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A FRACTIONAL ORDER COVID-19 EPIDEMIC MODEL WITH MITTAG–LEFFLER KERNEL

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We consider a nonlinear fractional-order Covid-19 model in a sense of the Atagana–Baleanu fractional derivative used for the analytic and computational studies. The model consists of six classes of persons, including susceptible, protected susceptible, asymptomatic infected, symptomatic infected, quarantined, and recovered individuals. The model is studied for the existence of solution with the help of a successive iterative technique with limit point as the solution of the model. The Hyers–Ulam stability is also studied. A numerical scheme is proposed and tested on the basis of the available literature. The graphical results predict the curtail of spread within the next 5000 days. Moreover, there is a gradual increase in the population of protected susceptible individuals.

1. Introduction

The coronavirus infection 2019 (Covid-19) is a communicable respiratory disease. SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus) is a disease caused by a newly discovered virus strain [1]. In Wuhan, China, Covid-19 was first identified in December, 2019, and quickly spread over the next four months. Within a short period, more than 2.9 million inhabitants in 185 nations throughout the world were infected and 206 thousand persons passed away [2]. On March 11, 2020, "The World Health Organization" announced that this coronavirus infection is a pandemic [3]. This disease can spread primarily by small droplets via coughing, sneezing, or person-to-person conversations. By contacting polluted surfaces, prone individuals can also be compromised. The most prevalent signs of this disease are fever, nausea, dry coughing, fatigue, breath shortages. All these signs are parts of Covid-19 [4]. Some patients may also have joint pain, nasal stuffiness, runny nose, sore throat, or diarrhea. The symptoms are typically mild but can slowly occur. In order to prevent infection, hand washing, nose covering, and/or mouth covering are advised, while sneezing or coughing, as well as avoiding nose or mouth touching plus some preventive steps for the eyes, and keeping social distances.

Due to the seriousness of the Covid-19 pandemic, many states made drastic decisions in order to curb the distribution of Covid-19 infection. In addition, they checked and covered their healthcare systems. Hence, they ruled the cancellation of public events, closing of public events, schools, public places, borders, restrictions on travel, and lockouts, etc. While these measures were helpful, the indicated lockdown led to the socioeconomic

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damage, such as bankruptcy of numerous workplaces, loss of the respective positions of a part of the staff, and so on. Further, this shutdown also disrupted supply chains and decreased productivity. The shutdown of China's drug-producing plants, i.e., the shutdown of the second largest pharmaceutical products exporter delayed the deliveries of generic drug processing factories [5]. The sectors of tourism, air transport, and oil were visibly influenced. It is also clear that invisible impacts are expected irrespective of the duration of pandemic. According to "The International Monetary Fund," the worldwide economy is expected to shrink by 3% in 2020 [6].

Governments try to prevent the failures of economy, thinking about security measures in order to relax the lockdown. Some advanced countries intend to grant immunity passports, which show immunity to the illness. However, this technique was disapproved by "The World Health Organization", since there is a lack of adequate scientific proof that reinfection is not possible in this case. A risk balancing strategy was adopted by the South African government to lift the lockout restrictions progressively.

We refer the readers to some scientific works done on infectious diseases [7–9] and, in particular, to several developed mathematical models related to Covid-19 [10–12], as well as to some recent scientific works on various fractional mathematical models [10, 13–34].

In the present paper, we consider the following Covid-19 model for the existence, stability, and numerical simulations based on the use of the Atanga–Baleanu fractional derivative in Caputo's sense; for details, we refer the readers to [29, 35]:

$$ {}_0^{ABC}D_t^{\wp_1^*} S = \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S, $$

$$ {}_0^{ABC}D_t^{\wp_2^*} S_P = \alpha_1 S - \mu S_P, $$

$$ {}_0^{ABC}D_t^{\wp_3^*} I_A = \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu) I_A, $$

$$ {}_0^{ABC}D_t^{\wp *_4} I_S = \alpha_2 \rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta) I_S, $$

$$ {}_0^{ABC}D_t^{\wp_5^*} Q = (1-\alpha_2) I_A + (1-\alpha_3) I_S - (\gamma + r_3 + \mu + \delta) Q, $$

$$ {}_0^{ABC}D_t^{\wp_6^*} R = \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R. $$

The population is divided into six compartments. These are: susceptible class S, protected susceptible class S_P, asymptomatic infected but not quarantined class I_A, symptomatic infected not quarantined class I_S, quarantined class Q, and recovered class R. The fractional orders are denoted by $\wp_i^* \in (0,1]$. The parameters are as follows: α_1 is the fraction of protected susceptible class, α_2 is the fraction of unidentified asymptomatic infected, α_3 is the fraction of unidentified symptomatic infected, η_1 is the contact rate between S and I_A, η_2 is the contact rate between S and I_A, η_3 is the contact rate between S and Q, ρ is the disease progression rate from I_A to I_S, r_1 is the recovery rate of I_A, r_2 is the recovery rate of I_S, r_3 is the recovery rate of Q, δ is the death rate caused by the Covid-19 disease, γ is the proportion of nonaffected quarantine class, and μ is the natural mortality rate. As far as the ABC-fractional calculus is concerned, we highlight the following useful literature.

Definition 1. *The ABC-fractional differential operator on $\psi \in H^*(a,b)$, $b > a$, for $\wp_1^* \in [0,1]$ is defined as follows:*

$$^{ABC}{}_{a}\mathcal{D}_{\tau}^{\wp_1^*}\psi(\tau) = \frac{B(\wp_1^*)}{1-\wp_1^*}\int_a^{\tau}\psi'(s)E_{\wp^*}\left[\frac{-\wp^*(\tau-s)^{\wp^*}}{1-\wp^*}\right]ds, \tag{1}$$

where $B(\wp^*)$ satisfies the property $B(0) = B(1) = 1$.

Definition 2. *For $\psi \in H^*(a,b)$, $b > a$, and $\wp^* \in [0,1]$, the ABR-fractional derivative is defined as follows:*

$$^{ABR}{}_{a}\mathcal{D}_{\tau}^{\wp^*}\psi(\tau) = \frac{B(\wp^*)}{1-\wp^*}\frac{d}{d\tau}\int_a^{\tau}\psi(s)E_{\wp^*}\left[\frac{-\wp^*(\tau-s)^{\wp^*}}{1-\wp^*}\right]ds.$$

Definition 3. *The AB-integral of $\psi \in H^*(a,b)$, $b > a$, $0 < \wp_1^* < 1$ is given by*

$$^{AB}{}_{a}\mathcal{I}_{\tau}^{\wp_1^*}\psi(\tau) = \frac{1-\wp_1^*}{B(\wp_1^*)}\psi(\tau) + \frac{\wp_1^*}{B(\wp_1^*)\Gamma(\wp_1^*)}\int_a^{\tau}\psi(s)(\tau-s)^{\wp_1^*-1}ds.$$

Lemma 1. *The AB fractional derivative and the AB fractional integral of the function ψ satisfy the Newton–Leibniz formula*

$$^{AB}{}_{a}\mathcal{I}_{\tau}^{\wp_1^*}\left(^{ABC}{}_{a}\mathcal{D}_{\tau}^{\wp_1^*}\psi(\tau)\right) = \psi(\tau) - \psi(a).$$

