



Optimized modeling of energy and environmental metrics of mixed flow turbofan engine used regional aircraft

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Abstract

As a kind of gas turbine engines, turbofan engines have powered a number of aero-vehicles in aviation sector. The necessity of turbofan with higher energy efficiency has been greatly drawn attention since these are operating dependent to fossil fuels. In this study, energy and emission metrics of fifty-one mixed flow turbofan engines (MFTE) with different bypass ratio, overall pressure ratio and fuel flow are modeled with multi-regression (MR) method. The obtained models are subjected to metaheuristic approaches involving genetic algorithm (GA) and simulated annealing (SA) so as to decrease error of the models. According to MR findings, rated thrust of MFTEs is estimated with 1.4877 of minimum square error (MSE) whereas GA and SA make it lower as 1.3404 and 1.2524, respectively. On the other hand, NO_x emission index of MFTEs is predicted with relatively low coefficient of determination (R²) as 0.8620. However, its accuracy is enhanced to 0.8633 (with GA) and 0.8655 (with SA). Finally, exergy efficiency of MFTEs is estimated the highest model correctness with GA. Namely, R² of the model is computed as 0.9280 with GA and 0.9277 with SA. Without applying these methods, its R² is obtained as 0.9263 with MR. When considering these outcomes, thanks to modeling and optimization methods, prediction of performance and emission indexes of mixed flow turbofan engines could be performed with lower error values. It is thought that the study helps in prediction of environmental effect regarding turbofan engines that are utilized at busy airports.

Keywords Mixed flow turbofan · Optimization · Modeling · Aircraft emission

Abbreviations

CORSIA	Carbon offsetting and reduction scheme for international aviation	LHV	Lower heating value
CPR	Compressor pressure ratio	MAE	Mean absolute error
BPR	Bypass ratio	MLRM	Multiple linear regression model
EI	Emission index (g kg ⁻¹ fuel)	MR	Multi-regression
Ex	Exergy	MFTE	Mixed flow turbofan engine
ExEFF	Exergy efficiency	nMSE	Normalized mean square error
FF	Fuel flow	m	Mass flow rate (kg s ⁻¹)
GA	Genetic algorithm	OPR	Overall pressure ratio
IATA	International air transportation association	PMX	Partially mapping crossover
ICAO	International civil aviation organization	R ²	Coefficient of determination
LTO	Landing and take-off	SA	Simulated annealing
		SAF	Sustainable aviation fuel
		SFC	Specific fuel consumption (g kN ⁻¹ s ⁻¹)
		TIT	Turbine inlet temperature (K)
		τ	Thrust (kN)
		V	Velocity
		WExR	Wasted exergy ratio

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Greek letters

α	Bypass ratio
β	Coefficient
η	Efficiency

Introduction

As the world population as well as the welfare level in the economy increase attain high level, it is projected that energy demand is enormously to be high. The needs such as electrical and heat energy depending on fossil fuel have come upper level due to rising population in the world [1]. When considering depletion hazard of petroleum reserves at close future as well as the raised energy consumption at 17-fold in the last century [2], aviation sector could be subjected to the operational limits unless novel technologies are applied to the systems used in this sector. It is thought that as emission mitigation policies become stricter, design and manufacturing limits may be rechecked whether these comply with environmental goals determined by authorities [3]. According to IATA survey in 2018, the growth rate of civil aviation industry will attain to 3.5% rate for the next two decades, despite the fact that this prediction could deviate due to COVID-19 impact in momentarily [4]. When considering 2019 year, aircraft engines worldwide emitted 915 million tonnes of CO₂ emission [5]. About 2.5% rate of global CO₂ emissions originate from aviation, but this does not cover overall contribution since aircraft engine emits different harmful gaseous except CO₂ leading to the environmental effect. For this concern, the related people have tried to take several measures in their field where there exists system based on energy consumption. Also, it is well-known that to alleviate environmental impact is partly depending on the awareness level of the society against to this concern. It is important that energy generation from any sources is performed with minimal environmental impact. Namely, one of main goals is to determine fuel or energy source leading to the lowest environmental impact. To meet the need of transportation of people with airline, the number of aircraft as well as flight have been increased day by day. This increment forces to be performed the goals regarding environmental sustainability. Therefore, new alternatives to fossil fuel have been searched to deal with harmful effects of aviation industry. In this context, for possible solution, biofuels to decrease dependent of fossil fuels have been suggested to use in aviation engines since these are promising mitigation of CO₂ emissions as well as gas turbine efficiency. For instance biofuels obtained from biomass such as the plants reduce CO₂ emission due to decreasing biomass that produces CO₂ [2, 6]. In addition to adverse impacts of harmful emissions on local air quality, environmental disasters such as flooding and increased wildfire triggered by global warming have become widespread in the world. When considering burning products of kerosene used by the aviation industry, these consist of CO₂, nitrous oxides, water vapor, carbon monoxide, unburnt hydrocarbons and

particulate matter [7]. It is observed that air traffic volume has increased about 4% per annum, which could trigger operating costs to higher value due to possible shortage in the traditional fuel [7]. One of several ways to reduce emission concern in aviation is to enhance the gas turbine efficiency. In this way, an important effort has been made by the engineers so far. Due to the increased aviation activities, thereby fossil fuel demand, influences of these improvements do not completely have been seen in this sector. The reason why aviation emission is scrutinized is due to both environmental impact and human health. When aircraft operate on the ground, these emissions affect local air quality vicinity of airport whereas has impact on global climate at cruise phase [8]. Effects of these emissions, especially NO_x, change according to the altitude where aircraft operate. At low altitude, NO_x leads to formation of ozone that damages human health whereas at high altitude ozone is depleted by NO_x, hence raising radiation [8]. Moreover, these aviation emissions have started to be debated in global extent since it is observed that harmful gaseous lead to health concerns of people that especially live to vicinity of airports. To decrease fuel consumption, thereby harmful effects from aircraft, there are a number of attempts such as using renewable energy, applying novelties to the engine and airframe, higher efficiently air traffic management and new binding deal on emissions such as CORSIA regulated by ICAO [9]. More green environment and less fuel consumption by integrating different engineering areas have been hot topics of aviation sector. For this aim, the attempts have focused on improvement of energy conversion systems used in aircraft. Main pillars of alleviation of environmental effect for aircraft are to increase aerodynamic efficiency such as lift/drag ratio, to improve engine efficiency with novel technologies, to replace kerosene with alternative jet fuels such as biodiesel and to design aero-vehicle in concept of more electric aircraft [7].

Conventional emission and environmental impact studies regarding gas turbine engines

Kroyan et al. [10] established multiple linear regression for estimation of fuel consumption depending on viscosity, density, and calorific content, which results in model success of $R^2=0.993$. The obtained model was applied to five different sustainable aviation fuels (SAF), which shows that SAF is promising potential in terms of environmental sustainability. Balli and Caliskan [11] dealt with environmental effect of different auxiliary power units located in aft of aircraft. Among the APUs, GTCGP36-300 has the lowest total emission index, which could be attributed that it has the lowest power. However, its environmental impact index (mPts kWh⁻¹) is determined relatively high with 49.23 mPts kWh⁻¹.

Dinc et al. [12] elaborated flight cycle of a turboprop engine including five phases point of view environmental effect. Although take-off phase has relatively the highest fuel consumption, minimum environmental parameter and thereby minimum CO₂ price was found to be 73.08 kgCO₂ day_{cycle}⁻¹ and 8.48 € day_{cycle}⁻¹, respectively due to having the lowest time in mode. Moreover, Sohret et al. [13] investigated a thermo-environmental aspects of turboprop engine at different flight phases using five novel parameters. According to the authors, thermo-environmental function attains the highest with 0.338 kW per unit mPts at climb whereas it decreases to 0.23563 kW/mPts at cruise phase where the aircraft spends the most of flight time. Furthermore, Balli et al. [14] compared effect of using H₂ on turbofan performance metrics with kerosene fuelled conventional system. The H₂ fuelled system benefits lower fuel consumption by 63.83%, thereby lower environmental impact sacrificing slightly energy efficiency and specific thrust of the engine. Lastly, Aygun and Caliskan [15] addressed environmental and environmental economic modeling of two different families of turbofan engines. The model accuracy of global warming potential index is obtained as 0.9956 of R² for family 1 engines and 0.9664 of R² for family 2 engines where bypass ratio and overall pressure ratio are considered as model inputs.

