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A Pollution Sensitive Marxian Production Inventory Model with Deterioration Under Fuzzy System

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Abstract

This article deals with an economic production quantity (EPQ) model with deterioration under the effect of environmental pollution in fuzzy environment. First of all, we develop a pollution generation (PG) model with the help of existing initial pollution status of the environment, and then we use it as an essential constraint in the proposed EPQ model. The model has also been studied in two different scenarios: (a) when unit selling price is given and (b) when unit selling price is associated with marginal profit respectively. However, in this article, the concept of manpower exploitation, law of surplus value and their impacts on profit function has been discussed. In fact, after the invention of Marxian production theory (1867), not a single article has been developed for studying inventory management problems/operations research using this theory. Thus, in this study, focussing Marxian principle we have developed a new production inventory model named Marxian economic production quantity (M-EPQ) model incorporating the extensions of the models developed by Harris (Mag Manag 10(2):135–136, 1913) and Taft (Iron Age 101:1410–1412, 1918) exclusively. Moreover, to deal with the non-random uncertainties of several cost components of production process we have utilized fuzzy system explicitly. A case study has been performed for numerical illustrations and we have developed a solution algorithm for numerical computations. Finally, sensitivity analysis and graphical illustrations are made to validate the new M-EPQ model followed by a conclusion.

Keywords EPQ model · PG model · Marxian theory · Exploitation · Fuzzy system · Optimization

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1 Introduction

Harris [24] developed first the classical economic order quantity (EOQ) inventory model and the EPQ model was established by Taft [35]. Since then, numerous research articles have been developed by eminent researchers. The aim of those models was to control the order quantity by means of controlling the cycle time to achieve the maximum profit or to avail a minimum system cost. Moreover, the concept of cleaner production in EPQ model has been introduced lately by eminent researchers. The major part of pollution consists of CO_2 and CO gases which have been studied in the inventory management problems in the name of 'Carbon foot print', 'Carbon cap policy' or 'Carbon taxation policy' explicitly. In its continuation, some recent articles might keep significant contribution in the related domain. Chen et al. [9] and Battini et al. [1] analysed EOQ problem for sustainability-related to environmental pollution. Hovelaque and Bironneau [26] proposed an EOQ model of carbon-constrained with carbon emission-dependent demand. Pollution in production process through carbon trade and carbon tax regulation has been discussed by He et al. [25] and Xu et al. [39] respectively. Li [30] developed various policies and strategies for solving lost sale-based inventory model under stochastic environment. Wang et al. [38] invented an optimal strategy to solve step-shaped demand-dependent inventory model.

Moreover, to undertake the problems of non-random uncertainty Zadeh [41] invented fuzzy set theory. Since then, numerous research articles have been developed to solve real-world problems that include production manufacturing problems also. Problems of pollution-based sponge iron industry have been analysed by Karmakar et al. [27, 28] in which they employed dense fuzzy and lock fuzzy system explicitly. Indeed, the concept of dense fuzzy was coined by De and Beg [12] first and then its extension was discussed by De and Mahata [13] in the name of cloudy fuzzy set. De and Mahata [14] also solved an economic order quantity model using fuzzy monsoon demand. Subsequently, the concept of dense fuzzy lock set was coined by De [11]. Bhattacharya et al. [4, 5] developed a pollution-sensitive global crude steel production transportation model under the effect of corruption perception index and solved a fuzzy decision-making problem on global warming. Researchers like Garai et al. [16–20], Garai and Garg [22] developed various models on multi-objective multi-item inventory with different kinds of costs and demand via fuzzy possibility theory and generalised non-linear intuitionistic fuzzy number extensively. Garai et al. [21] studied multi-objective inventory model under the effect of probabilistic environment. Giri et al. [23] also established a price and quality-dependent inventory model and solved it via fuzzy possibility theory.

1.1 Specific Study

In last decades, researchers were basically getting themselves engaged through modelling with CO_2 emissions by means of carbon cap and trade policies. In fact, these concepts do not carry the whole estimation of our environment's pollution. Most of the research papers were written by prescribing the use of preservation technology for controlling pollution. But the source and sink relationship of pollution are still out of reach in the existing research. We know most of the production sectors are occupied by special economic zone (SEZ) in different countries where the governmental rules and regulations have been relaxed that ignoring the issues of human health environmental pollution. A study reveals that about 85% of total pollution of a country comes from all SEZ of that country. The soil, air and water are being polluted rapidly causing the increase of non-fertile land, scarcity of fresh drinking water and shortages of oxygen for breath. Long history of SEZ is available in the literature. But we may consider few of them which might serve the purpose of this study. Wang [37] studied the economic impact of special economic zones of Chinese municipalities. Parwez [33] discovered the constraints of labour welfare in SEZs of India with special reference to Gujarat. Policy analysis and reduction of regional disparities in China's SEZ are analysed by Crane et al. [10] extensively.

Indeed, Covid-19 pandemic outbreak lost the natural rhythm of human activities but gave us knowledge about how to clean environment. In particular, during lockdown period March–April 2020 on a global average, all industries were shut down and transportation was minimized. As a result, for example in the highly polluted city Kolkata, India having normal 175 pollution index before Covid-19, it was tremendously down to index 30 and even some days, it was 9 only during the said lockdown period [3]. But till date not a single research article has been studied yet expressing the idea of environmental pollution. On the other hand, the purchasing capacities of common people are becoming low due to job crisis of each individual throughout the globe. The consequence is that most of the industrial products have no demand; even if in some cases, there is demand but the profits are marginal. Getting motivation from these facts we wish to study an EPQ model based on Marxian production theory.

We knew Karl Heinrich Marx (1818–1883) was a German social thinker, political economist and philosopher from working-class intellectual and revolutionary. In his time, he gave many economic theories which are basically designed for a mass working-class people and they are put in several volumes of the book "Das Kapital". Marx's method of production is based on historical materialism that conceives the process of human socio-economics reproduction [32]. The source of surplus-value of a commodity lies in the unpaid labour time performed by the workers during the production time in the capitalist mode of production [31]. The value of labour is studied by Bellofiore [2]. The distribution of surplus-value and its growth was developed by Carter [6, 8]. Also, pool of profits was analysed by Carter [7] and falling rate of profit is discussed by Eltis [15]. The Concepts of absolute and relative wages towards modern theory of distribution is given by Levrero [29]. We know, modern global economy is nothing but the economy of political economy [36]. Thus, after meeting long discussion on Marx's theory it is seen that in today's world no one can get more profit [34]. Because of the volume/page limitation of this article we may consider those parts of Marxian principle which are associated with capitalistic mode of production, especially the 'surplus value' of a commodity. The basic notion of Marx's theory is to establish the theory of socialism and communism that focuses critiques of capitalism. One of the essential key points of this theory is 'value of labour power'. According to Marx, the actual wages paid in labour-time or socially necessary human labour time for the production of a commodity with the given highest level of technological set-up available in society is called 'value of labour power'. The 'surplus value' means the extra money earned (drawn) from a production system at its end after selling the whole commodities explicitly. This includes the unpaid labour power to produce the commodities. However, the cost that went to produce the commodities may include the cost of raw materials, its transportation and fixed charges of industrial set up known as 'production cost' which according to Marx; is constant for a particular time horizon. Thus, if s_p be the selling value of one unit of a commodity, c_p be the production cost of one unit, v be the value of labour-power to produce one unit and s be the surplus-value of one unit of that commodity then Marx's theory says $s_p = c_p + v + s$. Here c_p is called c_p-constant capital and v is called v-variable capital. Also, the rate of exploitation r is given by $r = \frac{s_p - (c_p + v)}{c_p + v}$ and is called the fundamental Marxian theorem.

From the above study, we came to know that, most of the researchers were involved to maximize the profit or minimize the system cost that excluded the notion of exploitation. Thus, in this study, we develop a new EPQ model named M-EPQ model having the exploitation rate as a decision variable that optimizes the average profit function and average cost function simultaneously. Due to parametric flexibilities, the model has also been studied into two different scenarios namely model for normal profit and the model of marginal profit via fuzzy system.

The organization of this article is developed as follows: section one is introduction followed by specific study. Section 2 includes preliminaries that focus definition of fuzzy set, its defuzzification, arithmetic operations and partial orderings of fuzzy sets. Section 3 discusses a mathematical formula of pollution measures and a case study. Section 4 indicates Marxian production inventory model and its mathematical foundation. Section 5 includes the fuzzy mathematical model and a solution algorithm; Sect. 6 indicates numerical illustration and sensitivity analysis. Sections 7 develops graphical illustrations; Sect. 8 represents the research findings and managerial insights and finally Sect. 9 keeps a conclusion.

2 Preliminaries

2.1 Definition of Fuzzy Set and Defuzzification [40, 41]

Let $\tilde{A}$ be a triangular fuzzy number drawn from a universal set X of the form $\tilde{A} = \langle a_1, a_2, a_3 \rangle$. Then the membership function of the fuzzy set $\tilde{A}$ is defined by

$$\mu\left(\tilde{A}\right) = \begin{cases} 0, & \text{if } a\langle a_1 \text{ and } a\rangle a_3 \\ \frac{a-a_1}{a_2-a_1}, & \text{if } a_1 \le a \le a_2 \\ \frac{a_3-a}{a_3-a_2}, & \text{if } a_2 \le a \le a_3 \end{cases} \tag{1}$$

Now, for the left and right α-cuts of $\mu(\tilde{A})$ namely $L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $R(\alpha) = a_3 - (a_3 - a_2)\alpha$ respectively, we may utilize the index value of $\tilde{A}$ defined as

$$I\left(\tilde{A}\right) = \frac{1}{2}\int_0^1 [L(\alpha) + R(\alpha)]\mathrm{d}\alpha = \frac{(a_1 + 2a_2 + a_3)}{4} \tag{2}$$

2.2 Arithmetic Operations on Triangular Fuzzy Numbers

Let $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ and $\tilde{B} = \langle b_1, b_2, b_3 \rangle$ be two triangular fuzzy numbers, then the usual arithmetic operations $+, -, \times, \div$, namely addition, subtraction, multiplication and division between $\tilde{A}$ and $\tilde{B}$ can be stated as below:

