfrak{M}, w \Vdash \Box A \iff \forall w' \in \mathcal{W} : \mathfrak{M}, w' \Vdash A$
- $\mathfrak{M}, w \Vdash [A]B \iff \forall w_1 (\mathcal{R}_A w w_1 \Rightarrow \mathfrak{M}, w_1 \Vdash B) \ \& \ T(B) \ll T(A)$

There are various candidates for the relation $\ll$ and each of these candidates defines a class of imagination models.¹⁴ I introduce the candidate relations in Section 3.4.

¹⁴Thanks to an anonymous reviewer for suggesting this way of presentation.

Following Berto, we also add the condition that all the $\mathcal{R}_A$ accessible worlds must be worlds where A is true:

Definition 23 (Admissible models) Let $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \{\mathcal{R}_A\}, \mathcal{I}, \text{Topics}, \cup, \text{T} \rangle$ be an IBTT-imagination model. It is admissible if it satisfies:

(BC') for every formula $A \in \text{FORM}_{\mathcal{L}}$ and for all worlds $w, w_1 \in \mathcal{W}$, if $\mathcal{R}_A w w_1$, then $\mathfrak{M}, w_1 \models A$.

This condition has the following consequence. Consider the case where we have A be Rc_1c_2 and c_1 is not rigid. Then in imagining Rc_1c_2 , the agent might access worlds via $\mathcal{R}_{Rc_1c_2}$ where Rc_1c_2 is true for different reasons because different objects stand in the relation R . For example, one can imagine that the president of the United States (POTUS) meets Angela Merkel. The accessed worlds can include a world where Trump meets Merkel but also one where Obama meets Merkel. Note that the converse of (BC) is not required, i.e. one doesn't have to $\mathcal{R}_A$ -access *all* the worlds where A is true. Hence, to imagine that the POTUS meets Merkel, one doesn't have to imagine for each POTUS that they meet with Merkel, although one could.

From here on, I simply speak of models, and mean IBTT-imagination models in the sense of Definition 21 and which satisfy (BC').

3.2 An Issue for IBTT

I have not yet specified the relation between the topics in the truth condition for $[A]B$. As I have argued before, inclusion is too draconian, and overlap validates the unwanted $[A]B \models [A](B \vee C)$. So, what else is at our disposal? Consider the following example, given a model $\mathfrak{M}$:¹⁵

Example 1 Consider:

- (S1) Gwenny is at her favourite lake.
- (S2) Gwenny swims in her favourite lake.
- (TS1) $\text{T}(\text{"Gwenny is at her favourite lake"}) =$
 $\{\langle \llbracket is (located) at \rrbracket^{\mathfrak{M}}, \llbracket Gwenny \rrbracket^{\mathfrak{M}}, \llbracket Gwenny's favourite lake \rrbracket^{\mathfrak{M}} \rangle\}$
- (TS2) $\text{T}(\text{"Gwenny swims in her favourite lake"}) =$
 $\{\langle \llbracket swims in \rrbracket^{\mathfrak{M}}, \llbracket Gwenny \rrbracket^{\mathfrak{M}}, \llbracket Gwenny's favourite lake \rrbracket^{\mathfrak{M}} \rangle\}$

Note that neither the *topic* of (S1) nor the topic of (S2) is included in the other and the *topics* do not overlap. The topics don't overlap because the issues included in them are different for they involve different general concepts.

Moreover, intuitively speaking, (S1) and (S2) are both *about Gwenny* (and her favourite lake). This can be accounted for in IBTT as follows: given an issue I in

¹⁵Here, the italics within the intension brackets indicate that it is an intension of an expression.

a model $\mathfrak{M}$ and an object o , we say that $I = \langle F, i_1, \dots, i_n \rangle$ *concerns o exactly* just in case there is a *rigid designator* t such that for some j , $i_j = \llbracket t \rrbracket^{\mathfrak{M}}$ and *for every* $w \in \mathscr{W}$, $\llbracket t \rrbracket_w^{\mathfrak{M}} = o$ [7, p. 17, notation adjusted].¹⁶ A sentence A is exactly about an object o if there is an issue in the topic of A that concerns o exactly.

Analogously, we can say that an issue $I = \langle F, i_1, \dots, i_n \rangle$ concerns a relation (or property), just in case there is a general term T such that $F = \llbracket T \rrbracket^{\mathfrak{M}}$. A topic $\mathbf{t}$ is about the relation or property R just in case some issue in $\mathbf{t}$ concerns R . A sentence is about R just in case its topic is.

In the example, we also have a non-rigid designator, “Gwenney’s favourite lake”. The definition from before can be extended as follows: given an issue I in a model $\mathfrak{M}$ and an object o , we say that $I = \langle F, i_1, \dots, i_n \rangle$ *concerns o somehow* just in case there is a term t such that for some j , $i_j = \llbracket t \rrbracket^{\mathfrak{M}}$ and *for some* $w \in \mathscr{W}$, $\llbracket t \rrbracket_w^{\mathfrak{M}} = o$. A sentence is somehow about o if there is an issue in the topic of A that concerns o somehow. A sentence is about o if it is exactly or somehow about o .

We can then say that (S1) and (S2) are both about Gwenney (or her favourite lake) because they both intersect with the topic T (“Gwenney”) (T (“Gwenney’s favourite lake”). Hence, sentences are about topics, terms are about topics, and sentences and terms can be about the same topics.

Which solution to choose? Based on linguistic intuition, it seems to me that (S1) and (S2) are about the same *object* but not about the same *topic*, in an intuitive understanding of “topic”. Although the robustness of this intuition should be tested empirically, it seems common among others working on aboutness. For instance, the approaches discussed by Hawke [7] all seem to share this intuition in one way or another. Yablo explains this by appealing to subject matters of singular terms. On his account, these are not objects but equivalence classes and so sentences can’t be about objects, strictly speaking [14]. This is revisionary of natural language uses. When a student passes an exam and I say “You passed, you’re a great student”, it matters to them that the sentence is *about them*. And it is entirely correct for them to say “My teacher said *about me* that I am a great student”. They feel proud because *they* passed and I said *about them* that they’re are a great student. If I said only something about an equivalence class, as it would be on Yablo’s account, it is not clear how this emotion can be explained in terms of aboutness. Finally, the fact that Hawke introduces the notion of “concerning an object exactly” shows that he also thinks that there is a notion of aboutness in natural language that corresponds to sentences being about the same object. So, overall, I think that the first solution is more appropriate to explain why S1 and S2 are both about Gwenney.

