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CORRECTION

Correction to: A New Abadie-Type Constraint Qualification for General Optimization Problems

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Abstract

The publication "J. Optim. Theory Appl. 186, 86–101 (2020). https://doi.org/10.1007/s10957-020-01691-0" requires minor modifications which are carried out.

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1 Introduction

In [1], some parts of the proofs of Theorems 3.1 and 3.2(ii) are needed to be revised. The conclusion of [1, Theorem 3.1] holds under an additional assumption, thus it is restated and proved. Also, the proof of [1, Theorem 3.2(ii)] is modified. With this method of proof, the USRC of the function $\tilde{d}(u)$ at $u = 0$ becomes a smaller set in comparison with its counterpart in [1, Theorem 3.2(ii)]. Thus we can say, the statement of [1, Theorem 3.2(ii)] is improved. Furthermore, [1, Lemmas 3.2 and 3.3] are not

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required and hence are omitted. Accordingly, the paragraph before these Lemmas is also removed. Further, [1, Theorem 3.3] gives a better form of the optimality condition and is updated. No other changes are required regarding the preliminaries, definitions, main conclusions and examples.

2 Modified Results

First, we update [1, Theorem 3.1] by adding the following additional assumption from [2]:

We say that the function F is calm at $\bar{x}$ with some modulus $l > 0$, if there exists a positive scalar δ satisfying $\|F(x) - F(\bar{x})\| \leqslant l\|x - \bar{x}\|$, for each $x \in \bar{x} + \delta\mathbb{B}_n$.

Theorem 2.1 ([1, Theorem 3.1] updated) *Assume that the function F is calm at $\bar{x}$ with some modulus $l > 0$ and d_Λ is directionally differentiable at $F(\bar{x})$. If EBCQ holds at $\bar{x}$ with a constant σ, then ACQ is satisfied at $\bar{x}$ with the same constant.*

Proof *(modified)* Let $u \notin T(\bar{x}; F^{-1}(\Lambda))$ (otherwise there is nothing to prove) and EBCQ be satisfied at $\bar{x}$ with $\sigma = 1$. Assume also that $0 \leqslant \tilde{d}(u) < \infty$ (if $\tilde{d}(u) = +\infty$, the ACQ obviously holds). Thus, there is a sequence $t_k \downarrow 0$ such that

$$\tilde{d}(u) = \lim_{k\to\infty} d_{T(F(\bar{x});\Lambda)}\left(\frac{F(\bar{x} + t_k u) - F(\bar{x})}{t_k}\right).$$

The closedness of $T(F(\bar{x}); \Lambda)$, gives us a sequence $\{w_k\}$ such that for each k,

$$d_{T(F(\bar{x});\Lambda)}\left(\frac{F(\bar{x} + t_k u) - F(\bar{x})}{t_k}\right) = \left\|\frac{F(\bar{x} + t_k u) - F(\bar{x})}{t_k} - w_k\right\|. \tag{1}$$

We assert that the sequence $\{w_k\}$ is bounded. Fixing $\varepsilon > 0$ and observing (1), we obtain the following inequalities for all k sufficiently large:

$$\|w_k\| \leqslant \left\|\frac{F(\bar{x} + t_k u) - F(\bar{x})}{t_k} - w_k\right\| + \left\|\frac{F(\bar{x} + t_k u) - F(\bar{x})}{t_k}\right\| < \tilde{d}(u) + \varepsilon + l\|u\|,$$

which shows the boundedness of $\{w_k\}$ and the assertion is proved. Thus by passing to a subsequence, without relabelling, $\{w_k\}$ converges to some vector $w \in T(F(\bar{x}); \Lambda)$. Now, By EBCQ one has

$$\begin{aligned} \frac{d_{F^{-1}(\Lambda)}(\bar{x} + t_k u)}{t_k} &\leqslant \frac{d_\Lambda(F(\bar{x} + t_k u))}{t_k} \\ &\leqslant \frac{d_\Lambda(F(\bar{x}) + t_k w_k)}{t_k} + \left\|\frac{F(\bar{x} + t_k u) - F(\bar{x})}{t_k} - w_k\right\|. \end{aligned} \tag{2}$$

Next, we claim that $\limsup_{k\to\infty} \frac{d_\Lambda(F(\bar{x})+t_k w_k)}{t_k} = 0$. From [1, Lemma 3.1] and the fact that d_Λ is directionally differentiable at $F(\bar{x})$, we get

$$\begin{aligned} 0 \leqslant \limsup_{k\to\infty} \frac{d_\Lambda(F(\bar{x})+t_k w_k)}{t_k} &\leqslant \lim_{k\to\infty} \left\{ \frac{d_\Lambda(F(\bar{x})+t_k w)}{t_k} + \|w_k - w\| \right\} \\ &= d'_\Lambda(F(\bar{x}); w) = 0, \end{aligned} \tag{3}$$

