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REMARKS ON A HAMILTONIAN

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Abstract. In the present paper, we give some remarks on a well-known theorem on a Hamiltonian.

Lemma. *The tangent space $T_pG^k(E)$ of the Grassmanian manifold $G^k(E)$ of k-dimensional subspaces of a linear space E is canonically isomorphic to the space* $\mathrm{Lin}(p, E/p)$.

Proof. Any point of $G^k(E)$ can be represented as the image of a monomorphism $p \to E$. We have a fibration $\mathrm{Mon}(p, E) \to G^k(E)$. A monomorphism f maps to $\mathrm{Im}\, f$. The group $\mathrm{GL}(p)$ is linear, so that we have an exact sequence

$$0 \to \mathrm{Lin}(p, p) \to \mathrm{Lin}(p, E) \to T_pG^k(E) \to 0.$$

But $\mathrm{Lin}(p, E)/\mathrm{Lin}(p, p) = \mathrm{Lin}(p, E/p)$, which proves the lemma. □

Consider the following three fibrations over $G^k(E)$: $P \to G^k(E)$, $E \times G^k(E) \to G^k(E)$, and $Q \to G^k(E)$. Fibers over p are p itself, E, and E/p. We have the exact sequence

$$0 \to P \to E \times G^k(E) \to Q \to 0.$$

If we choose q as the complement to p, then $\mathrm{Lin}(p, q)$ can be embedded as a neighborhood of p in $G^k(E)$, assigning to a map f its image $\mathrm{Im}\, f$. Each point $x \in p$ will give rise to a section over this neighborhood $f \mapsto f(x) \in \mathrm{Im}\, f$ and consequently to a representation $T_xP = p \times \mathrm{Lin}(p, E/p)$ as a tangent to the section.

Proposition. *The representation $T_xP = p \times \mathrm{Lin}(p, E/p)$ is independent of the choice of the complement.*

Proof. All constructed sections have in p the same tangent. □

The symplectic structure on T_xP is defined by the representation $T_xP = p \times \mathrm{Lin}(p, E/p)$, so that the following theorem holds.

Theorem. *In the tangent space T_xP, there is a canonical symplectic structure with values in Q.*

If one interprets P as the phase space with the symplectic structure defined above, then the Hamiltonian will be a differentiable map from P to Q.

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