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ERRATUM

To the paper "Estimates for deviations from exact solutions to plane problems in the Cosserat theory of elasticity" by S. Repin and M. E. Frolov, Vol. 181, No. 2, pp. 281–291, February, 2012. The symbol + was lost in some displayed formulas in the English version of the paper by technical reasons. The list of corrected formulas is given below.

$$(\lambda + 2\mu)(\operatorname{div} u)_{,1} + (\mu + \mu_c)(u_{1,22} - u_{2,12}) + 2\mu_c\omega_{,2} + f_1 = 0, \tag{2.1}$$

$$(\lambda + 2\mu)(\operatorname{div} u)_{,2} + (\mu + \mu_c)(u_{2,11} - u_{1,21}) - 2\mu_c\omega_{,1} + f_2 = 0, \tag{2.2}$$

$$4B\Delta\omega - 4\mu_c\omega + 2\mu_c(u_{2,1} - u_{1,2}) + g = 0; \tag{2.3}$$

$$\begin{aligned} J(u,\omega) = \int_\Omega \Big(\mu\Big(u_{1,1}^2 + u_{2,2}^2 + \frac{1}{2}(u_{2,1} + u_{1,2})^2\Big) + \frac{\lambda}{2}(u_{1,1} + u_{2,2})^2 + \frac{\mu_c}{2}(u_{2,1} - u_{1,2} - 2\omega)^2 \\ + 2B(\omega_{,1}^2 + \omega_{,2}^2)\Big)d\Omega - \int_\Omega (f\cdot u + g\omega)d\Omega = \int_\Omega \Big(\frac{1}{2}L\varepsilon(u) : \varepsilon(u) + \frac{\mu_c}{2}(u_{2,1} - u_{1,2} - 2\omega)^2 \\ + 2B|\nabla\omega|^2\Big)d\Omega - \int_\Omega (f\cdot u + g\omega)d\Omega; \end{aligned}$$

$$\varepsilon(u) = \frac{1}{2}(\nabla u + (\nabla u)^T), \quad L\varepsilon = \lambda \operatorname{tr}\varepsilon\mathbb{I} + 2\mu\varepsilon;$$

$$\int_\Omega \big(L\varepsilon(u) : \varepsilon(v^0) + y(v_{2,1}^0 - v_{1,2}^0) - f\cdot v^0\big)\, d\Omega = 0 \quad \forall\, v^0 \in \Upsilon^0; \tag{2.4}$$

$$J(u,\omega) \geqslant J_1(u) + J_2(\omega);$$

$$|\!|\!| e_{\widetilde{u}}; e_{\widetilde{y}}; e_{\widetilde{\omega}} |\!|\!|^2 := \int_\Omega \Big(\frac{1}{2}L\varepsilon(e_{\widetilde{u}}) : \varepsilon(e_{\widetilde{u}}) + \frac{1}{2\mu_c}e_{\widetilde{y}}^2 + 2B|\nabla e_{\widetilde{\omega}}|^2\Big)\, d\Omega;$$

$$\begin{aligned} J(\widetilde{u},\widetilde{\omega}) - J(u,\omega) = \int_\Omega \Big(\frac{1}{2}L\varepsilon(\widetilde{u}) : \varepsilon(\widetilde{u}) + \frac{1}{2\mu_c}\widetilde{y}^2 + 2B|\nabla\widetilde{\omega}|^2 - \frac{1}{2}L\varepsilon(u) : \varepsilon(u) - \frac{1}{2\mu_c}y^2 - 2B|\nabla\omega|^2 \\ + f\cdot e_{\widetilde{u}} + ge_{\widetilde{\omega}}\Big)d\Omega = \int_\Omega \Big(\frac{1}{2}L\varepsilon(e_{\widetilde{u}}) : \varepsilon(e_{\widetilde{u}}) + \frac{1}{2\mu_c}e_{\widetilde{y}}^2 + 2B|\nabla e_{\widetilde{\omega}}|^2\Big)d\Omega \\ + \int_\Omega \Big(f\cdot e_{\widetilde{u}} + ge_{\widetilde{\omega}} - L\varepsilon(u) : \varepsilon(e_{\widetilde{u}}) - \frac{1}{\mu_c}ye_{\widetilde{y}} - 4B\nabla\omega\cdot\nabla e_{\widetilde{\omega}}\Big)d\Omega; \end{aligned} \tag{3.1}$$

$$\mathscr{L}(v,\theta;\tau,q,s) = \int_\Omega \Big(\varepsilon(v) : \tau - \frac{1}{2}L^{-1}\tau : \tau + q(v_{2,1} - v_{1,2} - 2\theta) - \frac{1}{2\mu_c}q^2 + s\cdot\nabla\theta - \frac{1}{8B}|s|^2 - f\cdot v - g\theta\Big)d\Omega;$$