As was pointed out above, whenever Gwenney swims in the lake, she is at the lake. There is a conceptual relation between the general concepts involved in TS1 and TS2, namely $\llbracket \text{swims in} \rrbracket$ is set-theoretically included in $\llbracket \text{is at} \rrbracket$ because whenever a swims in b then a is also (located) at b . So, $\llbracket \text{swims in} \rrbracket \subseteq \llbracket \text{is at} \rrbracket$. This is the motivating idea for the various relations between issues, and, consequently, topics, about to be addressed in the next subsection.

¹⁶This is not exactly what Hawke defines because in his model, there is also a mereology defined on the domain of objects. I ignore this mereology here for simplicity.

3.3 Relations Between Issues

From here on, I first define several relations between issues and from that relations between topics. I show that the relations can be partially ordered, and that there is a strongest and a weakest relation. I also investigate some familiar properties of the relations, such as reflexivity, symmetry, and transitivity. This allows to interpret some of the relations as different similarity relations (i.e. reflexive, symmetric, non-transitive relations). I will only consider non-empty issues and non-empty topics.

Just like general concepts, individual concepts can be included in each other. For example, $\llbracket \text{Mark Twain} \rrbracket^{\mathfrak{M}} = \llbracket \text{Samuel Langhorne Clemens} \rrbracket^{\mathfrak{M}}$ if we assume that proper names are rigid designators. We can also have proper inclusions, for example, $\llbracket \text{the last woman on Earth} \rrbracket^{\mathfrak{M}} \subset \llbracket \text{the last human on Earth} \rrbracket^{\mathfrak{M}}$, assuming essentialism about (natural) kinds. Of course, there can also be mere overlap. For example, say Gwenny's favourite lake is Titicaca lake in the actual world @. Then the pair $\langle @, \text{Titicaca lake} \rangle$ is in both $\llbracket \text{Titicaca lake} \rrbracket^{\mathfrak{M}}$ and $\llbracket \text{Gwenny's favourite lake} \rrbracket^{\mathfrak{M}}$ but, clearly, Gwenny's favourite lake could change in the future to, say, Loch Lomond. Since issues can feature several individual concepts, it can be that, given two issues I and J , some individual concepts in I are included in some individual concepts in J , all individual concepts in I are included in some individual concepts in J , etc. and similarly for the case of overlap. This motivates the following two definitions:¹⁷

Definition 24 (Issues - Conceptual Inclusion) Let the tuples $I = \langle F, i_1, \dots, i_n \rangle$ and $J = \langle G, j_1, \dots, j_m \rangle$ be issues. Then the following notions of issue I being included in issue J , $I \sqsubseteq J$, are definable:

1. $F \subseteq G$ and $\exists k \exists l : i_k \subseteq j_l$
2. $F \subseteq G$ and $\forall k \exists l : i_k \subseteq j_l$
3. $F \subseteq G$ and $\exists k \forall l : i_k \subseteq j_l$
4. $F \subseteq G$ and $\forall k \forall l : i_k \subseteq j_l$
5. $F \subseteq G$ and $\forall k : i_k \subseteq j_k$

By $I \sqsubseteq_i J$, I mean that the issues I and J are related according to clause i of Definition 24.

Definition 25 (Issues - Conceptual Overlap) Let the tuples $I = \langle F, i_1, \dots, i_n \rangle$ and $J = \langle G, j_1, \dots, j_m \rangle$ be non-empty issues. We can define the following relations of overlap between I and J , $I \propto J$:

1. $F \cap G \neq \emptyset$ and $\exists k \exists l : i_k \cap j_l \neq \emptyset$
2. $F \cap G \neq \emptyset$ and $\forall k \exists l : i_k \cap j_l \neq \emptyset$.
3. $F \cap G \neq \emptyset$ and $\exists k \forall l : i_k \cap j_l \neq \emptyset$.
4. $F \cap G \neq \emptyset$ and $\forall k \forall l : i_k \cap j_l \neq \emptyset$.
5. $F \cap G \neq \emptyset$ and $\forall k : i_k \cap j_k \neq \emptyset$

¹⁷Of course, there are also mixed relations where we have overlap of general concepts and inclusion of individual concepts, etc. One might also define relations where either conjunct is negated. I leave those for further investigation.

By $I \infty_i J$ I mean that the issues are related according to clause i of Definition 25.

We summarize the definitions again in a table:

	Overlap $I \infty J$	Inclusion $I \sqsubseteq J$
1.	$F \cap G \neq \emptyset$ and $\exists k \exists l : i_k \cap j_l \neq \emptyset$	$F \subseteq G$ and $\exists k \exists l : i_k \subseteq j_l$
2.	$F \cap G \neq \emptyset$ and $\forall k \exists l : i_k \cap j_l \neq \emptyset$	$F \subseteq G$ and $\forall k \exists l : i_k \subseteq j_l$
3.	$F \cap G \neq \emptyset$ and $\exists k \forall l : i_k \cap j_l \neq \emptyset$	$F \subseteq G$ and $\exists k \forall l : i_k \subseteq j_l$
4.	$F \cap G \neq \emptyset$ and $\forall k \forall l : i_k \cap j_l \neq \emptyset$	$F \subseteq G$ and $\forall k \forall l : i_k \subseteq j_l$
5.	$F \cap G \neq \emptyset$ and $\forall k : i_k \cap j_k \neq \emptyset$	$F \subseteq G$ and $\forall k : i_k \subseteq j_k$

One limitation of the approach is the following:

Note 5 Given issues $I = \langle F, i_1, \dots, i_n \rangle$ and $J = \langle G, j_1, \dots, j_m \rangle$, $F \cap G \neq \emptyset$ implies that the expressions corresponding to F and G have the same arity, i.e. $n = m$.

