



Combined Lorentz Symmetry: Lessons from Superfluid ^3He

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Abstract

We consider the possibility of the scenario in which the P , T and Lorentz symmetry of the relativistic quantum vacuum are all the combined symmetries. These symmetries emerge as a result of the symmetry breaking of the more fundamental P , T and Lorentz symmetries of the original vacuum, which is invariant under separate groups of the coordinate transformations and spin rotations. The condensed matter vacua (ground states) suggest two possible scenarios of the origin of the combined Lorentz symmetry, and both are realized in the superfluid phases of liquid ^3He : the $^3\text{He-A}$ scenario and the $^3\text{He-B}$ scenario. In these scenarios, the gravitational tetrads are considered as the order parameter of the symmetry breaking in the quantum vacuum. The $^3\text{He-B}$ scenarios applied to the Minkowski vacuum lead to the continuous degeneracy of the Minkowski vacuum with respect to the $O(3, 1)$ spin rotations. The symmetry breaking leads to the corresponding topological objects, which appear due to the nontrivial topology of the manifold of the degenerate Minkowski vacua, such as torsion strings. The fourfold degeneracy of the Minkowski vacuum with respect to discrete P and T symmetries suggests that the Weyl fermions are described by four different tetrad fields: the tetrad for the left-handed fermions, the tetrad for the right-handed fermions, and the tetrads for their antiparticles. This may lead to the gravity with several metric fields, so that the parity violation may lead to the breaking of equivalence principle. Finally, we considered the application of the gravitational tetrads for the solution of the cosmological constant problem.

Keywords Topological superfluid · Symmetry breaking · Lorentz symmetry

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1 Introduction

Topological superfluid phases of liquid ${}^3\text{He}$ provide many connections with the Standard Model of particle physics and gravity [1, 2]. Here, we consider the symmetries and degeneracy of the Minkowski vacuum from the point of view of the symmetry breaking in the B-phase of superfluid ${}^3\text{He}$ [3].

In the Standard Model, the discrete symmetries P and T are broken. It is possible that these symmetries are restored at some ultraviolet energy scale, which is below the Planck scale $E_{\text{UV}} \ll E_{\text{p}}$. However, even if T and P are restored above E_{UV} , they still have the signatures of the broken symmetry, since each of them represents the combined symmetry. The fermionic action is not invariant under the pure transformations of coordinates, $P_c \mathbf{r} = -\mathbf{r}$ and $T_c t = -t$. It becomes invariant only if these coordinate transformations are accompanied by the corresponding transformations of Dirac or Weyl spinors, P_s and T_s . This means that each of these two discrete symmetry operations is the product of two operations, $P = P_c P_s$ and $T = T_c T_s$. Since P_c and T_c are not the symmetries of the fermionic quantum vacuum even in the high-energy scale, it is natural to suggest that the Minkowski vacuum is degenerate with respect to these discrete symmetries. Such degeneracy of the quantum vacuum has been suggested by Vergeles [4].

The natural extension of this idea is the application to the continuous Lorentz symmetry L . The fermionic action lacks the separate symmetries, such as symmetry with respect to the only coordinate transformations L_c , but is invariant under the combined operation, $L = L_c L_s$, i.e., when the space rotations are accompanied to the spin rotations of the Dirac or Weyl spinors. One may suggest the symmetry breaking pattern $L_c \times L_s \rightarrow L$, which follows from the consideration of similar symmetry breaking in superfluid ${}^3\text{He-B}$ [5]. The original vacuum of normal liquid ${}^3\text{He}$ is invariant under separate spin and orbital (coordinate) rotations, $G = SO(3)_s \times SO(3)_c$. In the B-phase, this symmetry is reduced to the diagonal group $G \rightarrow H = SO(3)_J$ of the combined rotations. Such symmetry breaking is known as the broken relative symmetry [6], the vacuum becomes degenerate with respect to the separate coordinate or spin rotations. Extension to the Lorentz groups suggests the continuous degeneracy of the Minkowski quantum vacuum.

The same take place with the discrete parity symmetry, $P_c \times P_s \rightarrow P_c P_s$, which leads to the additional discrete degeneracy of the ${}^3\text{He-B}$ and correspondingly of the Minkowski vacuum. Here, we consider some consequences of this symmetry breaking pattern in the Minkowski vacuum. The paper is organized as follows.

In Sect. 2, the topological scenario of emergent combined symmetry is discussed, which follows from the analogy with the chiral superfluid ${}^3\text{He-A}$ with emergent Weyl fermions.