(i) $\tilde{A} + \tilde{B} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$
(ii) $\tilde{A} - \tilde{B} = \langle a_1 - b_3, a_2 - b_2, a_3 - b_1 \rangle$
(iii) $\widetilde{A} \times \widetilde{B} = \mathrm{Min}(a_i b_j),\ \mathrm{Max}(a_i b_j)\ \ \forall i, j = 1, 2, 3$
(iv) $\widetilde{A}/\widetilde{B} = \mathrm{Min}(a_i/b_j),\ \mathrm{Max}(a_i/b_j)$ for $b_j \neq 0, i, j = 1, 2, 3$
(v) $\mu\widetilde{A} = \mu a_1,\ \mu a_2,\ \mu a_3$ if $\mu \geq 0$ and $\mu\tilde{A} = \mu a_3,\ \mu a_2,\ \mu a_1$ if $\mu < 0$

2.3 Partial Order Relations

Let $A_j = \langle s_j, l_j, r_j \rangle$ and $B_i = \langle t_i, u_i, v_i \rangle$ for $i, j = 1, 2, 3, \ldots n$ be two fuzzy numbers, their summation and multiplication are the arithmetic operations on fuzzy numbers, and the partial order $\leq$ is defined by $A \leq B$ iff $\mathrm{MAX}(A, B) = B$. Moreover, in particular if $A = \langle s_1, l_1, r_1 \rangle$ and $B = \langle s_2, l_2, r_2 \rangle$ will satisfy $A \leq B$ then $s_1 \leq s_2, s_1 - l_1 \leq s_2 - l_2$ and $s_1 + r_1 \leq s_2 + r_2$.

3 A Pollution Generation Model

Here we shall discuss the story behind environmental pollution and its assessment. Traditionally, researchers are engaged in developing various models of producing commodities at different industrial sectors; its transportation and supply chains. We know, for any kind of industrial production there corresponds a considerable amount of pollution which are adding to the environment day by day. Such kind of 'pollution production' has no outlet to discharge/transfer or in other words, this produced item has no demand. However, in practical situation, we found a little bit of its reduction by means of absorption by nature (environment). For example, small particulate matters of air are getting washed out during rainfall, the green vegetation absorbs CO_2 during photosynthesis, the harmful pollutants like SO_2, CO, SO_3 etc. are converted to some other substances with the use of technological advancement associated with production industries itself.

Therefore, we may design a pollution production model under some assumptions and notations exclusively.

Assumptions

(i) The environment itself has a minimum level of pollution whether human intervention upon environment takes place or not.
(ii) 100% pollution reduction is not possible; rather a little part of total pollution can be reducible with the help of advanced technology and by the vegetation of environment and hence large amount of pollution is getting piled up over time.
(iii) Amount of pollution adding to the environment follows geometric progression.
(iv) At the time of infinity, the pollution level reaches a maximum level, called life limit.

(v) All kinds of pollutions are harmful to animal and plant kingdom.
(vi) Amount of pollution means the average of n successive amount of pollution coming from n production cycles of industry.
(vii) The term 'life limit' means a certain amount of environmental pollution for which about 80% each of the animal and plant species might extinct.

Notations

J_i: Pollution level at any time t of the i-th production cycle.
p: Rate of pollution absorption by the nature.
p_0^{i-1}: The threshold amount of pollution at the i-th production cycle.
γ: Scale parameter of current pollution
γ': Scale parameter of initial pollution
φ: Pollution absorbance rate by the nature
w_n: Total amount of pollution after n production cycle.
T: The industrial production cycle time (weeks).
λ', λ: Scale parameters of pollution.
p^i: Pollution rate at the i-th production cycle.

3.1 Formation of Pollution Generation (PG) Model

Let the pollution rate λp^i is adding to the environment during the i-th industrial production cycle of a single industrial setup. The pollution is reduced due to environmental absorption only and the major parts are adding to the environment over time. Then considering the above assumptions and for n cycles of production the governing differential equation of the pollution production model (shown in Fig. 1) is given by

$$\begin{aligned} &\frac{\mathrm{d}J_i(t)}{\mathrm{d}t} + \varphi J_i(t) = \gamma p^i, \qquad 0 \le t < \infty \\ &\text{Subject to } J_\mathrm{i}(0) = \gamma' p_0^{i-1},\ i = 1, 2, 3, \ldots n \end{aligned} \tag{3}$$

Solving (3) we get

$$J_i(t) = \gamma' p_0^{i-1} e^{-\varphi t} + \gamma p^i \left(1 - e^{-\varphi t}\right) \tag{4}$$

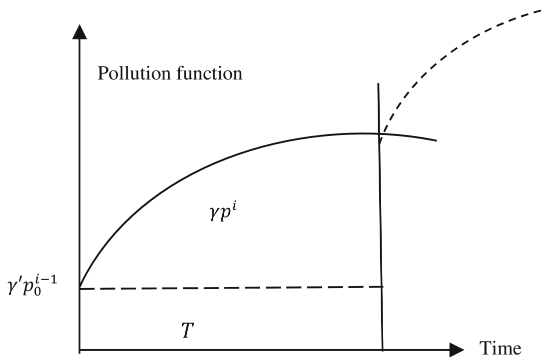


Fig. 1 Pollution generating model at ith production process

Now utilizing (4) the total amount of pollution after n cycles is given by

$$w_n = \sum_{i=1}^{n} \int_{0}^{nT} J_i(t)\mathrm{d}t = \int_{0}^{nT} \left\{ \gamma' e^{-\varphi t} \sum_{i=1}^{n} p_0^{i-1} + \gamma \left(1 - e^{-\varphi t}\right) \sum_{i=1}^{n} p^i \right\} \mathrm{d}t$$

So average amount of cumulative pollution per cycle over n cycles is

$$\overline{w} = \frac{w_n}{n} = \left[\left\{ T - \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi} \right\} \frac{\gamma p(p^n - 1)}{p - 1} + \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi} \frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1} \right] \quad (5)$$

3.1.1 Special Cases

(i) When the natural absorption rate $\varphi \to 0$ then $\overline{w} \to T \frac{\gamma^{/}(p_0^n - 1)}{p_0 - 1}$
[Since, $\lim\limits_{\varphi \to 0} \overline{w} = \lim\limits_{\varphi \to 0} \frac{1}{n} \left[\left\{ nT - \frac{(1 - e^{-\varphi nT})}{\varphi} \right\} \frac{\gamma p(p^n - 1)}{p - 1} + \frac{(1 - e^{-\varphi nT})}{\varphi} \frac{\gamma'(p_0^n - 1)}{p_0 - 1} \right]$

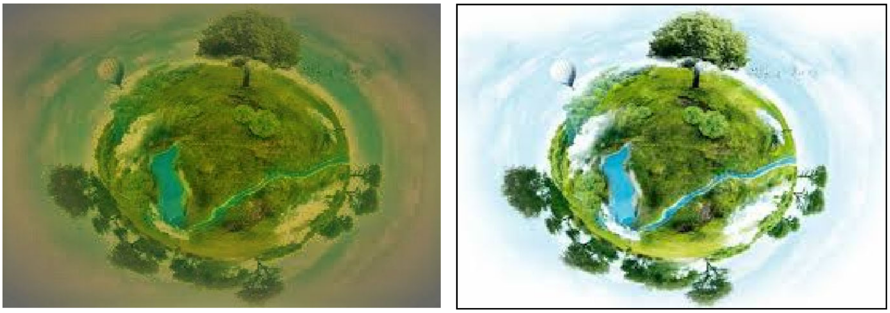
$$= \lim_{\varphi \to 0} \frac{1}{n} \left[\left\{ nT - \frac{nT\left(e^{-\varphi nT} - 1\right)}{-n\varphi T} \right\} \frac{\gamma p(p^n - 1)}{p - 1} + \frac{\left(e^{-\varphi nT} - 1\right)nT}{-n\varphi T} \frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1} \right]$$
$$= \frac{T\gamma'\left(p_0^n - 1\right)}{p_0 - 1}$$

so, $\overline{w} \to T \frac{\gamma^{/}(p_0^n - 1)}{p_0 - 1}$] (Shown in Fig. 2a).

(ii) When the natural absorption rate $\varphi \to \infty$ then $\overline{w} \to T \frac{\gamma p(p^n - 1)}{p - 1}$
[Since, $\lim\limits_{\varphi \to \infty} \overline{w} = \lim\limits_{\varphi \to \infty} \frac{1}{n} \left[\left\{ nT - \frac{(1 - e^{-\varphi nT})}{\varphi} \right\} \frac{\gamma p(p^n - 1)}{p - 1} + \frac{(1 - e^{-\varphi nT})}{\varphi} \frac{\gamma'(p_0^n - 1)}{p_0 - 1} \right]$.
Now, $\lim\limits_{\varphi \to \infty} \frac{(1 - e^{-\varphi nT})}{\varphi} = \lim\limits_{\varphi \to \infty} \frac{(e^{\varphi nT} - 1)}{\varphi e^{\varphi nT}} = \left(\frac{\infty}{\infty}\right)$form So, applying L' Hospital's rule we get $\lim\limits_{\varphi \to \infty} \frac{(1 - e^{-\varphi nT})}{\varphi} = \lim\limits_{\varphi \to \infty} \left[\frac{nTe^{\varphi nT}}{e^{\varphi nT} + \varphi nTe^{\varphi nT}} \right] = \lim\limits_{\varphi \to \infty} \left[\frac{nT}{1 + \varphi nT} \right] = 0$]
(Shown in Fig. 2b).

