Species and thermal radiation on micropolar hydromagnetic dusty fluid flow across a paraboloid revolution

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Abstract

Hydromagnetic flow of energy and species transfer behaviour of micropolar dusty fluid across a paraboloid revolution has been investigated. Heat and mass transfer phenomena are inspected through radiation, joule heating and chemical reaction. The boundary layer equations are modelled and transformed to a system of ODE'S with the aid of similarity transformations and solved numerically by utilizing Runge–Kutta integration scheme. The graphical analysis has been emphasized for the fluid and dust phase velocity, angular velocity, energy and species fields to the influence of sundry dynamical flow quantities. In addition, friction factor, Nusselt number and Sherwood number are presented in plots and tables and discussed elaborately and nice validation is emphasized. The obtained numerical results are checked with the earlier published articles. The boundary layers of angular velocity, temperature and concentration distributions increase for escalating values of magnetic parameter. For escalating values of material parameter, velocity profile increases for both momentum velocity and dusty velocity and opposite trend is seen in angular velocity profile. Concentration and boundary layer thickness diminish for escalating values of kr . Local friction factor declines for boost up values of magnetic parameter and porosity permeability parameter. The rate of heat transfer decreases for increasing values of Eckert number and radiation parameter, and rate of mass transfer increases for increasing values of Schmidt number and chemical reaction parameter.

Keywords Magnetic field · Porosity · Radiation · Micropolar dusty fluid · Heat transfer · Paraboloid revolution

List of symbols

A	Non-negative number
B_0	Magnetic field strength
Bi	Biot number
C	Concentrations of fluid
C_f	Friction factor
C_p	Concentrations of dust particles
C_{mf}	Specific heat of dust particles
C_∞	Ambient fluid concentration
D_m	Mass diffusivity coefficient
Ec	Eckert number
F	Dusty fluid

Gr_b	Thermal buoyancy parameter
Gr_c	Solutal buoyancy parameter
Kr^*	Chemical reaction coefficient
M	Magnetic parameter
N	Microrotation velocity
Nr	Radiation parameter
N_{ux}	Nusselt number
Pr	Prandtl number
R	Material parameter
Re_x	Reynolds number
Sc	Schmidt number
Sh_x	Sheer wood number
T	Temperature of fluid
T_f	Convective fluid Temperature
T_p	Temperature of dust particles
T_∞	Ambient fluid temperature
a	Stretching rate
b	Non-negative number
f	Fluid
g	Gravitational acceleration
h_f	Convective heat transfer
k	Vertex viscosity

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k_1	Stokes drag constant
k_f	Thermal conductivity of the fluid
k_p	Porous permeability parameter
kr	Chemical reaction parameter
j	Microinertia per unit mass
l	Mass concentration of dust particle
m	Velocity power law index
m_0	Boundary parameter
q_r	Radiative heat flux
r	Mass of dust particle
s	Number density of dust particles
u	Velocity components of liquid fluid along x axis
u_p	Velocity components of dusty fluid along x axis
u_w	Stretching velocity
v	Velocity components of fluid along y axis
v_p	Velocity components of dusty fluid along y axis
α	Upper horizontal sphere of paraboloid revolution
β	Thermal expansion coefficient
β^*	Solutal expansion coefficient
β_v	Fluid particle interaction parameter for velocity
β_t	Fluid particle interaction parameter for temperature
β_c	Fluid particle interaction parameter for concentration
γ	Specific heat coefficient
γ^*	Spin gradient viscosity
σ	Electrical conductivity
ρ	Effective density of fluid
ρ_p	Density of dust particles
μ	Dynamic viscosity of fluid
ν	Kinematic viscosity of fluid
τ_c	Concentration equilibrium time
τ_t	Thermal equilibrium time
τ_v	Relaxation time of dust particle for velocity
ξ	Stream function for fluid phase velocity
ξ_p	Stream function for dust phase velocity
θ	Dimension less temperature for fluid phase
θ_p	Dimension less temperature dust phase
ϕ	Dimension less concentration for fluid phase
ϕ_p	Dimension less concentration dust phase

Introduction

Non-Newtonian fluids have been one of the most challenging and interesting areas in research for engineers, physicists, mathematicians and numerical simulates. Navier–Stokes theory cannot illuminate effectively the flow premises of polyamide fluids, emulsion suspensions, and liquids comprise definite additives. Out of these, the micropolar fluid is first presented in Eringen [1]. Micropolar fluid is a fluid which consists of microstructure applicable to a group of fluids along unsymmetrical stress tensor. A review of the subject and applications of micropolar fluid mechanics was

mentioned by Peddison and McNitt [2], Khonsari and Brewe [3], Qukaszewicz [4]. Das [5] invoked the impact of hydro-magnetic micropolar fluid across a rotating frame. Dulal pal and Sewli Chatterjee [6] studied the 2D flow of a micropolar fluid through a permeable stretching sheet, and the equations are employed numerically with finite difference method using quasi-linearization technique. Asma Khalid et al. [7] studied hydromagnetic micropolar couple stress fluid in a porous medium. By obtaining solutions, they are using perturbation technique. Hayat et al. [8] scrutinized steady 2D micropolar nanofluid across a stretching sheet. Ganeswara Reddy and Rama Subba Reddy [9] studied heat transfer effects on MHD micropolar fluid over a vertical stretching surface. An unsteady two-dimensional MHD micropolar fluid flow over vertical plate analysed Ganeswara Reddy [10]. They are resolved analytically by using perturbation method.