Optimization studies of emission and performance regarding gas turbine engines

When considering the literature, there are a number of the studies dealing on aircraft engine. It is tried that novel studies are addressed by choosing the papers directly related to the current study. Ekralesian et al. [16] dealt with turbojet engine in terms of energetic and exergetic optimization. By determining three variables such as turbine inlet temperature, compressor pressure ratio and Mach number, multi objectives optimization was performed. The TIT, CPR and Mach points giving maximum thermal efficiency with 65.89% and minimum exergy destruction with 12.51 GJ/h are 1241.72 K, 22.91 and 0.792, respectively. Patel et al. [17] carried out multi-objective heat transfer search (MOHTS) algorithm to turbojet engine to determine minimum specific fuel consumption and maximum specific thrust and efficiency. Initially, each objective function is considered alone by comparing with genetic algorithm. When considering all objectives together, the obtained specific thrust drops from 1.1666 to 0.9499 kNs/kg (obtained by fuzzy method). Similarly, SFC increases from 0.0162 to 0.0225 kg/kNs. Fawal and Codal [18] assessed turbojet engine using several objective functions such as maximum power, power density, ecological coefficient of performance (ECOP) and ecological function (ECOL). The authors stated that thanks to ECOP and ECOL conditions, optimum compression ratio and maximum cycle temperature ratio as well as the burner,

turbine and nozzle size reductions could be achieved for civil applications. However, MP and MPD functions benefit SFC on supersonic conditions. Farahani et al. [19] performed effect analysis for turbofan engine by applying optimization technique called as teaching-learning-based optimization method (TLBO). In the study, altitude and Mach of flight are considered as variables. According to this, maximum exergy efficiency is obtained as 32.64% where altitude and Mach are 11,236 m and 1.994, respectively. Lee and Sung [20] examined performance metrics of turbofan engine called GE90 utilizing multi-objective particle swarm optimization. The analysis was performed without constraint and with constraints such as OPR and TIT. The authors expressed that for optimal solution, LPC PR remains low and HPC PR becomes high. However, HPC PR starts to diminish after specific thrust exceeds 160 Ns/kg. Kaba et al. [21] estimated thrust of turbojet engine, which different RPM values are known. For this aim, three mathematical models involving polynomial, Fourier and power expansion series were used. The P4 algorithm among all linear algorithms gives the best solution with the highest R² that is 0.9993. With nonlinear algorithm, R² of thrust modeling attends to 0.9996. Oruc et al. [22] modeled exergetic indexes of turboprop engine using cuckoo search algorithm. For this aim, six inputs including torque, power, gas generator speed, air&fuel mass flows and air to fuel ratio were used. Mean square error for EEF and ESI metrics were found as 4.327×10^{-9} and 5.1839×10^{-9} , which correspond 0.9999 of R². Piskin et al. [23] performed component matching and off-design calculations of turbojet by proposing hybrid ant colony and particle swarm optimization method. Thanks to the proposed algorithm, air mass flow difference between test measurement and the optimized calculation decreases to 1.4% by characteristic curve and 0.7% by turbine map. Aygun and Turan [24] carried out modeling of several exergetic metrics of turbofan engine using the least square method and its optimized version. The author stated that the model accuracy of exergy efficiency increases from 0.9974 of R² (by LSM) to 0.9999 of R² (by optimized GA). The other metrics experienced enhancement by the proposed method. Moreover, Dinc [25] conducted optimization of loiter time and specific power regarding turboprop engine using single objective and multiple objectives genetic algorithm. The author expressed that loiter time and specific power experience relatively less improvement when considering more than one objective. Namely, when compared baseline operation, these increase 14.2% and 9.7%, respectively, which have higher value if these are considered, separately. Demir [26] estimated the CO₂ emission in air transport based on flight and passenger statistics using GA. Zhao and Zhang [27] developed a solution methodology based on neural network optimization including Levenberg-Marquardt-GA back propagation for planes to complete the landing process more safely.

According to the literature, no studies about concentrating on the modeling of performance, emission and exergetic metrics for mixed flow turbofan engines have appeared to the best of the author's knowledge. This study differs from previous ones in several respects, which are (i) implementing two different metaheuristic methods for enhancement of conventional modeling, (ii) modeling both emission and exergy metrics related to mixed flow turbofan engines on real data basis (iii) considering bypass ratio, overall pressure ratio and fuel flow as inputs for modeling of the engine performance parameters. Main motivation of this study can be summarized as follows:

- To apply multi-regression method to rated thrust, NO_x emission index, exergy efficiency and wasted exergy ratio of MFTE
- To optimize coefficients of the model so as to minimize the model error with genetic algorithm and simulated annealing
- To compare the findings of conventional model and the proposed model regarding MFTEs

System description

Gas turbine engines have been main power tool for especially aero-vehicles due to their unique features such as high power to mass ratio and reliability compared with piston engines. Using turbofan engines in commercial aircraft has continued for last six decades. It can be recognized that the early version of those is Pratt Whitney JT3D used in Boeing 707. To clearly understand types of turbofan engines could be possible with Fig. 1. Being a kind of commercial turbofan engines, mixed flow turbofan engines have powered regional aircraft as well as commercial aircraft for three decades. At early years which turbojets were applied to aero-vehicles, mixed flow turbofan engines were only used at military aircraft whereas application area of these concepts have been

extended to commercial aircraft by increasing their bypass ratio. Main difference from turbofan with separate flow is that fan air leaves the engine by mixing with hot air incoming from core section through common exhaust assembly.

Figure 2 illustrates typical mixed flow turbofan engine with key components such as fan, low-pressure compressor, high-pressure compressor, combustor, high-pressure turbine and low-pressure turbine. As mentioned before, in this study, specifications of fifty-one mixed flow turbofan engine, most of whose consist of series of AE3007 (Rolls-Royce) and PW306-308, PW814-815 (Pratt&Whitney), BR700 (Rolls-Royce) as well as RB211 (Rolls-Royce) and Trend 772 (Rolls-Royce) were utilized to model their several parameters [29].

Figure 3 manifests design variables incorporating bypass ratio, overall pressure ratio and fuel flow according to fifty-one engines, most of them have powered to regional jet aircraft. As can be seen, bypass ratio of the engines changes between 4.05 and 5.7, whereas the mean BPR is 4.65. Moreover, overall pressure ratio varies between 15.8 and 36.33 while fuel flow considered the engines is measured between 0.301 and 3.139 kg s⁻¹. Mean values of OPR and FF regarding MFTEs are 20.27 and 0.57 kg s⁻¹, respectively.

Figure 4 gives emission index of HC, CO and NO_x throughout landing and take-off (LTO) cycle including idle, take-off,

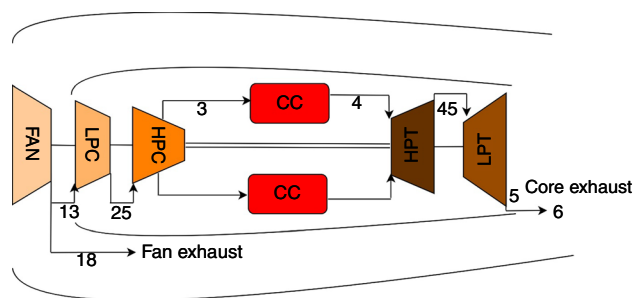


Fig. 2 Typical mixed flow turbofan engine

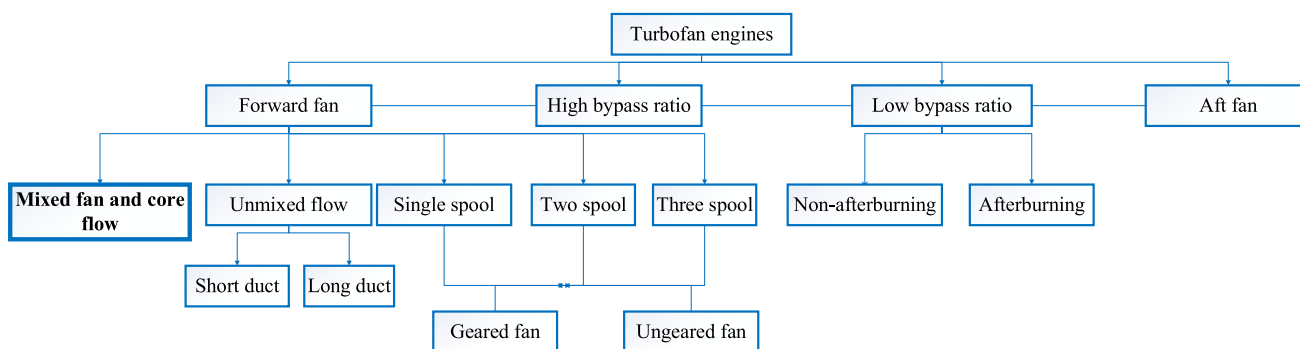


Fig. 1 Classification of turbofan engines according to design type (Adopted from [28])

Fig. 3 Specifications of fifty-one mixed flow turbofan engines [29]