(iii) If we wish to study the pollution status for the limit at $n \to \infty$ then we get

$$\lim_{n \to \infty} \overline{w} = \lim_{n \to \infty} \frac{w_n}{n}$$
$$= \lim_{n \to \infty} \left[\left\{ T - \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi} \right\} \frac{\gamma p(p^n - 1)}{p - 1} + \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi} \frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1} \right]$$
$$= \left\{ T - \lim_{n \to \infty} \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi} \right\} \gamma \lim_{n \to \infty} \frac{p(p^n - 1)}{p - 1}$$
$$+ \lim_{n \to \infty} \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi} \lim_{n \to \infty} \frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}$$



(a) View of globe under complete pollution **(b)** View of globe under no pollution

Fig. 2 **a** View of globe under complete pollution. **b** View of globe under no pollution. *Source*: Figure **a**, **b** copyright: www.twrecruitment.com

$$= \{T - 0\}\gamma \lim_{n\to\infty} \frac{p(p^n - 1)}{p - 1} + 0 \times \lim_{n\to\infty} \frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}$$

$$= T\gamma \lim_{n\to\infty} \frac{p(p^n - 1)}{p - 1} \Rightarrow \frac{T\gamma p}{1 - p} \text{ for } 0 < p < 1$$

$$\text{and } \lim_{n\to\infty} \overline{w} = T\gamma \lim_{n\to\infty} \frac{p(p^n - 1)}{p - 1} \to \infty \text{ for } p \geq 1.$$

3.2 Case Study

We visited a special economic zone (SEZ) situated at the bank of Sabarmati River, Gandhinagar (Lat. 23° 14′ 15.22″ N, Long. 72° 38′ 52.01″ E), Gujarat, India (The Google map shown in Fig. 3) on January 2020. It manufactures electronics goods in particular DVD player, home theatre, Memory card and Pen drive player etc. The industry pollutes to highest extent upon surrounding environments including Sabarmati River by the piles of garbage during production run time. The workers are forced to do over-duty (without wages) to secure more profit from the production management. During production, some items deteriorate and the production efficiency never attains 100%. All expenditures are mourned weekly basis. The demands are exhausted throughout the whole cycle time and the cost related to pollutions is estimated over the average pollution gathered for the duration of at least 10 successive production cycles of the industry. The data available for this study is set in Table 1 given below.

The research problems are developed as follows:

(i) What will be the optimum cycle time and optimum production run time so as to minimize the average pollution?
(ii) What will be the profit enhancement if industrial pollution is ignored?
(iii) What will happen when the marginal profit (no profit) is considered?
(iv) What will be the rate of exploitation (due to Marxian fundamental principle) of manpower with respect to profit maximization?

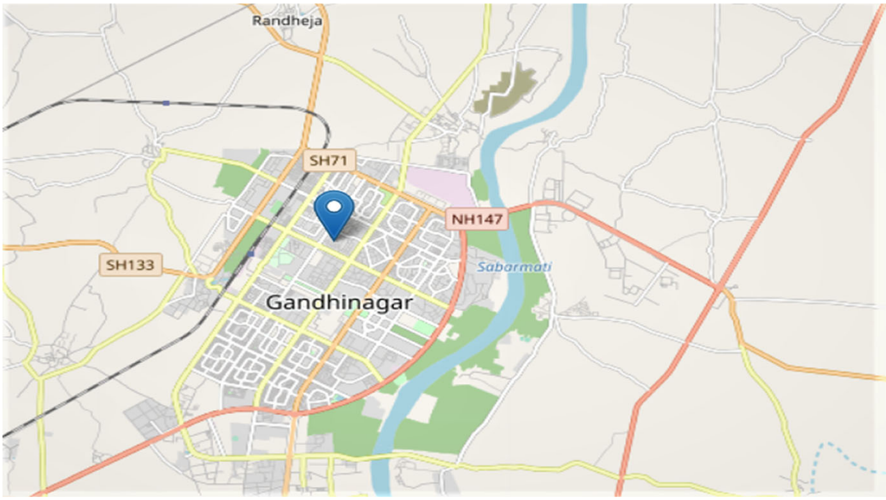


Fig. 3 Location of Gandhinagar Electronic SEZ. *Source*: Copyright of Figure: https://maps.google.com

Table 1 Data available for the proposed study

Labour cost $v = \$0.5$ to produce one unit of item	Demand rate $d = 1500$ units	Production rate $K = 2000$ units	Rate of current pollution generation $p = 2.5$
Set up cost $c_s = \$2000$	Production recovery rate $\delta = 0.9$	Threshold amount of pollution rate $p_0 = 1.5$	Purchasing cost of one unit of raw material $p_c = \$15$
Scale parameter of current pollution $\gamma = 10$	Scale parameter of initial pollution $\gamma' = 5$	Holding cost $c_h = \$5$	Deterioration cost per one unit of item $c_\theta = \$2.5$
Deterioration rate of produced items $\theta = 0.005$	Pollution absorbance rate by the nature $\varphi = 0.001$	Pollution cost per unit pollution $c_{pol} = \$5$	Unit selling price $s_p = \$50$

4 Formulation of Marxian EPQ Model

In this section, we are going to formulate the Marxian EPQ model according to the above case study. For the model formulation, we need some assumptions and notations. Therefore, the assumptions, notations and decision variables used throughout the paper are listed below.

Assumptions

(i) The production is pollution sensitive.
(ii) Replenishments are instantaneous.
(iii) 100% production capacity is not possible.
(iv) Lead time is zero.

(v) Shortages are not allowed.
(vi) Pollution cost incurs on the total estimated pollution observed in n successive production cycle.
(vii) The production system follows Marxian fundamental principle.
(viii) The produced items deteriorate with on-hand inventory during production period.
(ix) To establish profit function, the 'surplus value' and 'value of labour power' is considered.

Notations

K: Production rate (units per week)
δ: Rate of actual production per week
d: Demand rate (units per week).
θ: Deterioration rate per unit production per week.
Q: Maximum order quantity per week.
Y: Total selling price per cycle ($)
c_θ : Unit deterioration cost ($)
v : Value of labour cost to produce one unit of item ($)
Q_{T}: Total production per cycle (units)
c_{p} : Unit cost price to produce one unit of item ($)
c_{pol} : Unit pollution cost ($)
p_{c} : Unit purchasing cost of raw material ($)
c_{h} : Holding cost per unit item ($)

Decision Variables

T: Production cycle time (weeks)
T_1: Production run time (weeks)
Z: Total inventory cost per cycle ($)
Y_1: Total inventory profit per cycle ($)
s_{p} : Unit selling price ($)
r: Rate of exploitation (%)
n: Number of inventory cycles.
$\overline{w}$: Average total pollution for n cycles

Using these assumptions, notations and decision variables, we shall formulate the Marxian EPQ model in the next subsection.

4.1 Formulation of M-EPQ Model

Let the production start with zero inventory with an imperfect production rateδK, δ being the imperfect fraction of actual production rate K. Also, the inventory depletes with deterioration rate θ of on-hand inventory level at any time t and the demand departure rate is d. The production stops at time T_1 keeping the stock Q (shown in Fig. 4) and the inventory runs up to cycle time ($> T_1$). Thus, the governing differential

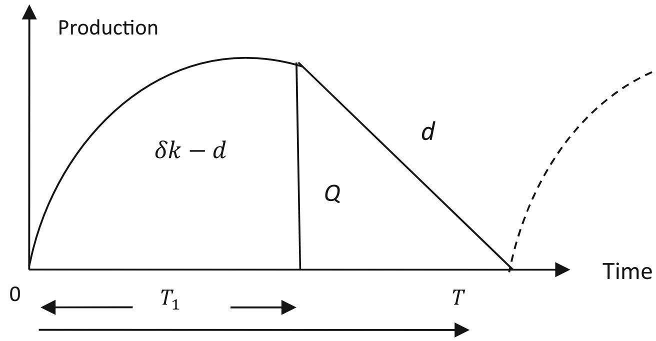


Fig. 4 Pollution sensitive EPQ model

equation of the industrial production model is given by

$$\begin{aligned}&\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} + \theta I_1(t) = \delta k - d, && \text{for } 0 \le t < T_1\\ &\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = -d, && \text{for } T_1 \le t < T\\ &\text{Subject to } I_1(0) = 0,\ I_1(T_1) = I_2(T_1),\ \text{and } I_2(T) = 0\end{aligned} \tag{6}$$

Solving (6) we get

$$\begin{aligned}I_1(t) &= \tfrac{\delta k - d}{\theta}\left[1 - e^{-\theta t}\right]\\ I_2(t) &= d(T - t)\end{aligned} \tag{7}$$

$$Q = d(T - T_1) = \frac{\delta k - d}{\theta}\left[1 - e^{-\theta T_1}\right] \tag{8}$$

Utilizing (7), the inventory holding cost is given by

$$\begin{aligned}\mathrm{HC} &= c_\mathrm{h}\left\{\int_0^{T_1} I_1(t)\mathrm{d}t + \int_{T_1}^{T} I_2(t)\mathrm{d}t\right\} = c_\mathrm{h}\left[\frac{\delta k - d}{\theta}\int_0^{T_1}\left(1 - e^{-\theta t}\right)\mathrm{d}t + \int_{T_1}^{T} d(T - t)\mathrm{d}t\right]\\ &= c_\mathrm{h}\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{\left(1 - e^{-\theta T_1}\right)}{\theta}\right\} + \frac{1}{2}d(T - T_1)^2\right]\end{aligned} \tag{9}$$

Deterioration cost is given by

$$\mathrm{DC} = c_\theta\left[\frac{\delta k - d}{\theta}\int_0^{T_1}\left(1 - e^{-\theta t}\right)\mathrm{d}t - dT_1\right] = c_\theta\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{\left(1 - e^{-\theta T_1}\right)}{\theta}\right\} - dT_1\right] \tag{10}$$

Using (5), Pollution cost per cycle is given by $\mathrm{PC} = c_{\mathrm{pol}}\overline{w}$

$$= c_{\mathrm{pol}}\left[\left\{T - \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi}\right\}\frac{\gamma p(p^n - 1)}{p - 1} + \frac{\left(1 - e^{-\varphi nT}\right)}{n\varphi}\frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}\right] \tag{11}$$

Since the amount of raw materials $= KT_1$,

$$\text{the purchasing cost of raw materials (PRC)} = p_{\mathrm{c}}KT_1 \tag{12}$$

$$\text{Set up cost (SC)} = A \tag{13}$$

$$\text{Total quantity produced is } Q_{\mathrm{T}} = Q + dT_1 = \left(\frac{\delta k - d}{\theta}\right)\left[1 - e^{-\theta T_1}\right] + dT_1 = dT \tag{14}$$

$$\text{Cost of labour power (LC)} = vQ_{\mathrm{T}} \tag{15}$$

Total cost including cost of labour-power to produce Q_{T} amount per cycle is given by

$$\begin{aligned} Z &= (\mathrm{PRC} + \mathrm{HC} + \mathrm{DC} + \mathrm{LC} + \mathrm{SC} + \mathrm{PC}) \\ &= \left[\begin{array}{l} p_{\mathrm{c}}KT_1 + c_{\mathrm{h}}\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{(1-e^{-\theta T_1})}{\theta}\right\} + \frac{1}{2}d(T - T_1)^2\right] + vQ_{\mathrm{T}} + A \\ +c_{\theta}\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{(1-e^{-\theta T_1})}{\theta}\right\} - dT_1\right] + c_{\mathrm{pol}}\overline{w} \end{array}\right] \end{aligned} \tag{16}$$