2. Existence Criteria

By the AB-fractional integral and the Covid-19 model (1), we have

$$\begin{aligned}
S(t) - S(0) = {} & \frac{1-\wp_1^*}{\beta(\wp_1^*)}\left[\Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S\right] \\
& + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma\wp_1^*}\int_0^t (t-s)^{\wp_1^*-1}\Bigg[\Lambda_1 + \gamma Q - \alpha_1 S \\
& - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S\Bigg]ds,
\end{aligned}$$

$$S_p(t) - S_p(0) = \frac{1-\wp_2^*}{\beta(\wp_2^*)}[\alpha_1 S - \mu S_P] + \frac{\wp_1^*}{\beta(\wp_2^*)\Gamma\wp_2^*}\int_0^t (t-s)^{\wp_2^*-1}[\alpha_1 S - \mu S_P]ds,$$

$$I_A(t) - I_A(0) = \frac{1-\wp_3^*}{\beta(\wp_3^*)}\Bigg[\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R}$$

$$- (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A\Bigg] + \frac{\wp_1^*}{\beta(\wp_3^*)\Gamma\wp_3^*}\int_0^t (t-s)^{\wp_3^*-1}$$

$$\times \left[\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R}\,(\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A\right] ds,$$

$$I_s(t) - I_s(0) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\Big[\alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S\alpha_2\rho I_A$$

$$- (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S\Big] + \frac{\wp_4^*}{\beta(\wp_4^*)(\Gamma\wp_4^*)}$$

$$\times \int_0^t (t-s)^{\wp_4^*-1}\left[\alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S\right] ds,$$

$$Q(t) - Q(0) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\left[(1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q\right]$$

$$+ \frac{\wp_5^*}{\beta(\wp_5^*)(\Gamma\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\left[(1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q\right] ds,$$

$$R(t) - R(0) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\left[\alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R\right]$$

$$+ \frac{\wp_6^*}{\beta(\wp_6^*)(\Gamma\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\left[\alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R\right] ds.$$

Assume the functions Y_i, $i = 1, \dots, 6$, are given below:

$$Y_1(t, S) = \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S,$$

$$Y_2(t, S_p) = \alpha_1 S - \mu S_P,$$

$$Y_3(t, I_A) = \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu)I_A,$$

$$Y_4(t, I_s) = \alpha_2\rho I_A - (1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta)I_S,$$

$$Y_5(t,Q) = (1-\alpha_2)I_A + (1-\alpha_3)I_S - (\gamma + r_3 + \mu + \delta)Q,$$

$$Y_6(t,R) = \alpha_2 r_1 I_A + \alpha_3 r_2 I_S + R_3 Q - \mu R,$$

$$\begin{cases} \psi_1 = \alpha_1 + k_1 + \mu, \\ \psi_2 = \mu, \\ \psi_3 = k_2 + (\alpha_2\rho + \alpha_2 r_1 + 1 - \alpha_2 + \mu), \\ \psi_4 = \mu_c, \\ \psi_5 = 1 - \alpha_3 + \alpha_3 r_2 + \mu + \delta, \\ \psi_6 = \mu. \end{cases}$$

Assumption ($\mathcal{B}$). *Assume that, for $S(t)$, $S^*(t)$, $S_p(t)$, $S_p^*(t)$, $I_A(t)$, $I_A^*(t)$, $I_s(t)$, $I_s^*(t)$, $Q(t)$, $Q^*(t)$, $R(t)$, $R^*(t) \in L[0,1]$, there exists constants $\kappa_i > 0$, $i = 1, \dots, 6$, such that*

$$\|S(t)\| \le \kappa_1, \quad \|S_p(t)\| \le \kappa_2, \quad \|I_A(t)\| \le \kappa_3, \quad \|I_s(t)\| \le \kappa_4, \quad \|Q(t)\| \le \kappa_5, \quad \|R(t)\| \le \kappa_6, \quad \xi_1, \xi_2 > 0,$$

and

$$\|S(t) + I_A(t) + Q(t)\| \le \xi_1,$$

$$\|I_s(t) + R(t)\| \le \xi_2.$$

Theorem 1. *The functions Y_i, $i \in N_1^6$, satisfy the Lipschitz condition provided that Assumption ($\mathcal{B}$) is obeyed.*

Proof. For Y_1, we obtain

$$\begin{aligned} \|Y_1(t,S) - Y_1(t,S^*)\| &= \Bigg\| \Lambda_1 + \gamma Q - \alpha_1 S - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S}{S + S_P + I_A + I_S + Q + R} - \mu S \\ &\quad - \left(\Lambda_1 + \gamma Q - \alpha_1 S^* - \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S^*}{S^* + S_P + I_A + I_S + Q + R} - \mu S^*\right)\Bigg\| \\ &\le \left\| \alpha_1 + \frac{(1-\alpha_1)(\alpha_2\eta_1 I_A + \alpha_3\eta_2 I_S + \eta_3 Q)S^*}{S^* + S_P + I_A + I_S + Q + R} + \mu \right\| \|S - S^*\| \\ &\le [\alpha_1 + k_1 + \mu]\|S_c - S_c^*\| = \psi_1\|S - S^*\|. \end{aligned} \tag{2}$$

For $Y_2(t, E_c)$, we get

$$\left\|Y_2(t,S_p)-Y_2\left(t,S_p^*\right)\right\| = \left\|(\alpha_1 S-\mu S_P)-\left(\alpha_1 S-\mu S_P^*,\right)\right\|$$

$$\leq [\mu]\left\|S_c-E_c^*\right\| \leq \psi_2\|E_c-E_c^*\|. \tag{3}$$

Further, for $Y_3(t,I_A^*)$, we find

$$\begin{aligned}\left\|Y_3(t,I_A)-Y_3\left(t,I_A^*\right)\right\| &= \Bigg\|\Bigg(\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A+I_S+Q+R}\\
&\quad -(\alpha_2\rho+\alpha_2 r_1+1-\alpha_2+\mu)I_A\Bigg)\\
&\quad -\Bigg(\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A^*+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A^*+I_S+Q+R}\\
&\quad -(\alpha_2\rho+\alpha_2 r_1+1-\alpha_2+\mu)I_A^*\Bigg)\Bigg\|\\
&\leq \Bigg\|\frac{(1-\alpha_1)(\alpha_2\eta_1 I_A+\alpha_3\eta_2 I_S+\eta_3 Q)S}{S+S_P+I_A+I_S+Q+R}\\
&\quad +(\alpha_2\rho+\alpha_2 r_1+1-\alpha_2+\mu)\Bigg\|\left\|I_A-I_A^*\right\|\\
&\leq [k_2+(\alpha_2\rho+\alpha_2 r_1+1-\alpha_2+\mu)]\left\|I_c-I_c^*\right\|\\
&= \psi_3\left\|I_A-I_A^*\right\|\end{aligned} \tag{4}$$

For $Y_4(t,I)$, we obtain

$$\begin{aligned}\left\|Y_4(t,I_s)-Y_4\left(t,I_s^*\right)\right\| &= \Big\|(\alpha_2\rho I_A-(1-\alpha_3+\alpha_3 r_2+\mu+\delta)I_S)\\
&\quad -\left(\alpha_2\rho I_A-(1-\alpha_3+\alpha_3 r_2+\mu+\delta)\,I_S^*\right)\Big\|\\
&\leq \|1-\alpha_3+\alpha_3 r_2+\mu+\delta\|\left\|I_s-I_s^*\right\|\\
&\leq \Big[1-\alpha_3+\alpha_3 r_2+\mu+\delta\Big]\|I_s-I_s^*\| \leq \psi_4\left\|I_s-I_s^*\right\|\end{aligned} \tag{5}$$

For $Y_5(t,Q)$, we get

$$\left\|Y_5(t,Q)-Y_5\left(t,Q^*\right)\right\| = \Big\|\,((1-\alpha_2)I_A+(1-\alpha_3)I_S-(\gamma+r_3+\mu+\delta)Q)$$

$$-\left((1-\alpha_2)I_A+(1-\alpha_3)I_S-(\gamma+r_3+\mu+\delta)Q^*\right)\Big\|$$

$$\leq \|\gamma+r_3+\mu+\delta\|\left\|Q-Q^*\right\|=\psi_5\left\|Q-Q^*\right\|. \tag{6}$$

Further, for $Y_6(t,R)$ we have

$$\left\|Y_6(t,R)-Y_6\left(t,R^*\right)\right\|=\Big\|(\alpha_2 r_1 I_A+\alpha_3 r_2 I_S+r_3 Q-\mu R,)$$

$$-\left(\alpha_2 r_1 I_A+\alpha_3 r_2 I_S+r_3 Q-\mu R^*\right)\Big\|$$

$$\leq \|\mu\|\left\|R-R^*\right\|=\psi_6\left\|R-R^*\right\|. \tag{7}$$

Thus, it follows from (2)–(7) that Y_i, $i=1,\ldots,6$, satisfy the Lipschitz condition.
This completes the proof.