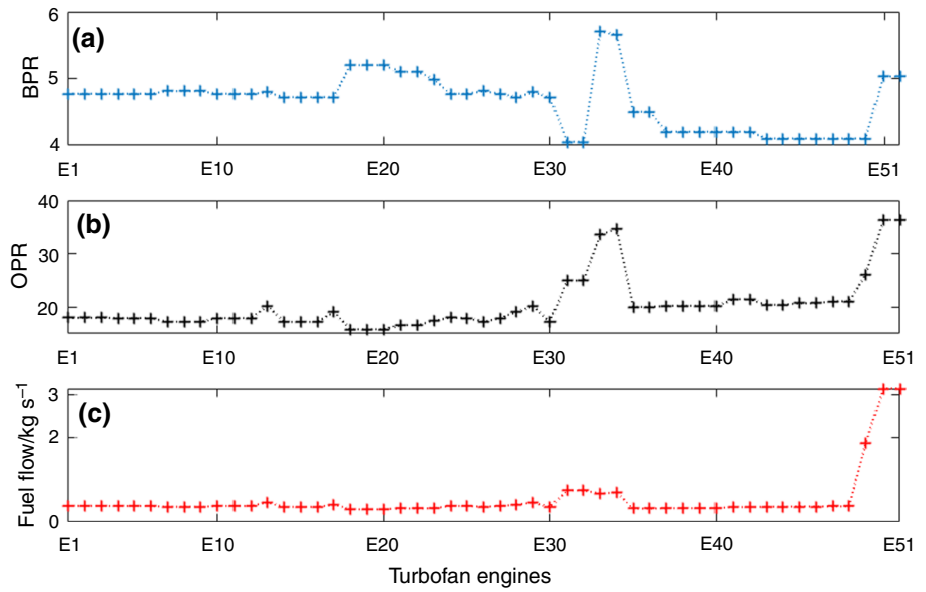
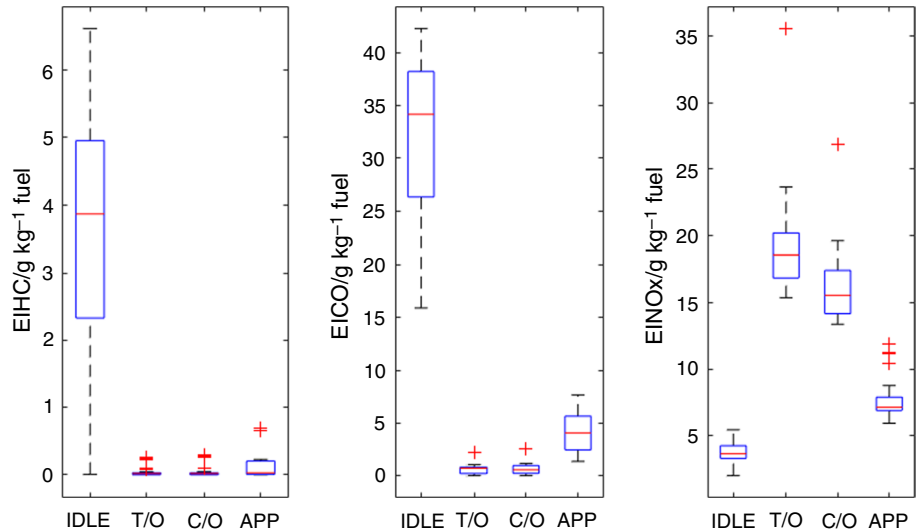


Fig. 4 Emission indexes of fifty-one MFTEs at LTO phases [29]



climb-out and approach phases. It could be understood that among phases, EI NO_x is observed the greatest degree at take-off phase whereas the biggest HC and CO emission indexes are measured at idle phase. Therefore, scrutinizing EI NO_x for take-off phase is of high importance. In this context, at take-off phase, EI NO_x of the MFTEs changes between 15.4 and 35.36 g/kg fuel while EIHC and EICO vary between, 0 and 2.27 g/kg fuel and 0 and 0.254 g/kg fuel, respectively.

Figure 5 shows how much fuel flow is consumed throughout LTO cycle on all engine bases. As expected, take-off phase is the first in the highest fuel consumption. It is followed by climb-out phase. Idle phase has the lowest fuel usage. Namely, fuel flow is observed between 0.131 and 3.139 kg s⁻¹ at take-off, 0.2527 and 2.53 kg s⁻¹ at climb-out, 0.0943 and 0.821 kg s⁻¹ at approach and 0.0377 and 0.27 kg s⁻¹ at idle phase.

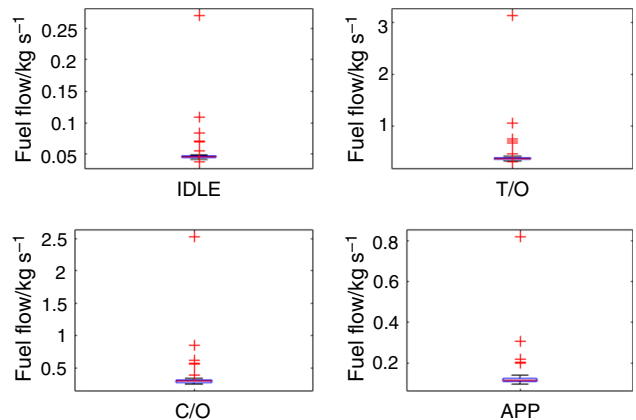


Fig. 5 Fuel flow of fifty-one MFTEs at LTO phases [29]

Lastly, total emission (g) including HC, CO, NO_x, CO₂ emissions and total fuel consumption (kg) of mixed flow turbofan considered in the present study are addressed for LTO cycle in Fig. 6. As can be recognized, the highest emission values belong to CO emission except CO₂, which changes between 1695 and 10,606 g. As for NO_x emission, it varies between 1050 and 17,668 g according to the engine. Moreover, HC emission is the lowest, varying between 1 and 1048 g throughout LTO cycle. On the other hand, total fuel consumption over LTO phases change between 127 and 1085 kg whereas total CO₂ emission varies between 401 and 3428 kg according to the engine types.

Solution methodology

Modeling parameters

Rated thrust

To propel aircraft for meeting the need of force at flight phases, especially take-off, the engine thrust is fundamental power source. Namely, it benefits the aero-vehicle to overcome drag force occurred from the air acting on its surface. For mixed flow turbofan engines, it can be expressed as follows [28]:

$$\tau = \dot{m}_a [(1 + f + \alpha)V_e - (1 + \alpha)V_i] \tag{1}$$

where *f* is fuel to air ratio, whereas *α* represents bypass ratio.

NO_x emission index

It can be computed with two ways, which are directly measurement from engine or empirical formula [30]. In this

study, ICAO data were utilized rather than improving the model depending on fuel flow. EI NO_x can be written as:

$$EI NO_x = \frac{NO_x}{\dot{m}_f} \tag{2}$$

where *m_f* is fuel flow injected to the combustor.

Exergy efficiency

The term exergy diagnoses the limits of maximum useful work obtained from energy conversion system such as gas turbine engines [31–33]. To calculate it, second law of thermodynamics is implemented by considering the surroundings conditions in where the system operates. For turbofan engines, the useful work is calculated by multiplying air velocity with thrust. Namely, exergy efficiency is ratio of useful or output power to input power imparted to the system.

$$\eta_{ex} = \frac{\dot{E}x_{out}}{\dot{E}x_{in}} = \frac{\tau * V_i}{\dot{m}_f * LHV * \gamma_f} \tag{3}$$

where *τ* and *V_i* denote rated thrust and inlet velocity of air, respectively. *γ_f* is fuel exergy grade, its value is 1.06789. Also, *LHV* represents lower heating value which is 42800 kJ kg⁻¹ for kerosene.