Thus, the unit production cost price $u_c\left(= c_p + v\right)$ to produce Q_T quantity is given by

$$u_{\mathrm{c}} = \frac{Z}{Q_{\mathrm{T}}} = \frac{1}{dT}\left[\begin{array}{l} p_{\mathrm{c}}KT_1 + c_{\mathrm{h}}\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{(1-e^{-\theta T_1})}{\theta}\right\} + \frac{1}{2}d(T - T_1)^2\right] + vQ_{\mathrm{T}} + A \\ +c_{\theta}\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{(1-e^{-\theta T_1})}{\theta}\right\} - dT_1\right] + c_{\mathrm{pol}}\overline{w} \end{array}\right] \tag{17}$$

Now, the total selling price after the sale of Q_{T} quantity is given by

$$Y = s_{\mathrm{p}}Q_{\mathrm{T}} = s_{\mathrm{p}}dT \tag{18}$$

and utilizing (18), the rate of exploitation is given by

$$r = \frac{s}{c_{\mathrm{p}} + v} = \frac{s_{\mathrm{p}} - \left(c_{\mathrm{p}} + v\right)}{u_{\mathrm{c}}} = \frac{s_{\mathrm{p}}dT - Z}{Z} \tag{19}$$

4.2 Scenario 1: When the Unit Selling Price of the Commodities are Given

Here, we consider the extended models of Harris [24] (the case of cost minimization) and Taft [35] (the case of profit maximization) to develop the new model. Initially, we may view the problem as a multi-objective constrained optimization problem (20) then rearranging these problems into a single problem we can get (21) which is called EPQ problem under Marxian principle.

$$\begin{aligned}
&\text{Cost function: Minimize } Z,\\
&\text{Profit function: Maximize } Y = s_{\mathrm{p}}dT - Z\\
&\text{Exploitation: Minimize } r = \tfrac{s_{\mathrm{p}}dT - Z}{Z}\\
&\text{Subject to } Q_{\mathrm{T}} = dT,\ T = T_1 + \left(\tfrac{\delta k - d}{d\theta}\right)\left[1 - e^{-\theta T_1}\right]
\end{aligned} \tag{20}$$

where the value of Z can be found from (16).

The above problem (20) is a multi-objective EPQ problem. The equivalent single objective problem of (20) may be written as

$$\begin{aligned}
&\text{Maximize } X = \tfrac{Y - Z}{r} = \tfrac{s_{\mathrm{p}}dT - Z}{r}\\
&\text{Subject to } Q_{\mathrm{T}} = dT,\ r = \tfrac{s_{\mathrm{p}}dT - Z}{Z}\\
&T = T_1 + \left(\tfrac{\delta k - d}{d\theta}\right)\left[1 - e^{-\theta T_1}\right] \text{ and Eq. (16)}
\end{aligned} \tag{21}$$

The problem (21) is called the problem of fundamental pollution sensitive Marxian EPQ (M-EPQ) model with deterioration.

Particular Cases:

(i) If we ignore the effect of environmental pollution then we put $c_{\mathrm{pol}} = 0$ in (16) then the revised problem (22) is considered as the problem of M-EPQ model with deterioration.

$$\begin{aligned}
&\text{Maximize } X = \tfrac{Y - Z}{r} = \tfrac{s_{\mathrm{p}}dT - Z}{r}\\
&\text{Subject to } Q_{\mathrm{T}} = dT,\ r = \tfrac{s_{\mathrm{p}}dT - Z}{Z}\\
&T = T_1 + \left(\tfrac{\delta k - d}{d\theta}\right)\left[1 - e^{-\theta T_1}\right]
\end{aligned} \tag{22}$$

Were,

$$Z = \begin{bmatrix} p_{\mathrm{c}}KT_1 + c_{\mathrm{h}}\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{(1 - e^{-\theta T_1})}{\theta}\right\} + \frac{1}{2}d(T - T_1)^2\right] + vQ_{\mathrm{T}} + A \\ + c_\theta\left[\frac{\delta k - d}{\theta}\left\{T_1 - \frac{(1 - e^{-\theta T_1})}{\theta}\right\} - dT_1\right] \end{bmatrix}$$

(ii) If we ignore the deterioration ($\theta \to 0$, $c_\theta = 0$) and pollution $\left(c_{\mathrm{pol}} = 0\right)$ both and assume full production ($\delta = 1$) then the problem (21) reduces to (23) and it is called the classical M-EPQ model.

$$\begin{aligned}&\text{Maximize } X = \frac{Y-Z}{r} = \frac{s_{\mathrm{p}}dT-Z}{r}\\&\text{Subject to } Q_{\mathrm{T}} = dT = KT_1,\ r = \frac{s_{\mathrm{p}}dT-Z}{Z}\\&Z = \left[\ p_{\mathrm{c}}KT_1 + \tfrac{1}{2}c_{\mathrm{h}}d(T-T_1)^2 + vQ_{\mathrm{T}} + A\ \right]\end{aligned} \tag{23}$$

4.3 When Unit Selling Price is Associated to Marginal Profit

Here we assume the unit selling price $s_{\mathrm{p}} = \frac{Z}{Q_{\mathrm{T}}} + \frac{\partial}{\partial T}\left(\frac{Z}{Q_{\mathrm{T}}}\right) = \frac{Z}{dT} + \frac{\partial}{\partial T}\left(\frac{Z}{dT}\right)$, where Z is the total cost incurred to produce Q_{T} items during the cycle time T.

Therefore, the pollution sensitive deteriorated M-EPQ problem under marginal profit is given by

$$\begin{aligned}&\text{Maximize } X = \frac{s_{\mathrm{p}}dT-Z}{r}\\&\text{Subject to } r = \frac{s_{\mathrm{p}}dT-Z}{Z},\ Q_{\mathrm{T}} = dT\\&T = T_1 + \frac{\delta k-d}{d\theta}\left[1-e^{-\theta T_1}\right]\\&s_{\mathrm{p}} = \frac{p_{\mathrm{c}}K+c_\theta d(T-T_1-1)}{\delta KT-d\theta T(T-T_1)} + c_{\mathrm{h}}\frac{(T^2-T_1^2)}{2T^2} - \frac{p_{\mathrm{c}}KT_1+(c_{\mathrm{s}}+v)}{dT^2} - \frac{c_{\mathrm{pol}}}{ndT^2}\left\{nT\frac{\gamma p(p^n-1)}{p-1} + \frac{\gamma'(p_0^n-1)}{\varphi(p_0-1)}\right\}\\&\quad+\left\{\left(1-e^{-n\varphi T}\right)\frac{\gamma p(p^n-1)}{p-1} + e^{-n\varphi T}\frac{\gamma'(p_0^n-1)}{p_0-1}\right\}\left(\frac{c_{\mathrm{pol}}+n\varphi T}{\varphi ndT^2}\right)\\&\quad+\frac{1}{dT^2}(c_{\mathrm{h}}+c_\theta)\frac{\delta k-d}{\theta}\left(T_1-\frac{1-e^{-\theta T_1}}{\theta}\right) + \frac{c_\theta T_1}{T^2}\end{aligned} \tag{24}$$

[For details calculation of s_{p}, see "Appendix" section].

5 Formulation of Fuzzy Mathematical Model

In crisp problem we see that all the cost components and unit selling prices are assumed to be deterministic; but in this changing real world, we are seeking such a model where all the cost and price components assume as flexible in nature. At this point of departure, we may express the proposed fuzzy model in the following:

$$\begin{aligned}&\widetilde{Maximize}\ X \cong \frac{\widetilde{s_{\mathrm{p}}}dT-\tilde{z}}{\tilde{r}}\\&\text{where } \tilde{z} = \widetilde{p_{\mathrm{c}}}KT_1 + \widetilde{c_{\mathrm{h}}}\left[\frac{\delta k-d}{\theta}\left\{T_1-\frac{(1-e^{-\theta T_1})}{\theta}\right\} + \frac{1}{2}d(T-T_1)^2\right] + \tilde{v}Q_{\mathrm{T}} + \tilde{A}\\&+\widetilde{c_\theta}\left[\frac{\delta k-d}{\theta}\left\{T_1-\frac{(1-e^{-\theta T_1})}{\theta}\right\} - dT_1\right] + \widetilde{c_{\mathrm{pol}}}\overline{w}\\&\text{and } \tilde{r} = \frac{\widetilde{s_{\mathrm{p}}}dT-\tilde{z}}{\tilde{z}}\end{aligned} \tag{25}$$

where the fuzzy parameters are obtained from (26) in the form of (1).

$$\begin{aligned}&\tilde{X} = \langle X_1, X_2, X_3\rangle,\ \tilde{r} = \langle r_1, r_2, r_3\rangle\\&\tilde{Z} = \langle Z_1, Z_2, Z_3\rangle,\ \text{where } Z_1 = Z|_{c_{i1}},\ Z_2 = Z|_{c_{i2}},\ Z_3 = Z|_{c_{i3}}\\&\widetilde{S_{\mathrm{p}}} = \langle s_1, s_2, s_3\rangle, \widetilde{c_i} = c_{i1}, c_{i2}, c_{i3}\\&\quad\text{where } \widetilde{c_i} = \left(\widetilde{p_{\mathrm{c}}}, \widetilde{c_{\mathrm{h}}}, \widetilde{c_\theta}, \tilde{A}, \widetilde{c_{\mathrm{pol}}}, \widetilde{v}\right),\quad i = 1, 2, \ldots, 6\end{aligned} \tag{26}$$

And the components of $\tilde{X} and \tilde{r}$ are defined as

$$\begin{array}{l} X_1 = \frac{s_1 dT - Z_3}{r_3},\ X_2 = \frac{s_2 dT - Z_2}{r_2},\ X_3 = \frac{s_3 dT - Z_1}{r_1} \\ r_1 = \frac{s_1 dT - Z_3}{Z_3},\ r_2 = \frac{s_2 dT - Z_2}{Z_2},\ r_3 = \frac{s_3 dT - Z_1}{Z_1} \end{array} \tag{27}$$

Now, to get the equivalent crisp problem of the fuzzy problem (25) we may utilize the defuzzification method studied at (2) and get the new problem developed in (28).

$$\begin{array}{l} \text{Maximize } I\left(\widetilde{X}\right) = \frac{X_1 + 2X_2 + X_3}{4}, \\ \text{Subject to } \ I\left(\widetilde{r}\right) = \frac{r_1 + 2r_2 + r_3}{4},\ \text{and Eqs.(16), (26), (27)} \end{array} \tag{28}$$

5.1 Solution Algorithm

Here we shall discuss a solution algorithm where the emphasis is given to maximize the total profit.

