The 3-Dimensional Fermi Liquid Description for the Iron-Based Superconductors

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Abstract The quasiparticles in the normal state of iron-based superconductors have been shown to behave universally as a 3-dimensional Fermi liquid. Because of interactions and the presence of sharp Fermi surfaces, the quasiparticle energy contains, as a function of the momentum $\mathbf{p}$, a term of the form $(p - p_0)^3 \ln(|p - p_0|/p_0)$, where $p = |\mathbf{p}|$ and p_0 is the Fermi momentum. The electronic specific heat coefficient, magnetic susceptibility (Knight shift), electrical resistivity, Hall coefficient and thermoelectric power divided by temperature follow, as functions of temperature T , the logarithmic formula $a - bT^2 \ln(T/T^*)$, a , b and T^* being constant; these formulae have been shown to explain the observed data for all iron-based superconductors. It is shown that the concept of non-Fermi liquids or anomalous metals which appears in the literature is not needed for descriptions of the present systems. When the superconducting transition temperature T_C and the b/a value for the resistivity are plotted as functions of the doping content x , there appear various characteristic diagrams in which regions of positive correlation and those of negative correlation between T_C and b/a are interconnected; from these diagrams, we may make speculations about the types of superconductivity and the crossover between them.

Keywords 3D-Fermi liquid · Fermi liquid effect · Logarithmic temperature dependence · Kink phenomenon · Electrical resistivity · Thermoelectric power · No non-Fermi liquid

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1 Introduction

Studies of the superconductivity of iron-based superconductors have been explored to a certain extent. Concerning the nature of the normal state, however, the conclusions have not yet been settled. Many authors [1–3] in this field describe the system of quasiparticles as a non-Fermi liquid. The present author [4], however, states that the normal state of the substances can be described as a truly normal 3-dimensional (3D) Fermi liquid. Because of the Fermi liquid effect which is caused by the presence of interactions and the sharp Fermi surfaces, most of thermodynamic and transport properties, e.g. the electrical resistivity, behave as $a - bT^2 \ln(T/T^*)$ as a function of temperature T , where a , b and T^* are constants.

According to the Green's function theory [5], the real part of the self-energy part, $\Sigma(\varepsilon)$, for the 3D Fermi liquid contain the $\varepsilon^3 \ln|\varepsilon|$ term, where ε is the quasiparticle energy measured from the Fermi energy μ ; this contributes the $\varepsilon^2 \ln|\varepsilon|$ term to the density-of-states function and the $T^2 \ln(T/T^*)$ term to the electronic specific heat coefficient, T^* being constant. The $\varepsilon^3 \ln|\varepsilon|$ dependence of Σ has been really observed in $\text{Ba}_{1-x}\text{K}_x\text{Fe}_2\text{As}_2$ [6]; thus, the 3-dimensionality of the system has been confirmed.

As has been discussed in the preceding paper [4], the b/a values for the resistivity are free from the nature of impurities and represent solely the inherent characters of the electron system. If we plot b/a and the superconducting transition temperature T_C as functions of the doping content x of the compounds, we obtain various kinds of characteristic diagrams for the arrangement of regions of positive correlation and those of negative correlation between b/a and T_C . By observing the diagrams, we may speculate about the types of superconductivity and the crossover between those.

Since the basic substance of the iron-based superconductors shows generally antiferromagnetism, the doped materials are believed also to be antiferromagnetic. Concerning $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ compounds, however, by analysing the susceptibility [7,8] and specific heat data [9], we find the system to be nearly ferromagnetic. The general view that antiferromagnetic fluctuations dominate these systems should not be taken as universal.

Concerning $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ compounds, Nakai et al. [10] have argued that this system exhibits the quantum critical point (QCP) near $x = 0.3$, since the effective mass determined by the de Haas–van Alphen effect [11] becomes very large. Shibauchi et al. [12] have also examined the QCP through measurements of the London penetration depth λ_L near absolute zero; λ_L is known as $\lambda_L^2 \propto m_s/n_s$, m_s and n_s being the mass and the number density of superconducting carriers. They have found that λ_L shows a steep peak near $x = 0.3$ and concluded that, because of the quantum fluctuations at absolute zero, m_s becomes very large and the system really undergoes the QCP. We have analysed the Knight shift experiment of the same system by Nakai et al. to find that the effective mass of carriers which is proportional to the density-of-states is not enhanced in the neighbourhood of $x = 0.3$; it is to be noted that how the interactions affect the effective mass varies with experiments.

We have already shown that the 3D Fermi liquid can possess the QCP in the presence of a magnetic field H . The free energy of the magnetized system has been shown to contain a term $M^4 \ln M$, M being the magnetization [13,14]. Thus, in terms of dimensionless H , M and T , the magnetization relation is given by

$$\tilde{H}/\tilde{M} = 1 + \lambda \tilde{M}^2 \ln(\tilde{M}^2 + \tilde{T}^2), \quad (1.1)$$

where λ is a positive constant representing the interaction strength. Because of this logarithmic form, the system is favourable for the appearance of a thermodynamic instability; the instability occurs when $\partial \tilde{H}/\partial \tilde{M} = 0$ has two solutions; at $\tilde{T} = 0$ this leads to $\lambda > \lambda_C = e^{5/3}/3 \approx 1.76$ for the condition of metamagnetic transitions. We have already shown that Laves phase compound $\text{Y}(\text{Co}_{0.92}\text{Al}_{0.08})_2$, $\lambda = 1.88$, [15] and MnSi under high pressures, $\lambda = 1.85$, [16] really undergo the transition, i.e. QCP.