Alloying of micrometre-sized particles (dust particles) in the base fluids is also used to improve the thermal conductivity of the base liquid which is called dusty fluid. It has several applications in the field of combustion, purification of crude oil, electrostatic precipitation and fluid droplet sprays, etc. Saffman [11] has described the heat transfer effects on particle suspension flow. Chamkha [12] investigated the unsteady two-dimensional magnetohydrodynamic dusty flow across a circular pipe. Influence of heat transfer effects on Marangoni convective Dusty Casson fluid was scrutinized by Mahanthesh and Gireesha [13]. Mamata et al. [14] analysed magnetohydrodynamic flow of Fourier heat flux over cylinder. They are plotting graphical representations for the combination of nanoparticles Water + Silver and Water + Graphene. Scrutinization of micropolar dusty nanofluid across a stretching sheet addressed Ghadikolaei et al. [15]. Ganeswara Reddy et al. [16] invoked the phenomena of heat transfer impacts on radiative hydromagnetic Oldroyd-B dusty fluid across two different geometries. Cross-diffusion effects on MHD nonlinear radiation in two non-Newtonian fluids have been examined by Ganeswara Reddy et al. [17]. Basma et al. [18] studied the slip flow of titanium and ferromagnetic nanoparticles along with dusty fluid.

Combined heat and mass transfer phenomena are found everywhere in nature, and it is important in all the branches of science and technology. In many practical applications, there exist significant temperature and concentration variations between the surface of the hot body and the free stream. It is a major area of interest, and it has assumed practical importance in engineering devices like heat exchangers, solar collectors, nuclear reactors and electronic equipments. Heat transfer effects on magnetohydrodynamic nanofluid were analysed by Sajid et al. [19]. They are obtaining solution by using homotopy analytical method. Dat et al. [20] examined the impact of Al_2O_3 nanoparticles with various shapes on thermal characteristics of nanofluid within a

permeable space. Plane Poiseuille nanofluid with heat and mass transfer in a channel is examined by Alamri et al. [21]. Bhatti et al. [22] have been examined effects of heat transfer on particle–fluid suspension induced by metachronal wave. Bhatti et al. [23] scrutinized coagulation and endoscopy analysis for a particulate fluid suspension. Ellahi et al. [24] investigated heat transfer impacts on Electro-Osmotic Couette–Poiseuille Flow of MHD Power Law Nanofluid with Entropy Generation. Khan et al. [25] deliberated by heat generation in peristalsis flow of MHD nanofluids filled in an asymmetric channel is proposed. Sheikholeslami et al. [26] analyse the thermal behaviour of alumina nanofluid in a duct. Raei [27] explored the stabilized γ - Al_2O_3 /water nanofluids have been examined at the concentrations of 0.05 and 0.15 vol% with variation of flow rates. Monge-Palacios et al. [28] presented that cluster representation for fluids is an effective method to gain more fundamental insight into the nature of heat transfer in complex chemical systems such as nanofluids. Sheikholeslami et al. [29] have reported Fe_3O_4 –water nanofluid flow in a cavity with constant heat flux is investigated 10 using a control volume-based finite element method. Sheikholeslami et al. [30] explored the heat transfer effects on magnetohydrodynamic nanoparticle through exergy analysis. Rafatiyo et al. [31] developed a relationship for the ratio of the frequencies of termolecular to bimolecular collisions in terms of the temperature, density, and collision times. Sheikholeslami [32] explored the effect of magnetic field in a porous thermal energy storage enclosure using CuO nanoparticles. Galerkin finite element method is employed for this unsteady problem.

The study of boundary layer fluid flow has broad range of applications in science, industry and aviation. Published information on horizontal stretching sheet, circular cylinder, and two different geometries has been reported in [33–36]. Mather [37] was first investigating the moment of viscous fluid across a paraboloid. There are so many practical applications in paraboloid revolution, mainly used in automobile headlights, solar furnaces, radar, and radio relay stations heater, satellite dishes, automobile headlights, McDonald's Arches Fountains path of a Bal. Lee [38] presented a boundary layer flow of thin needle over a paraboloid. Two-dimensional Blasius flow of Carreau flow on upper horizontal sphere of paraboloid revolution has been deliberated by Animasun and Pop [39]. They are obtaining numerically by using bvp5c MATLAB package. Kumaran et al. [40] developed a buoyancy model for two non-Newtonian fluids across a revolution of paraboloid. The researchers were plotted graphical representations for both the non-Newtonian (Casson and Carreau) fluids. Khan et al. [41] described bi-convection flow of Carreau nanofluid across an upper horizontal sphere of paraboloid revolution. It is concluded that for escalating values of Ha and Rb , there is an enhancement in the friction factor.