which proves the claim. Now, it follows from (2), (3) and (1) that

$$\begin{aligned} \liminf_{k\to\infty} \frac{d_{F^{-1}(\Lambda)}(\bar{x}+t_k u)}{t_k} &\leqslant \limsup_{k\to\infty} \left\{ \frac{d_\Lambda(F(\bar{x})+t_k w_k)}{t_k} \right. \\ &\quad \left. + \left\| \frac{F(\bar{x}+t_k u) - F(\bar{x})}{t_k} - w_k \right\| \right\} \\ &\leqslant \limsup_{k\to\infty} \frac{d_\Lambda(F(\bar{x})+t_k w_k)}{t_k} \\ &\quad + \lim_{k\to\infty} \left\| \frac{F(\bar{x}+t_k u) - F(\bar{x})}{t_k} - w_k \right\| \\ &= \tilde{d}(u). \end{aligned}$$

Using again [1, Lemma 3.1], the above especially implies that

$$d_{T(\bar{x};F^{-1}(\Lambda))}(u) = d^-_{F^{-1}(\Lambda)}(\bar{x}; u) \leqslant \tilde{d}(u),$$

and completes the proof of the theorem. □

In what follows, the proof of [1, Theorem 3.2(ii)] is modified. By this modification, USRC of the function $\tilde{d}(.)$ at $u = 0$ becomes a smaller set and gives a better result; hence its statement is also improved.

Theorem 2.2 ([1, Theorem 3.2(ii)] updated) *Assume that $\partial F(\bar{x})$ is an u.s.c. PJ of $F : \mathbb{R}^n \to \mathbb{R}^m$ at $\bar{x}$. Suppose also that $F(\bar{x}) \in \Lambda \subseteq \mathbb{R}^m$ and $\partial d_\Lambda(F(\bar{x}))$ is a bounded USRC of d_Λ at $F(\bar{x})$. Then the closure of the set*

$$\partial d_\Lambda(F(\bar{x})) \circ \{\mathrm{conv}\, \partial F(\bar{x}) \cup [(\partial F(\bar{x}))_\infty \setminus \{0\}]\}$$

is an USRC of the function $\tilde{d}$ at $u = 0$.

Proof *(revised)* Put $A := \partial d_\Lambda(F(\bar{x})) \circ \{\mathrm{conv}\partial F(\bar{x}) \cup [(\partial F(\bar{x}))_\infty \backslash \{0\}]\}$ and fix $u \in \mathbb{R}^n$. First, let us show that $\sup_{\eta\in A} \langle \eta, u\rangle \geqslant 0$. For given $M \in \mathrm{conv}\partial F(\bar{x}) \cup [(\partial F(\bar{x}))_\infty \backslash \{0\}]$, we have

$$\sup_{\xi\in\partial d_\Lambda(F(\bar{x}))} \langle \xi, Mu\rangle \geqslant d^+_\Lambda(F(\bar{x}); Mu) \geqslant 0.$$

Thus, using the definition of A, we get

$$\begin{aligned}\sup_{\eta\in A}\langle\eta,u\rangle &= \sup_{\substack{\xi\,\in\,\partial d_\Lambda(F(\bar{x}))\\ M\,\in\,\mathrm{conv}\partial F(\bar{x})\,\cup\,[(\partial F(\bar{x}))_\infty\,\setminus\,\{0\}]}}\langle\xi M,u\rangle\\ &= \sup_{\substack{\xi\,\in\,\partial d_\Lambda(F(\bar{x}))\\ M\,\in\,\mathrm{conv}\partial F(\bar{x})\,\cup\,[(\partial F(\bar{x}))_\infty\,\setminus\,\{0\}]}}\langle\xi, Mu\rangle \geqslant 0.\end{aligned}$$

There are two possible cases: If $\tilde{d}^+(0;u)=0$, then trivially we obtain

$$\tilde{d}^+(0;u)\leqslant\sup_{\eta\in A}\langle\eta,u\rangle\,. \tag{4}$$

Hence, let $\tilde{d}^+(0;u)>0$. If the following inequality holds:

$$\sup_{\substack{\xi\,\in\,\partial d_\Lambda(F(\bar{x}))\\ M\,\in\,\mathrm{conv}\partial F(\bar{x})\,\cup\,[(\partial F(\bar{x}))_\infty\,\setminus\,\{0\}]}}\langle\xi, Mu\rangle > 0,$$

due to the cone property of $(\partial F(\bar{x}))_\infty\setminus\{0\}$, we get

$$\sup_{\substack{\xi\,\in\,\partial d_\Lambda(F(\bar{x}))\\ M\,\in\,\mathrm{conv}\partial F(\bar{x})\,\cup\,[(\partial F(\bar{x}))_\infty\,\setminus\,\{0\}]}}\langle\xi, Mu\rangle = +\infty,$$

and the inequality in (4) holds trivially. Finally, the following case remains

$$\sup_{\substack{\xi\,\in\,\partial d_\Lambda(F(\bar{x}))\\ M\,\in\,\mathrm{conv}\partial F(\bar{x})\,\cup\,[(\partial F(\bar{x}))_\infty\,\setminus\,\{0\}]}}\langle\xi, Mu\rangle = 0. \tag{5}$$