In Sect. 3, the B-phase symmetry breaking is discussed. The spontaneous breaking of the separate rotational symmetries is demonstrated together with the spontaneous breaking of discrete symmetry $P_c \times P_s \rightarrow P = P_c P_s$.

In Sect. 4, the symmetry breaking scenario, which takes place in ${}^3\text{He-B}$, is extended to the relativistic Minkowski vacuum. Two independent Lorentz groups of the coordinate and spin rotations are broken to the diagonal subgroup of the

degenerate Minkowski vacuum. The broken discrete symmetries and the corresponding degeneracy of the Minkowski vacuum are also discussed.

The topological defects arising from the breaking of relative symmetry, both in $^3\text{He-B}$ and in the relativistic vacuum, are discussed in Sect. 5. They are obtained from the homotopy groups of the manifold of the degenerate states, $\pi_n(G/H)$. In Sect. 6, the particular topological object emerging due to the degeneracy of Minkowski vacuum is discussed—the torsion string.

In Sect. 7, the mirror reflected vacuum is discussed, which is obtained by the discrete symmetry operation acting on our Minkowski vacuum. We discuss the magnetic moment of electron in the mirror reflected vacuum.

In Sect. 8, we discuss the fermionic action for Weyl fermions instead of the Dirac fermions, which are the secondary composite objects. The broken discrete symmetries and the corresponding fourfold degeneracy of the Minkowski vacuum suggest that the Weyl fermions can be described by four tetrad fields: each for left-handed and right-handed fermions and for their antiparticles. Though the considerations in this paper are mainly restricted by the Minkowski vacua and their degeneracy, in Sect. 9, we consider the possible bi-metric gravity, where different metric can be composed from different tetrad fields.

In Sects. 10 and 11, we considered the possible application of the tetrad fields for the solution of the cosmological constant problem.

2 Topological A-Phase Scenario: Emergent Combined Symmetry

The condensed matter vacua suggest two possible scenarios of the origin of the combined Lorentz symmetry, which are realized in the superfluid phases of liquid ^3He : the $^3\text{He-A}$ scenario and the $^3\text{He-B}$ scenario. The latter scenario corresponds to the phase transition with spontaneous breaking of separate Lorentz symmetries L_c and L_s , see Sect. 3.

$^3\text{He-A}$ is the chiral superfluid, which has the topologically protected Weyl points in the quasiparticle spectrum [2]. In the vicinity of the Weyl point, quasiparticles behave as Weyl fermions moving in the effective dynamical gauge and tetrad fields. The combined Lorentz symmetry emerges together with the tetrads and Weyl fermions. In this scenario, the separate Lorentz symmetries, which independently act on coordinates and on fermions, are absent, since on the microscopic atomic scale (the analog of Planckian scale in quantum liquids) the physics is Galilean.

The effective gravity for Weyl fermions, provides an example of degeneracy of quantum vacuum with respect to the discrete symmetry—the P symmetry. The quantum transition is possible in the A-phase, at which some components of the tetrad change sign [7]. At the transition, the tetrad is degenerate, and two Weyl points of the A-phase transform to the Dirac nodal line in the quasiparticle spectrum of the polar phase [8]. The superfluid polar phase, which is time reversal invariant, connects two degenerate vacua with opposite chiralities—analogs of spacetime and antispacetime.

3 B-Phase Scenario: Combined Symmetry from Broken Symmetry

The $^3\text{He-B}$ is the fully gapped superfluid, which is obtained in the following scenario of the spontaneous symmetry breaking [3]. The original (non-superfluid) state of liquid ^3He can be described by the following relevant Hamiltonian:

$$\begin{aligned} \mathcal{H} - \mu\mathcal{N} = & \sum_{\mathbf{p}\alpha} \left(\frac{p^2}{2m} - \mu \right) a_{\mathbf{p}\alpha}^\dagger a_{\mathbf{p}\alpha} - \\ & - \frac{\lambda}{2} \sum_{\mathbf{p}'\alpha\mathbf{p}\beta} (\mathbf{p}' \cdot \mathbf{p}) a_{-\mathbf{p}'\beta}^\dagger a_{\mathbf{p}'\alpha}^\dagger a_{\mathbf{p}\alpha} a_{-\mathbf{p}\beta}. \end{aligned} \quad (1)$$

Here, $\alpha = (\uparrow, \downarrow)$ and $\beta = (\uparrow, \downarrow)$ denote the spin projections of an atom of ^3He , and λ is the small parameter of the relevant interaction. Here, we ignored the $U(1)$ symmetry breaking, which leads to the phase factor and makes the order parameter complex [5].