Wasted exergy ratio

It can be calculated from ratio between wasted exergy and input exergy. It could be expressed as [31–33]

Fig. 6 Total emission and total fuel consumption of fifty-one MFTEs at LTO phases [29]

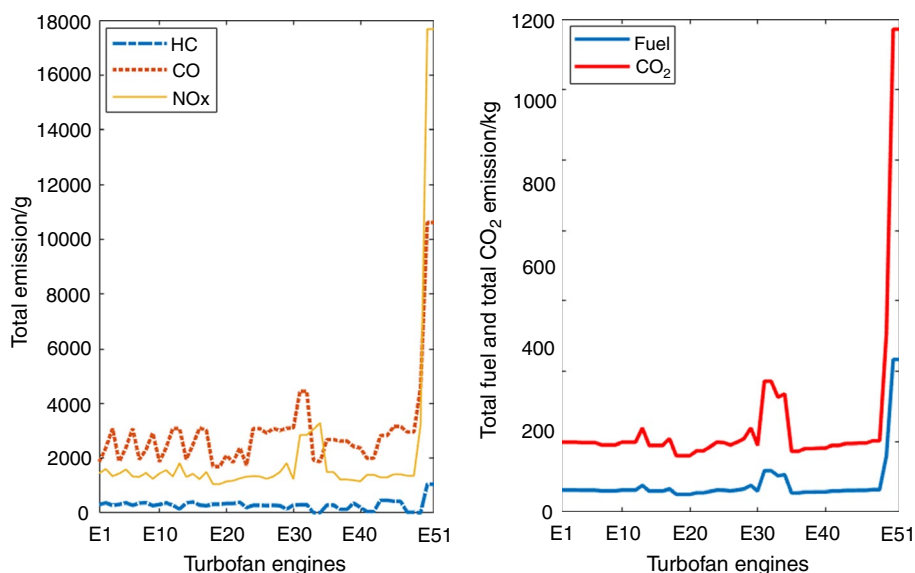


Table 1 Ranges of input data and modeling parameters

Input-data	Minimum	Maximum
Bypass ratio	4.05	5.7
Overall pressure ratio	15.8	36.33
Fuel flow (\dot{m}_f)/kg s ⁻¹	0.301	3.139
Modeling parameters	Minimum	Maximum
Rated thrust/kN	26.87	320.3
EI NO _x /g kg ⁻¹ fuel	15.4	35.56
Exergy efficiency/%	25.78	32.05
Wasted exergy ratio/%	67.94	74.21

$$\begin{aligned}
 WExR &= \frac{\sum \dot{E}x_{waste}^{eng}}{\sum \dot{E}x_{in}^{eng}} \\
 &= \left(\sum \dot{E}x_{in}^{eng} - \sum \dot{E}x_{useful}^{eng} - \sum \dot{E}x_{dest}^{eng} \right) / (\dot{m}_f * LHV * \gamma_f) \tag{4}
 \end{aligned}$$

Table 1 shows minimum and maximum value of input data and the modeling parameters throughout fifty-one mixed flow turbofan engines.

The multiple linear regression model

Multiple linear regression model (MLRM, hereafter is called MR) is a statistical estimation tool that provides explanation of a dependent variable with more than one independent

variable. In the literature, there are many studies on the application of the MLRM method on aviation problems [34]. Thanks to the model, the parameters to be used in the estimation are determined and the effect of the independent variables on the dependent variable is found. The least squares method is used to compute the coefficients in the MR method. In this way, it is aimed to minimize the difference between estimated values and actual values [35]. A classical multiple regression model is expressed in Eq. 5.

$$Y_i = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \dots + \beta_n * X_n + \epsilon \tag{5}$$

In Eq. 1, “ β_0 ” refers the regression coefficient, “ β_i ” represents the prediction coefficient of i . independent variable and “ ϵ ” also states the error term. One of the most important outputs in multiple regression analysis is the coefficient of determination (R^2). This value shows the ratio of the independent variables explaining the dependent variables and takes values between 0 and 1. The fact that the coefficient of determination is close to “0” indicates that the independent variables cannot adequately explain the dependent variables. Furthermore,

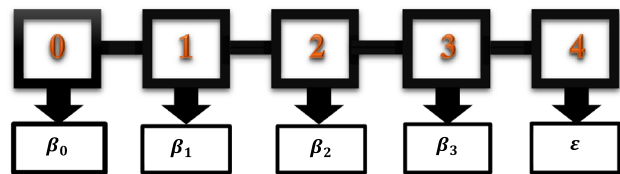
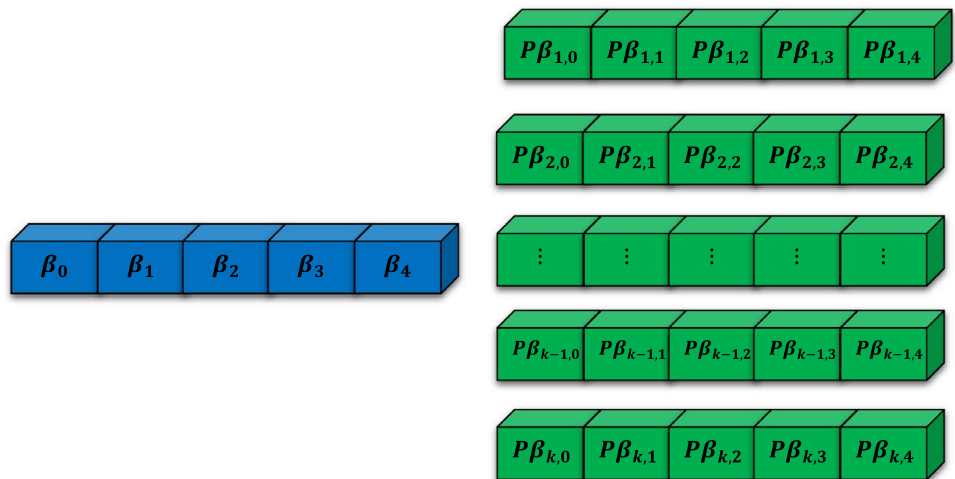


Fig. 7 Solution representation used in Genetic Algorithm

Fig. 8 Initial population determination mechanism



this coefficient is close to “1” indicates that the model is quite good in explaining the dependent variable [35]. The following formula is used to determine the R^2 value.

the selection of new individuals, the crossover and mutation operators are the parameters that must be determined in the

$$R^2 = \frac{\beta_1 * \left(\sum Y_i * X_i - \frac{\sum X_i * \sum Y_i}{n} \right) + \beta_2 * \left(\sum Y_2 * X_2 - \frac{\sum X_2 * \sum Y_2}{n} \right) + \dots + \beta_n * \left(\sum Y_n * X_n - \frac{\sum X_n * \sum Y_n}{n} \right)}{\sum (Y_i - \bar{Y})^2} \tag{6}$$

Genetic algorithm

Genetic Algorithm is a metaheuristic solution method developed by John Holland in 1975, inspired by the evolutionary process in nature [36]. In the algorithm, an initial population is formed based on different solutions and the algorithm progresses with iterative steps. In each step, the initial solution is improved by using certain solution mechanisms and the optimal solution is tried to be reached. In Genetic Algorithm, there are many parameters that make up the solution mechanism and affect the solution quality. The representation of the solution,

algorithm [37]. Information on the determination of these parameters is given in the following sections.

Solution representation

The structure of chromosomes and genes in the population may vary depending on the type of problem. Binary, integer, float and permutation are some of the solution representations used in the algorithm [37]. Float solution representation is used in this article and the related chromosome structure is shown in Fig. 7 is also shown.

<i>Chromosome Index</i>	<i>Old Generation</i>	<i>Fitness Value</i>	<i>Tournament</i>	<i>New Generation</i>
C1	$P\beta_{1,0} \quad P\beta_{1,1} \quad P\beta_{1,2} \quad P\beta_{1,3} \quad P\beta_{1,4}$	18	$\min\{C1, C3, C9\}$	C3
C2	$P\beta_{2,0} \quad P\beta_{2,1} \quad P\beta_{2,2} \quad P\beta_{2,3} \quad P\beta_{2,4}$	10	$\min\{C2, C6, C9\}$	C6
C3	$P\beta_{3,0} \quad P\beta_{3,1} \quad P\beta_{3,2} \quad P\beta_{3,3} \quad P\beta_{3,4}$	5	$\min\{C4, C5, C7\}$	C4
C4	$P\beta_{4,0} \quad P\beta_{4,1} \quad P\beta_{4,2} \quad P\beta_{4,3} \quad P\beta_{4,4}$	1	$\min\{C3, C8, C10\}$	C10
C5	$P\beta_{5,0} \quad P\beta_{5,1} \quad P\beta_{5,2} \quad P\beta_{5,3} \quad P\beta_{5,4}$	21	$\min\{C6, C8, C9\}$	C6
C6	$P\beta_{6,0} \quad P\beta_{6,1} \quad P\beta_{6,2} \quad P\beta_{6,3} \quad P\beta_{6,4}$	6	$\min\{C1, C2, C3\}$	C3
C7	$P\beta_{7,0} \quad P\beta_{7,1} \quad P\beta_{7,2} \quad P\beta_{7,3} \quad P\beta_{7,4}$	9	$\min\{C2, C4, C6\}$	C4
C8	$P\beta_{8,0} \quad P\beta_{8,1} \quad P\beta_{8,2} \quad P\beta_{8,3} \quad P\beta_{8,4}$	14	$\min\{C3, C5, C8\}$	C3
C9	$P\beta_{9,0} \quad P\beta_{9,1} \quad P\beta_{9,2} \quad P\beta_{9,3} \quad P\beta_{9,4}$	12	$\min\{C7, C8, C9\}$	C7
C10	$P\beta_{10,0} \quad P\beta_{10,1} \quad P\beta_{10,2} \quad P\beta_{10,3} \quad P\beta_{10,4}$	3	$\min\{C1, C5, C10\}$	C10