Assume that

$$S(0)=S_p(0)=I_A(0)=I_S(0)=Q(0)=R(0)=0,$$

This yields

$$S(t)=\frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t,S(t))+\frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t(t-s)^{\wp_1^*-1}\mathcal{Y}_1(s,S(s))ds, \tag{8}$$

$$S_p(t)=\frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t,S_p(t))+\frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t(t-s)^{\wp_2^*-1}\mathcal{Y}_2(s,S_p(s))ds, \tag{9}$$

$$I_A(t)=\frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t,I_A(t))+\frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t(t-s)^{\wp_3^*-1}\mathcal{Y}_3(s,I_A(s))ds, \tag{10}$$

$$I_s(t)=\frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t,I_s(t))+\frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t(t-s)^{\wp_4^*-1}\mathcal{Y}_4(s,I_s(s))ds, \tag{11}$$

$$Q(t)=\frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t,Q(t))+\frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t(t-s)^{\wp_5^*-1}\mathcal{Y}_5(s,Q(s))ds, \tag{12}$$

$$R(t)=\frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t,R(t))+\frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t(t-s)^{\wp_6^*-1}\mathcal{Y}_6(s,R(s))ds. \tag{13}$$

For the iterative scheme of the model (1), we define

$$S_n(t) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t, S_{n-1}(t)) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, S_{n-1}(s))ds,$$

$$S_{p_n}(t) = \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t, S_{p_{n-1}}(t)) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s, S_{p_{n-1}}(s))ds,$$

$$I_{An}(t) = \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t, I_{An-1}(t)) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3(s, I_{An-1}(s))ds,$$

$$I_{sn}(t) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t, I_{sn-1}(t)) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s, I_{sn-1}(s))ds,$$

$$Q_n(t) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t, Q_{n-1}(t)) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s, Q_{n-1}(s))ds,$$

$$R_n(t) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t, R_{n-1}(t)) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s, R_{n-1}(s))ds.$$

Theorem 2. *The fractional-order Covid-19 model (1) has a solution provided that*

$$\Delta = \max\{\Psi_i\} < 1, \quad i \in N_1^6.$$

Proof. We define the functions

$$\mathcal{K}1_n(t) = S_{n+1}(t) - S(t), \quad \mathcal{K}2_n(t) = S_{p_{n+1}}(t) - S_p(t), \quad \mathcal{K}3_n(t) = I_{An+1}(t) - I_A(t),$$

$$\mathcal{K}4_n(t) = I_{sn+1}(t) - I_s(t), \quad \mathcal{K}5_n(t) = Q_{n+1}(t) - Q(t), \quad \mathcal{K}6_n(t) = R_{n+1}(t) - R(t).$$

Thus, by using the above equations, we conclude that

$$\|\mathcal{K}1_n\| \leq \frac{1-\wp_1^*}{\beta(\wp_1^*)}\|\mathcal{Y}_1(t, \mathcal{S}_n(t)) - \mathcal{Y}_1(t, S_{n-1}(t))\|$$

$$+ \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\|\mathcal{Y}_1(s, S_n(s)) - \mathcal{Y}_1(t, S_{n-1}(t))\|ds$$

$$\leq \left[\frac{1-\wp_1^*}{\beta(\wp_1^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\right]\psi_1\|S_n - S\|$$

$$\leq \left[\frac{1-\wp_1^*}{\beta(\wp_1^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\right]^n \Delta^n\|S_1 - S\|$$

and

$$\|\mathcal{K}2_n\| \leq \frac{1-\wp_2^*}{\beta(\wp_2^*)}\|\mathcal{Y}_2(t, S_{p_n}(t)) - \mathcal{Y}_2(t, S_{p_{n-1}}(t))\|$$

$$+ \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t (t-s)^{\wp_2^*-1}\|\mathcal{Y}_2(s, S_{p_n}(s)) - \mathcal{Y}_2(t, S_{p_{n-1}}(t))\|ds$$

$$\leq \left[\frac{1-\wp_2^*}{\beta(\wp_2^*)} + \frac{1}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\right]\psi_2\|S_{p_n} - S_p\|$$

$$\leq \left[\frac{1-\wp_2^*}{\beta(\wp_2^*)} + \frac{1}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\right]^n \Delta^n\|S_{p_1} - S_p\|.$$

Similarly,

$$\|\mathcal{K}3_n\| \leq \frac{1-\wp_3^*}{\beta(\wp_3^*)}\|\mathcal{Y}_3(t, I_{An}(t)) - \mathcal{Y}_3(t, I_{An-1}(t))\|$$

$$+ \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t (t-s)^{\wp_3^*-1}\|\mathcal{Y}_3(s, I_{An}(s)) - \mathcal{Y}_3(t, I_{An-1}(t))\|ds$$

$$\leq \left[\frac{1-\wp_3^*}{\beta(\wp_3^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_3^*)}\right]\psi_3\|I_{An} - I_A\|$$

$$\leq \left[\frac{1-\wp_3^*}{\beta(\wp_3^*)} + \frac{1}{\beta(\wp_1^*)\Gamma(\wp_3^*)}\right]^n \Delta^n\|I_{A1} - I_A\|,$$

$$\|\mathcal{K}4_n\| \leq \frac{1-\wp_4^*}{\beta(\wp_4^*)}\|\mathcal{Y}_4(t, I_{sn}(t)) - \mathcal{Y}_4(t, I_{sn-1}(t))\|$$

$$+ \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t (t-s)^{\wp_4^*-1}\|\mathcal{Y}_4(s, I_{sn}(s)) - \mathcal{Y}_4(t, I_{sn-1}(t))\|ds$$

$$\leq \left[\frac{1-\wp_4^*}{\beta(\wp_4^*)} + \frac{1}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\right]\psi_4\|I_{sn} - I_s\|$$

$$\leq \left[\frac{1-\wp_4^*}{\beta(\wp_4^*)} + \frac{1}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\right]^n \Delta^n \|I_{s1} - I_s\|,$$

$$\|\mathcal{K}5_n\| \leq \frac{1-\wp_5^*}{\beta(\wp_5^*)}\|\mathcal{Y}_5(t, Q_n(t)) - \mathcal{Y}_5(t, Q_{n-1}(t))\|$$

$$+ \frac{\wp_5^*}{\beta(\wp_1^*)\Gamma(\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\|\mathcal{Y}_5(s, Q_n(s)) - \mathcal{Y}_5(t, Q_{n-1}(t))\|ds$$

$$\leq \left[\frac{1-\wp_5^*}{\beta(\wp_5^*)} + \frac{1}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\right]\psi_5\|Q_n - Q\|$$

$$\leq \left[\frac{1-\wp_5^*}{\beta(\wp_5^*)} + \frac{1}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\right]^n \Delta^n \|Q_1 - Q\|,$$

$$\|\mathcal{K}6_n\| \leq \frac{1-\wp_6^*}{\beta(\wp_6^*)}\|\mathcal{Y}_6(t, R_n(t)) - \mathcal{Y}_6(t, R_{n-1}(t))\|$$

$$+ \frac{\wp_6^*}{\beta(\wp_1^*)\Gamma(\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\|\mathcal{Y}_6(s, R_n(s)) - \mathcal{Y}_5(t, R_{n-1}(t))\|ds$$

$$\leq \left[\frac{1-\wp_6^*}{\beta(\wp_6^*)} + \frac{1}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\right]\psi_6\|R_n - R\|$$

$$\leq \left[\frac{1-\wp_6^*}{\beta(\wp_6^*)} + \frac{1}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\right]^n \Delta^n \|R_1 - R\|.$$

Thus, we get $\mathcal{K}(t)_n \to 0$, $i \in 1, \ldots, 6$, as $n \to \infty$ for $\Delta < 1$, which is the required proof.

3. Uniqueness of Solution

For our suggested model (1), we now analyze the problem of uniqueness of the solution.