This is a limitation because “Gweny runs around the house” and “Gweny runs” are not only related in terms of topic but also in terms of concepts. But since “runs around” and “runs” have different arity, there is no such connection on the formal level.¹⁸ Since issues are n -tuples, this is also a problem for Hawke’s original account, i.e., topic s is included in topic t only if for each issue I in s there is an issue J in t of the same arity (and J is identical to I).

The relations between the issues can be ordered amongst each other by their respective strength. Before considering those, it is worth noting that we can define the initial notions of topic inclusion and topic overlap in terms of the conceptual inclusion relations. First note that:

Note 6 $I \sqsubseteq_5 J$ and $J \sqsubseteq_5 I$ together imply that $I = J$.

This allows to prove the following:

Proposition 1 Let s and t be topics. $s \subseteq t$ iff $\forall I \in s \exists J \in t : I \sqsubseteq_5 J$ and $J \sqsubseteq_5 I$.

Proof Suppose the right side. Then every issue I in s is such that there is some issue J in t , such that the general concepts of I and J include each other. Moreover, each of the k -th individual concept in I will be identical to the k -th in J and vice versa. Hence, the issues must be identical because they have the same members.

¹⁸The limitation can be overcome by allowing to represent n -ary relations ($n > 1$) as unary relations by not replacing all singular terms with constants. Alternatively, we could work with a λ -operator and turn each formula into an atomic formula before considering its issues. I leave this for a different paper and accept the limitation, while allowing there to be more than unary relations/general concepts.

Suppose the left side. Then each issue in s is identical to some issue in t . Let I and J be such an arbitrary pair. Since they are identical, their general concepts are identical, and each k -th individual concept of I will be identical to the k -th individual concept of J and vice versa. Thus $\sqsubseteq_5$ holds in both directions between I and J . $\square$

Similarly, it holds that:

Proposition 2 *Let s and t be topics. Then $s \bowtie t$ iff $\exists I \in s \exists J \in t I \sqsubseteq_5 J$ and $J \sqsubseteq_5 I$*

Proof Suppose the right hand side. Then by Note 6, $J = I$. Hence, $s \cap t \neq \emptyset$. Suppose $s \bowtie t$. Then $\exists I : I \in s$ and $I \in t$. But then $\exists I \in s \exists J \in t : I = J$, which entails the respective inclusions. $\square$

There is an order among the various relations between issues, which can be lifted to an order among the various relations between topics. This reduces the amount of cases needed to be checked in some proofs below. As for the various relations among issues, the following holds:

Proposition 3 *Let $\circ_i \in \{\infty_i, \sqsubseteq_i \mid i \in \{1, 2, 3, 4, 5\}\}$. Then the following relations hold if we fix $\circ$. An arrow from i to j indicates that relation i implies relation j , i.e., whenever issues I and J are i -related, they are also j -related:*

Proof Each case follows from applying the definitions, simple first-order logic, and simple set theory. The important thing here is that issues are assumed to be non-empty, which guarantees that $\forall i \dots$ implies $\exists i \dots$. $\square$

So, relation $\infty_4 / \sqsubseteq_4$ is the strongest, and relation $\infty_1 / \sqsubseteq_1$ is the weakest.

Note 7 Note that $\sqsubseteq_4 \subseteq \infty_4$ and thus $\sqsubseteq_4$ is the strongest relation.

3.4 Relations Between Topics

Given the various relations between issues, it is possible to define various relations among topics. These relations between topics have/lack certain properties, e.g., reflexivity, symmetry, transitivity. This indicates whether there is, e.g., a relation of similarity between topics. Moreover, the relations among topics can also be ordered by strength.

Let $\mathbf{s} = \{I_1, \dots, I_n\}$ and $\mathbf{t} = \{J_1, \dots, J_m\}$ be non-empty topics and each I_i and J_j be non-empty issues of the form $\langle F, i_1, \dots, i_n \rangle$ and $\langle G, j_1, \dots, j_m \rangle$ respectively.¹⁹ Then we can define the following relations between contents in terms of the relations between their issues:

Definition 26 (Topic-Relations) Let $\mathbf{s}$ and $\mathbf{t}$ be topics. Let $Q_1, Q_2 \in \{\forall, \exists\}$ and $\circ_i \in \{\subseteq_i, \infty_i \mid i \in \{1, 2, 3, 4, 5\}\}$. Then for all $i \in \{1, 2, 3, 4, 5\}$, we can define the relation $\mathbf{s} \ll_{Q_1 Q_2 \circ_i} \mathbf{t}$ iff $Q_1 I \in \mathbf{s} Q_2 J \in \mathbf{t} : I \circ_i J$.

So, “ $\ll_{\forall\forall\infty_1}$ ” refers to the relation defined by $\mathbf{s} \ll_{\forall\forall\infty_1} \mathbf{t} \Leftrightarrow \forall I \in \mathbf{s} \forall J \in \mathbf{t} : I \infty_1 J$. The relations between issues from Proposition 3 lift to the relations between topics, i.e.:

Proposition 4 The following inclusions hold if we fix $Q_1, Q_2, \circ_i$ (an arrow from i to j indicates that relation i is a subset of relation j):

Proof Again this follows from the definitions, simple first order logic, and simple set theory. Again, it is important that topics (and issues) are non-empty. $\square$

We can now characterise different topic models based on the relation we define on them. Let $Q_1, Q_2 \in \{\forall, \exists\}$ and $\circ_i \in \{\subseteq_i, \infty_i \mid i \in \{1, 2, 3, 4, 5\}\}$. Given topic model $\langle \text{Topics}, \cup, \text{T} \rangle$ on which we define the relation $\ll_{Q_1 Q_2 \circ_i}$, we call this a $Q_1 Q_2 \circ_i$ -topic model. So, Proposition 4 actually tells us, that, for instance, all $\forall\exists \subseteq_4$ -topic models are also $\forall\exists \subseteq_1$ -topic models. This also lifts to relations between classes of IBTT-imagination models. Note that the proposition does not tell us anything about the relation between models with different permutation in the quantifiers quantifying over issues or differing in the relation between issues, i.e., it doesn't tell us anything about the relation between $\forall\exists \subseteq_4$ -topic models and, say, $\exists\forall \subseteq_4$ -models, or $\forall\exists\infty_2$ -models.