For each fixed $M\in\mathrm{conv}\partial F(\bar{x})\cup[(\partial F(\bar{x}))_\infty\setminus\{0\}]$, one has

$$\begin{aligned}0\leqslant d_\Lambda^+(F(\bar{x});Mu) &\leqslant \sup_{\xi\in\partial d_\Lambda(F(\bar{x}))}\langle\xi,Mu\rangle\\ &\leqslant \sup_{\substack{\xi\,\in\,\partial d_\Lambda(F(\bar{x}))\\ M\,\in\,\mathrm{conv}\partial F(\bar{x})\,\cup\,[(\partial F(\bar{x}))_\infty\,\setminus\,\{0\}]}}\langle\xi, Mu\rangle = 0\end{aligned}$$

Utilizing [1, Lemma 3.1], we have

$$0\leqslant d_{T(F(\bar{x});\Lambda)}(Mu)=d_\Lambda^-(F(\bar{x});Mu)\leqslant d_\Lambda^+(F(\bar{x});Mu)=0,$$

which means that $Mu\in T(F(\bar{x});\Lambda)$, for all $M\in\mathrm{conv}\partial F(\bar{x})\cup[(\partial F(\bar{x}))_\infty\setminus\{0\}]$. Now, since $\tilde{d}^+(0;u)>0$, there exits some positive number c such that $c<\tilde{d}^+(0;u)=\tilde{d}(u)$. Thus for some sequence $t_k\downarrow 0$ and for all k sufficiently large, one has

$$c<d_{T(F(\bar{x});\Lambda)}\left(\frac{F(\bar{x}+t_k u)-F(\bar{x})}{t_k}\right). \tag{6}$$

Applying now the mean value Theorem in [1, Propostion 2.3], we have for each k,

$$F(\bar{x}+t_k u)-F(\bar{x})\in \text{clconv}\{\partial F[\bar{x}+t_k u,\bar{x}]t_k u\}.$$

Using the upper semicontinuity of $\partial F(.)$ at $\bar{x}$, for given sequence $r_s\downarrow 0$, there exits $k_s>k_{s-1}$ satisfying

$$\begin{aligned}F(\bar{x}+t_{k_s}u)-F(\bar{x})&\in \text{clconv}\{\partial F[\bar{x}+t_{k_s}u,\bar{x}]t_{k_s}u\}\\&\subseteq \text{clconv}\{\{\partial F(\bar{x})+\frac{r_s}{2}\mathbb{B}_{m\times n}\}t_{k_s}u\}\\&\subseteq \text{cl}\{\{\text{conv}\partial F(\bar{x})+\frac{r_s}{2}\mathbb{B}_{m\times n}\}t_{k_s}u\}.\end{aligned}$$

Thus, there exists $M_{k_s}\in \text{conv}\partial F(\bar{x})$ such that

$$\left\|\frac{F(\bar{x}+t_{k_s}u)-F(\bar{x})}{t_{k_s}}-M_{k_s}u\right\|<r_s\|u\|.$$

Choosing now subsequences $M_s:=M_{k_s}$ and $t_s:=t_{k_s}$, and using the inequality in (6), we deduce that

$$c<d_{T(F(\bar{x});\Lambda)}\left(\frac{F(\bar{x}+t_s u)-F(\bar{x})}{t_s}\right)<d_{T(F(\bar{x});\Lambda)}(M_s u)+r_s\|u\|.$$

Observing that $d_{T(F(\bar{x});\Lambda)}(M_s u)=0$ and taking limit as $s\to\infty$ in the latter inequality, we arrive at the contradiction $c\leqslant 0$, which shows the case $\tilde{d}^{+}(0;u)>0$ and the equality (5) do not occur together and the proof is completed. □

Since the USRC of the function $\tilde{d}$ is changed, the optimality condition in [1, Theorem 3.3] is improved and updated, accordingly.

Theorem 2.3 ([1, Theorem 3.3] updated) *Suppose that ACQ is satisfied at the local optimal point $\bar{x}$ of GOP. Let $\partial f(\bar{x})$ and $\partial F(\bar{x})$ are USRC and u.s.c. PJ of f and F at $\bar{x}$, respectively and $\partial d_\Lambda(F(\bar{x}))$ is a bounded USRC of d_Λ at $F(\bar{x})$. Then*

$$0\in \text{cl conv}\{\partial f(\bar{x})+l\sigma\partial d_\Lambda(F(\bar{x}))\circ\{\text{conv}\partial F(\bar{x})\cup[(\partial F(\bar{x}))_\infty\setminus\{0\}]\}\},$$

where σ is the positive constant of ACQ and l is the Lipschitz constant of the function f in a neighborhood of $\bar{x}$.

3 Conclusion

The proofs of [1, Theorems 3.1 and 3.2(ii)] are rectified and their statements are updated. Also, [1, Theorem 3.3] gives a better form of the optimality condition which is improved.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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