The symmetric state of the normal (non-superfluid) liquid ^3He has the symmetry of the Hamiltonian (1). It is symmetric under $SO(3)_c$ group of the orbital rotations and, if the tiny spin-orbit interaction is neglected, it is also symmetric under rotations $SO(3)_s$ in spin space.

In the superfluid state, the bilinear form $a_{\mathbf{p}\alpha} a_{-\mathbf{p}\beta}$ acquires a nonzero vacuum expectation value. As a result, for $^3\text{He-B}$, the Hamiltonian (1) transforms to the Hamiltonian with 4-component Bogolyubov–Nambu spinor $\chi = (\Psi, \Psi^\dagger)$ with the spectrum:

$$H_{\mathbf{p}} = \left(\frac{p^2}{2m} - \mu \right) \tau_3 + e_a^i \tau_1 \sigma^a p_i. \quad (2)$$

Here, τ_1, τ_3 and σ^a with $a = 1, 2, 3$ are the 2×2 Pauli matrices, and the matrix e_a^i is the order parameter, which is obtained as vacuum expectation value of the bilinear form:

$$\lambda \left\langle \sum_{\mathbf{p}} \mathbf{p} a_{\mathbf{p}\alpha} a_{-\mathbf{p}\beta} \right\rangle_{\text{vac}} = \mathbf{e}_a (i\sigma^a \sigma^{(2)})_{\alpha\beta}. \quad (3)$$

The Hamiltonian (2) is the analog of the Dirac Hamiltonian, and in the limit $m \rightarrow \infty$, it approaches the Hamiltonian for the Dirac spinors [9] with mass $M = \mu$:

$$H_{\mathbf{p}} = -\mu\tau_3 + e_a^i \tau_1 \sigma^a p_i, \quad (4)$$

and the matrices τ_3 and $\tau_1 \sigma^a$ become the corresponding γ matrices.

The order parameter e_a^i breaks both rotational symmetries of the original Hamiltonian, since it transforms under $SO(3)_c$ group of the coordinate rotations and under $SO(3)_s$ group of the spin rotations. This means that the vacuum in $^3\text{He-B}$ is degenerate with respect to spin or to orbital rotations. But the symmetry $SO(3)_J$ under the combined spin and orbital rotations is obeyed, i.e., the spin rotation can

be compensated by the opposite orbital rotation. That is why the symmetry breaking pattern is as follows:

$$SO(3)_c \times SO(3)_s \rightarrow SO(3)_J. \quad (5)$$

Such symmetry breaking is called the broken relative symmetry [6].

In ${}^3\text{He-B}$, the order parameter matrix e_a^i plays the role of the triad field in the emerging $3 + 0$ effective gravity. That is why the $SO(3)_c$ group of orbital rotations plays the role of the group of coordinate transformations, while $SO(3)_s$ group of spin rotations plays the role of the group of the transformations of triads in spin space.

Let us also consider the discrete symmetries on example of parity P , which is broken in ${}^3\text{He-B}$. The original non-relativistic Hamiltonian (1) for the normal liquid is invariant both under the coordinate transformation $P_c \Psi(\mathbf{r}, t) = \Psi(-\mathbf{r}, t)$, and under transformation P_s in the spin space, since the non-relativistic fermions are invariant under parity transformation, $P_s \Psi P_s = \Psi$.

In the broken symmetry state each of these two discrete symmetries is broken, and the Hamiltonian (2) for ${}^3\text{He-B}$ is invariant only under combined symmetry: $P = P_c P_s$, where $P_c \chi(\mathbf{r}, t) = \chi(-\mathbf{r}, t)$ is the coordinate inversion and $P_s \chi = \tau_3 \chi$ is the inversion in spin space. Taking parity into account, the symmetry breaking scheme in ${}^3\text{He-B}$ becomes

$$G = SO(3)_c \times P_c \times SO(3)_s \times P_s \rightarrow H = SO(3)_J \times P. \quad (6)$$

Note again, that we ignore the $U(1)$ symmetry breaking in superfluid ${}^3\text{He-B}$ and consider only the real components of the order parameter. We also did not consider here the time reversal symmetry, since it is not broken in ${}^3\text{He-B}$.