Theorem 3. *The Covid-19 model (1) has a unique solution if*

$$\left[\frac{1-\varphi_i}{\beta(\varphi_i)} + \frac{1}{\beta(\varphi_i)\Gamma(\varphi_i)}\right]\psi_i \leq 1, \quad i \in \mathcal{N}_1^6. \tag{14}$$

Proof. Assume that there exists another solution $\overline{S}(t)$, $\overline{S_c}(t)$, $\overline{I_A}(t)$, $\overline{i_s}(t)$, $\overline{Q}(t)$, $\overline{R}(t)$ such that

$$\overline{S}(t) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t, \overline{S}(t)) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, \overline{S}(s))ds,$$

$$\overline{S_p}(t)=\frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t,\overline{S_p}(t))+\frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t(t-s)^{\wp_2^*-1}\mathcal{Y}_2(s,\overline{S_p}(s))ds,$$

$$\overline{I_A}(t)=\frac{1-\wp_3^*}{\beta\left(\wp_3^*\right)}\mathcal{Y}_3(t,\overline{I_A}(t))+\frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t(t-s)^{\wp_3^*-1}\mathcal{Y}_3(s,\overline{I_A}(s))ds,$$

$$\overline{I_s}(t)=\frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t,\overline{I_s}(t))+\frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t(t-s)^{\wp_4^*-1}\mathcal{Y}_4(s,\overline{I_s}(s))ds,$$

$$\overline{Q}(t)=\frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t,\overline{Q}(t))+\frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t(t-s)^{\wp_5^*-1}\mathcal{Y}_5(s,\overline{Q}(s))ds,$$

$$\overline{R}(t)=\frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t,\overline{R}(t))+\frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t(t-s)^{\wp_6^*-1}\mathcal{Y}_6(s,\overline{R}(s))ds.$$

Thus,

$$\begin{aligned}\|S(t)-\overline{S}(t)\| &\leq \frac{1-\wp_1^*}{\beta(\wp_1^*)}\|\mathcal{Y}_1(t,S(t))-Y_1(t,\overline{S}(t))\| \\ &\quad+\frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t(t-s)^{\wp_1^*-1}\|Y_1(s,S(s))-Y_1(t,\overline{S}(t))\|ds \\ &\leq\left[\frac{1-\wp_1^*}{\beta(\wp_1^*)}+\frac{1}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\right]\psi_1\|S-\overline{S}\|,\end{aligned}$$

whence it follows that

$$\left[\frac{1-\wp_1^*}{\beta(\wp_1^*)}\psi_1+\frac{\psi_1}{\beta(\wp_1^*)\Gamma(\wp_1^*)}-1\right]\|S-\overline{S}\|\geq 0. \tag{15}$$

By virtue of (14), relation (15) is true if

$$\left\|S-\overline{S}\right\|=0,$$

which implies that $S(t)=\overline{S}(t)$. Similarly, we have

$$\left\| S_p(t) - \overline{S_p}(t) \right\| \leq \frac{1-\wp_2^*}{\beta(\wp_2^*)} \left\| \mathcal{Y}_2(t, S_p(t)) - Y_2\left(t, \overline{S_p}(t)\right) \right\|$$

$$+ \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)} \int_0^t (t-s)^{\wp_2^*-1} \left\| Y_2(s, S_p(s)) - Y_2(t, \overline{S_p}(t)) \right\| ds$$

$$\leq \left[\frac{1-\wp_2^*}{\beta(\wp_2^*)} + \frac{1}{\beta(\wp_2^*)\Gamma(\wp_2^*)} \right] \psi_2 \left\| S_P - \overline{S_p} \right\|,$$

$$\left[\frac{1-\wp_2^*}{\beta(\wp_2^*)} \psi_2 + \frac{\psi_2}{\beta(\wp_2^*)\Gamma(\wp_2^*)} - 1 \right] \left\| S_P - \overline{S_p} \right\| \geq 0, \tag{16}$$

which follows. By virtue of (14), inequality (16) is true for $\|S_p - \overline{S_p}\| = 0$, which implies that $S_p(t) = \overline{S_p}(t)$. Further, for I_A, we obtain

$$\left\| I_A(t) - \overline{I_A}(t) \right\| \leq \frac{1-\wp_3^*}{\beta(\wp_3^*)} \left\| \mathcal{Y}_3(t, I_A(t)) - Y_3(t, \overline{I_A}(t)) \right\|$$

$$+ \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)} \int_0^t (t-s)^{\wp_3^*-1} \left\| Y_3(s, I_A(s)) - Y_3(t, \overline{I_A}(t)) \right\| ds$$

$$\leq \left[\frac{1-\wp_3^*}{\beta(\wp_3^*)} + \frac{1}{\beta(\wp_3^*)\Gamma(\wp_3^*)} \right] \psi_3 \left\| I_A - \overline{I_A} \right\|.$$

This gives

$$\left[\frac{1-\wp_3^*}{\beta(\wp_3^*)} \psi_3 + \frac{\psi_3}{\beta(\wp_3^*)\Gamma(\wp_3^*)} - 1 \right] \left\| I_A - \overline{I_A} \right\| \geq 0. \tag{17}$$

Thus, in view of (14), inequality (17) is true if $\|I_A - \overline{I_A}\| = 0$, which means that $I_A(t) = \overline{I_A}(t)$ and, therefore,

$$\left\| I_s(t) - \overline{I_s}(t) \right\| \leq \frac{1-\wp_4^*}{\beta(\wp_4^*)} \left\| \mathcal{Y}_4(t, I_s(t)) - Y_4(t, \overline{I_s}(t)) \right\|$$

$$+ \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)} \int_0^t (t-s)^{\wp_4^*-1} \left\| Y_4(s, I_s(s)) - Y_4(t, \overline{I_s}(t)) \right\| ds$$

$$\leq \left[\frac{1-\wp_4^*}{\beta(\wp_4^*)} + \frac{1}{\beta(\wp_4^*)\Gamma(\wp_4^*)} \right] \psi_4 \left\| I_s - \overline{I_s} \right\|.$$

Hence, we get

$$\left[\frac{1-\wp_4^*}{\beta(\wp_4^*)} \psi_4 + \frac{\psi_4}{\beta(\wp_4^*)\Gamma(\wp_4^*)} - 1 \right] \left\| I_{s-}\overline{I_s} \right\| \geq 0. \tag{18}$$

By virtue of (14), inequality (18) is true if $\|I_s - \overline{I_s}\| = 0$. This yields $I_s(t) = \overline{I_s}(t)$. Further, for Q, we obtain

$$\begin{aligned}\left\|Q(t) - \overline{Q}(t)\right\| &\le \frac{1-\wp_5^*}{\beta(\wp_5^*)} \left\|\mathcal{Y}_5(t, Q(t)) - Y_5(t, \overline{Q}(t))\right\| \\ &\quad + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)} \int_0^t (t-s)^{\wp_5^*-1} \left\|Y_5(s, Q(s)) - Y_5\left(t, \overline{Q}(t)\right)\right\| ds \\ &\le \left[\frac{1-\wp_5^*}{\beta(\wp_5^*)} + \frac{1}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\right]\psi_5 \left\|Q - \overline{Q}\right\|.\end{aligned}$$

This yields

$$\left[\frac{1-\wp_5^*}{\beta(\wp_5^*)}\psi_5 + \frac{\psi_5}{\beta(\wp_5^*)\Gamma(\wp_5^*)} - 1\right]\left\|Q - \overline{Q}\right\| \ge 0,$$

$$\begin{aligned}\left\|R(t) - \overline{R}(t)\right\| &\le \frac{1-\wp_6^*}{\beta(\wp_6^*)} \left\|\mathcal{Y}_6(t, R(t)) - Y_6(t, \overline{R}(t))\right\| \\ &\quad + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)} \int_0^t (t-s)^{\wp_6^*-1} \left\|Y_6(s, R(s)) - Y_6(t, \overline{R}(t))\right\| ds \\ &\le \left[\frac{1-\wp_6^*}{\beta(\wp_6^*)} + \frac{1}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\right]\psi_6 \left\|R - \overline{R}\right\|.\end{aligned}$$

Therefore,

$$\left[\frac{1-\wp_6^*}{\beta(\wp_6^*)}\psi_6 + \frac{\psi_6}{\beta(\wp_6^*)\Gamma(\wp_6^*)} - 1\right]\left\|R - \overline{R}\right\| \ge 0. \tag{19}$$

By virtue of (14), this implies that relation (19) is true if $\|R - \overline{R}\| = 0$, which means that $R(t) = \overline{R}(t)$. Thus, model (1) has a unique solution.