Fig. 9 The tournament selection mechanism used in the algorithm

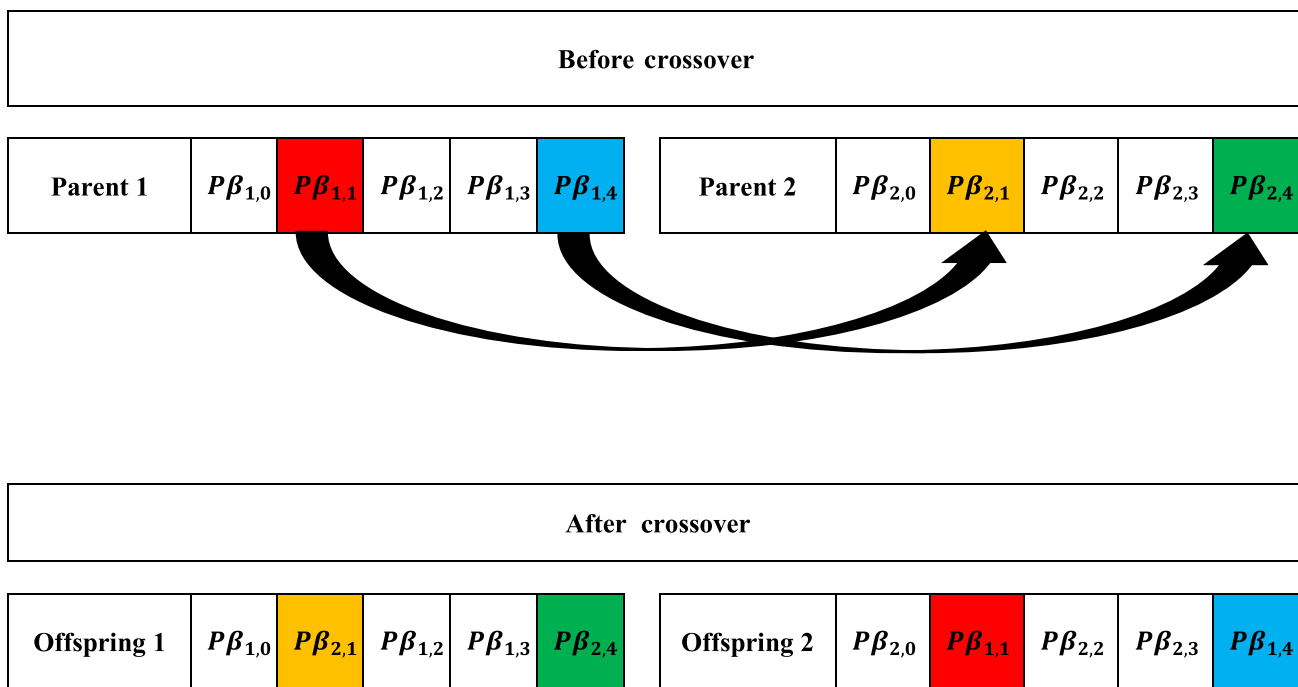


Fig. 10 The crossover mechanism used in the algorithm

The chromosome in Fig. 7 consists of five genes and each gene corresponds to the coefficients in the prediction model.

Generating the initial population

The initial population is an important parameter for the performance of the algorithm. Thanks to a good initial solution, the solution quality of the algorithm increases and it can reach the optimal in a shorter time. In the article, the estimation model obtained as a result of Multiple Regression Analysis was taken into account while determining the initial population. Starting from the coefficients in the estimation model, the initial population was determined by using the following Eq. 7.

$$P\beta_{i,j} = \beta_j + \beta_j^{ranint} * Rnd \tag{7}$$

where ‘ranint’ denotes random integer.

The starting population generated from the above formulation is shown in Fig. 8.

New population selection mechanism

In Genetic Algorithm, the new heap selection mechanism is used to transfer the strong individuals in the population to the next generations. Proportionate reproduction, ranking, tournament, and steady state or genitor are some of the selection mechanisms used in the GA.

The tournament selection mechanism is used in the algorithm developed in this article, since it prevents untimely convergence and is effective in minimization problems [37]. The solution steps of the algorithm are shown in Fig. 9.

In the figure, three different individuals from the old generation were randomly selected for each individual in the new generation and the chromosome with the minimum fitness value was transferred to the next generation.

Crossover

Different chromosome families are produced by using the crossover operator in the genetic algorithm in order to scan the solution space more broadly and to avoid the local optimum. In this article, the PMX (Partially Mapping Crossover) operator is used for the crossover operation [37]. The PMX crossover mechanism is shown in Fig. 10.

Mutation

The mutation process is employed in the genetic algorithm to provide diversity and increase the quality of the solution. Thanks to the mutation process, while obtaining new solutions, the copying of previous solutions is prevented. Thus, unexplored new solutions are found and better quality and even optimal solutions are reached faster. The mutation is carried out according to a certain acceptance probability in the genetic algorithm [38]. In order to concentrate on quality