It is still open what the right relation between topics should be in the truth condition for the imagination operator. Since set inclusion between topics is too strong, we need a weaker kind of relation between topics. The relations between topics might have some properties that make them more suitable than others.

Note 8 The following hold:

¹⁹In what follows, topics and issues are taken to be non-empty.

1. All relations in Definition 26 for overlap and inclusion are reflexive. (Obvious)
2. All relations defined in terms of overlap in Definition 26 are not transitive. (Since overlap between concepts isn't transitive)
3. All relations defined in terms of inclusion in Definition 26 are not symmetric. (Because inclusion is not symmetric)

Moreover, we have the following results, where “sym” stands for symmetry, “a-s” stands for anti-symmetry, and “trans” for transitivity (none of these require sophisticated proofs but just applying the respective definitions):

		$\forall\forall$	$\forall\exists$	$\exists\forall$	$\exists\exists$
∞	1	sym	not sym	not sym	sym
	2	sym	not sym	not sym	not sym
	3	not sym	not sym	not sym	not sym
	4	sym	not sym	not sym	sym
	5	sym	not sym	not sym	sym
$\sqsubseteq$	1	not trans	not trans	not trans	not trans
	2	trans	not trans	trans	not trans
	3	trans	trans	trans	not trans
	4	trans	trans, a-s	trans	not trans
	5	trans	trans, a-s	trans	not trans

3.5 Truth Condition for Imagination Operator

Which of the previously defined relations is the right one for the truth-condition of the imagination operator?²⁰ That is, which class of imagination models should we consider? This depends on what kind of use the imagination is put to. There are transcendent and instructive uses of the imagination, where transcendent uses are those where “[i]magination is [...] used to enable us to escape or look beyond the world as it is, as when we daydream or fantasize or pretend” and instructive uses are those where “imagination is [...] used to enable us to learn about the world as it is, as when we plan or make decisions or make predictions about the future” [10, p. 1].

Even in transcendental uses of the imagination, however, some connection must exist between the initial content and the output content. Even if we sometimes imagine something for which we have no concepts (or, at least, something we cannot describe with language) we still use language expressing familiar concepts to approximate it. Moreover, there is always a conceptual background upon which the imagining takes place. Consider, for example, works by H.P. Lovecraft whose short stories and novels often aim to describe the incomprehensible. They succeed by using stylistic devices and terminology we are familiar with. They present existing concepts

²⁰In the following when I consider, e.g., the relation $\ll_{\forall\forall\infty_1}$, I consider that $s \ll_{\forall\forall\infty_1} t$ holds where s is the output content and t is the initial content.

in new relations to each other in a particular context, thereby creating the uncanny feeling characteristic of these stories, and for which we often lack the concepts or language to express it directly. The access we have to these situations, for which we might have no exact concepts, is through approximation by familiar concepts. On the present account, if imagination is used transcendently, this is captured by connecting some of the concepts in some of the issues from the output content to some of the concepts of some of the issues in the initial content by overlap. That is, the contents are connected by the weakest relations among topics, that is, relations from $\ll\exists\exists\infty_i$.

Consider the simple case of initial content $\mathbf{t} = \{J\}$ and output content $\mathbf{s} = \{I\}$ where $I = \langle F, i_1, \dots, i_n \rangle$ and $J = \langle G, j_1, \dots, j_m \rangle$. Suppose that this is a transcendental use of imagination, for example, writing a story and coming up with some new characters. So $j_1, \dots, j_m$ are the individual concepts associated with the characters one has already imagined and who are involved in some action, associated with general concept G . For example, Harry and Ron meeting in the train to Hogwarts. Now one is imagining a new situation, possibly involving some of the previous characters but also new ones in a slightly different situation. For example, imagining Harry and Hermione meeting in the train to Hogwarts.²¹

Clearly the relation $\ll\exists\exists\infty_4$ would be too strong. It requires that all individual concepts overlap with each other, which is rarely the case for any use of the imagination and certainly not in the mentioned example. Also $\ll\exists\exists\infty_5$ seems too strong a requirement for transcendental uses of imagination because it excludes one from imagining something involving any new individual concepts. But this is exactly the case in the example. Requiring that some individual concept must overlap all the ones involved in the initial content ($\ll\exists\exists\infty_3$) also seems way too restrictive for transcendental uses. Clearly, the individual concepts associated with new characters might not overlap any of the already imagined ones, just like in our example. A similar case can be made for the relation $\ll\exists\exists\infty_2$. So, the weakest relation $\ll\exists\exists\infty_1$ suggests itself. Of course, if one introduces new characters, one can do so by considering a whole new scene in which there is no overlap in any of the concepts at all. This, however, is a case of choosing a new initial content.

If imagination is used instructively, the connection between the contents must be tighter. But even instructive uses of the imagination usually do not require that the output content is already fully contained in the initial content (contrary to Berto's requirement). Instructive uses do require, however, that the output content is not too far off from the initial content either [9, 13]. That is, whatever we end up imagining, must somehow be connected to what we started with. This suggests that instructive uses of the imagination require one of the relations between topics among $\ll\forall\exists\sqsubseteq_i$ or $\ll\forall\exists\sqsupseteq_i$. So, these relations seem natural candidates to be considered for the "rational imagination" Berto aims to model, and which he takes it to be reality-oriented mental simulation. Below, I investigate these relations with respect to the (in)validities Berto discusses. Based on these results I argue that relation $\ll\forall\exists\sqsubseteq_1$ seems to be the best

²¹Since the presented account is restricted to concepts with the same arity, there is a restriction on the kind of examples. I take this example as a simple but realistic one, however. Moreover, it seems an instance of a typical situation where new characters meet, which in turn is a typical example of transcendental uses of imagination.

candidate for Berto to be featured in the truth condition for the imagination operator because it is the only plausible candidate that also still satisfies (Sub) and (ST), see Proposition 8. Moreover, consider Example 1 again. It is easily checked that $TS2 \ll_{\forall\exists\sqsubseteq_1} TS1$.