4. Hyers–Ulam Stability

Definition 4. *The integral system (8)–(13) is Hyers–Ulam stable if, for $\Delta_i > 0$, $i \in \mathcal{N}_1^6$, and $\gamma_i > 0$, $i \in \mathcal{N}_1^6$, we have*

$$\left|S(t) - \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t, S(t)) - \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, S(s))ds\right| \le \gamma_1,$$

$$\left|S_p(t) - \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t, S_p(t)) - \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s, S_p(s))ds\right| \le \gamma_2,$$

$$\left| I_A(t) - \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t, I_A(t)) - \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3(s, I_A(s))ds \right| \leq \gamma_3,$$

$$\left| I_s(t) - \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t, I_s(t)) - \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s, I_s(s))ds \right| \leq \gamma_4,$$

$$\left| Q(t) - \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t, Q(t)) - \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s, Q(s))ds \right| \leq \gamma_5,$$

$$\left| R(t) - \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t, Q(t)) - \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s, R(s))ds \right| \leq \gamma_5.$$

Further, for $\dot{S}(t)$, $\dot{S}_p(t)$, $\dot{I}_A(t)$, $\dot{I}_s(t)$, $\dot{Q}(t)$, $\dot{R}(t)$, we get

$$\dot{S}(t) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t, \dot{S}(t)) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s, \dot{S}(s))ds,$$

$$\dot{S}_p(t) = \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t, \dot{S}_p(t)) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s, \dot{S}_p(s))ds,$$

$$\dot{I}_A(t) = \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t, \dot{I}_A(t)) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3\left(s, \dot{I}_A(s)\right)ds,$$

$$\dot{I}_s(t) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4\left(t, \dot{I}_s(t)\right) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4\left(s, \dot{I}_s(s)\right)ds,$$

$$\dot{Q}(t) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t, \dot{Q}(t)) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5\left(s, \dot{Q}(s)\right)ds,$$

$$\dot{R}(t) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6\left(t, \dot{R}(t)\right) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6\left(s, \dot{R}(s)\right)ds$$

such that

$$\left|S(t) - \dot{S}(t)\right| \leq \delta_1\gamma_1,$$

$$\left|S_p(t) - \dot{S}_p(t)\right| \leq \delta_2\gamma_2,$$

$$\left|I_A(t) - \dot{I}_A(t)\right| \leq \delta_3\gamma_3,$$

$$\left|I_s(t) - \dot{I}_s(t)\right| \le \delta_4\gamma_4,$$

$$\left|Q(t) - \dot{Q}(t)\right| \le \delta_5\gamma_5,$$

$$\left|R(t) - \dot{R}(t)\right| \le \delta_6\gamma_6.$$

Theorem 4. *If Assumption* ($\mathcal{B}$) *is satisfied, then model (1) is Hyers–Ulam-stable.*

Proof. By Theorem 3, the Covid-19 model (1) has a unique solution, say, $S(t)$, $S_p(t)$, $I_A(t)$, $I_s(t)$, $Q(t)$, $R(t)$. Let $\left(\dot{S}(t), \dot{S_p}(t), \dot{I_A}(t), \dot{I}_s(t), \dot{Q}(t), \dot{R}(t)\right)$ be an approximate solution of (1) satisfying (8)–(13). Thus, we get

$$\begin{aligned}\left\|S(t) - \dot{S}(t)\right\| &\le \frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)} \left\|Y_1(t, S(t)) - Y_1\left(t, \dot{S}(t)\right)\right\| \\ &\quad + \frac{\wp_1^*}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)} \int_0^t (t-s)^{\wp_1^*-1} \left\|Y_1(s, S(s)) - Y_1\left(t, \dot{S}(t)\right)\right\| ds \\ &\le \left[\frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)} + \frac{1}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)}\right]\psi_1 \|S - \dot{S}\|.\end{aligned}$$

If we take $\gamma_1 = \psi_1$ and

$$\Delta = \frac{1-\wp_1^*}{\beta\left(\wp_1^*\right)} + \frac{\wp_1^*}{\beta\left(\wp_1^*\right)\Gamma\left(\wp_1^*\right)},$$

then we get

$$\left\|S(t) - \dot{S}(t)\right\| \le \gamma_1\Delta_1.$$

Similarly, for $S_p(t)$, $\dot{S_p}(t)$, $I_A(t)$, $\dot{I_A}(t)$, $I_s(t)$, $\dot{I}_s(t)$, $Q(t)$, $\dot{Q}(t)$, $R(t)$, $\dot{R}(t)$, we get

$$\left\|S_p(t) - \dot{S_p}(t)\right\| \le \gamma_2\Delta_2,$$

$$\left\|I_A(t) - \dot{I_A}(t)\right\| \le \gamma_3\Delta_3,$$

$$\left\|I_s(t) - \dot{I}_s(t)\right\| \le \gamma_4\Delta_4,$$

$$\left\|Q(t) - \dot{Q}(t)\right\| \le \gamma_5\Delta_5,$$

$$\left\|R(t) - \dot{R}(t)\right\| \le \gamma_6\Delta_6.$$

This implies that system (1) is Hyers–Ulam stable, which ultimately ensures the stability of (1).

This completes the proof.

5. Numerical Scheme

By using (2)–(7), we produce the following numerical scheme:

$$\begin{cases} {}_0^{ABC}\mathcal{D}_t^{\wp_1^*} S(t) = Y_1(t,S), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_2^*} S_p(t) = Y_2(t,S_p), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_3^*} I_A(t) = Y_3(t,I_A), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_4^*} I_s(t) = Y_4(t,I_s), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_5^*} Q(t) = Y_5(t,Q), \\ {}_0^{ABC}\mathcal{D}_t^{\wp_6^*} R(t) = Y_6(t,R). \end{cases} \tag{20}$$

With the help of the fractional AB-integral operator, relations (20) take the following form:

$$S(t) - S(0) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t,S) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\int_0^t (t-s)^{\wp_1^*-1}\mathcal{Y}_1(s,S)ds,$$

$$S_p(t) - S_p(0) = \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t,I_s) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)}\int_0^t (t-s)^{\wp_2^*-1}\mathcal{Y}_2(s,S_p)ds,$$

$$I_A(t) - I_A(0) = \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_2(t,I_A) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)}\int_0^t (t-s)^{\wp_3^*-1}\mathcal{Y}_3(s,I_A)ds,$$

$$I_s(t) - I_s(0) = \frac{1-\wp_4^*}{\beta(\wp_4^*)}\mathcal{Y}_4(t,I_s) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)}\int_0^t (t-s)^{\wp_4^*-1}\mathcal{Y}_4(s,I_s)ds,$$

$$Q(t) - Q(0) = \frac{1-\wp_5^*}{\beta(\wp_5^*)}\mathcal{Y}_5(t,Q) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)}\int_0^t (t-s)^{\wp_5^*-1}\mathcal{Y}_5(s,Q)ds,$$

$$R(t) - R(0) = \frac{1-\wp_6^*}{\beta(\wp_6^*)}\mathcal{Y}_6(t,R) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)}\int_0^t (t-s)^{\wp_6^*-1}\mathcal{Y}_6(s,R)ds.$$

Dividing the assumed interval $[0,t]$ into subintervals with the help of points t_{m+1}, for $m = 0,1,2\ldots$, we obtain

$$S(t_{m+1}) - S(0) = \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t_m,S) + \frac{\wp_1^*}{\beta(\wp_1^*)\Gamma(\wp_1^*)}\sum_{k=0}^{n}\int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_1^*-1}\mathcal{Y}_1(s,S)ds,$$