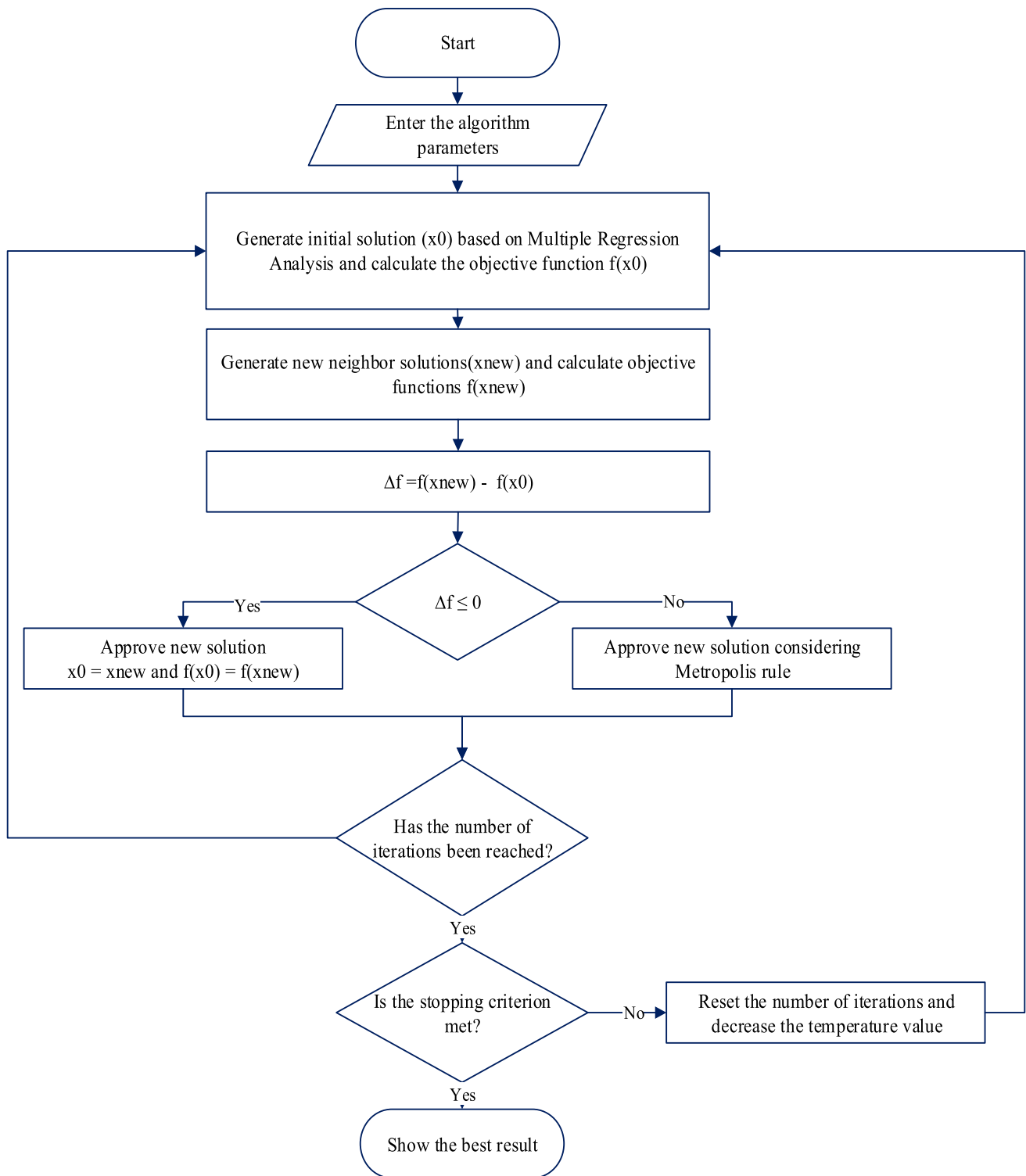
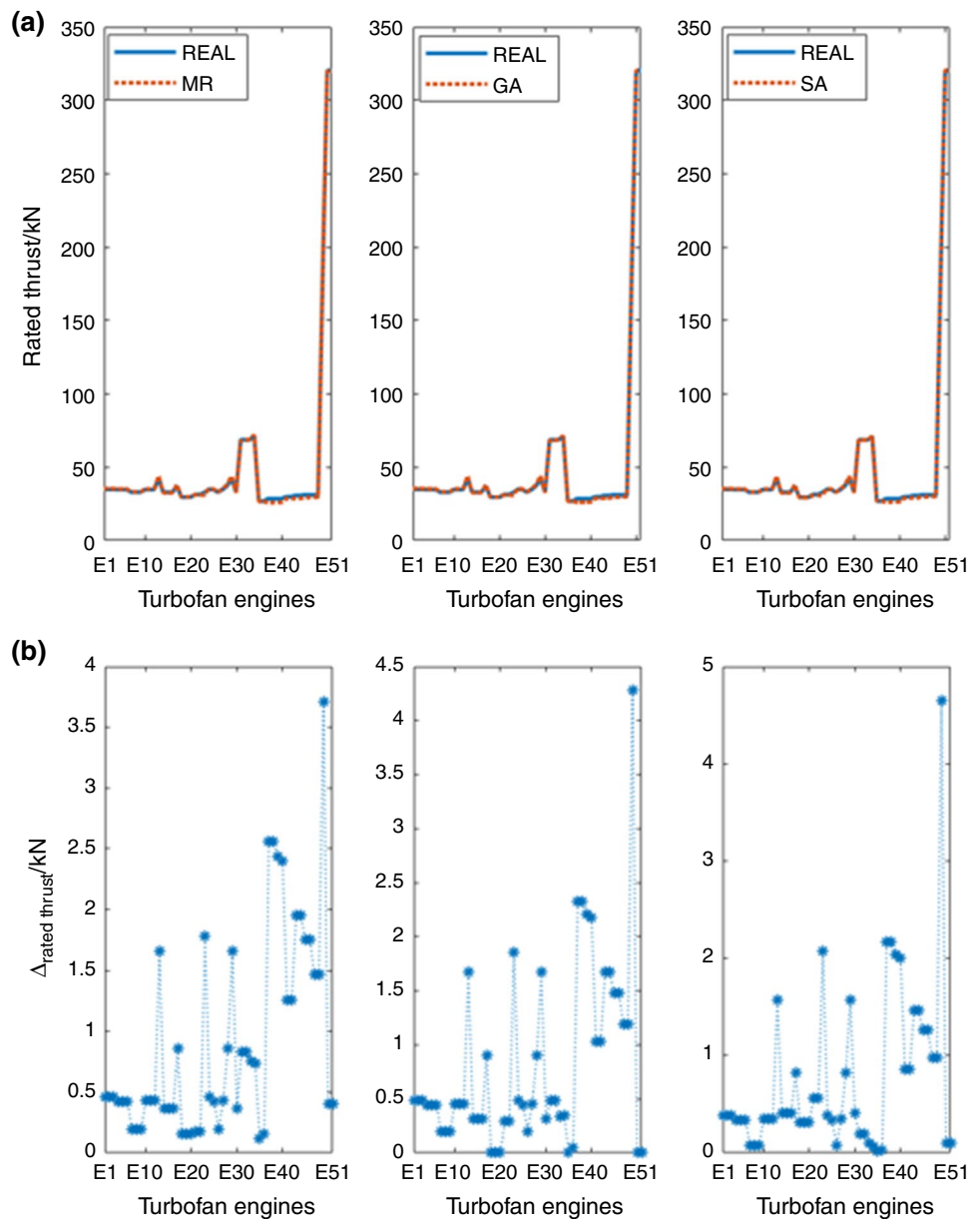


Fig. 11 Simulated Annealing Algorithm

Fig. 12 Real and prediction of rated thrust and the differences between them



solutions, changing the genes in the chromosome according to a certain acceptance probability is called mutation in the genetic algorithm. Different mutation operators can be used according to the problem type and solution representation. In this article, the mutation process is performed by changing the value of the randomly selected gene in the chromosome [37].

Fitness function

The fitness function is used to determine whether the solutions in the chromosomes meet the problem constraints and how close they are to the determined objective [37]. In this article, the error due to estimation is tried to be minimized. The fitness function used in the article is shown in Eq. 8.

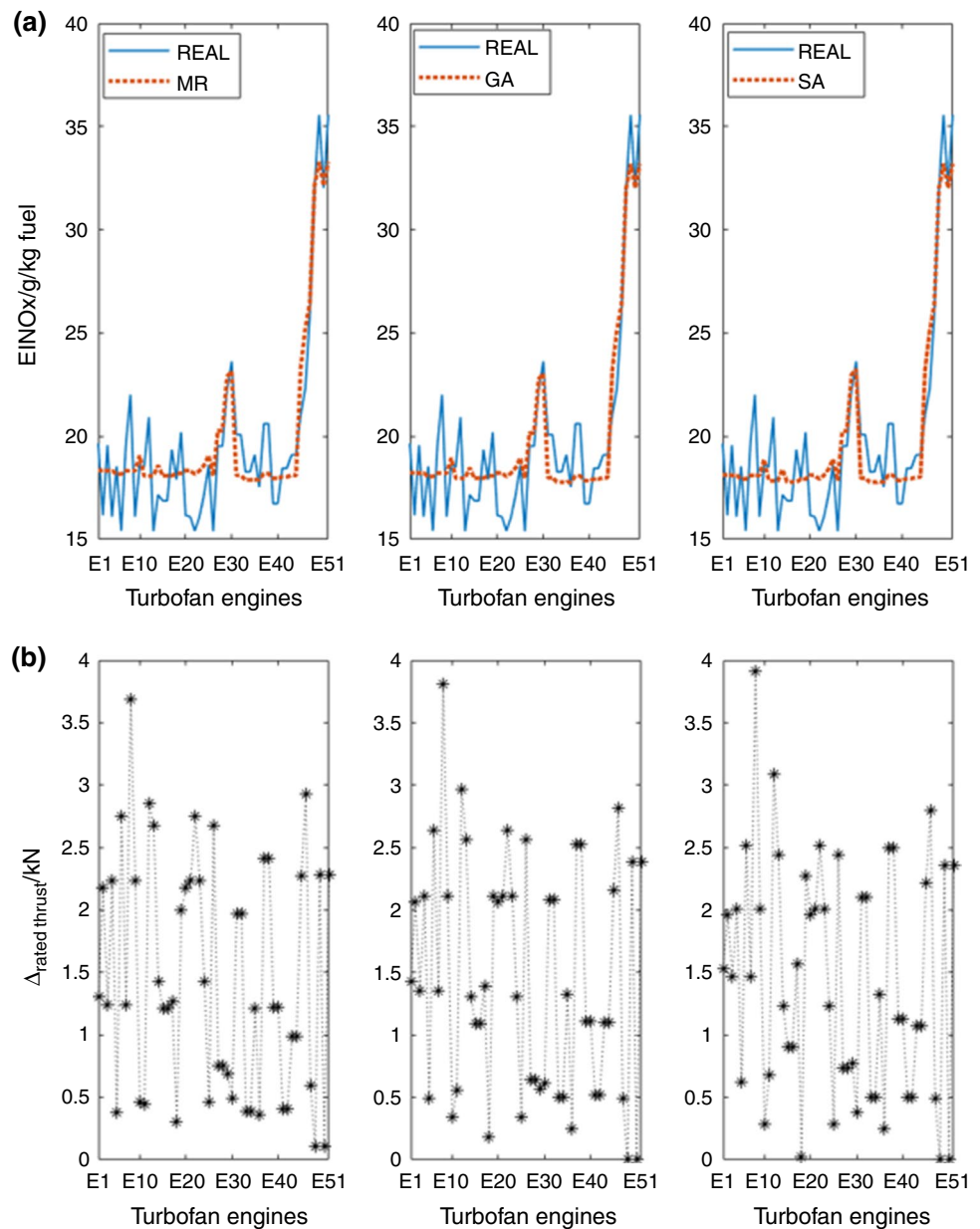
$$\min \{atae\} \rightarrow atae = \frac{\sum_{i=1}^n |Y'_i - Y_i|}{n} \tag{8}$$

where ‘atae’ means average total absolute error. Moreover, Y_i and Y'_i represent actual and estimated values of i th dependent variable.