Before I go on, a disclaimer: the distinction between transcendental and instructive uses is a vague one. While we have clear cases of transcendental uses like free-floating creative imagination and clear cases of instructive uses like rational planning, many uses of the imagination seem to be both. For example, the initial content of a story can be highly transcendental and take us to a world very different from ours. Nevertheless, the story might teach us much about human nature or interpersonal relations that we consider universal and thus relevant to our world. This points to a further distinction. Namely that we can consider the input or output contents themselves as transcendental or instructive but also the way we got from the initial to the output content can be transcendental or instructive. The discussion in the previous paragraphs of this section was concerned with the latter. Since the distinction between transcendental and instructive uses is vague, giving *the* truth condition for the imagination operator is not possible. We have to consider the respective use of the imagination, and see whether it complies with a plausible connection between topics. I take it that the selection of candidates I have presented offers a first starting point.

Now, I present how the respective relations from $\ll_{\forall\exists\sqsubseteq_i}$ or $\ll_{\forall\exists\infty_i}$ fare with respect to the (in)validities and principles Berto discusses.

Proposition 5 *For each member j of $\ll_{\forall\exists\sqsubseteq_i}$, if the relation in the truth condition for $[A]B$ is defined by relation j and for each member j of $\ll_{\forall\exists\infty_i}$ if the relation in the truth condition for $[A]B$ is defined by relation j , the following hold:*

1. $[A](B \wedge C) \models [A]B$ and $[A](B \wedge C) \models [A]C$
2. $[A]B \not\models [A](B \vee C)$
3. $[A]B, [A]C \models [A](B \wedge C)$
4. $[A](B \vee C) \not\models [A]B \vee [A]C$
5. $[A]B \not\models [A \wedge C]B$
6. $A \rightarrow B \not\models [A]B$
7. $\not\models [A \wedge \neg A]B$
8. $\not\models [A](B \rightarrow B)$
9. $[A](B \rightarrow C), [A]B \models [A]C$

Proof Here, I am only concerned with the set of relations defined by the $\forall\exists$ ordering of quantifiers. Because of Proposition 4 and Note 7 it suffices to show that the above validity claims hold if the truth condition for $[A]B$ is defined using the relation $\ll_{\forall\exists\infty_1}$. For the invalidity claims, it suffices to provide a counterexample in which the relation between topics in the truth condition for $[A]B$ is defined in terms of $\ll_{\forall\exists\sqsubseteq_4}$. Thus, when proving validity claims, it is assumed that the truth condition for $[A]B$ is given in terms of $\ll_{\forall\exists\infty_1}$. In the case of the counterexamples, it is assumed that the truth condition for $[A]B$ is given in terms of $\ll_{\forall\exists\sqsubseteq_4}$.

1. The case for the first conjunct of the truth condition is obvious. Suppose $T(B \wedge C) \ll_{\forall\exists\infty_1} T(A)$. Then $T(B) \ll_{\forall\exists\infty_1} T(A)$ because $T(B) \subseteq T(B \wedge C)$.

2. It is straightforward to construct a counterexample based on the one by [1, p. 1880], adjusting it for the case of a predicate language.
3. The case for the first conjunct of the truth condition is obvious. We suppose that $T(B) \ll_{\forall\exists\infty_1} T(A)$ and $T(C) \ll_{\forall\exists\infty_1} T(A)$. Since $T(B \wedge C) = T(B) \cup T(C)$, it follows by simple set theory that $T(B \wedge C) \ll_{\forall\exists\infty_1} T(A)$.
4. It is straightforward to construct a counterexample based on the one by [1, p. 1881]. The invalidity holds due to the first conjunct of the truth condition.
5. It is straightforward to construct a counterexample based on the one by [1, p. 1881]. The invalidity holds because the accessibility relation changes from $\mathcal{R}_A$ to $\mathcal{R}_{A \wedge C}$.
6. It is straightforward to construct a counterexample based on the one by [1, p. 1882]. The invalidity holds because the strict conditional imposes no constraints on the relation between topics of antecedent and consequent.
7. It is straightforward to construct a counterexample based on Berto's, adjusting it for the case of a predicate language.
8. It is straightforward to construct a counterexample based on the one by [1, p. 1882], adjusting it for the case of a predicate language.
9. Suppose $\mathfrak{M}, w \Vdash [A](B \rightarrow C)$ and $\mathfrak{M}, w \Vdash [A]B$. Let w' be such that $\mathcal{R}_A w w'$. By classical logic, it follows that $\mathfrak{M}, w' \Vdash C$. Now for the topic condition.

From the first supposition, we have $T(B \rightarrow C) = T(B) \cup T(C) \ll_{\forall\exists\infty_1} T(A)$. We have that for every issue I in $T(B) \cup T(C)$ there is an issue J in $T(A)$ such that $I \infty_1 J$. Since $T(C) \subseteq T(B) \cup T(C)$, it follows that for every issue I in $T(C)$ there is an issue J in $T(A)$ such that $I \infty_1 J$. So, $T(C) \ll_{\forall\exists\infty_1} T(A)$. Thus, $\mathfrak{M}, w \Vdash [A]C$. $\square$

So, since none of the relations affects any of these (in)validities, we haven't yet found a basis upon which to choose the right relation for improving Berto's account. On the other hand, Berto considers all of these (in)validities to be plausible. Berto discusses two further principles of Substitutivity and Special Transitivity:

- (Sub) $[A]B, [B]A, [A]C \models [B]C$
(ST) $[A]B, [A \wedge B]C \models [A]C$

It is easy to construct counterexamples in the new framework based on Berto's counterexamples because both principles do not hold due to the first conjunct of the truth condition. Berto adds the following Principle of Imaginative Equivalents to validate (Sub) and (ST) in his semantics:

- (PIE) If $f_A(w) \subseteq \llbracket B \rrbracket$ and $f_B(w) \subseteq \llbracket A \rrbracket$, then $f_A(w) = f_B(w)$.

In the present framework, however, we can invalidate (Sub) and (ST), even if (PIE) is around. Of course, this is only true if we consider certain relations among the $\ll_{\forall\exists\infty_i}$ and $\ll_{\forall\exists\infty_j}$, and thus specific classes of models. This will be the basis on which a decision can be made.

Proposition 6 *If the truth condition for $[A]B$ is given in terms of any of the members from $\ll_{\forall\exists\infty_i}$, (Sub) and (ST) fail, even if (PIE) holds.*

Proof By Proposition 4, it suffices to show that the proposition holds if we define the truth condition in terms of the strongest relation $\ll_{\forall\exists\infty_4}$.

