Dedication to Benson

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This special issue is dedicated to Professor Harold P. Benson, a pioneering scholar and celebrated researcher in mathematical programming, optimization, management science, multiple criteria decision-making and global optimization, among other areas.

Harold Benson attended The University of Michigan, Ann Arbor, where he graduated Phi Beta Kappa with a BS in mathematics with highest honours in 1971. He then studied in the Industrial Engineering/Management Sciences Department of Northwestern University, where he obtained the MS and PhD degrees in 1973 and 1976, respectively.

Professor Benson worked at the General Motors Research Laboratories in Warren, Michigan, from 1976 through 1979, where he was Associate Senior Research Engineer.

From 1979 through 2013, Professor Benson was a professor at The Warrington College of Business Administration of The University of Florida.

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In 2004, Professor Benson received the Georg Cantor Award from the International Society on Multiple Criteria Decision Making, their highest form of recognition for a researcher. In 2008, he was recognized in a study by his peers as being among the forty most productive researchers in Multiple Criteria Decision Making in the world. In the 2011 book *Multiple Criteria Decision Making—From Early History to the twenty-first century*, Professor Benson was deemed to be among 42 leading scholars to have ever engaged in Multiple Criteria Decision Making research.

During his academic years, Professor Benson was Associate Editor of the *Journal of Optimization Theory and Applications*, the *Journal of Global Optimization*, *Naval Research Logistics*, the *Journal of Mathematical Analysis and Applications* and *Operations Research Letters*. He also served on the Editorial Boards of *Mathematical Reviews* and of four additional international journals. He was a founding member of the INFORMS Section on Multiple Criteria Decision Making in 2010.

Professor Benson is a patient, caring and thorough teacher, advisor and mentor. He received various teaching awards, including the Outstanding Teacher of the Year Award, from his department in the College of Business Administration at the University of Florida.

Professor Benson's research has mainly been in the areas of global optimization, multiple criteria decision making, mathematical programming, sensitivity analysis, parametric programming, and applications of optimization to problems ranging from citrus rootstock selection to surgery scheduling.

Most of Professor Benson's contributions to multiple criteria decision making and to global optimization are in the areas of delineating the structures of these problems, the nature of solutions to these problems, methods for probing the existence of solutions to these problems, showing where in the feasible regions of these problems solutions must lie, and devising practical algorithms for finding these solutions.

For example, in multiple criteria decision making, he showed that the efficient outcome set of a multiple objective convex program with linear criterion functions often has a significantly simpler structure than the efficient decision set. This helped to change the focus of research to generating the efficient outcome set or subsets thereof, rather than generating the more complicated, computationally taxing efficient decision set. In particular, as a result of this research, he and others developed several algorithms for finding efficient points in the outcome sets of multiple objective convex and linear programs. Among these are his global shooting procedure, his finite, simplicial partitioning-outer approximation method, and, subsequently, a finite, outer approximation algorithm in outcome space that generates the entire weakly efficient set and the set of efficient extreme points of the outcome polyhedron of a multiple objective linear program. The latter algorithm, in particular, has generated significant attention. For instance, studies based upon this algorithm have been made that tweak it computationally to improve its performance, to create a dual variant of the algorithm and to adapt it to solving, or approximately solving, multiple objective integer programming problems, multiple objective convex programming problems and even multiple objective nonconvex programming problems. It has also been used to perform polyhedral calculus on convex polyhedra and on polyhedral convex functions. In the practical realm, this algorithm or its variants have been applied to solving multiple objective linear programs that arise in problems in mathematics, finance, probability, microbial

ecosystems, and entropy, for example. A publicly available code, called BENSOLVE, that executes this algorithm and its variants is available on the Internet.

In multiple criteria decision making, Professor Benson also helped to introduce, define and study the class of proper efficient solutions, a class which must be taken into account in both theory and practice. To help study the nature and existence of efficient, proper efficient and improper efficient points, he formulated a simple test.

Some of the other areas of multiple criteria decision making to which Professor Benson contributed include complete efficiency, the domination property, finding an initial efficient point for algorithmic purposes, optimization over an efficient set, and conditions for the closedness of efficient sets.

In global optimization, one of the areas of research to which Professor Benson has made key contributions is concave minimization. The minimization of a concave function over a convex set is a fundamental problem in global optimization. It has been the subject of intense interest for over 50 years. Motivated by this, Professor Benson created a branch-and-bound algorithm for solving the problem of minimizing a nonseparable concave function over a polyhedron that finds a globally optimal extreme point solution for the problem in a finite number of steps. A short time later, he and colleagues proposed a more general algorithm for globally minimizing a concave function over a compact convex set. The algorithm works in such a way that the computationally most expensive subroutines of previous methods are avoided. At approximately the same time, Professor Benson proposed a customized method for globally minimizing a separable, concave function over a compact convex set. Professor Benson has made several other important contributions to concave minimization, including, for example, his creation of improved gamma-valid cuts.