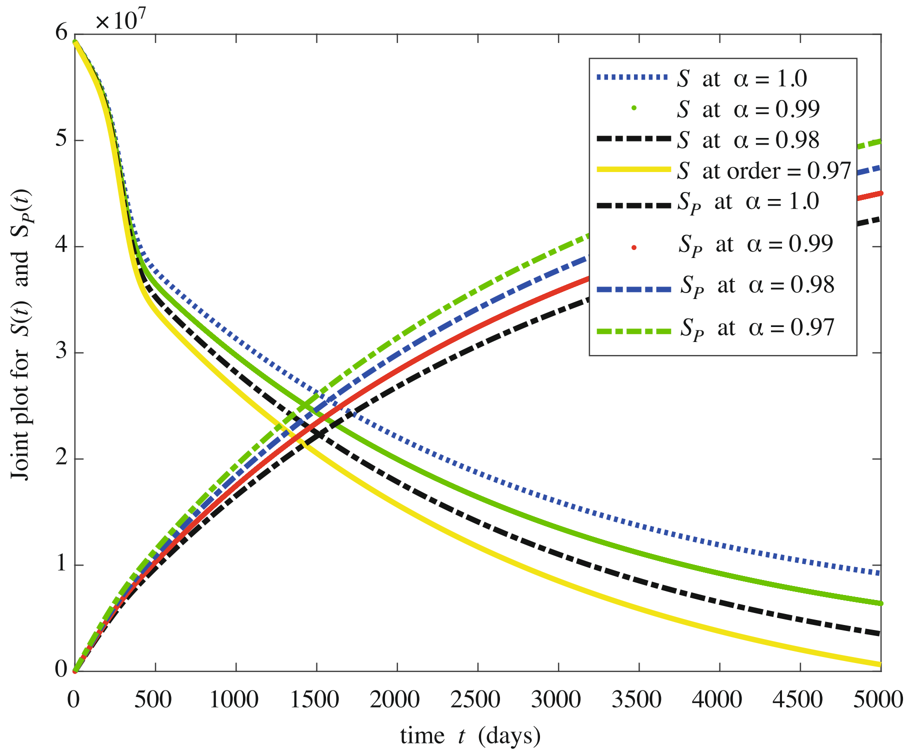

Fig. 1. Comparative analysis for the $S(t)$ and $S_P(t)$ and the following orders: 1.0, 0.99, 0.98, and 0.97.

$$
\begin{aligned}
S_p(t_{m+1}) - S_p(0) &= \frac{1-\wp_2^*}{\beta(\wp_2^*)} \mathcal{Y}_2(t_m, S_p) + \frac{\wp_2^*}{\beta(\wp_2^*)\Gamma(\wp_2^*)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_2^*-1} \mathcal{Y}_2(s, S_p)ds, \\
I_A(t_{m+1}) - I_c(0) &= \frac{1-\wp_3^*}{\beta(\wp_3^*)} \mathcal{Y}_3(t_m, I_A) + \frac{\wp_3^*}{\beta(\wp_3^*)\Gamma(\wp_3^*)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_3^*-1} \mathcal{Y}_3(s, I_A)ds, \\
I_s(t_{m+1}) - I_s(0) &= \frac{1-\wp_4^*}{\beta(\wp_4^*)} \mathcal{Y}_4(t_m, I_s) + \frac{\wp_4^*}{\beta(\wp_4^*)\Gamma(\wp_4^*)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_4^*-1} \mathcal{Y}_4(s, I_s)ds, \\
Q(t_{m+1}) - Q(0) &= \frac{1-\wp_5^*}{\beta(\wp_5^*)} \mathcal{Y}_5(t_m, Q) + \frac{\wp_5^*}{\beta(\wp_5^*)\Gamma(\wp_5^*)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_5^*-1} \mathcal{Y}_5(s, Q)ds, \\
R(t_{m+1}) - R(0) &= \frac{1-\wp_6^*}{\beta(\wp_6^*)} \mathcal{Y}_6(t_m, R) + \frac{\wp_6^*}{\beta(\wp_6^*)\Gamma(\wp_6^*)} \sum_{k=0}^{n} \int_{t_k}^{t_{k+1}} (t_{m+1}-s)^{\wp_6^*-1} \mathcal{Y}_6(s, R)ds. \qquad (21)
\end{aligned}
$$

Now, by using Lagrange's interpolation, we get

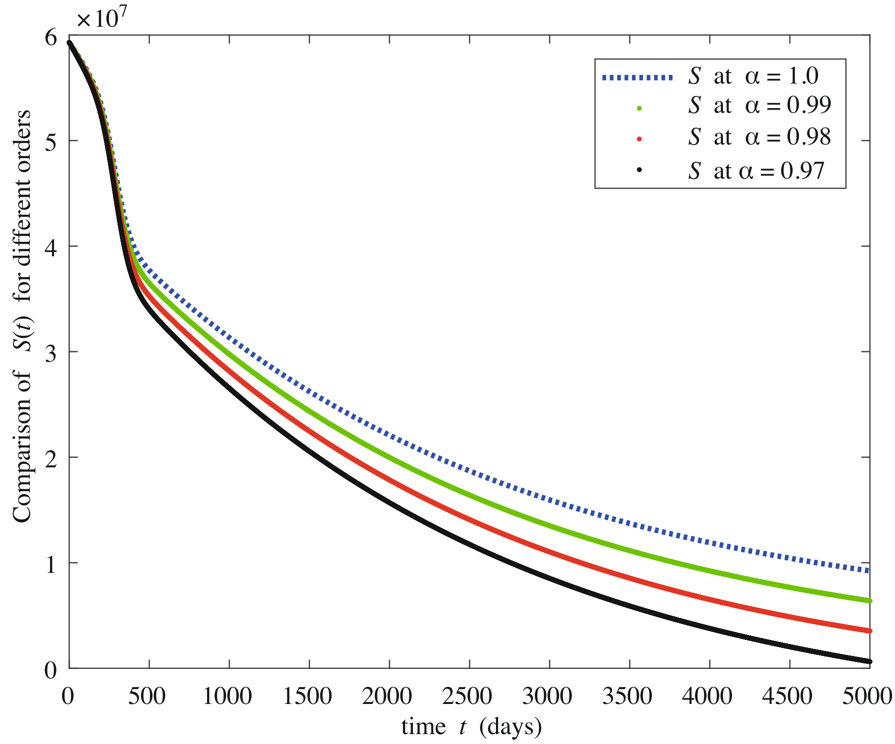

Fig. 2. Comparative analysis of the $S(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

$$
\begin{aligned}
S(t_{m+1}) = S(0) + \frac{1-\wp_1^*}{\beta(\wp_1^*)}\mathcal{Y}_1(t_k, S) + \frac{\wp_1^*}{\mathcal{B}(\wp_1^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_1^*}\mathcal{Y}_1(t_k, S)}{\Gamma(\wp_1^*+2)} \\
\times\Big((m+1-k)^{\wp_1^*}\left(m-k+2+\wp_1^*\right) - (m-k)^{\wp_1^*}\left(m-k+2+2\wp_1^*\right)\Big) \\
-\frac{h^{\wp_1^*}\mathcal{Y}_1(t_{k-1}, S)}{\Gamma(\wp_1^*+2)}\Big((m+1-k)^{\wp_1^*} - (m-k)^{\wp_1^*}\left(m+1-k+\wp_1^*\right)\Big)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
S_p(t_{m+1}) = S_p(0) + \frac{1-\wp_2^*}{\beta(\wp_2^*)}\mathcal{Y}_2(t_k, S_p) + \frac{\wp_2^*}{\mathcal{B}(\wp_2^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_2^*}\mathcal{Y}_2(t_k, S_p)}{\Gamma(\wp_2^*+2)} \\
\times\Big((m+1-k)^{\wp_2^*}\left(m-k+2+\wp_2^*\right) - (m-k)^{\wp_2^*}\left(m-k+2+2\wp_2^*\right)\Big) \\
-\frac{h^{\wp_2^*}\mathcal{Y}_2(t_{k-1}, S_p)}{\Gamma(\wp_2^*+2)}\Big((m+1-k)^{\wp_2^*} - (m-k)^{\wp_2^*}\left(m+1-k+\wp_2^*\right)\Big)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
I_A(t_{m+1}) = I_A(0) + \frac{1-\wp_3^*}{\beta(\wp_3^*)}\mathcal{Y}_3(t_k, I_A) + \frac{\wp_3^*}{\mathcal{B}(\wp_3^*)}\sum_{k=0}^{n}\Bigg[\frac{h^{\wp_3^*}\mathcal{Y}_3(t_k, I_A)}{\Gamma(\wp_3^*+2)} \\
\times\Big((m+1-k)^{\wp_3^*}\left(m-k+2+\wp_3^*\right) - (m-k)^{\wp_3^*}\left(m-k+2+2\wp_3^*\right)\Big)
\end{aligned}
$$