$$\sup_{(\tau,q,s)\in\mathscr{H}(\Omega)} \mathscr{L}(v,\theta;\tau,q,s) = \int_\Omega \Big(\frac{1}{2}L\varepsilon(v) : \varepsilon(v) + \frac{\mu_c}{2}(v_{2,1} - v_{1,2} - 2\theta)^2 + 2B|\nabla\theta|^2\Big)d\Omega - \int_\Omega (f\cdot v + g\theta)d\Omega = J(v,\theta);$$

$$\begin{aligned} \inf_{(v,\theta)\in\Upsilon\times\Theta} \mathscr{L}(v,\theta;\tau,q,s) = I^*(\tau,q,s) := \int_\Omega (\varepsilon(u^\Gamma) : \tau + q(u_{2,1}^\Gamma - u_{1,2}^\Gamma - 2\omega^\Gamma) + s\cdot\nabla\omega^\Gamma \\ - (f\cdot u^\Gamma + g\omega^\Gamma))d\Omega - \int_\Omega \Big(\frac{1}{2}L^{-1}\tau : \tau + \frac{1}{2\mu_c}q^2 + \frac{1}{8B}|s|^2\Big)d\Omega; \end{aligned} \tag{4.1}$$

$$\int_\Omega \big(\varepsilon(v^0) : \tau + q(v_{2,1}^0 - v_{1,2}^0) - f\cdot v^0\big)\, d\Omega = 0 \quad \forall\, v^0 \in \Upsilon^0; \tag{4.2$_1$}$$

$$I^*(L\varepsilon(u), y, 4B\nabla\omega) = \int_\Omega (L\varepsilon(u) : \varepsilon(u^\Gamma) + y(u^\Gamma_{2,1} - u^\Gamma_{1,2} - 2\omega^\Gamma) + 4B\nabla\omega \cdot \nabla\omega^\Gamma) d\Omega - \int_\Omega (f \cdot u^\Gamma + g\omega^\Gamma) d\Omega$$
$$- \int_\Omega \Big(\frac{1}{2} L\varepsilon(u) : \varepsilon(u) + \frac{\mu_c}{2}(u_{2,1} - u_{1,2} - 2\omega)^2 + 2B|\nabla\omega|^2\Big) d\Omega = J(u, \omega);$$

$$J(\widetilde{u}, \widetilde{\omega}) - I^*(\tau, q, s) = \int_\Omega \left(\frac{1}{2} L\varepsilon(\widetilde{u}) : \varepsilon(\widetilde{u}) + \frac{1}{2\mu_c}\widetilde{y}^2 + 2B|\nabla\widetilde{\omega}|^2\right) d\Omega - \int_\Omega (f \cdot \widetilde{u} + g\widetilde{\omega}) d\Omega + \int_\Omega \Big(\frac{1}{2} L^{-1}\tau : \tau$$
$$+ \frac{1}{2\mu_c} q^2 + \frac{1}{8B}|s|^2\Big) d\Omega - \int_\Omega (\varepsilon(u^\Gamma) : \tau + q(u^\Gamma_{2,1} - u^\Gamma_{1,2} - 2\omega^\Gamma) + s \cdot \nabla\omega^\Gamma - f \cdot u^\Gamma - g\omega^\Gamma) d\Omega$$
$$= \int_\Omega \Big(\frac{1}{2}(L\varepsilon(\widetilde{u}) - \tau) : (\varepsilon(\widetilde{u}) - L^{-1}\tau) + \frac{1}{2\mu_c}|\widetilde{y} - q|^2 + 2B|\nabla\widetilde{\omega} - \frac{1}{4B}s|^2\Big) d\Omega$$
$$+ \int_\Omega \Big(\varepsilon(\widetilde{u} - u^\Gamma) : \tau + q\Big(\frac{1}{\mu_c}\widetilde{y} - (u^\Gamma_{2,1} - u^\Gamma_{1,2} - 2\omega^\Gamma)\Big)$$
$$+ s \cdot \nabla(\widetilde{\omega} - \omega^\Gamma) - f \cdot (\widetilde{u} - u^\Gamma) - g(\widetilde{\omega} - \omega^\Gamma)\Big);$$

$$\|e_{\widetilde{u}}; e_{\widetilde{y}}; e_{\widetilde{\omega}}\|^2 \leqslant \int_\Omega \left(\frac{1}{2}(L\varepsilon(\widetilde{u}) - \tau) : (\varepsilon(\widetilde{u}) - L^{-1}\tau) + \frac{1}{2\mu_c}(\widetilde{y} - q)^2 + 2B|\nabla\widetilde{\omega} - \frac{1}{4B}s|^2\right) d\Omega; \tag{4.3}$$