Step 0: Start
Step 1: Solve single objective M-EPQ model developed in (21) and get the optimal decision variables as $\left\{Y_1^*, Z_0^* : r_0^*, \overline{w}_0^*, T_{10}^*, T_0^*, S_{p0}^*\right\}$.
Step 2: Construct appropriate fuzzy membership functions of the fuzzy parameters using the optimal values obtained at **Step 1** as the initial approximations in (1).
Step 3: Fuzzify the problem (21), get its defuzzification using (2) and reset an equivalent crisp problem (28).
Step 4: Solve (28) by setting the production frequency $n = 1$ and store these optimal results as $X_n^{opt} = \left\{Y_{1n}^*, Z_{0n}^* : r_{0n}^*, \overline{w}_{0n}^*, T_{1n}^*, T_n^*, S_{pn}^*\right\}$.
Step 5: Set $n = n + 1$ and go to Step 4.
Step 6: Check whether $Z_{0n+1}^* - Z_{0n}^* > \in_1$, $r_{0n+1}^* - r_{0n}^* > \in_2$ and $\overline{w}_{0n+1}^* - \overline{w}_{0n}^* > \in_3$ hold where $\in_1$, $\in_2$ and $\in_3$ are the user defined positive number.
Step 7: Get optimal solution for $n = m$ as $X_m^{opt} = \left\{Y_{1m}^*, Z_{0m}^* : r_{0m}^*, \overline{w}_{0m}^*, T_{1m}^*, T_m^*, S_{pm}^*\right\}$.
Step 8: Go to **Step 5**.
Step 9: End.

6 Numerical Illustrations

To perform the numerical study, we use the data set from case study given in Table 1. Then using (21), (24) and (28) and utilizing the proposed solution algorithm developed at subsection 5.1 we compute numerical result and they are put in Tables 2, 3, and 4 respectively.

Table 2 shows the optimal total profit for the Marxian pollution model for 10 successive production cycles. It is seen that the maximum profit is \$ 750,114 with respect to the inventory expenditure \$ 20, 79,536 for the first production cycle getting

Table 2 Optimal solutions for pollution sensitive Marxian problem

n	$T_1{}^*$ Weeks	T^* Weeks	$\overline{w}^*$	r^* (%)	$Y^* = s_\text{p}\text{d}T^*$ (\$)	Z^* (\$)	$Y_1{}^* = Y^* - Z^*$ (\$)
1	31.84	37.73	202.70	36.1	2,829,650	2,079,536	750,114
2	31.80	37.68	574.86	36.0	2,825,993	2,077,197	748,796
3	31.71	37.57	1341.19	36.0	2,817,938	2,071,789	746,149
4	31.49	37.31	3089.68	36.0	2,798,357	2,058,148	740,209
5	30.91	36.64	7299.75	35.9	2,748,254	2,022,417	725,837
6	29.46	34.94	17,133.47	35.8	2,620,667	1,930,202	690,465
7	26.07	30.95	36,203.74	35.6	2,321,595	1,712,331	609,264
8	19.66	23.41	57,642.94	35.3	1,755,655	1,297,600	458,055
9	11.54	13.78	56,971.63	34.9	1,033,337	765,732	267,605
10	5.27	6.31	34,432.04	34.3	473,121	352,194	120,927

Table 3 Optimal solution for Marxian problem with marginal profit

n	$T_1{}^*$ Weeks	T^* Weeks	s_p^*(\$)	$\overline{w}^*$	r^*(%)	$Y^* = s_\text{p}^*\text{d}T^*$(\$)	Z^*(\$)	$Y_1{}^* = Y^* - Z^*$ (\$)
1	0.834	1.00	16.29	5.01	0.0065	24,435.00	24,433.40	1.60
2	0.834	1.00	16.32	13.52	0.015	24,480.00	24,476.33	3.67
3	0.834	1.00	16.35	24.08	0.00	24,530.34	24,530.34	0.00
4	0.834	1.00	16.42	41.81	0.0273	24,630.00	24,623.27	6.73
5	0.834	1.00	16.52	69.79	0.0141	24,780.00	24,776.51	3.49
6	0.834	1.00	16.70	115.73	0.0163	25,050.00	25,045.93	4.07
7	0.834	1.00	16.89	162.23	0.00	25,339.06	25,339.06	0.00
8	0.834	1.00	17.00	346.69	0.00	25,500.00	25,500.00	0.00
9	0.129	0.16	17.00	64.88	0.00	3953.22	3953.22	0.00
10	0.05	0.09	17.20	57.24	0.0466	2316.84	2315.76	1.08

36.1% exploitation, 31.84 weeks of production run time and 37.73 weeks inventory cycle time that contribute pollution index 202.70. The other cases are of decreasing rate of profits with varying optimal variables.

Table 3 shows the optimal marginal profit of the Marxian model for 10 successive production cycles. As per principle, the model has negligible profit or no profit no exploitation. In the whole table, we found the profit range is \$ (0–6.73) with equal production run time 0.834 week and most of the cases production cycle time is 1 week. The unit selling price range is \$ (16.29–17.20), the pollution range is 5.01–346.69 with % of exploitation range 0–0.05%. However, we compute the optimal solutions that include original and marginal profit for without pollution of the model and the solutions

Table 4 Optimal solutions for crisp with no pollution and fuzzy with pollution models

Problems	T_1* Weeks	T* Weeks	s_p^*($)	$\overline{w}$*	r* (%)	$Y^* = s_p^* dT^*$($)	Z* ($)	$Y_1^* = Y^* - Z^*$ ($)
Crisp normal Profit	31.86	37.75	50	–	36.10	2,831,528	2,080,681	750,847
Crisp marginal Profit	0.834	1.00	16.27	–	0.00	24,408.30	24,408.3	0.0
Fuzzy system normal profit	31.59	37.43	50	200.99	35.30	2,737,168	2,023,163	714,005

Initial fuzzy system parameters $(\rho_c, \sigma_c, \rho_s, \sigma_s) = (0.15, 0.09, 0.2, 0.1)$; $n^* = 1$ in every case

of fuzzy mathematical model with pollution respectively for 10 successive years each and they are put in Table 4.

Table 4 shows that optimal results are same for 10 successive years of study with the normal, marginal profit and fuzzy normal profit maximization problems. It is seen that no profit no exploitation is viewed for the marginal profit-seeking model. But fuzzy system is giving considerable amount of profit with minimum exploitation 35.30% than normal profit-seeking crisp model having the average pollution index 200.99.

6.1 Sensitivity Analysis

Here we perform the sensitivity analysis by changing the fuzzy deviation parameters on and from − 50%, − 25%, + 25% and + 50% for all unit cost components and the unit selling price of the fuzzy mathematical model and the results are put in Table 5.

Here we see that the rate of exploitation has the range (33.9, 50.2) % with pollution impact range 155.36–205.82 for the average production run time is about 31 weeks, production cycle time is about 37 weeks respectively. Moreover, the total profit varies on and from − 4.65% + 8.64% throughout for the changes of all fuzzy deviation parameters.

7 Graphical Illustrations

Here we shall draw several graphs for justification of the model. Figure 5 shows that, for normal profit-seeking Marxian EPQ model, the amount of average cumulative pollution is increasing up to 8 production cycles keeping value near 57,100 and after that, it began to fall down. Also, Fig. 6 indicates the total average cumulative pollution measures of the marginal profit-seeking production model, increases slowly up to 7th turnover and at 8th turn over it gets a peak near value 340. But at 9th production cycle

Table 5 Sensitivity analysis over left and right fuzzy deviation parameters

Fuzzy system parameters	% Change	$T_1{}^*$ (Weeks)	T^* (Weeks)	$\overline{w}^*$	r^* (%)	$Y^* = s_p dT^*$(\$)	Z^* (\$)	$Y_1{}^* = Y^* - Z^*$ (\$)	$R_e = \frac{Y_1^* - Y_{1*}}{Y_{1*}} \times 100\%$
ρ_c	+ 50	31.77	37.64	202.20	37.5	2,752,492	2,002,048	750,444	5.1
	+ 25	31.77	37.64	202.20	36.2	2,752,492	2,021,473	731,020	2.38
	– 25	30.83	36.54	195.89	35.8	2,671,974	1,967,030	704,943	– 1.27
	– 50	30.10	35.69	191.03	36.3	2,609,720	1,914,093	695,627	– 2.57
σ_c	+ 50	29.51	35.00	187.11	38.8	2,559,437	1,843,326	716,110	0.29
	+ 25	30.62	36.29	194.48	36.9	2,653,884	1,938,994	714,889	0.12
	– 25	31.76	37.63	202.13	35.7	2,751,610	2,028,226	723,384	1.31
	– 50	31.93	37.83	203.29	36.0	2,766,367	2,033,519	732,848	2.64
ρ_s	+ 50	24.71	29.36	155.36	50.2	2,092,185	1,393,106	699,079	– 2.09
	+ 25	28.22	33.49	178.52	42.0	2,417,232	1,702,113	715,118	0.16
	– 25	31.93	37.83	203.31	36.2	2,802,123	2,057,346	744,777	4.31
	– 50	32.31	38.27	205.82	37.0	2,870,431	2,094,752	775,679	8.64
σ_s	+ 50	31.71	37.58	201.83	36.7	2,783,035	2,035,468	747,567	4.7
	+ 25	31.65	37.51	201.42	36.0	2,760,161	2,029,419	730,742	2.34
	– 25	31.52	37.35	200.55	34.6	2,714,048	2,016,688	697,360	– 2.33
	– 50	31.45	37.28	200.10	33.9	2,690,796	2,009,985	680,811	– 4.65