For (Sub), define $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \emptyset, \mathcal{J}, \text{Topics}, \cup, \top \rangle$ with $\mathcal{W} = \{w, w'\}$, $\mathcal{D}(w) = \mathcal{D}(w') = \{o_1, o_2\}$, $\mathcal{J}(w, a) = \mathcal{J}(w, c) = o_1$, $\mathcal{J}(w, b) = \mathcal{J}(w', b) = o_2$, $\mathcal{J}(w, F) = \{o_1\}$. Let $A := Fa$, $B := Fb$, $C := Fc$. Note that (PIE) is true because $f_{Fa}(w) = f_{Fb}(w) = \emptyset$.

Then $[Fa]Fb$, $[Fb]Fa$, $[Fa]Fc \not\models [Fb]Fc$. The first conjunct of the truth condition is vacuously satisfied (at both worlds) for each of the formulas. Note that $\llbracket a \rrbracket = \{\langle w, a \rangle, \langle w', b \rangle\}$, $\llbracket b \rrbracket = \{\langle w, b \rangle, \langle w', b \rangle\}$, $\llbracket c \rrbracket = \{\langle w, a \rangle, \langle w', c \rangle\}$. So, $\llbracket a \rrbracket \cap \llbracket b \rrbracket \neq \emptyset$, $\llbracket a \rrbracket \cap \llbracket c \rrbracket \neq \emptyset$, and $\llbracket b \rrbracket \cap \llbracket c \rrbracket = \emptyset$. So, $\top(Fb) \ll_{\forall\exists\infty_4} \top(Fa)$, $\top(Fa) \ll_{\forall\exists\infty_4} \top(Fb)$, $\top(Fc) \ll_{\forall\exists\infty_4} \top(Fa)$, and $\top(Fc) \not\ll_{\forall\exists\infty_4} \top(Fb)$.

For (ST), we can use the same model and set $A := Fb$, $B := Fa$, $C := Fc$. Then $[Fb]Fa$, $[Fb \wedge Fa]Fc \not\models [Fb]Fc$. This follows from the facts about the topic relations in the previous paragraph. $\square$

For relation $\ll_{\forall\exists\subseteq_1}$, we have the following:

Proposition 7 *If the truth condition for $[A]B$ is defined by relation $\ll_{\forall\exists\subseteq_1}$, then (Sub) and (ST) fail, even if (PIE) holds.*

Proof Define $\mathfrak{M} = \langle \mathcal{W}, \mathcal{D}, \emptyset, \mathcal{J}, \text{Topics}, \cup, \top \rangle$ with $\mathcal{W} = \{w, w'\}$, $\mathcal{D}(w) = \mathcal{D}(w') = \{o_1, o_1\}$, $\mathcal{J}(w, a) = o_1$, $\mathcal{J}(w, b) = \mathcal{J}(w', b) = o_1$, $\mathcal{J}(w, c) = \mathcal{J}(w', c) = o_2$, $\mathcal{J}(w, F) = \mathcal{J}(w', F) = \{o_1, o_2\}$. Note that $\llbracket c \rrbracket \subseteq \llbracket a \rrbracket$, $\llbracket b \rrbracket \subseteq \llbracket a \rrbracket$. As for (Sub), let $A := Fab$, $B := Fbb$, $C := Fcc$. It is to be shown that $\mathfrak{M}, w \Vdash [Fab]Fbb$, $\mathfrak{M}, w \Vdash [Fbb]Fab$, $\mathfrak{M}, w \Vdash [Fab]Fcc$ and $\mathfrak{M}, w \not\Vdash [Fbb]Fcc$. The first conjunct of the truth condition of the formulas is vacuously satisfied. As for the second conjunct, $\top(Fbb) \ll_{\forall\exists\subseteq_1} \top(Fab)$ and $\top(Fab) \ll_{\forall\exists\subseteq_1} \top(Fbb)$ because $\llbracket b \rrbracket \subseteq \llbracket a \rrbracket$ and only inclusion among some individual concepts is required. Since $\llbracket c \rrbracket \subseteq \llbracket a \rrbracket$, we have $\top(Fcc) \ll_{\forall\exists\subseteq_1} \top(Fab)$. But since $\llbracket c \rrbracket \not\subseteq \llbracket b \rrbracket$, it follows that $\mathfrak{M}, w \not\Vdash [Fbb]Fcc$.

Concerning (ST), we use the same model but let $A := Faa$, $B := Fab$, $C := Fcc$. We need to show that $\mathfrak{M}, w \Vdash [Fbb]Fab$, $\mathfrak{M}, w \Vdash [Fbb \wedge Fab]Fcc$ and $\mathfrak{M}, w \not\Vdash [Fbb]Fcc$. Again, the first conjunct of the truth condition for the imagination operator holds vacuously. For the second condition, $\top(Fab) \ll_{\forall\exists\subseteq_1} \top(Faa)$ because $\llbracket b \rrbracket \subseteq \llbracket a \rrbracket$. Since $\llbracket c \rrbracket \subseteq \llbracket a \rrbracket$, we have $\top(Fcc) \ll_{\forall\exists\subseteq_1} \top(Fab) \subseteq \top(Fbb \wedge Fab)$. So, $\top(Fcc) \ll_{\forall\exists\subseteq_1} \top(Fbb \wedge Fab)$. But since $\llbracket c \rrbracket \not\subseteq \llbracket b \rrbracket$, it follows that $\mathfrak{M}, w \not\Vdash [Fbb]Fcc$. $\square$

As for the other relations we get this:

Proposition 8 *If the truth condition for $[A]B$ is defined by either of the relations 2, 3, 4 or 5 from $\ll_{\forall\exists\subseteq_i}$ and (PIE) holds, then (Sub) and (ST) hold.*

Proof Again, it suffices to prove the claim for the weaker relations 2 and 3. I first show (Sub) for both relations 2 and 3, and then (ST) for both relations. I do not prove the case for the first conjunct of the truth condition because it remains the same as in [1].