$$\|e_{\widetilde{u}}; e_{\widetilde{y}}; e_{\widetilde{\omega}}\|^2 \leqslant \frac{1}{2}\|L\varepsilon(\widetilde{u}) - \tau\|^2_* + \frac{1}{2\mu_c}\|\widetilde{y} - q\|^2 + \frac{1}{8B}\|4B\nabla\widetilde{\omega} - s\|^2 \leqslant (1+\beta)\Big(\frac{1}{2}\|L\varepsilon(\widetilde{u}) - \widetilde{\tau}\|^2_*$$
$$+ \frac{1}{2\mu_c}\|\widetilde{y} - \widetilde{q}\|^2 + \frac{1}{8B}\|4B\nabla\widetilde{\omega} - \widetilde{s}\|^2\Big) + (1+\beta^{-1})\Big(\frac{1}{2}\|\tau - \widetilde{\tau}\|^2_* + \frac{1}{2\mu_c}\|q - \widetilde{q}\|^2 + \frac{1}{8B}\|s - \widetilde{s}\|^2\Big);$$

$$\|e_{\widetilde{u}}; e_{\widetilde{y}}; e_{\widetilde{\omega}}\|^2 \leqslant (1+\beta) D(\widetilde{\tau}, \widetilde{q}, \widetilde{s}) + (1+\beta^{-1}) R(\widetilde{\tau}, \widetilde{q}, \widetilde{s}); \tag{5.1}$$

$$D(\widetilde{\tau}, \widetilde{q}, \widetilde{s}) := \frac{1}{2}\|L\varepsilon(\widetilde{u}) - \widetilde{\tau}\|^2_* + \frac{1}{2\mu_c}\|\widetilde{y} - \widetilde{q}\|^2 + \frac{1}{8B}\|4B\nabla\widetilde{\omega} - \widetilde{s}\|^2;$$

$$R(\widetilde{\tau}, \widetilde{q}, \widetilde{s}) := -\sup_{(\overline{\tau}, \overline{q}, \overline{s}) \in \mathscr{H}(\Omega)} \Big\{ -\int_\Omega \left(\frac{1}{2} L^{-1}\overline{\tau} : \overline{\tau} + \frac{1}{2\mu_c}\overline{q}^2 + \frac{1}{8B}|\overline{s}|^2\right) d\Omega \Big\};$$

$$\int_\Omega (\varepsilon(v^0) : \overline{\tau} + \overline{q}(v^0_{2,1} - v^0_{1,2}) - \overline{f} \cdot v^0) d\Omega = 0 \quad \forall\, v^0 \in \Upsilon^0;$$

$$\overline{f} := \begin{pmatrix} f_1 + \widetilde{\tau}_{11,1} + \widetilde{\tau}_{21,2} - \widetilde{q}_{,2} \\ f_2 + \widetilde{\tau}_{12,1} + \widetilde{\tau}_{22,2} + \widetilde{q}_{,1} \end{pmatrix}, \quad \overline{g} := g + 2\widetilde{q} + \operatorname{div} \widetilde{s};$$

$$\overline{J}(\overline{u}, \overline{\omega}) = \int_\Omega \Big(\frac{1}{2} L\varepsilon(\overline{u}) : \varepsilon(\overline{u}) + \frac{\mu_c}{2}(\overline{u}_{2,1} - \overline{u}_{1,2} - 2\overline{\omega})^2 + 2B|\nabla\overline{\omega}|^2\Big) d\Omega - \int_\Omega (\overline{f} \cdot \overline{u} + \overline{g\omega})\, d\Omega;$$

$$\|\overline{u}\|^2 + \|\overline{\omega}\|^2 \leqslant C_\Omega \int_\Omega (L\varepsilon(\overline{u}) : \varepsilon(\overline{u}) + \mu_c(\overline{u}_{2,1} - \overline{u}_{1,2} - 2\overline{\omega})^2 + 4B|\nabla\overline{\omega}|^2) d\Omega;$$

$$R(\widetilde{\tau}, \widetilde{q}, \widetilde{s}) \leqslant -\inf_{t \in R^+} (t^2/2 - at) = a^2/2, \quad a = \sqrt{\|\overline{f}\|^2 + \|\overline{g}\|^2};$$

$$r(\widetilde{\tau}, \widetilde{q}, \widetilde{s}) := \frac{1}{2}\left(\|f_1 + \widetilde{\tau}_{11,1} + \widetilde{\tau}_{21,2} - \widetilde{q}_{,2}\|^2 + \|f_2 + \widetilde{\tau}_{12,1} + \widetilde{\tau}_{22,2} + \widetilde{q}_{,1}\|^2 + \|g + 2\widetilde{q} + \operatorname{div} \widetilde{s}\|^2\right);$$

$$\begin{bmatrix} (\lambda + 2\mu)u_{1,1} + \lambda u_{2,2} & \mu(u_{1,2} + u_{2,1}) \\ \mu(u_{1,2} + u_{2,1}) & (\lambda + 2\mu)u_{2,2} + \lambda u_{1,1} \end{bmatrix}.$$