Many investigators concentrate all the above researches on analysing heat and mass transfer nature of flows by taking some physical effects with the normal substantial models. But in the present model, we study the flow of a micropolar dusty fluid across a paraboloid. Here we built up the influence of thermal radiation, chemical reaction, magnetic field, porosity parameter, joule heating. The boundary layer governing differential equations are modelled and transformed to a system of nonlinear ODE'S with the aid of similarity transformations. The final controlled equations along boundary restrictions are resolved numerically by Runge–Kutta method. The graphical results are elaborated for the fluid and dust phase velocity, angular velocity, temperature and concentration fields through the plots of non-dimensional quantities for sundry values of arising flow parameters. In addition, the physical quantities of interest namely friction factor, Nusselt and Sherwood numbers are presented in plots and tables and discussed elaborately to the impacts of multiple flow parameters.

Modelling

Consider a steady magnetohydrodynamic 2D flow of micropolar dusty fluid past a non-melting upper part of the revolving paraboloid. The flow is confined over the region $A(x+b)^{\frac{1-m}{2}} \leq y < \infty$, and the revolving paraboloid is stretched with the velocity $u_w = a(x+b)^m$; hence, starting point of the flow at the slot is a function $y = A(x+b)^{1-m/2}$.

Here horizontal surface is located along x -direction and y -axis is normal to it. A uniform magnetic field of strength B_0 is applied in the flow direction as displayed in Fig. 1. Magnetic Reynolds number as low as conceivable to disregard the induce magnetic field, radiation, joule heating along with

Fig. 1 Geometry of the problem

chemical reaction effects are taken into account; under these assumptions, the governing equations of the flow are

The governing equations with stream function ξ are as follows [40–42]:

$$\frac{\partial^2 \xi}{\partial x \partial y} - \frac{\partial^2 \xi}{\partial y \partial x} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial \xi}{\partial y} \frac{\partial^2 \xi}{\partial x \partial y} - \frac{\partial \xi}{\partial x} \frac{\partial^2 \xi}{\partial y^2} - \left[\frac{\partial}{\partial x} \left(g \beta x \left(\frac{m+1}{2} \right) (T - T_\infty) \right) \right. \\ \left. + \frac{\partial}{\partial x} \left(g \beta^* x \left(\frac{m+1}{2} \right) (C - C_\infty) \right) \right] \\ = \left(\nu + \frac{k}{\rho} \right) \frac{\partial^3 \xi}{\partial y^3} - \frac{\sigma B_0^2}{\rho} \frac{\partial \xi}{\partial y} + \frac{k_1 s}{\rho} \left(\frac{\partial \xi_p}{\partial y} - \frac{\partial \xi}{\partial y} \right) \\ + \frac{k}{\rho} \frac{\partial N}{\partial y} - \frac{\nu}{k_p} \frac{\partial \xi}{\partial y} \end{aligned} \quad (2)$$

$$\frac{\partial \xi}{\partial y} \frac{\partial N}{\partial x} - \frac{\partial \xi}{\partial x} \frac{\partial N}{\partial y} = \frac{\gamma^*}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j} \left(2N + \frac{\partial^2 \xi}{\partial y^2} \right), \quad (3)$$

$$\begin{aligned} \frac{\partial \xi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \xi}{\partial x} \frac{\partial T}{\partial y} = \frac{k_f}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{1}{c_p} \left(\nu + \frac{k}{\rho} \right) \left(\frac{\partial^2 \xi}{\partial y^2} \right)^2 \\ + \frac{\sigma B_0^2(x)}{\rho c_p} \left(\frac{\partial \xi}{\partial y} \right)^2 + \frac{\rho_p C_m}{\tau_t} (T_p - T) + \frac{\rho_p}{\tau_v} \left(\frac{\partial \xi_p}{\partial y} - \frac{\partial \xi}{\partial y} \right)^2 \end{aligned} \quad (4)$$

$$\rho c_p \left(\frac{\partial \xi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \xi}{\partial x} \frac{\partial C}{\partial y} \right) = D_m \frac{\partial^2 C}{\partial y^2} - \text{Kr}^* (C - C_\infty) + \frac{\rho_p C_p}{\tau_c} (C_p - C) \quad (5)$$

$$\frac{\partial^2 \xi_p}{\partial x \partial y} - \frac{\partial^2 \xi_p}{\partial y \partial x} = 0 \quad (6)$$

$$\rho_p \left(\frac{\partial \xi_p}{\partial y} \frac{\partial^2 \xi_p}{\partial x \partial y} - \frac{\partial \xi_p}{\partial x} \frac{\partial^2 \xi_p}{\partial y^2} \right) = k_1 s \left(\frac{\partial \xi}{\partial y} - \frac{\partial \xi_p}{\partial y} \right) \quad (7)$$

$$\rho_p C_m \left(\frac{\partial \xi_p}{\partial y} \frac{\partial T_p}{\partial x} - \frac{\partial \xi_p}{\partial x} \frac{\partial T_p}{\partial y} \right) = -\frac{\rho_p C_{pf}}{\tau_t} (T_p - T) \quad (8)$$

$$\rho_p C_m \left(\frac{\partial \xi_p}{\partial y} \frac{\partial C_p}{\partial x} - \frac{\partial \xi_p}{\partial x} \frac{\partial C_p}{\partial y} \right) = -\frac{\rho_p C_{pf}}{\tau_c} (C_p - C) \quad (9)$$