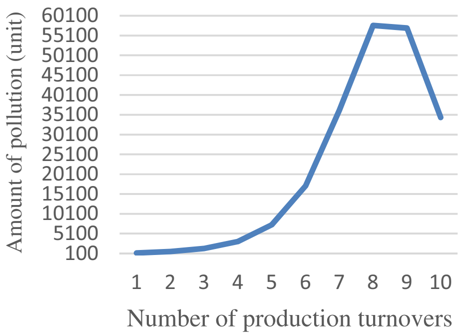


Fig. 5 Variation over pollution measures in original scenario

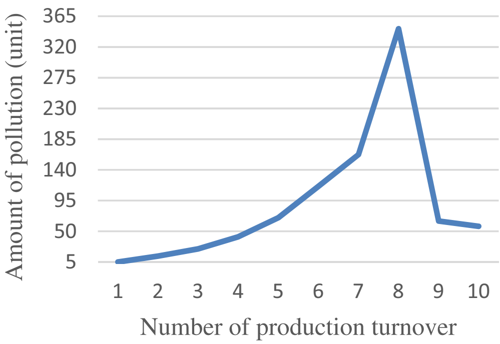


Fig. 6 Variation over pollution measures in marginal profit

that level has a sudden jump to value near 70 and after that, it began to decrease. We further observe that the pollution data obtained from the marginal profit model is about 100 times less than that of the data available from the normal profit-seeking model.

Figure 7 expresses the distribution of marginal cost and marginal profit in percentage for 10 successive years of study. The % of total profit assumes high at 4th production cycle by reaching nearly 37% by covering a zigzag path within 10 production cycle.

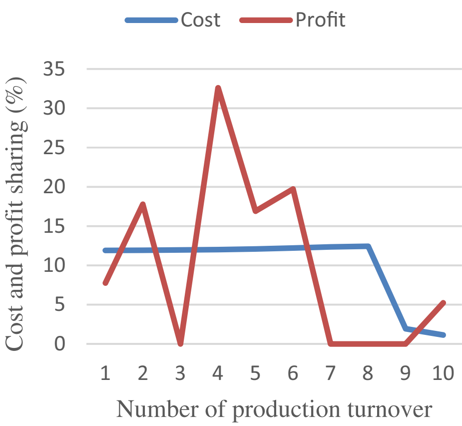


Fig. 7 Marginal cost and profit distribution in percentage

However, the % cost curve gets a horizontal line up to 8th cycle assuming value nearly 13% and no profit has been found for 7–9th cycle after that the cost curve began to decrease but profit curve began to increase.

Figure 8 reveals that for normal Marxian pollution model a large gap ($ 14 lakh) has been found between total cost and total profit up to 7th production cycle, after that these curves are getting much closure to 10th production cycle or more.

Figure 9 discusses the profit distribution for with and without pollution impacts over 10 successive years of studies in Marxian normal profit-seeking model. The maximum profit is nearly $ 75,000 for at least first 3 consecutive production cycles of both the models but after that, the amount of profit began to decrease rapidly for the pollution model but for without pollution model the total profit remains same.

Figure 10 gives a comparative study over the cumulative rate of exploitation (%) of normal and marginal pollution-sensitive profit-seeking model over 10 successive production cycles. The normal Marxian model always gives almost constant (near 10%) average exploitation but that for marginal model; it follows a zigzag path ranging

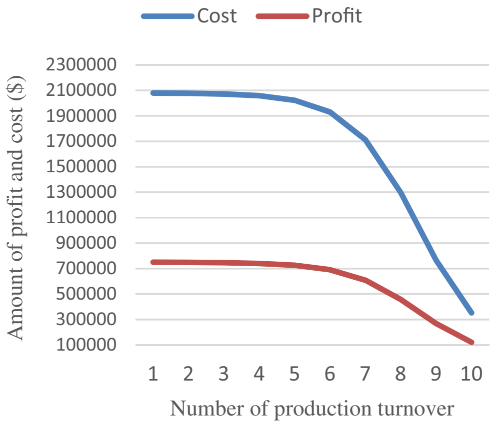


Fig. 8 Cost and profit curve of original scenario

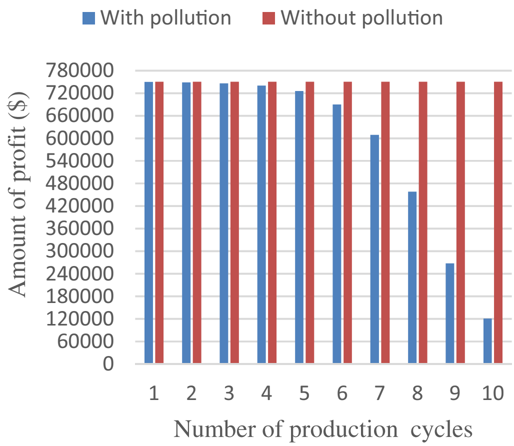


Fig. 9 Variation of profit due to pollution and no pollution in original scenario

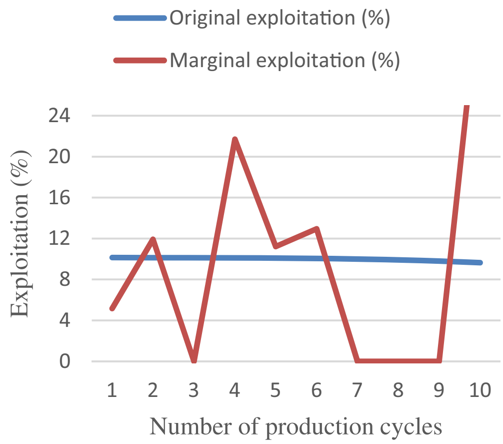


Fig. 10 Exploitation variation in original and marginal scenario

the values 0% to 24% on average up to 10th production cycle. At production cycles 3,7, 8 and 9th, no exploitation is found.

Figure 11 discusses the profit variation of normal profit-seeking exploitation-based Marxian production inventory model showing up to 35.9% exploitation the profit value is increasing but it becomes stable for more exploitation keeping the profit value near \$75,000. Figure 12 indicates the variation of marginal profit curve of the total %

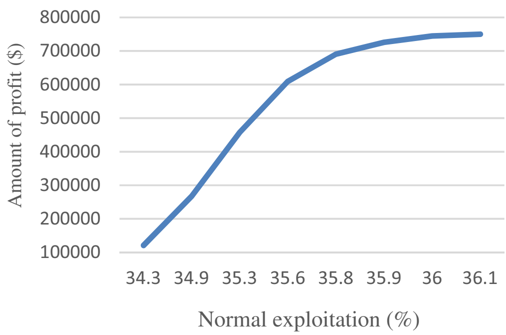


Fig. 11 Profit variation under normal exploitation

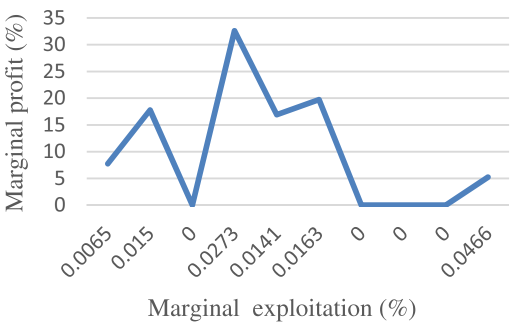


Fig. 12 Profit variation under marginal exploitation

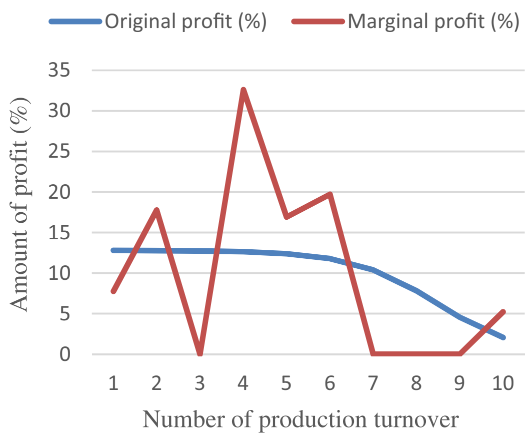


Fig. 13 Variation of profits in original and marginal scenario

variations over 10 successive cycles of exploitation. The exploitations are negligible and hence the profits are negligible followed by a zigzag path.

From Fig. 13 it is seen that, though the marginal profit (no profit) always gets minimum, on the basis of % gain over several successive production cycles, it becomes high (up to 32%) within the cycles 4th to 7th and beyond that, the normal Marxian model gives the higher % of profit not exceeding 13% always. Figure 14 discusses the comparison on % of exploitations made at subproblems by considering pollution and no pollution measures in the normal Marxian production inventory model exclusively. It is observed that up to 7th production cycles the % rates of exploitations are more than 10% and for more production cycles of pollution model, the exploitation began to decrease by reaching at 9.6% on sharp. However, without pollution model gives almost same amount of exploitation for all production cycles with 10% impact of exploitation.