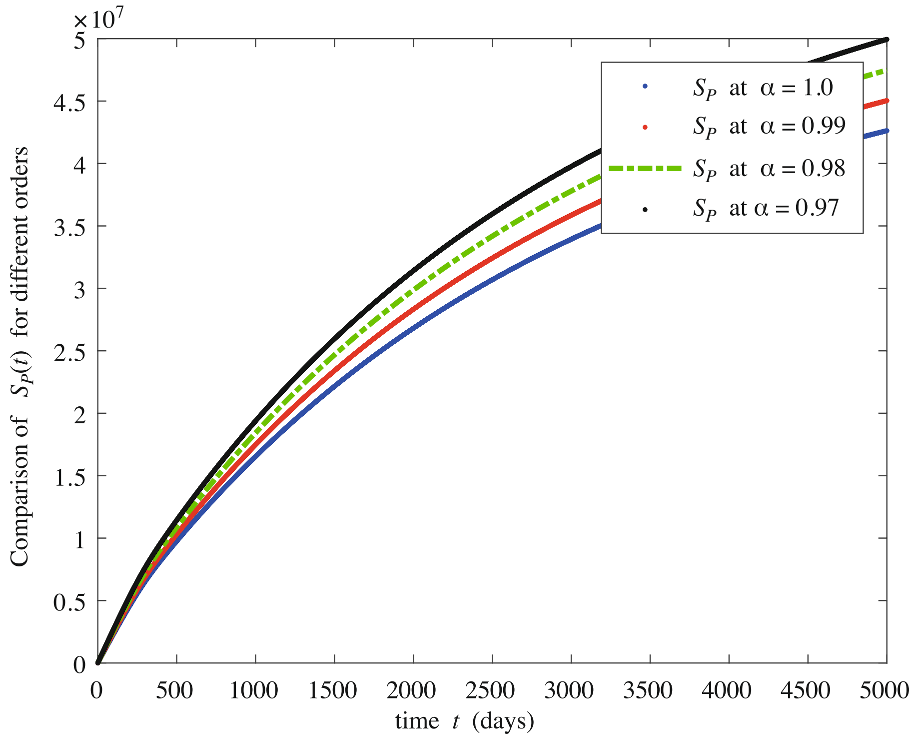

Fig. 3. Comparative analysis of the $S_P(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

$$
- \frac{h^{\wp_3^*} \mathcal{Y}_3(t_{k-1}, I_A)}{\Gamma\left(\wp_3^* + 2\right)} \left((m+1-k)^{\wp_3^*} - (m-k)^{\wp_3^*} \left(m+1-k+\wp_3^*\right) \right) \Bigg],
$$

$$
I_s(t_{m+1}) = I_s(0) + \frac{1-\wp_4^*}{\beta\left(\wp_4^*\right)} \mathcal{Y}_4(t_k, I_s) + \frac{\wp_4^*}{\mathcal{B}\left(\wp_4^*\right)} \sum_{k=0}^{n} \Bigg[\frac{h^{\wp_4^*} \mathcal{Y}_4(t_k, I_s)}{\Gamma\left(\wp_4^* + 2\right)}
$$

$$
\times \left((m+1-k)^{\wp_4^*} \left(m-k+2+\wp_4^*\right) (m-k)^{\wp_4^*} \left(m-k+2+2\wp_4^*\right) \right)
$$

$$
- \frac{h^{\wp_4^*} \mathcal{Y}_4(t_{k-1}, I_s)}{\Gamma\left(\wp_4^* + 2\right)} \left((m+1-k)^{\wp_4^*} - (m-k)^{\wp_4^*} \left(m+1-k+\wp_4^*\right) \right) \Bigg],
$$

$$
Q(t_{m+1}) = Q(0) + \frac{1-\wp_5^*}{\beta\left(\wp_5^*\right)} \mathcal{Y}_5(t_k, Q) + \frac{\wp_5^*}{\mathcal{B}\left(\wp_5^*\right)} \sum_{k=0}^{n} \Bigg[\frac{h^{\wp_5^*} \mathcal{Y}_5(t_k, Q)}{\Gamma\left(\wp_5^* + 2\right)}
$$

$$
\times \left((m+1-k)^{\wp_5^*} \left(m-k+2+\wp_5^*\right) (m-k)^{\wp_5^*} \left(m-k+2+2\wp_5^*\right) \right)
$$

$$
- \frac{h^{\wp_5^*} \mathcal{Y}_5(t_{k-1}, Q)}{\Gamma\left(\wp_5^* + 2\right)} \left((m+1-k)^{\wp_5^*} - (m-k)^{\wp_5^*} \left(m+1-k+\wp_5^*\right) \right) \Bigg],
$$

$$
R(t_{m+1}) = R(0) + \frac{1-\wp_6^*}{\beta\left(\wp_6^*\right)} \mathcal{Y}_6(t_k, R) + \frac{\wp_6^*}{\mathcal{B}\left(\wp_6^*\right)} \sum_{k=0}^{n} \Bigg[\frac{h^{\wp_6^*} \mathcal{Y}_6(t_k, R)}{\Gamma\left(\wp_6^* + 2\right)}
$$

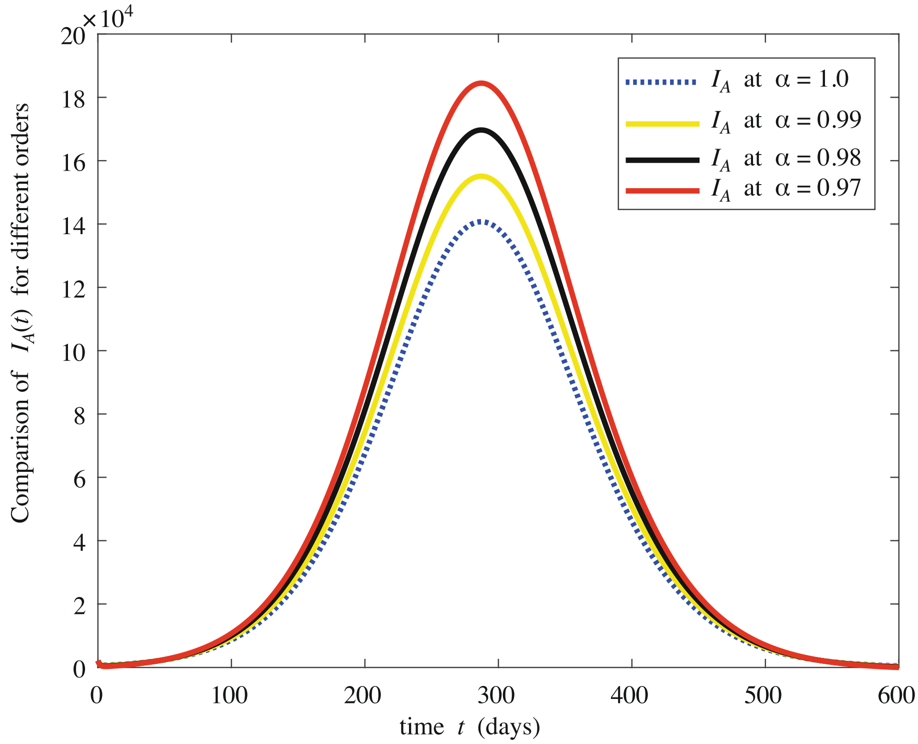

Fig. 4. Comparative analysis of the $I(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

$$\times\left((m+1-k)^{\wp_6^*}\left(m-k+2+\wp_6^*\right)(m-k)^{\wp_6^*}\left(m-k+2+2\wp_6^*\right)\right)$$

$$-\frac{h^{\wp_6^*}\mathcal{Y}_6(t_{k-1},R)}{\Gamma\left(\wp_6^*+2\right)}\left((m+1-k)^{\wp_6^*}-(m-k)^{\wp_6^*}\left(m+1-k+\wp_6^*\right)\right)\Bigg].$$

This numerical scheme helps us to predict the role of protected susceptible persons, which was practically exercised in various nations as a control strategy. Although this strategy has the worst effect on the economy of a nation, it is essential to curtail the process of spread of the infection of lethal Covid-19. The sensitivity analysis was given in [36]. It shows that the role of this strategy is very much effective in the curtail of the spread process.