4 Extension to Relativistic Physics

It is natural to extend the symmetry breaking scheme presented in Eq. (5) to the $3+1$ case of Minkowski vacuum [5]. The analogy from ${}^3\text{He-B}$ suggests that there is the original deep quantum vacuum, which is symmetric with respect to the group G , which contains two independent transformations: the vacuum has the symmetry under the Lorentz space-time rotations (which can be extended to the group of coordinate transformations), and the symmetry under the Lorentz $3+1$ transformations of the fermionic spins. This vacuum experiences the symmetry breaking of these two Lorentz symmetries to their diagonal subgroup:

$$L_c \times L_s \rightarrow L_J. \quad (7)$$

The order parameter of this symmetry breaking is the matrix e_a^μ , which plays the role of the tetrad of the (maybe emergent) $3 + 1$ gravity. The Minkowski vacuum becomes degenerate with respect to each of the two rotations, but remains invariant under the combined Lorentz transformation. The latter represents the rest symmetry H of the present degenerate vacuum.

Simultaneously, the original fermions acquire the form of the Dirac or Weyl fermions, which obey the Weyl or Dirac action containing the tetrad field. The gravity

interacting with fermions is described by the Einstein–Cartan–Sciama–Kibble theory and its possible extensions. The typical action for Dirac fermions is as follows:

$$U_{\Psi} = \frac{1}{6} e_{abcd} \int \Theta^a \wedge e^b \wedge e^c \wedge e^d, \quad (8)$$

$$\Theta^a = \frac{i}{2} [\bar{\Psi} \gamma^a D_{\mu} \Psi - D_{\mu} \bar{\Psi} \gamma^a \Psi] dx^{\mu}, \quad (9)$$

where the spin indices are raised with η^{ab} , the flat Minkowski metric. The state of the broken symmetry vacuum and the corresponding Dirac or Weyl action remain invariant only if the transformation of the coordinate x^{μ} is accompanied by the proper rotations of the tetrads e_{μ}^a in the spin space. These combined transformation may also include the operations of discrete symmetries. For example, the combined PT symmetry is the discrete coordinate transformation $(PT)_c x^{\mu} = -x^{\mu}$, which is accompanied by the corresponding transformation of the tetrads $(PT)_s e_{\mu}^a = -e_{\mu}^a$. The action is invariant under transformation $PT = (PT)_c (PT)_s$, but is not invariant under the separate symmetry operation $(PT)_s$, which transforms e^a and $-e^a$. This $(PT)_s$ symmetry is broken, and thus the vacua with e^a and $-e^a$ are different [4]. Just such scenario of the symmetry breaking takes place in the theories, where the tetrad field emerges as bilinear form of the fermionic fields [10–15].

Taking into account both the discrete and continuous symmetries, the natural extension of the symmetry breaking in Eq. (6) to the relativistic physics will be

$$G = L_c \times P_c \times T_c \times L_s \times P_s \times T_s \rightarrow H, \quad (10)$$

$$H = L \times P \times T, \quad (11)$$

$$L = L_c L_s, \quad P = P_c P_s, \quad T = T_c T_s. \quad (12)$$

The simplest model illustrating this symmetry breaking pattern $G \rightarrow H$ has the following action on the microscopic scale [13]:

$$U_{\text{micro}} = \frac{1}{24} e_{abcd} \int \Theta^a \wedge \Theta^b \wedge \Theta^c \wedge \Theta^d. \quad (13)$$

This action has the full symmetry G . The symmetry breaking is represented by the appearance of the Bogoliubov quasi-average:

$$e^a = \langle \Theta^a \rangle. \quad (14)$$

This order parameter e_{μ}^a plays the role of the emerging tetrad field. The zeroth term in the expansion gives the gravitational term corresponding to the cosmological constant:

$$U_0 = \frac{1}{24} e_{abcd} \int e^a \wedge e^b \wedge e^c \wedge e^d, \quad (15)$$

while the first-order term in expansion produces the action (8) for fermions with rest symmetry H .

5 Degenerate Minkowski Vacua and Topological Objects

In ${}^3\text{He-B}$, the symmetry breaking scheme in Eq. (6) leads to several topological objects—the topologically stable configurations of the order parameter e_a^i . They are described by the homotopy groups of the space $R = G/H$ —the manifold of degenerate states:

$$\pi_n(R) = \pi_n(G/H) = \pi_n(SO(3) \times Z_2). \quad (16)$$

The group $\pi_0(R) = Z_2$ gives rise to the domain walls discussed in Ref. [16]. When the $U(1)$ symmetry breaking is taken into account, these domain walls become the analogs of the Kibble–Lazarides–Shafi walls bounded by the Alice strings [17, 18] (half-quantum vortices [16, 19, 20]).

