

# Triplet Superconductivity in a Nutshell

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**Abstract** The triplet superconductors have been around us since 1980, when Jerome et al. discovered the Bechgaard salts  $(\text{TMTSF})_2\text{PF}_6$  and  $(\text{TMTSF})_2\text{ClO}_4$ . Now there are more than 20 or so triplet superconductors discovered. Recently we found that most of them with the known gap symmetries can be mapped to the superfluid phase of  $^3\text{He-A}$  and  $^3\text{He-A}_1$ . Further, in most of them,  $\hat{l}$  (the chiral vector, i.e. the quantization axis of the pair angular momentum) is fixed parallel to one of the crystal axes and all the topological defects are considered in terms of  $\hat{d}$ -textures, where  $\hat{d}$  is the spin vector. Then  $\text{Sr}_2\text{RuO}_4$ ,  $\text{UPt}_3$ ,  $\text{PrOs}_4\text{Sb}_{12}$  and  $(\text{TMTSF})_2\text{ClO}_4$  belong to type-A, which is an analog of superfluid  $^3\text{He-A}$ . In these superconductors, a vortex splits into a pair of half quantum vortices (HQVs) at low temperatures. On the other hand,  $\text{CePt}_3\text{Si}$ ,  $\text{CeIrSi}_3$ ,  $\text{CeRhSi}_3$ ,  $\text{UIr}$  and  $\text{Li}_2\text{Pt}_3\text{B}$  (those in non-centrosymmetric crystals) belong to type- $\text{A}_1$ , which is an analog of superfluid  $^3\text{He-A}_1$ . In all of these triplet superconductors, vortices harbor the zero mode or the Majorana fermions, the implications of which deserves further exploration.

**Keywords** Unconventional superconductivity · Half quantum vortex

## 1 Introduction

The first triplet superconductors have been discovered in 1980 by Jerome et al. [1] in Bechgaard salts  $(\text{TMTSF})_2\text{PF}_6$  and  $(\text{TMTSF})_2\text{ClO}_4$  [2, 3]. Since then many triplet superconductors have been discovered, which is concisely reviewed by Sigrist and Ueda [4]. Although it has been noticed that all of these triplet superconductors are nodal (i.e. non s-wave), the gap symmetry has not been addressed seriously until around 1994. We now have a proposal on the gap symmetry of  $\text{UPt}_3$ ,  $\text{Sr}_2\text{RuO}_4$  and  $\text{PrOs}_4\text{Sb}_{12}$  as shown in Fig. 1 [5, 6]. Here we understand that these triplet superconductors are characterized by the equal spin pairing (ESP) and with extremely small spin-orbit coupling energy  $E_{so} \sim 10^{-3}\Delta$ , where  $\Delta$  is the superconducting energy gap [7]. Therefore the superconducting order parameter is characterized by  $\hat{l}$  (the chiral vector) and  $\hat{d}$  (the spin vector) as in superfluid  $^3\text{He-A}$ , except for the gap symmetry [8, 9].

On the other hand, the discovery of a non-centrosymmetric superconductor  $\text{CePt}_3\text{Si}$  by Bauer et al. in 2004 [10] opened a new door. According to Anderson's argument [11], such a triplet superconductor cannot exist. In order to clarify the role of the parity-breaking term (e.g. the Rashba term [12]), Frigeri et al. [13, 14] considered a model with a Rashba term. They found, first of all, the Fermi surface is split into the one for up-spin and the another for down-spin, as shown in Fig. 2. Here we have introduced the Rashba Hamiltonian  $\mathcal{H}_R = \alpha k_z \sigma_z$ , different from the one used by Frigeri et al. [13, 14]. Secondly, they found the strong admixture of the singlet component. Later, many non-centrosymmetric superconductors have been discovered in