5.1. Numerical Results. In this section, we provide a detail of numerical results related to the model with the data available from the literature. The parameters and initial data were taken from the available literature. The initial values are as follows: $S(0)=59300000$, $S_P(0)=0$, $I_S(0)=0$, $Q(0)=0$, $I_A(0)=2079$, $R(0)=903$, and the values of parameters are as follows: $\alpha_1=0.0008$, $\alpha_2=0.1$, $eta_1=0.25$, $\rho=0.0001$, $\eta_2=0$, $\eta_3=0.385$, $r_1=0.2976$, $r_2=0$, $\gamma=0$, $r_3=0.2976$, $\mu=0.00236/90$, $\delta=0.017/90$, $\alpha_3=1$, and $\Lambda_1=296425.875/90$ [36].

In Fig. 1, we present a joint comparative simulation for the two classes $S(t)$ and $S_P(t)$ for the following orders: 1.0, 0.99, 0.98, 0.97. In Fig. 2, we present a graphical study of the $S(t)$ class for various orders 1.0, 0.99, 0.98, and 0.97 for a long period of 5000 days. There is a decrease in the population of the class. Moreover, as the order decreases, a relatively large decrease is observed in the population while the behavior of the class remains similar. Figure 3 shows the comparative analysis of the $S_P(t)$ for the indicated orders and a gradual increase can be seen in the graph.

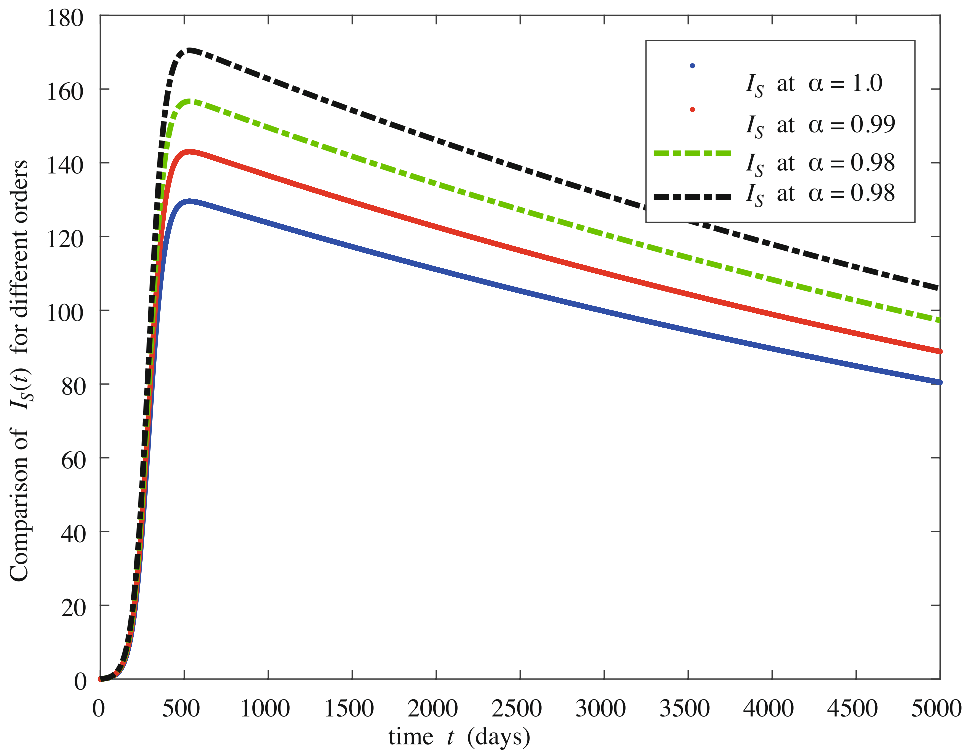

Fig. 5. Comparative analysis of the $I_S(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

In Fig. 4, we present the plots for the infected population, which show an increase detected up to 300 days and a decrease observed after 300–600 days. In Fig. 5, we show a numerical representation of the class $I_S(t)$ for various orders, whereas Fig. 6 displays the plots for the $R(t)$ class.

6. Conclusions

In the present article, we focus on the theoretical and computational studies of the fractional-order Covid-19 model in the ABC-sense of derivative. The existence and uniqueness results were carried out with the help of an iterative sequential approach with limit point as the solution of the suggested model (1). We also estimated the Hyers–Ulam stability and a numerical scheme was obtained on the basis of Lagrange's interpolation. The numerical scheme was then tested and very similar results, like the integer order, were obtained. The numerical results were interpreted with the help of six graphs. The details are as follows: In Fig. 1, we present a joint comparative simulation for the two classes $S(t)$ and $S_P(t)$ and the orders 1.0, 0.99, 0.98, and 0.97. In Fig. 2, we present a graphical study of the $S(t)$ class for various orders 1.0, 0.99, 0.98, and 0.97 for a long time of 5000 days. There is a decrease in the population of the analyzed class. Moreover, as the order decreases, we observe a relatively large decrease in the population, while the behavior of the class remains similar. In Fig. 3, we show the comparative analysis of the class $S_P(t)$ for the mentioned orders and a gradual increase can be seen in the graph. Figure 4 is for the infected population and shows an increase for up to 300 days followed by a decrease observed after 300–600 days. Figure 5 shows a numerical representation of the class $I_S(t)$ for the various orders. Finally, Fig. 6 shows the behavior of the $R(t)$ class. The reader of the paper can work on the comparative analysis of different fractional operators for higher accuracy and better results.

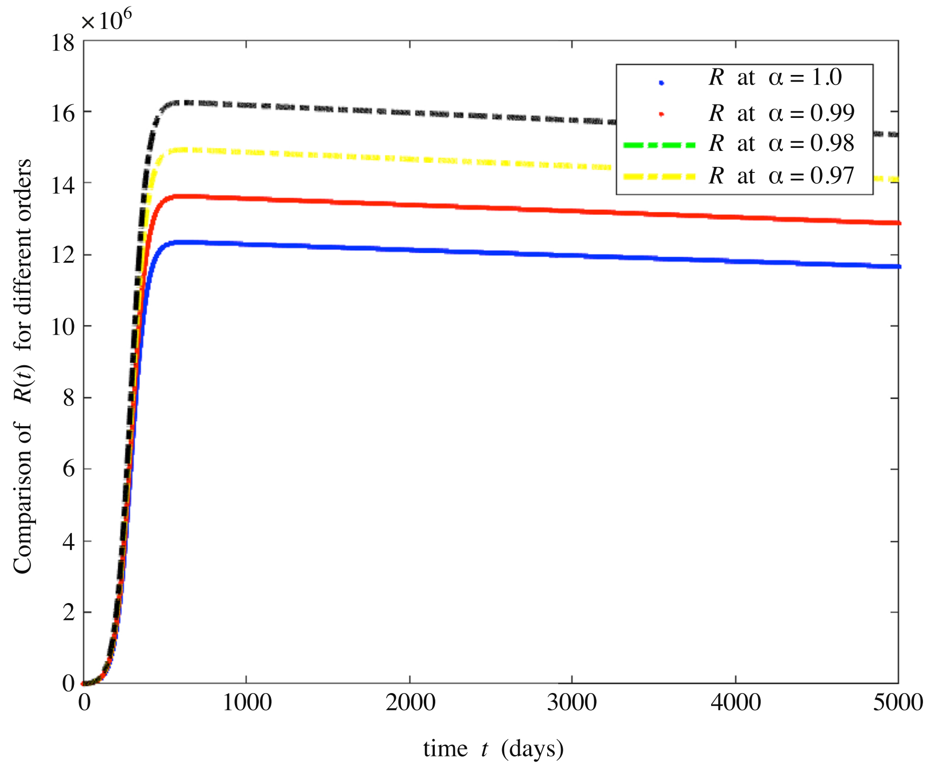

Fig. 6. Comparative analysis of the $R(t)$ for the orders 1.0, 0.99, 0.98, and 0.97.

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