The fundamental group of the manifold of the degenerate vacuum states in ${}^3\text{He-B}$ is $\pi_1(G/H) = Z_2$. The corresponding topological objects are the Z_2 spin-orbital vortices [21]. They have been identified in NMR experiments on ${}^3\text{He-B}$ [22].

Similar topological objects may exist in the 3 + 1 tetrad gravity. As follows from Eqs. (10) and (11), the homotopy groups of the manifold of the degenerate Minkowski vacua are

$$\pi_n(G/H) = \pi_n(O(3, 1)) = \pi_n(SO(3, 1) \times Z_2 \times Z_2), \quad (17)$$

where we took into account the broken discrete symmetries: parity $P_c \times P_s \rightarrow P$ and time reversal $T_c \times T_s \rightarrow T$.

The group $\pi_0(G/H) = Z_2 \times Z_2$ gives rise to three types of cosmic domain walls, in which either the space components of tetrads change sign, or time component changes sign, or the whole tetrad changes sign.

The fundamental group of the manifold of the Minkowski vacuum states is $\pi_1(G/H) = Z_2$. It is the same as in ${}^3\text{He-B}$, where it gives rise to the spin-orbital vortex. Its analog in Minkowski vacuum is the torsion string, in which the curvature has δ -function singularity on the axis, see Sect. 6.

6 Torsion String in the Broken Symmetry Vacuum

Here, we consider the torsion string following discussion with Jacobson [23]. Let us consider the tetrad 1-forms e^a and the $SO(3,1)$ spin connection 1-form ω_b^a as independent variables. The action for pure GR without matter is

$$\int e^a \wedge e^b \wedge R^{cd} \epsilon_{abcd}. \quad (18)$$

The equations of motion obtained by variation over ω_b^a and e^a are correspondingly

$$D(e^{1a} \wedge e^{b1}) = 0, \quad (19)$$

$$e^b \wedge R^{cd} \epsilon_{abcd} = 0. \quad (20)$$

We are interested in such the torsion strings, where tetrads have the following axisymmetric form:

$$e^0 = dt, \quad e^3 = dz, \quad e^1 = dr, \quad e^2 = r d\phi. \quad (21)$$

This structure corresponds to the vortex in chiral ${}^3\text{He-A}$ and also to the spin vortex in ${}^3\text{He-B}$ —the Z_2 defect in the order parameter matrix R_a^i .

The metric does not feel the topology of the space dependent tetrad field, the space part of the metric is as follows:

$$g^{ik} = -\delta^{ik}. \quad (22)$$

Thus, the Riemann curvature, calculated directly from the metric, is zero. However, the curvature 2-form R_b^a calculated from the tetrads has singularity. The first equation of motion, Eq. (19) has solution for the spin connection

$$\omega_a^0 = 0, \quad \omega_a^3 = 0, \quad \omega^1_2 = \frac{1}{r} e^2, \quad (23)$$

which gives the singularity in the curvature on the vortex axis:

$$R_{12ik} = e_{ikl} \overset{\circ}{\zeta}^l \delta_2(\mathbf{r}). \quad (24)$$

The other topological objects in tetrad gravity are the topological instantons [24, 25]. In principle, the Big Bang can be also considered as the topological defect, in which the tetrads change sign [26–28].

7 Sign of Gravitational Tetrads and Magnetic Moment of Electron

Let us consider some consequences of the broken discrete symmetries, such as $(PT)_c \times (PT)_s \rightarrow PT$. We start with the anomalous magnetic dipole moment of the electron, which is determined by the following 3+1 term in the Lagrangian:

$$L \propto ie^{\mu}_a e^{\nu}_b \bar{\psi} (\gamma^a \gamma^b - \gamma^b \gamma^a) \psi F_{\mu\nu}, \quad (25)$$

where $F_{\mu\nu}$ is the electromagnetic field. We consider the magnetic field $\mathbf{B} \parallel \hat{z}$. The projection on z axis of the magnetic moment of electron, which is the variation of the action over \mathbf{B} , is:

$$\mu_z = \gamma e^x_1 e^y_2 \sigma_z, \quad (26)$$

Let us make the following transformation of the tetrad fields:

$$e_1^x \rightarrow -e_1^x, \quad e_2^y \rightarrow e_2^y. \quad (27)$$

Then, the electron magnetic moment changes sign with respect to the spin:

$$\mu_z \rightarrow -\gamma e_1^x e_2^y \sigma_z, \quad (28)$$

The Zeeman energy $\mu_z B_z$ also changes sign after transformation (27). The Zeeman energy of the electron is gravitating as any other energy. That is why the gravitational interaction of the body, in which the electrons are polarized in the external magnetic field, with the other body, where electrons are not polarized, will be different for different sign of e_1^x . The Newton gravitational attraction will be increased after the change of sign. The original value of the gravitational interaction will be restored only after the electrons in the body will finally reorient their spins to reduce the Zeeman energy.