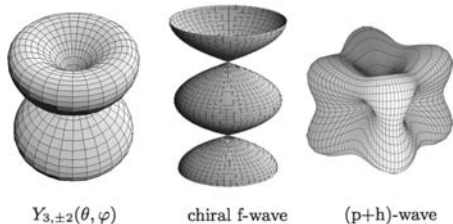
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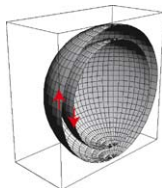
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**Fig. 1** Our proposal on gap functions of  $Y_{3,\pm 2}(\theta, \varphi)$  for UPt<sub>3</sub> and CePt<sub>3</sub>Si, chiral f-wave for Sr<sub>2</sub>RuO<sub>4</sub>, (p+h)-wave for PrOs<sub>4</sub>Sb<sub>12</sub>

**Fig. 2** The split Fermi surface for each spin due to the parity-breaking term



CeIrSi<sub>3</sub>, CeRhSi<sub>3</sub>, UIr and Li<sub>2</sub>Pt<sub>3</sub>B [15–18]. Also the spin-orbit coupling energy  $E_{so}$  or the Rashba term is found invariably extremely large [15, 18], i.e.  $E_{so} \sim 10^3$  K in these systems. Recently in [19], we have proposed that the triplet superconductors in non-centrosymmetric crystals should be like the one in <sup>3</sup>He-A<sub>1</sub>, in contrast to the previous proposals. The superconductivity occupies the Fermi surface associated with one spin component (say, up-spin) while the other Fermi surface remains in the normal state. With this assumption, we can describe many unusual experimental results of CePt<sub>3</sub>Si.

## 2 The Zero Mode and the Majorana Fermion

In this section, we shall summarize the bound-state spectrum around an Abrikosov's vortex in triplet superconductors. In both type-A and type-A<sub>1</sub> triplet superconductors, we conclude that vortices harbor the zero mode which behaves as the Majorana fermion.

First of all, we note that  $\hat{l}$  (the chiral vector) of many triplet superconductors are fixed parallel to the crystal  $\hat{c}$ -axis. Secondly, in the absence of a magnetic field,  $\hat{d} \parallel \hat{l}$  is due to the spin-orbit coupling. However, when the magnetic field  $\vec{H}$  (parallel to  $\hat{l}$ ) is applied in the vicinity of  $H_{c2}(T)$  (the upper critical field),  $\hat{d} \perp \hat{l}$  is realized. In this particular situation, Ivanov [20, 21] have shown that the BdG equations for the triplet superconductor decouple into the one for the up-spin component and the other for the down-spin component. Then these BdG equation have the same structure as the one for s-wave superconductor first written down in [22, 23]. Therefore, first let us consider a single vortex.

Following Caroli, de Gennes and Matricon [22, 23], the bound state energy formula is given as follows (see, for detail [24–26])

$$\epsilon_n = n\omega_0 \quad \text{with } n = 0, \pm 1, \pm 2, \pm 3, \dots, \quad (1)$$

where  $\omega_0 \simeq \Delta(p_F \xi)^{-1} [\ln(\frac{2}{\delta})]^{-1}$  ( $\delta$  is the cut-off parameter) and  $\delta \simeq \sqrt{\Gamma/\Delta}$  ( $\Gamma$  is the quasiparticle scattering rate in the normal state) in the very clean system. So  $\omega_0$  is essentially the same as in s-wave superconductors except for the log-correction due to the nodal structure. Note that the bound-state spectrum is very different from the one for the singlet superconductors (s-wave, d-wave etc.). For the singlet superconductors,  $\epsilon_n = (n + \frac{1}{2})\omega_0$  with  $n = 0, \pm 1, \pm 2, \pm 3, \dots$  with a gap  $\omega_0/2 \sim \Delta^2/E_F$ . On the other hand, in triplet superconductors, there is the zero mode or the Majorana fermion [9, 27, 28] with zero energy (right on the Fermi surface). The Majorana fermion wavefunction associated with the zero mode is given by

$$\phi_0(r, \hat{k}) \sim [\text{sech}(r/\xi)]^{C|f|} \quad (2)$$

and  $C = \Delta \xi v^{-1} \sim 1$ . These wavefunctions should be accessible at low temperatures.