Figure 15 studied comparison of profit variation (range \$6,80,000–\$ 7,75,000) on Marxian normal pollution model with the fuzzy parametric changes on and from − 50%, − 25%, + 25%and + 50% respectively of several unit cost parameters and the

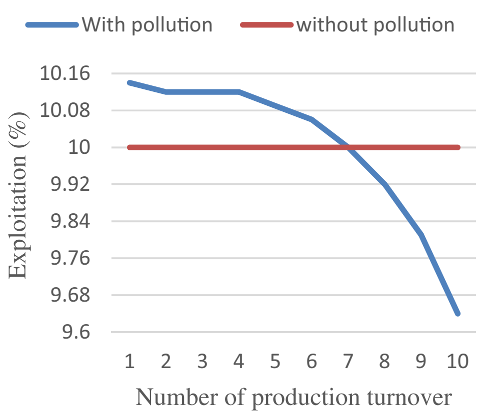


Fig. 14 Exploitation (%) due to pollution and no pollution in original scenario

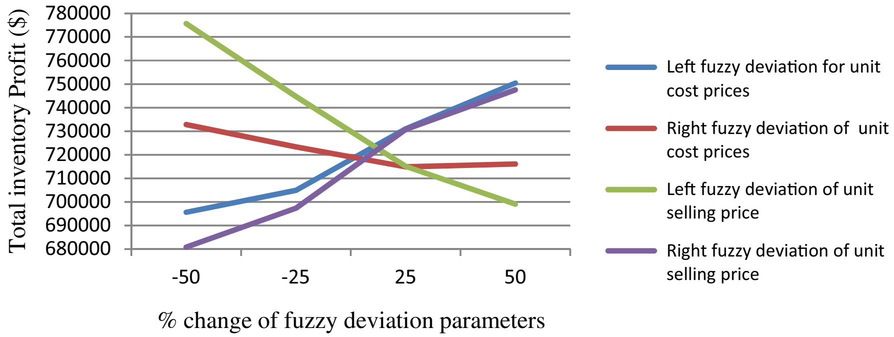


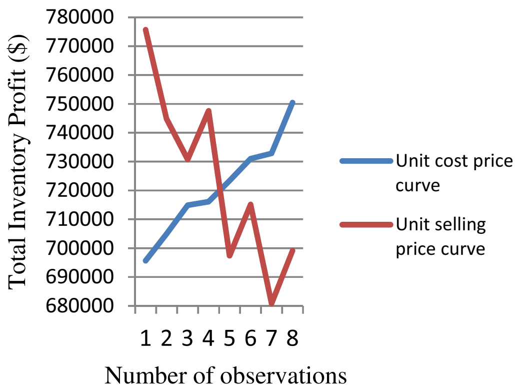
Fig. 15 Profit variation under fuzzy parametric changes (%)

parameter of unit selling price also. For unit selling price, the right fuzzy deviation parameter is increasing and that for left fuzzy it is decreasing and they meet around 10% change. Moreover, the reverse conditions are found for the fuzzy deviations of unit cost parameters also.

Figure 16 gives the total profit variation whenever unit cost prices increase slowly the unit selling price curve decreases with a Shaw's tooth-like path under 8 observations. Figure 17 discusses the % change of exploitation over the variation of unit cost price and unit selling price parameter over 8 observations. For the variation of unit selling price, the rate of exploitation gets a range within 34% to 50%, whereas, for the unit cost price it was only 35.5–39% around.

Figure 18 reveals the joint effect of exploitation hike (%) made by the variations of production run time and production cycle time in weeks. We see within production runtime 26 weeks and cycle time 32 weeks the % rate of exploitation reaches 48% but beyond that the curve assumes downstairs like structures. Moreover, within 30–32 weeks of production run time and more than 38 weeks of cycle time, the exploitation increases up to 35% approximately.

Fig. 16 Profit variation under unit cost and unit selling price



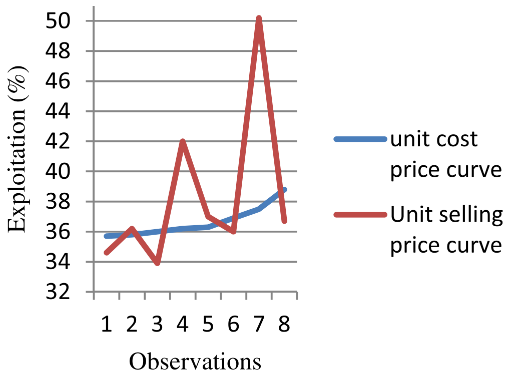


Fig. 17 Exploitation variation over unit cost price and unit selling price

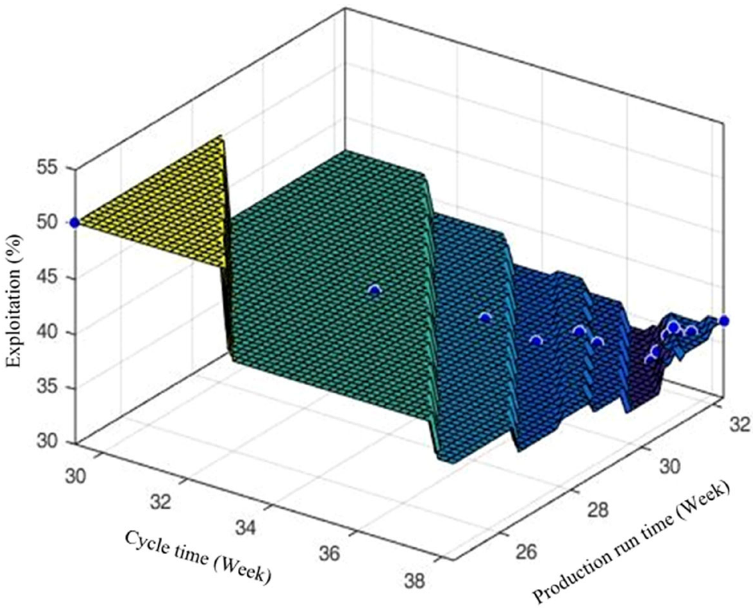


Fig. 18 Variation of exploitation (%) due to cycle time and production run time

8 Research Findings and Managerial Insights

In this section we shall discuss our research findings over the case study and the managerial insights as follows:

8.1 Research Findings

(i) Optimum total cost and total profit of the model are \$2,079,536.00 and \$750,114.00 with respect to the production run time 31.84 weeks, the cycle time 37.73 weeks and pollution index 202.70 for basic M-EPQ model.

(ii) If the effect of pollution is ignored then the expected profit goes to 0.135% more with respect to that of the pollution model.
(iii) In case of marginal profit and no exploitation, the pollution index will come down to 5.01 only for production run time 0.834 week where the inventory cost and profits become $ 24,433.40 and $ 1.60 respectively.
(iv) The maximum exploitation gets 36.1% corresponding to the maximum profit $ 750,114.

8.2 Managerial Insights

(a) The normal pollution model gives maximum exploitation.
(b) The marginal (no) profit model gives less (no) exploitation.
(c) Percentage of profits and costs vary with the number of production turnovers.
(d) For marginal M-EPQ model, the total cost and profit functions follow saw teeth curve.
(e) Profits and cost curves are getting closure with more production turnovers.

9 Conclusions

In this article, we have developed Marxian EPQ model incorporating industrial pollution, deterioration, partial production capacity, manpower exploitation with the help of a profit maximization single objective function under fuzzy environment. The traditional concept of considering pollution parameter is replaced by a new pollution function via the modelling of a separate pollution generation function.

However, two different scenarios under Marxian fundamental principles of production having normal profit and marginal profit are considered and then we split them into with and without effect of pollutions. In each stage, we have calculated the total profit earned, total cost incurred, and the amount of exploitation generated in the whole entire production process exclusively. Fuzzy system is utilised because of the flexibilities of the different cost components involved in the production process and variation of selling prices of the commodities explicitly. Though the present production system (on SEZ and others) is solely depending upon automation and technology where the role of labour is quite passive but still the working-class people are exploited differently in this production manufacturing world. The great novelties of this study are:

(i) Marxian production inventory (M-EPQ) model has been developed first time after the invention of Taft's EPQ model studied in 1918.
(ii) Discovery of pollution generation model is another pioneering work.
(iii) Manpower exploitation in production farm is analysed first time.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest regarding publication of this article.

Appendix

For the case of Marginal profit, we have $\frac{\partial}{\partial T}\left[c_p + v\right] = \frac{\partial}{\partial T}\left[\frac{Z}{Q_T}\right] = \frac{\frac{\partial}{\partial T}(Z)Q_T - Z\frac{\partial}{\partial T}(Q_T)}{Q_T^2}$.

We have $= T_1 + \frac{\delta K - d}{d\theta}\left(1 - e^{-\theta T_1}\right)$. So, differentiating both sides with respect to T we get.

$\frac{\partial T}{\partial T_1} = \left[1 + \frac{\delta K - d}{d} e^{-\theta T_1}\right]$ and therefore, $\frac{\partial T_1}{\partial T} = \frac{1}{\left[1 + \frac{\delta K - d}{d} e^{-\theta T_1}\right]} = \frac{e^{\theta T_1}}{\left[\frac{\delta K - d}{d} + e^{\theta T_1}\right]}$.

Again, we have, $Q_T = dT$ So, $\frac{\partial Q_T}{\partial T} = d$. Thus from $\frac{\delta K - d}{d\theta}\left(1 - e^{-\theta T_1}\right) = T - T_1$ we get $\frac{\partial T_1}{\partial T} = \frac{d}{\delta K - d\theta(T - T_1)}$.

Now,

$$
\begin{aligned}
\frac{\partial}{\partial T}(Z) &= p_c K \frac{\partial T_1}{\partial T} + c_h\left[\frac{\delta K - d}{\theta}\left\{\frac{\partial T_1}{\partial T} + \frac{1}{\theta}(-\theta)e^{-\theta T_1}\frac{\partial T_1}{\partial T}\right\} + \frac{d}{2} \times 2(T - T_1)\left(1 - \frac{\partial T_1}{\partial T}\right)\right] \\
&\quad + vd + c_\theta\left[\frac{\delta K - d}{\theta}\left\{\frac{\partial T_1}{\partial T} + \frac{1}{\theta}(-\theta)e^{-\theta T_1}\frac{\partial T_1}{\partial T}\right\} - d\frac{\partial T_1}{\partial T}\right] \\
&\quad + \frac{c_{\text{pol}}}{n}\left[\left\{n + \frac{1}{\varphi}(-n\varphi)e^{-n\varphi T}\right\}\frac{\gamma p(p^n - 1)}{p - 1} + \frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}\left(-\frac{1}{\varphi}\right)(-n\varphi)e^{-n\varphi T}\right] \\
&= \frac{\partial T_1}{\partial T}\left[p_c K + c_h\left\{\frac{\delta K - d}{\theta}\left(1 - e^{-\theta T_1}\right) - d(T - T_1)\right\} + c_\theta\left\{\frac{\delta K - d}{\theta}\left(1 - e^{-\theta T_1}\right) - d\right\}\right] \\
&\quad + vd + c_h d(T - T_1) + c_{\text{pol}}\left\{\left(1 - e^{-n\varphi T}\right)\frac{\gamma p(p^n - 1)}{p - 1} + e^{-n\varphi T}\frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}\right\} \\
&= \frac{e^{\theta T_1}}{\left[\frac{\delta K - d}{d} + e^{\theta T_1}\right]}\left[p_c K + c_h\{d(T - T_1) - d(T - T_1)\} + c_\theta\{d(T - T_1) - d\}\right] \\
&\quad + vd + c_h d(T - T_1) + c_{\text{pol}}\left\{\left(1 - e^{-n\varphi T}\right)\frac{\gamma p(p^n - 1)}{p - 1} + e^{-n\varphi T}\frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}\right\} \\
&= \frac{d}{\delta K - d\theta(T - T_1)}\left[p_c K + c_\theta\{d(T - T_1) - d\}\right] + vd + c_h d(T - T_1) \\
&\quad + c_{\text{pol}}\left\{\left(1 - e^{-n\varphi T}\right)\frac{\gamma p(p^n - 1)}{p - 1} + e^{-n\varphi T}\frac{\gamma'\left(p_0^n - 1\right)}{p_0 - 1}\right\}
\end{aligned}
$$