along the boundary conditions

$$u = u_w = a(x+b)^m, \quad v = 0,$$

$$N = -m_0 \frac{\partial u}{\partial y}, \quad -k_f \frac{\partial T}{\partial y} = h_f (T_f - T),$$

$$C = C_w \text{ at } y = A(x+b)^{\frac{1-m}{2}}$$

$$u \rightarrow 0, \quad N \rightarrow 0, \quad u_p \rightarrow 0, \quad v_p \rightarrow v,$$

$$T \rightarrow T_\infty, \quad T_p \rightarrow T_\infty, \quad C \rightarrow C_\infty,$$

$$C_p \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (10)$$

in which the components u, u_p are the velocity components of liquid and dusty fluid along x axis, respectively, v, v_p are the velocity components of liquid and dusty fluid along y axis, respectively. ν is kinematic viscosity of the fluid, k is vertex viscosity, ρ is effective density of the fluid, σ is the electrical conductivity, B_0 is magnetic field strength, $k_1 = 6\pi\mu_f a$ is the stokes drag constant, $\rho_p = rs$ is density of dust particles, m is velocity power law index, s is number density of the dust particles, r is the mass of the dust particle, g is the gravitational acceleration, β is the thermal expansion coefficient, and j, γ^* and k are the microinertia per unit mass, spin gradient viscosity and vertex viscosity, respectively.

Spin gradient viscosity γ^* can be defined as

$$\gamma^* = \left(\mu + \frac{k}{2} \right) j = \mu \left(1 + \frac{R}{2} \right) \quad (11)$$

where μ dynamic viscosity of the fluid, $R = \frac{k}{\mu}$ is the dimensionless viscosity ratio and is called the material parameter, and we take $j = \frac{\nu}{a}$ a reference length. m_0 is the boundary parameter where T and T_p are the temperatures of the fluid and dust particles, respectively. C_{mf} is the specific heat of the dust particles, τ_t is the thermal equilibrium time, τ_v is the relaxation time of the dust particle for velocity, and C and C_p are the concentrations of fluid and dust particles.

Define the following similarity transformations and variable quantities:

$$\zeta = \sqrt{\frac{a(m+1)}{2\nu}} (x+b)^{\frac{m-1}{2}} y, \quad \xi = \sqrt{\frac{2}{n+1}} \nu a (x+b)^{m+1} h(\zeta),$$

$$\xi_p = \sqrt{\frac{2}{m+1}} \nu a (x+b)^{m+1} H(\zeta),$$

$$N = a(x+b)^m \sqrt{\frac{a(m+1)}{2\nu}} (x+b)^{\frac{m-1}{2}} G(\zeta),$$

$$\Theta(\zeta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \Theta_p(\zeta) = \frac{T_p - T_\infty}{T_w - T_\infty},$$

$$\Phi(\zeta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \Phi_p(\zeta) = \frac{C_p - C_\infty}{C_w - C_\infty} \quad (12)$$

Table 1 Comparison values of $f''(0)$ for various values of m

m	Animasun and Sandeep [42]	Kumaran et al. [40]	Present results
0.1	-0.8671009	-0.86710092	-0.86710093
0.2	-0.8624053	-0.8624053	-0.8624051
0.3	-0.8584863	-0.8584863	-0.8584863
0.4	-0.8481543	-0.8481543	-0.8481544

By making use of Eq. (12), Eqs. (2)–(8) are transformed as

$$(1+R)h''' + hh'' - \frac{2m}{m+1}h'^2 + RG' + l\beta_v(H' - h') - \frac{2}{m+1}\left(M + \frac{1}{k_p}\right)h' + Gr_b\Theta - Gr_c\Phi = 0 \quad (13)$$

$$\left(1 + \frac{R}{2}\right)G'' + hG' - \frac{3m-1}{m+1}h'G - \frac{2R}{m+1}(2G + h'') = 0 \quad (14)$$

$$\left(1 + \frac{4}{3}Nr\right)\Theta'' + \frac{2}{m+1}Prh\Theta' + \frac{2}{m+1}Pr l\beta_t(\Theta_p - \Theta) + \frac{2}{m+1}Pr lEc\beta_v(H' - h')^2 + Pr Ec(1+R)h''^2 + Pr EcM^2h'^2 = 0 \quad (15)$$

$$\Phi'' + Sc h\Phi' + Sc \beta_c(\Phi_p - \Phi) + Sc kr\Phi = 0 \quad (16)$$

$$HH'' - \frac{2m}{m+1}H'^2 + \beta_v(h' - H') = 0 \quad (17)$$

$$\Theta'_p H - \gamma \beta_t(\Theta_p - \Theta) = 0 \quad (18)$$

$$\Phi'_p H - \gamma \beta_c(\Phi_p - \Phi) = 0 \quad (19)$$

Along with boundary conditions

$$\begin{aligned} h(\alpha) &= \alpha \frac{1-m}{m+1}, \quad h'(\alpha) = 1, \quad G(\alpha) = -m_0 h''(\alpha), \\ \Theta'(\alpha) &= -Bi(1 - \Theta(\alpha)), \quad \Phi(\alpha) = 1 \quad \text{at } \alpha = 0 \\ h' &\rightarrow 0, \quad H' \rightarrow 0, \quad H \rightarrow h, \quad G \rightarrow 0, \quad \Theta \rightarrow 0, \\ \Theta_p &\rightarrow 0, \quad \Phi \rightarrow 0, \quad \Phi_p \rightarrow 0 \quad \text{as } \alpha \rightarrow \infty \end{aligned} \quad (20)$$