## 3 Half-Quantum Vortices

Half-quantum vortices are generated in type-A triplet superconductors, which correspond to the  $\hat{d}$ -soliton or the  $\hat{d}$ -domain wall [24–26]. Since  $\hat{l}$ -vector is fixed parallel to the crystalline  $\hat{c}$ -axis, we limit ourselves to the texture free energy associated with the  $\hat{d}$  vector. Especially for UPt<sub>3</sub>, Sr<sub>2</sub>RuO<sub>4</sub> and PrOs<sub>4</sub>Sb<sub>12</sub> in a magnetic field parallel to  $\hat{l}$ , the texture free energy simplifies as [29]

$$\mathcal{F} = \frac{1}{2} \chi_N C^2 \int dx dy \left[ K (\nabla \Phi)^2 + \sum_{i,j} |\partial_i \hat{d}_j|^2 + \xi_D^{1-2} (\hat{d}_x^2 + \hat{d}_y^2) \right], \quad (3)$$

where  $\chi_N$  and  $C$  are the spin susceptibility and the spin-wave velocity, and  $\xi_D^{1-2} = \xi_D^{-2} - \xi_H^{-2}$ . Here

$$K(t) = \frac{\rho_s(t)}{\rho_{sp}(t)} = \frac{1 + \frac{F_1}{3} \frac{1 + \frac{F_1^a}{3} (1 - \rho_s^0(t))}{1 + \frac{F_1^a}{3} \frac{1 + \frac{F_1}{3} (1 - \rho_s^0(t))}{1 + \frac{F_1^a}{3} (1 - \rho_s^0(t))}}, \quad (4)$$

where  $t = T/T_c$ ,  $\rho_s(t)$ ,  $\rho_{sp}(t)$  and  $\rho_s^0(t)$  are the superfluid density, the spin superfluid density and the bare superfluid density. In PrOs<sub>4</sub>Sb<sub>12</sub>,  $\rho_s^0(t)$  is well approximated by  $\rho_s^0(t) = 1 - 0.460572t - 0.53428t^{1.729}$  [30]. Experimentally,  $m^*/m = 50$  in PrOs<sub>4</sub>Sb<sub>12</sub> and, if we ignore  $F_1^a$ ,

$$K(t) = \left[ 1 - \left( 1 - \frac{m}{m^*} \right) \rho_s^0(t) \right]^{-1}. \quad (5)$$

Here we note that the temperature dependence of  $K(t)$  in UPt<sub>3</sub> and Sr<sub>2</sub>RuO<sub>4</sub> is very similar to that of PrOs<sub>4</sub>Sb<sub>12</sub> [24, 25].

Here we shall compare the free energy of an Abrikosov’s vortex and a bound pair of half-quantum vortices by the  $\hat{d}$ -soliton. For this purpose, we consider the free energy of an Abrikosov’s vortex in a unit circular cell with radius  $a$  which encloses a flux quantum. Then  $a$  is given by

$$a = \left( \frac{\phi_0}{\pi H} \right)^{1/2}, \tag{6}$$

where  $H$  is the field strength. Note also a similar relation due to de Gennes:

$$H_{c2}(t) = \frac{\phi_0}{2\pi\xi^2(t)}, \tag{7}$$

where  $\xi(t)$  is the coherence distance. This indicates, for  $H < H_{c2}(t)$ ,  $a > \sqrt{2}\xi(t)$ . The free energy for an Abrikosov’s vortices is readily given by

$$\mathcal{F}_A = \pi \chi_N C^2 K \ln(a/\xi). \tag{8}$$

For a bound pair of HQVs separated by a distance  $R$  and inside the unit cell,

$$\begin{aligned} \mathcal{F}_{BP} = \frac{\pi}{2} \chi_N C^2 \left\{ K \ln \left( \frac{a + \sqrt{a^2 - R^2/4}}{2\xi} \right) + \ln(a/\xi) \right. \\ \left. - \frac{R}{4a} \sin^{-1} \left( \frac{R}{\sqrt{a^2 - R^2/4}} \right) \right. \\ \left. + \frac{1}{4} \left( \frac{R}{\xi'_D} \right)^2 \ln \left( \frac{2\xi'_D}{a} \frac{K+1}{\sqrt{2K+1}} \right) \right\}. \tag{9} \end{aligned}$$

