Correction to: An Energy Conservative hp -method for Liouville's Equation of Geometrical Optics

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The original version of the article unfortunately contained mistakes. It has been corrected in this correction. The original article has also been corrected.

1. Arrow above symbols q and p were missing in Section 2.1 and Section 3. Correct symbols are $\vec{q}$ and $\vec{p}$. For example, equation (7) should read:

$$\vec{p} = (p, p_z) = n (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta),$$

2. Missing ' $<$ ' in ' < 0 ' and incorrect spacing in equation (48). Equation (48) should read

$$\tilde{F} = \frac{\Delta l}{2} (\mathbf{u} \cdot \hat{\mathbf{n}}) \begin{cases} \rho_L & \text{if } \mathbf{u} \cdot \hat{\mathbf{n}} \geq 0, \\ \rho_R & \text{if } \mathbf{u} \cdot \hat{\mathbf{n}} < 0, \end{cases}$$

3. Missing arrow above unit normal $\mathbf{v}$ in Section 4 and Section 5.2. For example, equation (51a) should read:

$$\vec{p} = S(p; n_1, n_2, \vec{v}) := \begin{cases} p - (\psi + \operatorname{sgn}(n_2)\sqrt{\delta})v_q & \text{if } \delta \geq 0, \\ p - 2\psi v_q & \text{if } \delta < 0, \end{cases}$$

4. Completely missing a symbol in equation (60) and text on page 15. For example, equation (60) should read (where $\vec{v} = (-1, 0)$):

$$\vec{p} = S(p; n_1, n_2, \vec{v}) = \begin{cases} \sqrt{n_2^2 - n_1^2 + p^2} & \text{if } p \geq p_c, \\ -p & \text{if } p < p_c, \end{cases}$$

The original article can be found online at <https://doi.org/10.1007/s10915-021-01612-x>.

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5. Placement of the '0' in the block matrices should be in the bottom-right block. This error occurred in equations (77), (78), (88), (105) and (106b). For example, equation (78) should read:

$$A := \begin{pmatrix} \text{diag}(\mathbf{w}) & \boldsymbol{\alpha}^R \circ \mathbf{w} \\ (\boldsymbol{\alpha}^R \circ \mathbf{w})^T & 0 \end{pmatrix}.$$

6. Equation (99) starts with $e_{\text{DG}} = e_{\text{DG}} = \dots$. Instead, equation (99) should read:

$$e_{\text{DG}} = C_{\text{DG}} K^{-\gamma_{\text{DG}}/2},$$

7. Spacing in equation (101a) was incorrect. The corrected version of equation (101a) reads:

$$\rho(z, q, p) = \begin{cases} \rho_0 \left(q - z \frac{p}{\sqrt{n_1^2 - p^2}}, p \right) & \text{if } q < 0, p \geq 0, \\ \rho_0 \left(z \frac{p}{\sqrt{n_1^2 - p^2}} - q, -p \right) & \text{if } q < 0, -p_c < p < 0, \\ \rho_0 \left((\delta z - z) \frac{\bar{p}}{\sqrt{n_1^2 - \bar{p}^2}}, \bar{p} \right) & \text{if } q > 0, p \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

8. Layout of table 2 is different from table 1. The corrected version of Table 2 reads:

Table 2 Bucket of water: convergence data

K	$\frac{e_{\text{DG}}}{N=1}$		$\frac{\gamma_{\text{DG}}}{N=2}$		$\frac{e_{\text{DG}}}{N=3}$	
480	4.93e-02		1.71e-02		8.70e-03	
1920	1.82e-02	1.44	4.90e-03	1.80	1.23e-03	2.83
7680	6.25e-03	1.54	6.61e-04	2.89	8.07e-05	3.92
30720	1.56e-03	2.00	5.82e-05	3.50	3.71e-06	4.44
	$N=4$		$N=5$		$N=6$	
480	4.15e-03		2.03e-03		1.03e-03	
1920	3.55e-04	3.55	1.08e-04	4.24	3.36e-05	4.94
7680	1.17e-05	4.93	1.98e-06	5.77	3.79e-07	6.47
30720	3.08e-07	5.24	3.10e-08	6.00	3.37e-09	6.81