(Sub) and relation 2: $\mathfrak{M}, w \Vdash [A]B$, $\mathfrak{M}, w \Vdash [B]A$ and $\mathfrak{M}, w \Vdash [A]C$. These entail, respectively, 1) $f_A(w) \subseteq \llbracket B \rrbracket$ and $T(B) \ll_{\forall \exists \subseteq_2} T(A)$, 2) $f_B(w) \subseteq \llbracket A \rrbracket$ and $T(A) \ll_{\forall \exists \subseteq_2} T(B)$, 3) $f_A(w) \subseteq \llbracket C \rrbracket$ and $T(C) \ll_{\forall \exists \subseteq_2} T(A)$. From 3) we know that there is $J_A = \langle G_A, j_{A1}, \dots, j_{An} \rangle \in T(A)$ such that $F \subseteq G_A$ and for all k and some $l, i_k \subseteq j_{Al}$. From 2) it follows that there is an issue $J_B = \langle G_B, j_{B1}, \dots, j_{Bn} \rangle \in T(B)$ such that $G_A \subseteq G_B$ and for all k' and some $l', j_{Ak'} \subseteq j_{Bl'}$. So, in particular, for some l and some $l', j_{Al} \subseteq j_{Bl'}$. It follows that $F \subseteq G_A \subseteq G_B$. Moreover, for all k , there are l and l' such that $i_k \subseteq j_{Al} \subseteq j_{Bl'}$. So, $T(C) \ll_{\forall \exists \subseteq_2} T(B)$.

(Sub) and relation 3. Suppose $\mathfrak{M}, w \Vdash [A]B$, $\mathfrak{M}, w \Vdash [B]A$ and $\mathfrak{M}, w \Vdash [A]C$. These entail, respectively, 1) $f_A(w) \subseteq \llbracket B \rrbracket$ and $T(B) \ll_{\forall \exists \subseteq_3} T(A)$, 2) $f_B(w) \subseteq \llbracket A \rrbracket$ and $T(A) \ll_{\forall \exists \subseteq_3} T(B)$, 3) $f_A(w) \subseteq \llbracket C \rrbracket$ and $T(C) \ll_{\forall \exists \subseteq_3} T(A)$. Consider an issue $I = \langle F, i_1, \dots, i_n \rangle \in T(C)$. From 3) we know that there is $J_A = \langle G_A, j_{A1}, \dots, j_{An} \rangle \in T(A)$ such that $F \subseteq G_A$ and for some k and each $l, i_k \subseteq j_{Al}$. From 2) it follows that there is an issue $J_B = \langle G_B, j_{B1}, \dots, j_{Bn} \rangle \in T(B)$ such that $G_A \subseteq G_B$ and for some k' and each $l, j_{Ak'} \subseteq j_{Bl}$. Then $F \subseteq G_A \subseteq G_B$ and for some k, k' and each $l, i_k \subseteq j_{Ak'} \subseteq j_{Bl}$. So, $T(C) \ll_{\forall \exists \subseteq_3} T(B)$.

(ST) and relation 2. Suppose $\mathfrak{M}, w \Vdash [A]B$ and $\mathfrak{M}, w \Vdash [A \wedge B]C$. From this it follows that 1) $f_A(w) \subseteq \llbracket B \rrbracket$ and $T(B) \ll_{\forall \exists \subseteq_2} T(A)$, 2) $f_{A \wedge B}(w) \subseteq \llbracket C \rrbracket$ and $T(C) \ll_{\forall \exists \subseteq_2} T(A \wedge B)$. Suppose that $I = \langle F, i_1, \dots, i_n \rangle \in T(C)$. From 2) we get that there is an issue $J = \langle G, j_1, \dots, j_n \rangle \in T(A \wedge B)$ such that $F \subseteq G$ and for all k there is some l such that $i_k \subseteq j_l$. Since $T(A \wedge B) = T(A) \cup T(B)$, $J \in T(A)$ or $J \in T(B)$. If $J \in T(A)$, we are done. Suppose $J \in T(B)$. From 1), it follows that there is an issue $J_A = \langle G_A, j_{A1}, \dots, j_{An} \rangle \in T(A)$ such that $F \subseteq G \subseteq G_A$ and for each k' there is some l' such that $j_{k'} \subseteq j_{Al'}$. So, in particular, for j_l it follows that for some l' that $i_k \subseteq j_l \subseteq j_{Al'}$ for each k . Thus, $T(C) \ll_{\forall \exists \subseteq_2} T(A)$.

(ST) and relation 3. Suppose $\mathfrak{M}, w \Vdash [A]B$ and $\mathfrak{M}, w \Vdash [A \wedge B]C$. From this it follows that 1) $f_A(w) \subseteq \llbracket B \rrbracket$ and $T(B) \ll_{\forall \exists \subseteq_3} T(A)$, 2) $f_{A \wedge B}(w) \subseteq \llbracket C \rrbracket$ and $T(C) \ll_{\forall \exists \subseteq_3} T(A \wedge B)$. Suppose $I = \langle F, i_1, \dots, i_n \rangle \in T(C)$. From 2) we get that there is an issue $J = \langle G, j_1, \dots, j_n \rangle \in T(A \wedge B)$ such that $F \subseteq G$ and for some k every l is such that $i_k \subseteq j_l$. Since $T(A \wedge B) = T(A) \cup T(B)$, $J \in T(A)$ or $J \in T(B)$. If $J \in T(A)$, we are done. Suppose $J \in T(B)$. From 1), it follows that there is an issue $J_A = \langle G_A, j_{A1}, \dots, j_{An} \rangle \in T(A)$ such that $F \subseteq G \subseteq G_A$ and for some k' every l' is such that $j_{k'} \subseteq j_{Al'}$. So, in particular, for $j_{k'}$ for some k and for each l' it follows that $i_k \subseteq j_{k'} \subseteq j_{Al'}$. Thus, $T(C) \ll_{\forall \exists \subseteq_3} T(A)$. $\square$