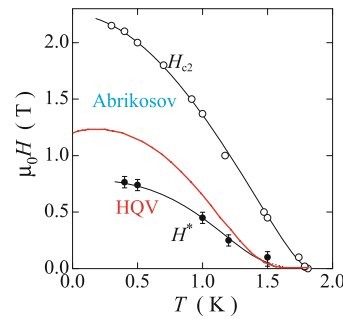
Also, for  $K(t) \gg 1$ , this equation is optimized at  $R/2a = \sqrt{2K+1}/K+1$  and [31]

$$\begin{aligned} \mathcal{F}_{BP} = \frac{\pi}{2} \chi_N C^2 \left\{ K \ln \left( \frac{a}{2\xi} \frac{2K+1}{K+1} \right) + \ln \left( \frac{a}{\xi} \right) \right. \\ \left. - \frac{1}{2} \frac{\sqrt{2K+1}}{K+1} \sin^{-1} \left( \frac{2\sqrt{2K+1}}{K+1} \right) \right. \\ \left. + \frac{2K+1}{(K+1)^2} \left( \frac{a}{\xi'_D} \right)^2 \ln \left( \frac{2\xi'_D}{a} \frac{K+1}{\sqrt{2K+1}} \right) \right\}. \tag{10} \end{aligned}$$

Now from  $\mathcal{F}_A = \mathcal{F}_{BP}$ , we find that

$$\begin{aligned} H^*(t)/H_{c2}(t) \\ = \left[ 2^{-(K-2)} (2K+1)^{(2K+1)} (K+1)^{-2(K+1)} \right]^{-1/(K-1)}, \tag{11} \end{aligned}$$

where  $H^*(t)$  is the phase boundary below which HQVs are stable. For PrOs<sub>4</sub>Sb<sub>12</sub>, we show the stability region of HQVs in Fig. 3 [26], which is close to the phase boundary determined experimentally by Izawa et al. [32]. According to the our theoretical picture, the B phase discovered in [32] is



**Fig. 3** For PrOs<sub>4</sub>Sb<sub>12</sub>, the stability region of HQV (inside the line without circles) is shown, based on our theoretical result [26]. It is close to the phase (whose boundary is given by *black circles*) determined experimentally by Izawa et al. [32]

the vortex state filled with HQVs. Also we have got a similar result for the B phase in UPT<sub>3</sub> [24, 25]. On the other hand, as to Sr<sub>2</sub>RuO<sub>4</sub>, there is no clear hallmark of such a phase transition. However, we speculate that the anomalous angle-dependent magneto-specific heat found by Deguchi et al. [33] below 300 mK can be due to HQVs.

Finally, it is to be noted that we can access the HQVs by scanning tunneling microscopy (STM) [34], micromagnetometry [35] and small angle neutron scattering (SANS) at ultra-low temperature.

#### 4 Concluding Remarks

We have proposed a new paradigm for triplet superconductivity.

- (1) Most of triplet superconductors, which were discovered before 2004, belong to type-A. Their topological structure is very similar to superfluid <sup>3</sup>He-A. At low temperatures, a vortex splits into a pair of half-quantum vortices (HQVs).
- (2) A new class of superconductors in non-centrosymmetric crystals, which were discovered after 2004, belong to type-A<sub>1</sub>. The topological structure is the same as of superfluid <sup>3</sup>He-A<sub>1</sub>.
- (3) All these triplet superconductors have the zero mode/Majorana fermion attached to a vortex, the implications of which should be explored further.

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