Here $\alpha = A\sqrt{\frac{a(m+1)}{2v}}$ denotes UHSPR. To transform the equations and boundary conditions $h(\zeta - \alpha) = f(\eta)$, $G(\zeta - \alpha) = g(\eta)$, $H(\zeta - \alpha) = F(\eta)$, $\Theta(\zeta - \alpha) = \theta(\eta)$, $\Theta_p(\zeta - \alpha) = \theta_p(\eta)$ which gives

The governing equations are reduced to the form

$$(1+R)f'''' + ff'' - \frac{2m}{m+1}f'^2 + Kg' + l\beta_v(h' - f') - \frac{2}{m+1}\left(M + \frac{1}{k_p}\right)f' + Gr_b\theta + Gr_c\phi = 0 \quad (21)$$

$$\left(1 + \frac{R}{2}\right)g'' + fg' - \frac{3m-1}{m+1}f'g - \frac{2R}{m+1}(2g + f'') = 0 \quad (22)$$

$$\left(1 + \frac{4}{3}Nr\right)\theta'' + \frac{2}{m+1}Prf\theta' + \frac{2}{m+1}Pr l\beta_t(\theta_p - \theta) + \frac{2}{m+1}Pr lEc\beta_v(h' - f')^2 + Pr Ec(1+R)f'^2 + Pr EcM^2f'^2 = 0 \quad (23)$$

$$\phi'' + Sc f\phi' + Sc \beta_c(\phi_p - \phi) + Sc kr\phi = 0 \quad (24)$$

$$FF'' - F'^2 + \frac{2m}{m+1}\beta_v(f' - F') = 0 \quad (25)$$

$$\theta'_p F - \gamma \beta_t(\theta_p - \theta) = 0 \quad (26)$$

$$\phi'_p F - \gamma \beta_c(\phi_p - \phi) = 0 \quad (27)$$

and relevant boundary conditions

$$\begin{aligned} f(0) &= \alpha \frac{1-m}{m+1}, \quad f'(0) = 1, \quad g(0) = -m_0 f''(0), \\ \theta'(0) &= -Bi(1 - \theta(0)), \quad \phi(0) = 1 \quad \text{at } \eta = 0 \\ f' &\rightarrow 0, \quad f'' \rightarrow 0, \quad g \rightarrow 0, \quad F' \rightarrow 0, \quad F \rightarrow f, \\ \theta &\rightarrow 0, \quad \theta_p \rightarrow 0, \quad \phi \rightarrow 0, \quad \phi_p \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (28)$$

in which $R = \frac{k}{\mu}$ is material parameter, $Gr_b = \frac{g\beta(T_w - T_\infty)}{a^2(x+b)^{2m-1}}$ thermal buoyancy parameter, $Gr_c = \frac{g\beta^*(C_w - C_\infty)}{a^2(x+b)^{2m-1}}$ solutal buoyancy parameter, $Re_x = u_w(x+b)/\nu$ Reynolds number, $M = \frac{\sigma B^2}{a\rho(x+b)^{m-1}}$ magnetic parameter, $k_p = \frac{ak_p}{v(x+b)^{m-1}}$ porous permeability parameter, $l = \frac{\tau_s}{\rho}$ mass concentration of the dust particle, $\beta_v = \frac{1}{a\tau_v(x+b)^{m-1}}$ the fluid particle interaction parameter for velocity, $\gamma = \frac{c_m}{c_p}$ specific heat coefficient, $Pr = \frac{\nu}{\alpha}$ the Prandtl number, $Nr = \frac{4\sigma^*T_\infty^3}{3k_t k^*}$ radiation parameter, $Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}$ the Eckert number, $\beta_t = \frac{1}{a\tau_t(x+b)^{m-1}}$ the fluid particle interaction parameter for temperature, $Sc = \frac{\nu}{D_m}$ Schmidt number, $kr = \frac{Kr^*(C_w - C_\infty)}{a\tau_c(x+b)^{m-1}}$ chemical reaction parameter, $\beta_c = \frac{1}{a\tau_c(x+b)^{m-1}}$ the fluid particle interaction parameter for concentration, and $Bi = \frac{h_f}{k_f} \sqrt{\frac{\nu}{a}}(x+b)^{\left(\frac{m-1}{2}\right)}$ the Biot number.