This is the example of the symmetry breaking in the vacuum, which is caused by the existence of tetrads: vacua with different signs of the tetrad elements are not equivalent.

There are the other examples related to spin, in particular related to the rotating black hole. If the black hole is formed by the spin-polarized matter, then the transformation (27) transforms the black hole to the black hole with the opposite direction of rotation.

Also, it is not excluded the existence of the spin current in the vacuum in the presence of electric field [29], which is analogous to spin supercurrent in $^3\text{He-B}$ [30].

8 Weyl Fermions and Multiple Tetrads

Let us now exploit the fact that the Weyl fermions are more fundamental than Dirac fermions. The Dirac fermions are the secondary composite objects, which appear in the electroweak phase transition. Since the Weyl fermions are primary objects, the action for gravity interacting with fermions should be written in terms of the Weyl fermions. The interaction of gravity with left and right fermions and with their anti-particles requires 4 different tetrads e_{aA}^μ , $A = 1, 2, 3, 4$:

$$\mathcal{L}_A = \Psi_A^+ e_{aA}^\mu p_\mu \sigma^a \Psi_A, \quad A = 1, 2, 3, 4, \quad \sigma^a = (1, \boldsymbol{\sigma}), \quad (29)$$

where $\boldsymbol{\sigma}$ are the Pauli matrices. These four tetrads can be obtained from each other by P_s and T_s transformations. Choosing one of the 4 tetrads as the reference frame, for example the tetrad e_{aR}^μ for right particles, one obtains the other three tetrads:

$$e_{aR}^\mu \equiv e_a^\mu, \quad e_{aL}^\mu = P_s e_a^\mu, \quad e_{a\bar{R}}^\mu = P_s T_s e_a^\mu, \quad e_{a\bar{L}}^\mu = T_s e_a^\mu. \quad (30)$$

This demonstrates the breaking of discrete symmetries P_s , T_s and $P_s T_s$ in the Minkowski vacuum.

Discussion of different tetrads for left and right Weyl fermions in Weyl materials can be found in Ref. [31, 32]. The topological invariants for different types of the Weyl points are discussed in Ref. [33, 34].

In our approach, four tetrads in Eq. (30) are connected by symmetry operations, which means that the corresponding metric field is unique:

$$g_{\mu\nu}^L \equiv g_{\mu\nu}^R \equiv g_{\mu\nu}. \quad (31)$$

9 Possible Bi-Metric Gravity from Multiple Tetrads

At low energies (well below the Planck scale), the original symmetry between the fermions is violated. For example, in the Standard Model, the symmetry between the left and right fermions is violated. Could this lead to the violation of Eq. (31), i.e., to the splitting of the metric field? The bi-metricity does not take place, if on the fundamental level the metric is unique. However, one can try the scenarios, when the broken symmetry causes the small splitting between the metrics experienced by left and right fermions. In this case, this could lead to the bi-metric gravity at low energy.

Let us for example consider the scenario, in which the metric fields of different fermions are equal due to terms in action describing diffeomorphism invariant interaction between the tetrads (or between the metrics, $U(g_{\mu\nu}^L, g_{\mu\nu}^R)$), and the minimum of this potential U corresponds to $g_{\mu\nu}^L = g_{\mu\nu}^R$. When the symmetry between the left and right fermions is violated, the potential U has corrections, $U = U_{\min} + \delta U$, where δU contains the diffeomorphism invariant corrections of first and second order in perturbation $g_{\mu\nu}^L - g_{\mu\nu}^R$:

$$\delta U \propto \epsilon (g_{\mu\nu}^L - g_{\mu\nu}^R) (g^{\mu\nu L} + g^{\mu\nu R}) + (g_{\mu\nu}^L - g_{\mu\nu}^R) (g^{\mu\nu L} - g^{\mu\nu R}), \quad (32)$$

where $\epsilon \ll 1$. The metric fields for left and right fermions become slightly different:

$$g^{\mu\nu L} - g^{\mu\nu R} = -\epsilon g^{\mu\nu}. \quad (33)$$

The ϵ correction to the potential comes from the integration over fermions. If the left and right fermions are different, the result will be different. This would mean that the Newton constant of gravitational interaction between fermions depends on their chiralities: there are 3 gravitational couplings, G_{RR}, G_{LL}, G_{LR} , i.e., the broken parity may lead to breaking of equivalence principle.

