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CORRECTION

Correction to: Indefinite Abstract Splines with a Quadratic Constraint

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1 Introduction

In [1] the statement of Lemma 4.1 is incorrect. In fact, if the dimension of $\mathcal{H}$ is infinite it is always possible to find a sequence in $\mathcal{C}_V$ which converges weakly to a vector not contained in $\mathcal{C}_V$, see Proposition 2.1 below. We apologize for this mistake. If the dimension of $\mathcal{H}$ is finite, Lemma 4.1 is not necessary to prove Theorem 4.1. In this erratum, we provide the correct statement for Theorem 4.1 for the finite dimensional case, as well as its proof.

2 Corrected Result

Let us start by proving that $\mathcal{C}_V$ is not necessarily weakly closed if $\mathcal{H}$ is infinite dimensional. Recall that $(\mathcal{H}, \langle \cdot, \cdot \rangle)$ is a separable Hilbert space, $(\mathcal{E}, [\,\cdot, \cdot\,])$ is a Krein space

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and $V \in \mathcal{L}(\mathcal{H}, \mathcal{E})$ is surjective. If $\dim N(V)^\perp$ is finite the problem can be treated as if $\mathcal{H}$ were finite dimensional, simply considering $N(V)^\perp$ instead of $\mathcal{H}$.

Proposition 2.1 *If* $\dim N(V)^\perp$ *is infinite then* $\mathcal{C}_V$ *is not weakly closed.*

Proof Assume that $\dim N(V)^\perp$ is infinite. Then, $\mathcal{E}$ is also infinite dimensional (and separable) because V is surjective. Let $\mathcal{E} = \mathcal{E}_+[\dot{+}]\mathcal{E}_-$ be a fundamental decomposition of $\mathcal{E}$. Hence, $(\mathcal{E}_+, [\,\cdot,\cdot\,])$ and $(\mathcal{E}_-, -[\,\cdot,\cdot\,])$ are Hilbert spaces. Without loss of generality, we can assume that $\dim \mathcal{E}_+$ is infinite.

Let us consider an orthonormal basis $(e_n^+)_{n\geq 1}$ of $\mathcal{E}_+$. As a consequence of Bessel's inequality we have that $e_n^+ \xrightarrow{w} 0$, i.e.

$$\left[\, e_n^+, z \,\right] \to 0 \qquad \text{for every } z \in \mathcal{E}^+.$$

For each $n \geq 1$ there exists a unique $x_n \in N(V)^\perp$ such that $Vx_n = e_n^+$. Also, choose $e^- \in \mathcal{E}_-$ such that $\left[\, e^-, e^- \,\right] = -1$ and the unique $x_0 \in N(V)^\perp$ such that $Vx_0 = e^-$. Then, define

$$y_n = x_n + x_0, \qquad n \geq 1.$$

Below we show that the sequence $(y_n)_{n\geq 1}$ is contained in $\mathcal{C}_V$ and it weakly converges to $x_0 \notin \mathcal{C}_V$. On the one hand, if $n \geq 1$,

$$[\, Vy_n, Vy_n \,] = \left[\, e_n^+ + e^-, e_n^+ + e^- \,\right] = \left[\, e_n^+, e_n^+ \,\right] + \left[\, e^-, e^- \,\right] = 1 - 1 = 0,$$

i.e. $y_n \in \mathcal{C}_V$. On the other hand, given $x \in \mathcal{H}$,

$$\left|\, \langle\, y_n - x_0, x \,\rangle \,\right| = \left|\, \langle\, x_n, x \,\rangle \,\right| = \left| \left\langle\, V^\dagger e_n^+, x \,\right\rangle \right| = \left| \left[\, e_n^+, (V^\dagger)^\# x \,\right] \right| \xrightarrow[n\to\infty]{} 0.$$

Therefore, $y_n \xrightarrow{w} x_0$ and $x_0 \notin \mathcal{C}_V$ since $[\, Vx_0, Vx_0 \,] = \left[\, e^-, e^- \,\right] = -1 \neq 0$. □

Now we give the correct statement and proof of [1, Theorem 4.1], which establishes sufficient conditions for the existence of indefinite interpolating splines for every $z_0 \in \mathcal{E}$ in the finite dimensional setting.

Theorem 2.1 [1, Theorem 4.1] *Suppose that* $\mathcal{H}$ *is a finite dimensional space. If* $R(L)$ *is a uniformly positive subspace of* $(\mathcal{K} \times \mathcal{E}, [\,\cdot,\cdot\,]_\rho)$ *for some* $\rho \neq 0$ *then* $\mathcal{S}_{z_0} \neq \varnothing$ *for every* $z_0 \in \mathcal{E}$.

Proof In order to prove the theorem, we apply [1, Proposition 4.2]. To this end, we first show that $T^\# T x \in R(L^\# L)$ for every $x \in \mathcal{H}$.

By [1, Proposition A.1], $R(L)$ is a regular subspace. Then, for every $(y, z) \in \mathcal{K} \times \mathcal{E}$ there exists (a unique) $x \in \mathcal{H}$ such that $Lx - (y, z) \in R(L)^{[\perp]}$, or equivalently, $L^\# Lx = L^\#(y, z)$. Since T and V are surjective, for each $(y, z) \in \mathcal{K} \times \mathcal{E}$ there exist $u, w \in \mathcal{H}$ such that $y = Tu$ and $z = Vw$. Therefore, there exists $x \in \mathcal{H}$ such that

$$T^\# Tu + \rho V^\# Vw = L^\#(Tu, Vw) = (T^\# T + \rho V^\# V)x,$$

and consequently $R(L^{\#}L) = R(T^{\#}T + \rho V^{\#}V) = R(T^{\#}T) + R(V^{\#}V)$. Thus, $R(T^{\#}T) \subseteq R(L^{\#}L)$.

Given $z_0 \in \mathcal{E}$, let $x_0, u_0 \in \mathcal{H}$ be such that $V x_0 = z_0$ and $T^{\#}T x_0 = L^{\#}L u_0$. Since $\mathcal{H}$ is a finite dimensional space, $(R(L^{\#}L), (\cdot, \cdot))$ is a Hilbert space and $\mathcal{C}_V \cap \mathcal{B}_L$ is compact. Then $\mathrm{d}(u_0, \mathcal{C}_V \cap \mathcal{B}_L)$ is attained. In this case, [1, Proposition 4.2] ensures that $\mathcal{S}_{z_0} \neq \varnothing$. □

Reference

1. Gonzalez Zerbo, S., Maestripieri, A., Martínez Pería, F.: Indefinite abstract splines with a quadratic constraint. J. Optim. Theory Appl. **186**, 209–225 (2020)