Fig. 2 Impact of Magnetic parameter on **a** velocity profile. **b** Dust phase velocity profile. **c** Angular velocity profile. **d** Temperature profile. **e** Dust phase temperature profile. **f** Concentration profile $\phi(\eta)$. **g** Dust phase concentration profile

For physical quantities of engineering interest [39, 40]:

$$C_{fx} = \frac{\tau_w}{\rho u_w^2} \left(\frac{m+1}{2} \right)^{-0.5}, \quad Nu_x = \frac{(x+b)q_w}{k_f(T_w - T_\infty)} \left(\frac{m+1}{2} \right)^{-0.5},$$

$$Sh_x = \frac{(x+b)j_w}{k_f(C_w - C_\infty)} \left(\frac{m+1}{2} \right)^{-0.5} \quad (29)$$

where τ_w the surface shear stress q_w is the surface heat flux and j_w is the mass flux; they are given by:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{\eta=0}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{\eta=0} \quad \text{and} \quad j_w = -D_m \left(\frac{\partial C}{\partial y} \right)_{\eta=0} \quad (30)$$

Using the similarity transformations, we obtain

$$C_f Re_x^{1/2} = \sqrt{\frac{m+1}{2}} f''(0),$$

$$Nu_x Re_x^{-1/2} = -\sqrt{\frac{m+1}{2}} \theta'(0),$$

$$Sh_x Re_x^{-1/2} = -\sqrt{\frac{m+1}{2}} \phi'(0) \quad (31)$$

where $Re_x = u_w(x+b)/\nu$ is the local Reynolds number.

Method of solution

The nonlinear coupled ODEs (21)–(27) with the boundary conditions (28) are solved numerically using *R-K*-based shooting method. Initially, the set of nonlinear ordinary

Fig. 3 Impact of permeability parameter on **a** velocity profile. **b** Dust phase velocity profile. **c** Angular velocity profile

Fig. 4 Impact of mass concentration of the dust particle on **a** velocity profile. **b** Dust phase velocity profile. **c** Dust particle on angular velocity profile. **d** Dust particle on temperature profile. **e** Dust particle on dust phase temperature profile. **f** Dust particle on concentration profile. **g** Dust particle on dust phase concentration profile

$$f = f(1), \quad f' = f(2), \quad f'' = f(3)$$

$$f'' = -\frac{1}{(1+R)} \left[f(1)f(3) - \frac{2m}{m+1} [f(2)]^2 + Kf(5) + l\beta_v(f(11) - f(2)) - \frac{2}{m+1} \left(M + \frac{1}{k_p} \right) f(2) + Gr_b f(6) + Gr_c f(8) \right] \quad (32)$$

differential Eqs. (21)–(27) are converted to first-order differential equations, by using the following procedure

Fig. 5 Impact of material parameter on **a** velocity profile. **b** Dust phase velocity profile. **c** Angular velocity profile

Fig. 6 Impact of fluid particle interaction parameter for velocity on **a** velocity profile. **b** Dust phase velocity profile. **c** Temperature on temperature profile. **d** Temperature on dust phase temperature profile. **e** Concentration on concentration profile. **f** Dust phase concentration profile

Fig. 7 Impact of thermal buoyancy parameter on **a** velocity profile. **b** Dust phase velocity profile. **c** Angular velocity profile

$$\begin{aligned}
 g &= f(4), \quad g' = f(5) \\
 g'' &= -\frac{1}{\left(1 + \frac{R}{2}\right)} \left[f(1)f(5) - \frac{3m-1}{m+1} f(2)f(4) - \frac{2R}{m+1} (2f(4) + f(3)) \right] \\
 F &= f(10), \quad F' = f(11) \\
 F'' &= \frac{1}{f(10)} \left[[f(11)]^2 - \frac{2m}{m+1} \beta_v (f(2) - f(11)) \right]
 \end{aligned} \quad (33)$$

$$\begin{aligned}
 \theta &= f(6), \quad \theta' = f(7) \\
 \theta' &= -\frac{1}{\left(1 + \frac{4}{3}Nr\right)} \left[\begin{aligned} &\frac{2}{m+1} \Pr f(1)f(7) + \frac{2}{m+1} \Pr l \beta_t (f(12) - f(6)) \\ &+ \frac{2}{m+1} \Pr l Ec \beta_v (f(11) - f(2))^2 + \Pr Ec (1 + R) [f(3)]^2 \\ &+ \Pr Ec M^2 [f(2)]^2 \end{aligned} \right]
 \end{aligned} \quad (34)$$

$$\begin{aligned}
 \phi &= f(8), \quad \phi' = f(9) \\
 \phi'' &= -[Sc f(1)f(9) + Sc \beta_c (f(13) - f(8)) + Sckrf(8)] \\
 \theta_p &= f(12) \\
 \theta'_p &= \frac{1}{f(10)} \gamma \beta_t (f(12) - f(6))
 \end{aligned} \quad (35) \quad (37)$$