Simulated annealing

The Simulated Annealing algorithm is a metaheuristic algorithm developed by Kirkpatrick et al., in 1938 [39]. The development of the algorithm was inspired by the physical annealing process of solids. The algorithm, which has a

Fig. 13 Real and prediction of NO_x emission index and the differences between them



stochastic search method, acts iteratively and various solution mechanisms are used to improve the solution quality in each iteration. Quality solutions are carried over to the next iteration to improve the solution. Bad solutions, on the other hand, are accepted according to certain acceptance probability and transferred to the next iteration in order not to get stuck in the local optimum. Boltzmann probability factor $P = e^{(-\Delta/T)}$ is used to determine this acceptance probability.

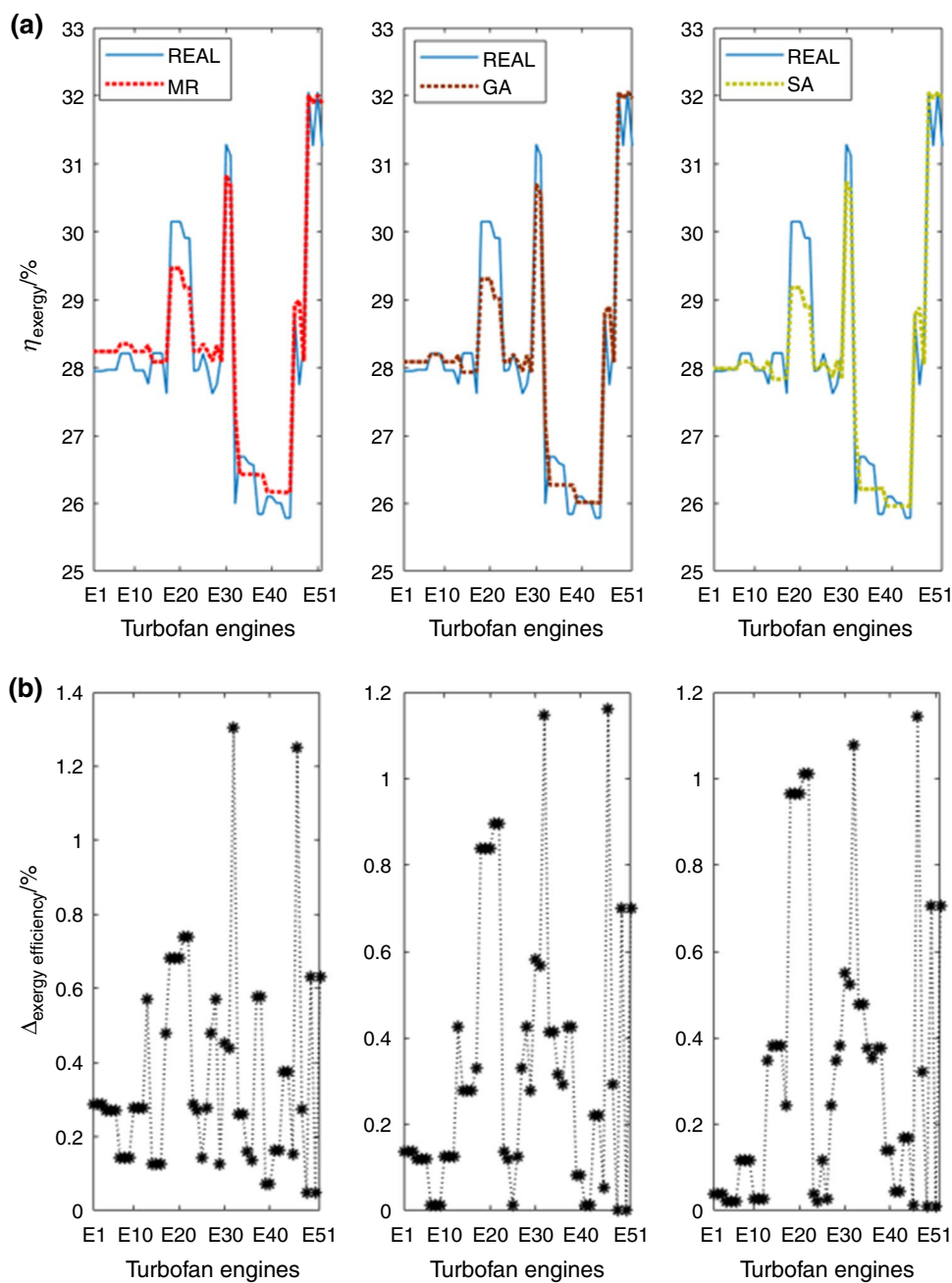
The Simulated Annealing algorithm is frequently utilized in optimization problems because it can use the information and scan the solution space well. In this article, the estimation equation obtained by multiple regression analysis is employed in the initial solution of the Simulated Annealing

algorithm, as in the genetic algorithm [37]. The Simulated Annealing algorithm developed for the problem is shown in Fig. 11 [40].

Results and discussion

This section includes real and prediction values of performance and energy indicators of fifty-one mixed flow turbofan engines (MFTEs) with three different approaches such as multi-regression, genetic algorithm and simulated annealing. In this study, the engines except three ones, have moderate thrust ranges answered the needs of regional aircraft. To

Fig. 14 Real and prediction of exergy efficiency and the differences between them



establish model of the engine parameters, bypass ratio, overall pressure ratio and fuel flow are selected as independent variable that are mentioned above.

Figure 12 presents rated thrust of mixed flow turbofan engines at take-off phase. As can be seen in, with all three methods, the converge to real thrust values exists with high accuracy shown in Fig. 12a. Real rated thrust of the MFTE changes between 26.87 and 320.3 kN whereas the predicted thrust is computed to change between 25.93 and 319.89 kN by the MR technique. Being one of metaheuristic method, GA estimates thrust values to vary between 26.16 and 320.3

kN while thanks to SA, it is found between 26.32 and 320.4 kN.

When comparison is made between these methods, the mean difference between real and prediction thrust occurs the lowest at SA approach with 0.7471 kN. It is followed by GA with 0.8045 kN. As for multi-regression method, it gives the biggest difference with 0.8998 kN. When considering optimization enhancement, thanks to metaheuristic methods, the model error could be minimized.

Figure 13 shows another important parameter that is NO_x emission index (EI NO_x) at take-off phase. Aircraft

Fig. 15 Real and prediction of wasted exergy ratio and the differences between them