Thus, if one considers (Sub) and (ST) plausible, which Berto seems to do, one of the relations 2, 3, 4, 5 from $\ll_{\forall \exists \subseteq_i}$ should be used in the truth condition for the imagination operator. Relation 4 seems way too strong since it requires that *all* individual concepts from the output content are included in *all* the individual concepts from the input content. This is a very restrictive notion of imagination, even if one considers rational imagination. Also relation 3 seems implausible for it requires that some individual concept from the output content is included in *all* individual concepts from the input content. Again, this is too limiting, even for rational imagination. It seems to me that relation 2 and 5 are good candidates. Relation 5 requires a point-wise inclusion of the individual concepts. This constrains the imagination implausibly for it excludes that we end up imagining the individuals being related in a different order

than in the initial content. If we consider $\forall\exists \sqsubseteq_2$ -models, then whatever individual concepts are featured in the output content must already have been featured somehow in the initial content. That is, which individuals we end up imagining about is constrained by which individuals we started to imagine about. But also this is still too strong. For example, consider an instructive use of imagination expressed by “In imagining Gwenny at the lake, Helena imagines Chris at the lake”²² This introduces an individual in the output that’s not present in the input. Only $\forall\exists \sqsubseteq_1$ -models allow this to happen. This would require giving up (Sub) and (ST). Since it is easy to see that Example 1 is dealt with also within $\forall\exists \sqsubseteq_1$ -models, I suggest that we consider the class of $\forall\exists \sqsubseteq_1$ -models, and thus $\ll_{\forall\exists \sqsubseteq_1}$ in the truth-condition of the imagination operator. In that way, the problematic example is dealt with and we have content-assignments based on plausible semantic considerations. So, the dilemma facing Berto’s account has been solved. It does, however, raise the question about (Sub) and (ST). Since considerations of legitimate instructive uses of imaginative episodes require $\forall\exists \sqsubseteq_1$ -models and these do not validate (Sub) and (ST), we have independent reason to doubt their adequacy.

4 Conclusion

This paper has extended Berto’s semantics for aboutness in imagination to a semantics for a language with predicates and constants to solve a dilemma for Berto’s original account. The role of contents is played by topics in the sense of Hawke’s issue-based theory of topics. Several new relations between initial content and output content have been introduced, extending Hawke’s initial approach. This allows to account for previously wrong predictions of Berto’s account concerning intuitively legitimate imaginative episodes. It is suggested how different uses of the imagination, namely transcendent and instructive uses, can be modelled by considering different classes of models, defined by the various relations.

There are various subjects for further research. While I have allowed constant symbols to be non-rigid, there is no fine-grained model of definite descriptions. Since the relation symbols featuring in the descriptions can influence our imaginings (qua designating properties or relations), and also seem to have “aboutness”, this is a promising extension.

One limitation of the present approach is that relations between general concepts are only definable if the concepts are of the same arity. Consequently, issues can only be related if their general concepts have the same arity. It is conjectured that adding a λ -operator can circumvent this problem.

Moreover, adding variables and quantifiers is a natural extension. While there is an axiomatisation for Berto’s initial account [6], there is one needed for the present extension. Given Berto’s initial content condition is definable in the present paper, it is conjectured that Giordani’s axiomatisation can be extended in such a way that his completeness proof can be complemented with methods from proving completeness

²²Thanks to an anonymous reviewer for bringing up such an example.

for systems with contingent identity for this particular relation between topics. As for other relations among topics, especially the ones discussed in this paper, it is not clear whether Giordani's strategy can be tweaked accordingly.

Acknowledgements Open Access funding enabled and organized by Projekt DEAL. Various versions of this paper have been presented at the workshop *Logic(s) for Imagination* (2018), the conference *Logic in Bochum V* (2019), both at the Ruhr University Bochum, Germany; the workshop *Fiction and Proper Names* (2018) at the Institute of Philosophy of the Slovak Academy of Sciences in Bratislava, Slovakia; the workshop *On What There Isn't* (2019) at the Heinrich-Heine University Düsseldorf, Germany; and at the *Logic of Conceivability* (2019) seminar at the ILLC, Amsterdam. I thank all participants of these events for their valuable feedback, especially Peter Hawke for his helpful comments on a previous version of this paper. Moreover, I am grateful to Franz Berto and Heinrich Wansing. Without their support and extensive feedback, this paper would have remained imaginary.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

1. Berto, F. (2018). Aboutness in imagination. *Philosophical Studies*, 175, 1871–1886. <https://doi.org/10.1007/s11098-017-0937-y>.
2. Ferguson, T.M. (2014). A computational interpretation of conceptivism. *Journal of Applied Non-Classical Logics*, 24(4), 333–367. <https://doi.org/10.1080/11663081.2014.980116>.
3. Fine, K. (2017). A theory of truthmaker content i: conjunction, disjunction and negation. *Journal of Philosophical Logic*, 46(6), 625–674.
4. Fine, K. (2017). A theory of truthmaker content ii: subject-matter, common content, remainder and ground. *Journal of Philosophical Logic*, 46(6), 675–702.
5. Fitting, M., & Mendelsohn, R.L. (1998). *First-Order Modal logic* springer.
6. Giordani, A. (2018). Axiomatizing the logic of imagination. *Studia Logica* pp 1–19.
7. Hawke, P. (2017). Theories of Aboutness. *Australasian Journal of Philosophy*, 96(4), 697–723. <https://doi.org/10.1080/00048402.2017.1388826>.
8. Hawke, P., & Berto, F. (2019). Knowability relative to information. *Mind* <https://doi.org/10.1093/mind/fzy045>.
9. Kind, A. (2016). Imagining under constraints. In Kind, A., & Kung, P. (Eds.) *Knowledge through imagination* (pp. 145–160). Oxford: Oxford University Press.
10. Kind, A., & Kung, P. (2016). Introduction: the puzzle of imaginative use. In Kind, A., & Kung, P. (Eds.) *Knowledge through imagination* (pp. 1–41). Oxford: Oxford University Press.
11. Lewis, D. (1988). Relevant implication. *Theoria*, 54(3), 161–174. <https://doi.org/10.1111/j.1755-2567.1988.tb00716.x>.
12. Perry, J. (1989). Possible worlds and subject matter. In Alto, P. (Ed.) *The problem of the essential indexical and other essays, CSLI publications* (pp. 145–160).
13. Williamson, T. (2016). Knowing by imagining. In Kind, A., & Kung, P. (Eds.) *Knowledge through imagination* (pp. 113–124). Oxford: Oxford University Press.
14. Yablo, S. (2014). *Aboutness*. Princeton: Princeton University Press.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.