Fig. 8 Impact of solutal buoyancy parameter on **a** velocity profile. **b** Dust phase velocity profile. **c** Angular velocity profile

$$\begin{aligned}\phi_p &= f(13) \\ \phi'_p &= \frac{1}{f(10)} \gamma \beta_c (f(13) - f(8))\end{aligned}\quad (38)$$

and relevant boundary conditions

$$\begin{aligned}f(1) &= \alpha \frac{1-m}{m+1}, \quad f(2) = 1, \quad f(4) = -m_0 f(3), \quad f(7) = -\text{Bi}(1 - f(6)), \quad f(8) = 1 \quad \text{at } \eta = 0 \\ \left. \begin{aligned}f(2) &\rightarrow 0, \quad f(3) \rightarrow 0, \quad f(4) \rightarrow 0, \quad f(11) \rightarrow 0, \quad f(10) \rightarrow f(1), \\ f(6) &\rightarrow 0, \quad f(12) \rightarrow 0, \quad f(8) \rightarrow 0, \quad f(12) \rightarrow 0\end{aligned} \right\} \text{ as } \eta \rightarrow \infty\end{aligned}\quad (39)$$

To solve Eqs. (29)–(35), we guess the values of $y_4, y_6, y_8, y_9, y_{11}$ and y_{12} which are not given at the initial conditions. Once all initial conditions are found, then we solve Eqs. (29)–(35) which are integrated by using *R-K*

fourth-order method with the successive iterative step size 0.01. Table 1 represents the comparison of $f''(0)$ for various values of m with existing work. It is concluded that the present results are in good agreement with the previous studies.

Graphical discussion

For summarizing the effects of several flow quantities $f', g, F', \theta, \theta_p, \phi$ and ϕ_p . Here we fix the non-dimensional variables for numerical calculations as

Fig. 9 Impact of Prandtl number on **a** temperature profile. **b** Dust phase temperature profile. **c** Concentration profile. **d** Dust phase concentration profile

$M = 0.5, R = 1.2, Ec = 0.5, Pr = 0.6, Gr_b = 0.8, Gr = 0.8c, \alpha = 0.5, l = 1.2, m = 0.6, kr = 0.3, kp = 0.4, \beta_v \neq \beta_t = 1.2 = \beta_c, Nr = 0.5$; throughout the study these values have been not changed.

Figure 2a–g displays the effects of magnetic parameter M on $f', g, F', \theta, \theta_p, \phi$ and ϕ_p . We can reveal that velocity diminishes for both the phases with boost up values of M and the opposite behaviour is seen in other profiles. This declares that magnetic field repel the liquid transfer due to boost up values in M tends to increment in Lorentz force, which counter the fluid flow. This slows down the motion of the fluid. Magnetic field can regulate boundary layer division. This is due to the fact that magnetic field builds a resistance force on the fluid in the boundary layer. This

results the sink in the thinness of the thermal and solutal boundary layer. Moreover, impact of M on dusty phase flow in F', θ_p and ϕ_p is almost enclosed within the dynamic region $[0, 0.23], [0.41, 0.69], [0.17, 0.37]$.

Figure 3a–c demonstrates the impacts of permeability parameter k_p on velocity and angular velocity profiles for both the phases. It is apparently that the occurrence of a porous medium effect higher restraint to the liquid flow which in twirl, slows its motion. Consequently, with escalating permeability parameter, the resistance to the fluid motion also increases. This affects the fluid velocity, dusty fluid velocity increases, and opposite behaviour is seen in angular velocity.

Fig. 10 Impact of radiation parameter on **a** temperature profile. **b** Dust phase temperature profile

Figure 4a–g is aimed to the impact of mass concentration of the dust particle parameter l on velocity, angular velocity, thermal and species profiles for both the phases. The profiles f' , g , F' , θ , θ_p , ϕ and ϕ_p decline with boost up values of l . Physically, the drag force between the liquid and dusty phase is intensified by escalating the mass concentration of dust particles. As a result, the momentum, thermal and solutal of the fluid delayed rapidly. Consequently, the particle phase momentum, thermal and solutal are lower too, whereas the particle phase is being dragged along by the fluid. Further, the more dust particles suck up the heat from the fluid when they come into contact, but the solitary particles will receive less power from the liquid phase.

The impact of material parameter R on velocity and angular velocity is illuminated in Fig. 5a–c. Figure 5a–b can be seen that with escalating values of R , the velocity profile increases for both the fluids (fluid phase, dust phase). Due

Fig. 11 Impact of Eckert number on **a** temperature profile. **b** Dust phase temperature profile

to a reason that escalating values of R which corresponds to low viscosity and magnifies the velocity. And the reverse behaviour is seen in Fig. 5c. Material parameter denotes the fragment of vertex viscosity to dynamic viscosity. The dynamic viscosity declines by escalating values of R .

Figure 6a–f illustrates to see the variations of β_v , β_t and β_c on the dimensionless distributions in the both the phases. Figure 6a, c, e shows that the $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ functions decline in micropolar fluid phase with escalating values of β_v , β_t and β_c , respectively. But an opposite behaviour is seen in particle phases, and particle phase velocity, temperature, concentration are almost covered within the dynamic region [0.28, 0.58], [0.18, 0.38], [0.15, 0.29] (see Fig. 6b, d, f).