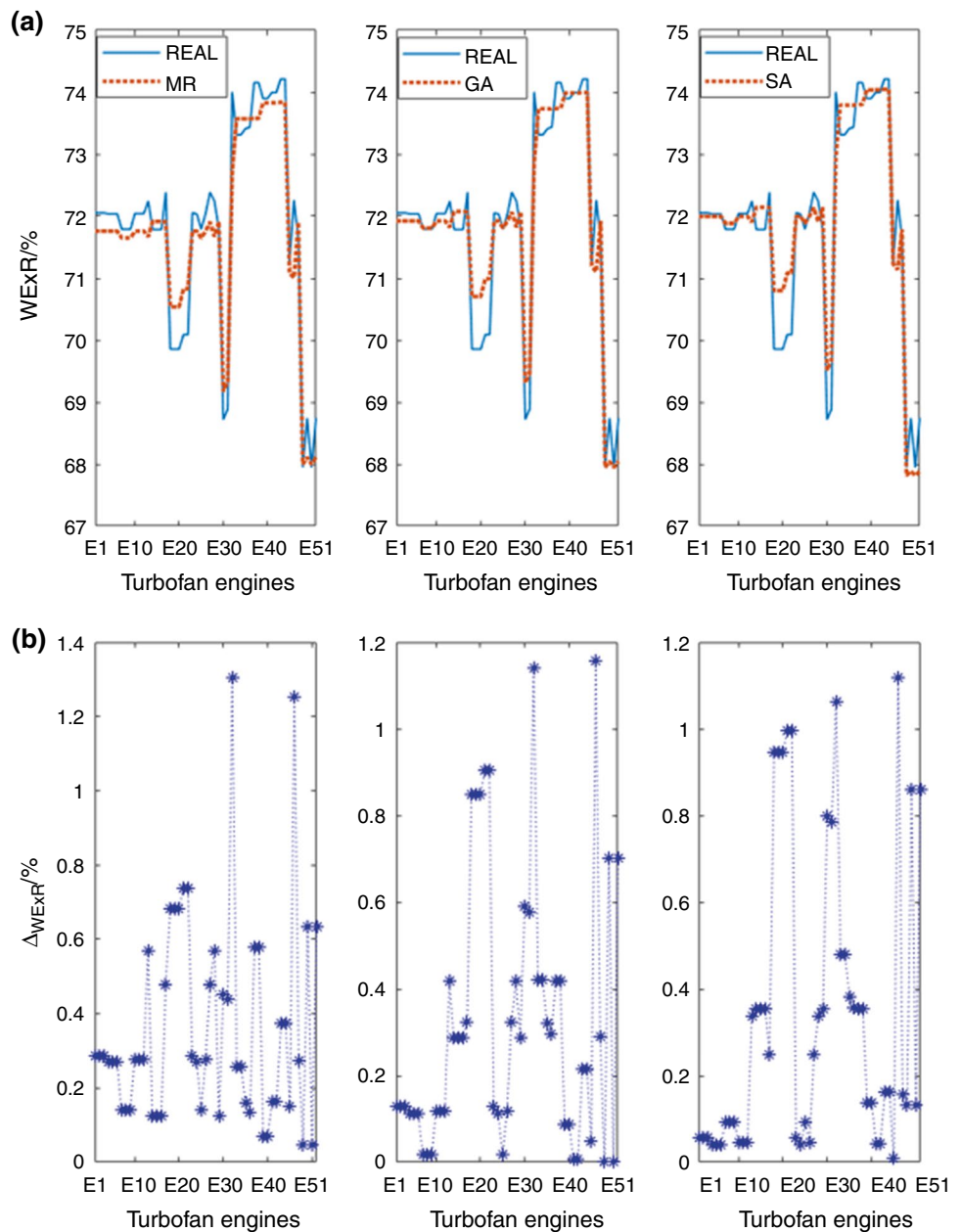


Table 2 The errors and correlation of coefficient regarding rated thrust and EI NO_x

	Rated thrust			EI NO_x		
	MR	GA	SA	MR	GA	SA
MSE	1.4877	1.34041	1.2524	2.9766	2.9497	2.9025
MAE	0.8998	0.80455	0.7471	1.4642	1.4530	1.4383
nMSE	0.000605	0.000544	0.000508	0.00749	0.00746	0.00736
R^2	0.99957	0.99961	0.99964	0.8620	0.8633	0.8655

NO_x emission consists of NO and NO_2 , related to engine power. The higher the power, the higher the NO_x , whereas it holds for vice versa for CO and HC emissions. Therefore,

modeling NO_x emission index at high power such as take-off and cruise is the one of main motivations of the study. According to Fig. 13, real EI NO_x of MFTE changes between

Table 3 The errors and correlation of coefficient regarding exergy efficiency and wasted exergy ratio

	Exergy efficiency			Wasted exergy ratio		
	MR	GA	SA	MR	GA	SA
MSE	0.2044	0.1998	0.2012	0.2041	0.2008	0.2015
MAE	0.3619	0.3259	0.3264	0.3615	0.3258	0.3601
nMSE	0.000256	0.000252	0.000253	3.968×10^{-5}	3.891×10^{-5}	3.935×10^{-5}
R ²	0.9263	0.9280	0.9277	0.9265	0.9276	0.9270

15.4 and 35.6 g/kg fuel while prediction values obtained by MR method remain between 17.88 and 33.28 g/kg fuel. Moreover, GA estimates EI NO_x of the engines between 17.77 and 33.17 g/kg fuel whereas with SA approach, it is found between 17.78 and 33.20 g/kg fuel. To clarify the differences between real and predicted EI NO_x, Fig. 13b is plotted the EI NO_x difference according to each engine. In this context, the mean difference changes with respect to the method. The lowest mean difference belongs to SA technique with 1.4383 g/kg fuel whereas GA slightly obtains higher difference with 1.4530 g/kg fuel. As expected, the highest difference is obtained with 1.4642 g/kg fuel by MR method.

Figure 14 illustrates modeling of exergetic parameters regarding fifty-one MFTEs. Initially, real exergy efficiency of the engines is observed to change between 25.78% and 32.05%. However, the predicted ExEFF of the engines throughout the MR method varies between 26.16% and 32% whereas the GA method predicts ExEFF varying between 26 and 32.05% whereas with SA, it is found to remain between 25.95% and 32.04%. To look the results closer, Fig. 14b gives the difference real and predicted ExEFF on the engine basis. Contrast to the previous analyses, GA method provides the lowest the mean difference with 0.3259% whereas it is followed by SA approach with 0.3264%. At the same way, the highest difference happens with MR method with 0.3619%. It could be inferred that metaheuristic methods could change superiority according to each other; however, these benefits to improve conventional modeling methods.

Lastly, wasted exergy ratio of the MFTEs is given in Fig. 15. It is recognized that real WExR changes between 67.94% and 74.21% while the estimated WExR vary between 67.99% and 73.83% by MR method. The predicted values of WExR are found between 67.94% and 73.99% when implementing GA technique whereas it is seen that these change between 67.81% and 74.04% by SA approach. On the other hand, there is relatively low difference between the real and predicted WExR shown in Fig. 15b. Namely, it is calculated that the mean difference becomes 0.3619% by MR, 0.3258% by GA and 0.3404% by SA.

To delicately evaluate the several findings of model, some error values as well as coefficient of determination (R²) are computed for modeling of the engine parameters.

According to Table 2, the model errors of rated thrust and EI NO_x by multi-regression are improved at the highest degree by applying simulated annealing. Namely, minimum square error decreases from 1.4877 (obtained by MR) to 1.2524 (obtained by SA) whereas R² of the thrust modeling attains from 0.99957 to 0.99964. Furthermore, EI NO_x modeling is obtained relatively lower accuracy. This situation shows that EI NO_x depends more variables than rated thrust, namely this factor could be type of combustor. In this regard, MSE of the EI NO_x modeling diminishes from 2.9766 to 2.9025 while its R² changes from 0.8620 to 0.8655 due to implementation of SA technique.

Table 3 presents the model errors pertinent to exergy efficiency and wasted exergy ratio of the MFTEs. In these models, genetic algorithm improves findings of multi-regression more when compared with SA method. Model of exergy efficiency has 0.2044 of MSE by MR method whereas thanks to GA it poses the lower MSE with 0.1998. Namely, R² of the exergy efficiency modeling increases from 0.9263 to 0.9280. Lastly, MSE of model of wasted exergy ratio regarding the MFTE diminishes from 0.2041 to 0.2008 owing to implementation of GA. Moreover, R² of the WExR varies from 0.9265 to 0.9276 thanks to genetic algorithm method.

Conclusions

Modeling of aircraft engine parameters showing energy efficiency and environmental impact has been drawn attention for scientists since alleviating aircraft emissions has become a hot topic for across the world. In this study, multi-regression is performed regarding performance, emission and exergy metrics of fifty-one mixed flow turbofan engines while considering their bypass ratio, overall pressure ratio and fuel flow as independent variables. Moreover, the models obtained by MR method are exposed to metaheuristic techniques such as genetic algorithm and simulated annealing. After that, the model findings regarding the engine parameter are compared with each other. For mixed flow turbofan engines, modeling of energy, emission and exergy parameters are performed at take-off condition where fuel flow and NO_x emission index emerge relatively higher. Main findings of the present study could be ranked as follows:

- With knowing of bypass ratio, overall pressure ratio and fuel flow of mixed flow turbofan engines, rated thrust, exergy efficiency and wasted exergy ratio of MFTEs could be estimated higher fidelity than NO_x emission index regarding MFTEs. It could be partly attributed that EI NO_x is affected from environmental condition, type of combustor and the engine age.
- It is observed that to apply metaheuristic methods to improve conventional regression method benefits to the model accuracy. Namely, thanks to GA and SA methods, the model error decreases when compared with multi-regression method for each parameter.
- Among the MFTE metrics, R^2 of rated thrust is modeled at the highest accuracy, which is more than 0.999 by each techniques.
- Normalized minimum square error of rated thrust of MFTEs experiences decrement from 0.000605 (by MR) to 0.000508 (by SA). Similarly, nMSE of EI NO_x diminishes from 0.00749 to 0.00736 thanks to implementation of metaheuristic method.
- It is comprehended that exergy efficiency and wasted exergy ratio considerably depend on the determined variables. Namely, R^2 of ExEFF and WExR is computed as 0.9280 and 0.9276, respectively.
- For NO_x emission index, if type of combustor type as a variable could be added to modeling of EI NO_x , its R^2 could be enhanced.

When considering the outcomes obtained from the current analyses, environmental effect and the engine efficiency for mixed flow turbofan engines that are utilized most of regional jet aircrafts could be estimated by knowing their three specifications with multi-regression methods. Thanks to several metaheuristic techniques, the model error of these engine metrics could be minimized. As a next study, aircraft emissions of mixed flow turbofan engines for all LTO phases could be predicted by machine learning approach. Also, the applied methods in this study could be implemented to other types of gas turbine engines so as to estimate emissions values. Lastly, HC and CO emissions regarding MFTEs can be estimated several phases such as taxiing, approach and landing where these emissions are more dominant compared with take-off phase.

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