Different metrics for the left-handed and right-handed fermions has been also considered by Chadha and Nielsen [35], see also the so-called chiral gravity [36], and the bi-metric theory for different particles in Ref. [37]. Although the developing of such theories often suffers the ghost problem (Boulware–Deser ghost instability), see, e.g., Ref. [38], it is not excluded that due to the small value of the splitting in Eq. (33), this problem can be avoided. Anyway, the possible scenario of emergent bi-metricity requires the detailed study.

10 Tetrads and Cosmological Constant Problem

Finally, let us mention that the approach based on tetrads may have direct relation to the solution of the cosmological constant problem. The original approach developed in Ref. [39, 40], which leads to the nullification of the cosmological constant in the Minkowski vacuum, was based on the nonlinear extension of the Hawking consideration of the vacuum variable as the four-form gauge field. Here, we show that the role of the vacuum variable can be played by the tetrad determinant, $e \equiv \det e^a_\mu$. It is also the 4-form, but it is more natural for Standard Model than the 4-form gauge field. The simplest Einstein action in terms of the tetrad determinant is as follows:

$$S = - \int_{\mathbb{R}^4} d^4x e \left(\epsilon(e) + \frac{R}{16\pi G(e)} + \mathcal{L}^M(e, \psi) \right). \quad (34)$$

This action is invariant under coordinate transformations preserving the 4-volume. But it is not the unimodular gravity where $e = \text{const}$. Similar generalization of unimodular gravity has been considered in Ref. [41].

If one ignores the dependence of the matter action \mathcal{L} and gravitational coupling G on e , one obtains the Einstein equations with the cosmological term $\rho_{\text{vac}}(e)g_{\mu\nu}$, where the vacuum energy density is

$$\rho_{\text{vac}}(e) = \frac{\delta S}{\delta e} = \epsilon(e) + e \frac{de(e)}{de}. \quad (35)$$

The fully equilibrium Minkowski vacuum (without matter) corresponds to the extremum of the action:

$$\frac{\delta S}{\delta e} = 0. \quad (36)$$

According to Eq. (35), the equilibrium value $e = e_{\text{eq}}$ in Minkowski vacuum is determined by equation

$$\rho_{\text{vac}}(e_{\text{eq}}) = 0, \quad (37)$$

which in turn corresponds to the nullification of that vacuum energy, which enters Einstein equations. In this approach, the cosmological constant in the equilibrium Minkowski vacuum is zero without fine tuning: the Planck scale quantity $\epsilon(e)$ is compensated by $e de/de$.

Till now we considered positive determinant, $e > 0$. But this can be extended to negative e . In this case, one has two equilibrium states with opposite signs of e . The degenerate vacua are connected by the PT symmetry operation [4]. That is why the appearance of nonzero e corresponds to the symmetry breaking transition, at which the PT symmetry is broken [4] (together with other symmetries). The scale invariance is also violated in this transition.

The simplest example is

$$e\epsilon(e) = m^4 e \left(\frac{1}{5} \frac{e^4}{e_{\text{eq}}^4} - 1 \right), \quad \rho_{\text{vac}}(e) = m^4 \left(\frac{e^4}{e_{\text{eq}}^4} - 1 \right), \quad (38)$$

where the scale m could be the Planck energy scale.

11 Extension to Thermodynamics

One can extend the determinant e to the imaginary values. This corresponds to Euclidean metric in the 4D space (in condensed matter the transition between Minkowski and Euclidean metric for Goldstone modes see in Ref. [42, 43]). On the imaginary axis $e = i\beta$ the quantity β becomes the inverse temperature in the 4+0 thermodynamics. The path integral in quantum mechanics is formally identical to the partition function e^{-S} of a statistical mechanical system, with Hamiltonian $\epsilon(\beta)$ equal to the free energy:

$$S = \int_{\text{Eu}} d^4x \beta \left(\epsilon(\beta) + \frac{R}{16\pi G(\beta)} + \mathcal{L}^{\text{M}}(\beta, \psi) \right). \quad (39)$$