So,

$$
\begin{aligned}
\frac{\partial}{\partial T}\left[\frac{Z}{Q_{\text{T}}}\right] &= \frac{1}{d^2 T^2}\left[\frac{d^2 T}{\delta K - d\theta(T - T_1)}\left[p_c K + c_\theta\{d(T - T_1) - d\}\right] + vd^2 T\right. \\
&\quad \left. + c_h d^2 T(T - T_1) + c_{\text{pol}} dT\left\{\left(1 - e^{-n\varphi T}\right)\frac{\gamma p(p^n - 1)}{p - 1} + e^{-n\varphi T}\frac{\gamma'^{(p_0^n - 1)}}{p_0 - 1}\right\}\right.
\end{aligned}
$$

$$
-d\Bigg[p_cKT_1+c_s+c_h\left\{\frac{\delta K-d}{\theta}\left(T_1-\frac{\left(1-e^{-\theta T_1}\right)}{\theta}\right)+\frac{d}{2}(T-T_1)^2\right\}
$$

$$
+c_\theta\left\{\frac{\delta K-d}{\theta}\left(T_1-\frac{\left(1-e^{-\theta T_1}\right)}{\theta}\right)-dT_1\right\}
$$

$$
+\upsilon dT+\frac{c_{pol}}{n}\left\{\left(nT-\frac{1-e^{-n\varphi T}}{\varphi}\right)\frac{\gamma p(p^n-1)}{p-1}+\frac{1-e^{-n\varphi T}}{\varphi}\frac{\gamma'(p_0^n-1)}{p_0-1}\right\}\Bigg]\Bigg]
$$

$$
=\frac{p_cK+c_\theta d(T-T_1-1)}{\delta KT-d\theta T(T-T_1)}+\frac{\upsilon}{T}+c_h\frac{(T-T_1)}{T}
$$

$$
+\frac{c_{pol}}{dT}\left\{\left(1-e^{-n\varphi T}\right)\frac{\gamma p(p^n-1)}{p-1}+e^{-n\varphi T}\frac{\gamma'\left(p_0^n-1\right)}{p_0-1}\right\}-\frac{p_cKT_1}{dT^2}
$$

$$
-\frac{c_h}{dT^2}\left\{\frac{\delta k-d}{\theta}\left(T_1-\frac{1-e^{-\theta T_1}}{\theta}\right)+\frac{1}{2}d(T-T_1)^2\right\}
$$

$$
-\frac{c_\theta}{dT^2}\left\{\frac{\delta k-d}{\theta}\left(T_1-\frac{1-e^{-\theta T_1}}{\theta}\right)-dT_1\right\}
$$

$$
-\frac{c_{pol}}{ndT^2}\left\{\left(nT-\frac{1-e^{-n\varphi T}}{\varphi}\right)\frac{\gamma p(p^n-1)}{p-1}+\frac{1-e^{-n\varphi T}}{\varphi}\frac{\gamma'\left(p_0^n-1\right)}{p_0-1}\right\}-\frac{(c_s+\upsilon dT)}{dT^2}
$$

$$
=\frac{p_cK+c_\theta d(T-T_1-1)}{\delta KT-d\theta T(T-T_1)}+\frac{\upsilon}{T}+c_h\frac{(T-T_1)}{T}-\frac{p_cKT_1+(c_s+\upsilon dT)}{dT^2}
$$

$$
-\frac{c_{pol}}{ndT^2}\left\{nT\frac{\gamma p(p^n-1)}{p-1}+\frac{\gamma'\left(p_0^n-1\right)}{\varphi(p_0-1)}\right\}
$$

$$
+\left\{\left(1-e^{-n\varphi T}\right)\frac{\gamma p(p^n-1)}{p-1}+e^{-n\varphi T}\frac{\gamma'\left(p_0^n-1\right)}{p_0-1}\right\}\left\{\frac{c_{pol}}{dT}+\frac{c_{pol}}{\varphi ndT^2}\right\}
$$

$$
-\frac{1}{dT^2}(c_h+c_\theta)\frac{\delta k-d}{\theta}\left(T_1-\frac{1-e^{-\theta T_1}}{\theta}\right)-\frac{c_h}{2T^2}(T-T_1)^2+\frac{c_\theta T_1}{T^2}
$$

$$
=\frac{p_cK+c_\theta d(T-T_1-1)}{\delta KT-d\theta T(T-T_1)}+\frac{\upsilon}{T}+c_h\frac{\left(T^2-T_1^2\right)}{2T^2}-\frac{p_cKT_1+c_s+\upsilon dT}{dT^2}
$$

$$
-\frac{c_{pol}}{ndT^2}\left\{nT\frac{\gamma p(p^n-1)}{p-1}+\frac{\gamma'\left(p_0^n-1\right)}{\varphi(p_0-1)}\right\}
$$

$$
+c_{pol}\left\{\left(1-e^{-n\varphi T}\right)\frac{\gamma p(p^n-1)}{p-1}+e^{-n\varphi T}\frac{\gamma'\left(p_0^n-1\right)}{p_0-1}\right\}\left(\frac{1+n\varphi T}{\varphi ndT^2}\right)
$$

$$
-\frac{1}{dT^2}(c_h+c_\theta)\frac{\delta k-d}{\theta}\left(T_1-\frac{1-e^{-\theta T_1}}{\theta}\right)+\frac{c_\theta T_1}{T^2}
$$

Subject to,

$$
s_p=\frac{p_cK+c_\theta d(T-T_1-1)}{\delta KT-d\theta T(T-T_1)}+\frac{\upsilon}{T}+c_h\frac{\left(T^2-T_1^2\right)}{2T^2}-\frac{p_cKT_1+c_s+\upsilon dT}{dT^2}
$$

$$
\begin{aligned}
&- \frac{c_{\mathrm{pol}}}{ndT^2}\left\{nT\frac{\gamma p(p^n-1)}{p-1}+\frac{\gamma'\left(p_0^n-1\right)}{\varphi(p_0-1)}\right\}\\
&+c_{\mathrm{pol}}\left\{\left(1-e^{-n\varphi T}\right)\frac{\gamma p(p^n-1)}{p-1}+e^{-n\varphi T}\frac{\gamma'\left(p_0^n-1\right)}{p_0-1}\right\}\left(\frac{1+n\varphi T}{\varphi ndT^2}\right)\\
&-\tfrac{1}{dT^2}(c_{\mathrm{h}}+c_\theta)\tfrac{\delta k-d}{\theta}\left(T_1-\tfrac{1-e^{-\theta T_1}}{\theta}\right)+\tfrac{c_\theta T_1}{T^2}\\
&+\frac{1}{dT}\left[p_{\mathrm{c}}KT_1+c_{\mathrm{s}}+c_{\mathrm{h}}\left\{\frac{\delta K-d}{\theta}\left(T_1-\frac{\left(1-e^{-\theta T_1}\right)}{\theta}\right)+\frac{d}{2}(T-T_1)^2\right\}\right.\\
&+c_\theta\left\{\frac{\delta K-d}{\theta}\left(T_1-\frac{\left(1-e^{-\theta T_1}\right)}{\theta}\right)-dT_1\right\}\\
&\left.+vdT+\frac{c_{\mathrm{pol}}}{n}\left\{\left(nT-\frac{1-e^{-n\varphi T}}{\varphi}\right)\frac{\gamma p(p^n-1)}{p-1}+\frac{1-e^{-n\varphi T}}{\varphi}\frac{\gamma'\left(p_0^n-1\right)}{p_0-1}\right\}\right]
\end{aligned}
$$

$$
\begin{aligned}
s_{\mathrm{p}}=&\frac{p_{\mathrm{c}}K+c_\theta d(T-T_1-1)}{\delta KT-d\theta T(T-T_1)}+\frac{v(1+T)}{T}\\
&+c_{\mathrm{h}}\frac{\left(T^2-T_1^2\right)+T(T-T_1)^2}{2T^2}-\frac{p_{\mathrm{c}}KT_1-p_{\mathrm{c}}KTT_1+c_{\mathrm{s}}-c_{\mathrm{s}}T+vdT}{dT^2}\\
&+\frac{c_{\mathrm{pol}}(T-1)}{ndT^2}\left\{nT\frac{\gamma p(p^n-1)}{p-1}+\frac{\gamma'(p_0^n-1)}{\varphi(p_0-1)}\right\}\\
&+c_{\mathrm{pol}}\left\{\left(1-e^{-n\varphi T}\right)\frac{\gamma p(p^n-1)}{p-1}+e^{-n\varphi T}\frac{\gamma'(p_0^n-1)}{p_0-1}\right\}\left(\frac{1+n\varphi T-T}{\varphi ndT^2}\right)\\
&-\tfrac{1}{dT^2}(c_{\mathrm{h}}-c_{\mathrm{h}}T+c_\theta-c_\theta T)\tfrac{\delta k-d}{\theta}\left(T_1-\tfrac{1-e^{-\theta T_1}}{\theta}\right)+\tfrac{c_\theta T_1-c_\theta TT_1}{T^2}
\end{aligned}
$$

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