The effect of thermal Grashof number is presented in Fig. 7a–c. Figure 7a, b exhibits the fluid phase and dust

Fig. 12 Impact of Schmidt number on **a** concentration profile. **b** Dust phase concentration profile

Fig. 13 Impact of chemical reaction parameter on **a** concentration profile. **b** Dust phase concentration profile

phase velocity profiles for different values of Gr_b . We observed a significant increase in $f'(\eta), F'(\eta)$ when Gr_b rises. Reverse trend is seen in Fig. 7c. We noticed as $g(\eta)$ diminishes near UHSPR for accumulating values of Gr_b . Figure 8a, b declares the fluid phase and dust phase velocity profiles for different values of Gr_c . We observed that $f'(\eta), F'(\eta)$ diminishes near the surface when Gr_c advances. Figure 8c represents the effect of Gr_c on $g(\eta)$. We noticed as $g(\eta)$ diminishes near upper horizontal sphere of paraboloid revolution for accumulating values of Gr_c .

The impact of Prandtl number Pr is depicted in Fig. 9a–d. An escalating value of Pr temperature and concentration profiles diminishes for both the fluids. Prandtl number is a material property. It varies from fluid to fluid. Prandtl number is large when thermal conductivity is small and viscosity is large. Prandtl number's growing values reduce the thermal diffusivity, and as a result the heat flows below the fluid and the thermal boundary layer thickness reduces with increase in the values of Prandtl number. Furthermore, the effects of

Pr on particle phase temperature and species phase temperature are almost covered within the dynamic region [0.42, 0.55], [0.18, 0.28].

The impact of radiation parameter Nr is illustrated in Fig. 10a, b. For boost up values of Nr , it shows an increase in temperature for both the fluids. Due to the reason that intensifying radiation parameter releases the heat energy to the flow, it helps to enhance the temperature profiles. Thus, Nr impacts play a main role in amplifying the rate of heat transfer. This performance is qualitatively agreed with earlier published work [19].

Figure 11a, b shows the temperature profile for different values of Ec . Eckert number defines the ratio of the adjective mass transfer to the heat dissipation. It is the ratio of the kinetic energy to the enthalpy (or the dynamic temperature to the temperature) driving force for heat transfer. The results show the impact of escalating values of Ec ; the temperature increases for both the fluids.

Fig. 14 **a** Impact of permeability parameter versus magnetic parameter on C_{fx} . **b** Impact of fluid particle interaction parameter for velocity versus mass concentration of the dust particle on C_{fx} . **c** Impact of Eckert number versus radiation parameter on Nu_x . **d** Impact of fluid particle interaction parameter for temperature versus mass concentra-

tion of the dust particle on Nu_x . **e** Impact of chemical reaction parameter versus Schmidt number on Sh_x . **f** Impact of fluid particle interaction parameter for concentration versus mass concentration of the dust particle on Sh_x

Fig. 15 Impact of thermal buoyancy parameter on velocity profile

Figure 12a, b shows the impact of Sc on $\phi(\eta)$, $\phi_p(\eta)$. For mount up values of Sc , the concentration rises for both the phases. Schmidt number is defined as the ratio of momentum diffusivity and mass diffusivity. Schmidt number is used to describe fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Schmidt number relates the relative thickness of the hydrodynamic layer and mass transfer boundary layer. Raise of the values indicates strong molecular motions which rise the fluid temperature. In case of higher values, shorter penetration depth of temperature occurs. It exhibits that the dimensionless concentration profile declines as the Schmidt number rises. The impact of chemical reaction parameter kr on $\phi(\eta)$, $\phi_p(\eta)$ is drawn in Fig. 13a, b; the concentration and boundary layer thickness diminish for escalating values of kr .

Figure 14a, b is sketched to interrogate Cf_x for contrasting values of M , k_p , β_v and l . It conveys that due to the rise in M , k_p , β_v and l decline the Cf_x . Effect of Nr , Ec , β_t and l parameters on Nu_x is sketched in Fig. 14c, d. It is clearly observed that rising values of Nr and Ec declines the Nu_x , and opposite trend is seen in β_t and l . Fluctuation in Sh_x for kr , Sc , β_c and l can be seen in Fig. 14e. Rising values of kr , Sc , β_c and l enhance Sh_x . Figure 15 shows the impact of Gr_b on $f'(\eta)$. It is concluded that the present results are in good agreement with the previous studies.

Conclusions

Heat and mass transfer phenomena are inspected through radiation, joule heating, chemical reaction. The boundary layer equations are modelled and transformed to a system of

ODE'S with the aid of similarity transformations and solved numerically by utilizing R-K-F integration scheme. The graphical analysis has been emphasized for the fluid and dust phase velocity, angular velocity, energy and species fields to the influence of sundry dynamical flow quantities. In addition, friction factor, Nusselt number and Sherwood number are presented in plots and tables and discussed elaborately, and nice validation is emphasized.

1. Velocity profile diminishes for higher values of M
2. Thermal and species profiles enhance for rising vales of M
3. Compactness of momentum layer shrinkage for exaggerated values of Gr_b , Gr_c and reverse behaviour is seen in angular velocity
4. Escalating values of Eckert number, radiation parameter temperature profile enhances.
5. Compactness of thermal and solutal layer decays for larger Pr , Sc and kr
6. Local friction factor declines for boost up values of magnetic parameter and porosity permeability parameter.
7. Rate of heat transfer decreases for increasing values of Eckert number and radiation parameter.
8. Rate of mass transfer increases for increasing values of Schmidt number and chemical reaction parameter

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