Then, the vacuum energy density becomes:

$$\Lambda \equiv \rho_{\text{vac}}(T) = \epsilon - T \frac{d\epsilon}{dT} = \epsilon + \beta \frac{d\epsilon(\beta)}{d\beta} = \frac{d(\beta\epsilon(\beta))}{d\beta} = \frac{dS(\beta)}{d\beta}. \quad (40)$$

The extremum of partition function suggests that in thermodynamic equilibrium and in the absence of external pressure, the cosmological constant is nullified:

$$\Lambda_{\text{equilibrium}} = \frac{dS_{\text{vac}}(\beta)}{d\beta} = 0. \quad (41)$$

Equation (41) determines the equilibrium value T_{eq} of temperature (and correspondingly the nonzero equilibrium value $e = e_{\text{eq}}$ in Minkowski vacuum). Example in Eq. (38) extended to imaginary $e = i\beta$ gives

$$\beta\epsilon(\beta) = m^4 \beta \left(\frac{1}{5} \frac{\beta^4}{\beta_{\text{eq}}^4} - 1 \right), \quad \rho_{\text{vac}}(\beta) = m^4 \left(\frac{\beta^4}{\beta_{\text{eq}}^4} - 1 \right). \quad (42)$$

So, the nonzero temperature of the 4+0 system gives rise to gravity in the 3+1 system. This is another connection between gravity and thermodynamics in addition to that suggested by Jacobson [44]. The original 4D quantum vacuum has $\beta = 0$, and this infinite temperature corresponds to complete chaos, where gravity is absent. The finite T_{eq} gives rise to the Einstein action and is responsible for the equilibrium Minkowski vacuum with equilibrium value of the Newton constant $G(e_{\text{eq}})$ and zero value of the cosmological constant. It is also related to the existence of the elementary (Planck scale) 4-volume.

While in thermodynamics the nullification of the cosmological constant in the full equilibrium is natural, in dynamics the de Sitter attractors may prevent the approach to the equilibrium state of the quantum vacuum [40].

12 Conclusion

We considered two scenarios of the formation of the combined symmetry describing the tetrad gravity interacting with fermions: the scenario of emergent symmetry suggested by analogy with the chiral superfluid ${}^3\text{He-A}$ and the scenario of symmetry breaking, which is suggested by the symmetry breaking pattern in topological superfluid ${}^3\text{He-B}$. In the first one, the topology of the Weyl points leads to the tetrads emerging in the vicinity of the Weyl point. In the second one, the gravitational tetrads e_a^μ appear as the order parameter of the symmetry breaking phase transition, which is the bilinear form of the fermionic fields [10–15].

Such broken symmetry phase transition gives rise to the degeneracy of the Minkowski vacua, the particular example—the broken PT symmetry in the Minkowski vacuum—was discussed by Vergeles [4]. In this scenario, the original trans-Planckian vacuum was invariant under coordinate transformations: the Lorentz L_c coordinate transformation and the space and time reversal transformations, $P_c \mathbf{r} = -\mathbf{r}$ and $T_c t = -t$. The tetrad order parameter, which leads to formation of gravity, spontaneously breaks these symmetries. The degenerate quantum vacua obtained after the symmetry breaking lose the invariance under the pure coordinate transformations. The broken symmetry vacua obey only the combined symmetries: $P = P_c P_s$ and $T = T_c T_s$ symmetries as well as combined Lorentz symmetry $L = L_c L_s$, where the coordinate transformation is accompanied by the corresponding rotation or reflection of the order parameter—the tetrad—in spin space.

Such mechanism of the formation of gravity leads to many interesting effects, including the topological defects formed by this symmetry breaking due to Kibble–Zurek mechanism, and to the four tetrad fields for four types of Weyl fermions: for left-handed fermions, for right-handed fermions and for their corresponding antiparticles. In principle, in some scenarios, the parity violation in Standard Model may lead to the breaking of equivalence principle.

The discussed symmetry breaking, $L_c \times L_s \rightarrow L = L_c L_s$, $P_c \times P_s \rightarrow P = P_c P_s$ and $T_c \times T_s \rightarrow T = T_c T_s$, probably takes place at the Planck or higher energy scale. At the lower energy scale, where the Standard Model operates, the combined symmetry of the degenerate vacuum is further reduced by the breaking of the discrete P and T symmetries. Which of these symmetry breaking transitions are responsible for the observed baryon asymmetry of the Universe and for the strong CP problem, is an open question raised in Ref. [4].

Finally, we considered the application of the tetrad fields for the solution of the cosmological constant problem.

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