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FORUM

A Note On the Weak Convergence of the Extragradient Method for Solving Pseudo-Monotone Variational Inequalities

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Abstract
A corrigendum in Vuong (J Optim Theory Appl 176:399–409, 2018) is given.

Keywords Variational inequality · Extragradient method · Pseudo-monotonicity · Weak convergence

Mathematics Subject Classification 47J20 · 49J40 · 49M30

1 Introduction

The weak convergence of the extragradient method for solving pseudo-monotone variational inequalities in infinite dimensional Hilbert spaces was studied recently in [1]. The convergence analysis requires the sequential weak continuity of the associated operator. The illustrated example given in [1, Section 4] unfortunately does not satisfy this assumption. We provide in this note new examples satisfying all assumptions required.

2 Updating of Reference 1

Let $H = \ell_2$, the real Hilbert space, whose elements are the square-summable sequences of real numbers. Let $\beta > 0$ and

$$F_\beta(u) := (\beta - \|u\|)\, u, \quad \forall u \in H. \tag{1}$$

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It is proved in [1] that the operator F_β is pseudo-monotone and Lipschitz continuous. However, the sequential weak continuity of F_β was not checked. Indeed, this assumption is not satisfied: let $\{e_n\}$ be the standard basic of H, i.e., $e_n = (0, \ldots, 1, 0, \ldots, 0)$ with 1 at the n-th position. Then $e_1 + e_n$ converges weakly to e_1 but $F_\beta(e_1 + e_n) = (\beta - \sqrt{2})(e_1 + e_n)$ converges weakly to $(\beta - \sqrt{2})e_1$ and $(\beta - \sqrt{2})e_1 \neq (\beta - 1)e_1 = F_\beta(e_1)$.

A correct example can be found in [2, Example 2.1], where the operator F is pseudo-monotone, Lipschitz continuous and sequentially weakly continuous.

3 Conclusions

We give a corrigendum and new example to illustrate the main results obtained in [1]. The author thanks Radu Boţ for fruiful discussion in finding new examples and Heinz Bauschke for his awesome lectures at the University of Vienna in Dec, 2018, where the author learnt that the projection operator is not weakly continuous [3].

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