The temperature dependence of the Hall coefficient [1], magnetoresistance [1] and thermoelectric power [17] has also been shown to follow logarithmic formulae; these formulae explain clearly the experimental facts. When Wilson wrote the book, “The Theory of Metals” [18], the theory could not account for the experiment of the thermoelectric power. Now, the (inverse) v -shape curves for Pt and iron-pnictide compounds can be explained by $\alpha T + \beta T^3 \ln(T/T^*)$ law, where α , β and T^* are constants.

Among the literature related to the Fermi liquid description of the iron-based superconductors, we may list the works by Popovich et al. [19] and by Charnukha et al. [20]. These authors have shown that the specific heat and optical properties of high purity $\text{Ba}_{0.68}\text{K}_{0.32}\text{Fe}_2\text{As}_2$ single crystal can be analysed without invoking the non-Fermi liquid concept. For description of the physics for highly correlated electron systems, we should list the work by Yamada and his colleagues [21, 22]. They have shown that the normal state of high- T_C -cuprates can be thoroughly described and clarified on the basis of the Fermi liquid theory. Matsuda [23] has pointed out that “the breakdown of the Fermi liquid theory” which appears frequently in the literature can be avoided through proper recognition to the differences of effective masses.

2 The Logarithmic Quasiparticle Energy in 3D Systems

We shall describe here the self-energy part of the quasiparticles in the normal state, $\Sigma(\mathbf{p}, \varepsilon)$, as required for the 3D Fermi liquid, where $\mathbf{p}$ is the momentum.

In 1958, Galitsky [24] showed that the quasiparticle energy of interacting fermions contains a $(p - p_0)^3 \ln(|p - p_0|/p_0)$ term, where $p = |\mathbf{p}|$, p_0 being the Fermi momentum. Subsequently, Migdal [5] discussed comprehensively the analytic form of the self-energy part in the Green function $G(\mathbf{p}, \varepsilon)$. Examining the lowest order diagrams for Σ , he showed that $\text{Im } \Sigma$ takes the form

$$\text{Im } \Sigma(\varepsilon) = \gamma \varepsilon |\varepsilon| + \alpha \varepsilon^3 + \alpha' \varepsilon^2 |\varepsilon|, \quad (2.1)$$

where γ , α and α' are constants. Here $\Sigma(\varepsilon)$ is not an analytic function of ε ; the analytic function in the upper half plane, $\sigma(\varepsilon)$, is given by

$$\text{Im } \sigma(\varepsilon) = \gamma \varepsilon^2 + \alpha \varepsilon^2 |\varepsilon| + \alpha' \varepsilon^3, \quad (2.2)$$

which states that, because of the presence of the Fermi surface, $\text{Im } \sigma$ has a discontinuity equal to $2\alpha \varepsilon^3$ when the sign of ε changes. Using the dispersion relation for $\sigma(\varepsilon)$, he derived

$$\Sigma(\varepsilon) = \Sigma_0(\varepsilon) + (2/\pi)\alpha\varepsilon^3 \ln(|\varepsilon|/\mu), \quad (2.3)$$

where μ is the Fermi energy (chemical potential), $\text{Re } \Sigma_0(\varepsilon)$ is expressed in a power series of ε , and $\text{Im } \Sigma_0(\varepsilon)$ is given by Eq. (2.1). The $\varepsilon^3 \ln |\varepsilon|$ term in Eq. (2.3) confirms the result of Galitsky. It should be noted that the logarithmic $\alpha\varepsilon^3 \ln(|\varepsilon|/\mu)$ term arises exclusively from the presence of the sharp Fermi surfaces in 3D systems.

From the $\varepsilon^3 \ln |\varepsilon|$ dependence of the quasiparticle energy, we have $\varepsilon^2 \ln |\varepsilon|$ dependence of the density-of-states function, and the $T^2 \ln T$ dependence of the electronic specific heat coefficient $\gamma(T)$. The $T^2 \ln T$ dependence of $\gamma(T)$ has been confirmed in liquid He^3 [25, 26], heavy fermion compounds [27, 28], and iron-based superconductors [4]. Here it should be remarked that, for heavy fermion CeAl_3 [27], an extremely clear $T^2 \ln T$ term of $\gamma(T)$ has been observed.

3 The Kink Phenomenon of the 3D Quasiparticle Self-Energy

The quasiparticle behaviours, energies and states around Fermi surfaces of $(\text{Ba}, \text{K})\text{Fe}_2\text{As}_2$ and $(\text{Sr}, \text{K})\text{Fe}_2\text{As}_2$ were thoroughly examined by Wray et al. [6] through the angle-resolved-photoemission spectroscopic (ARPES) experiments. They measured the self-energy of quasiparticles in the superconducting and normal states and found characteristic kinks in $\text{Re } \Sigma(\varepsilon)$ around $\varepsilon = 40 \text{ meV}$. They expected that the kinks might be associated with some collective modes such as phonons and spin fluctuations. They have found, however, that phonons are inadequate and the relation to the collective spin modes cannot be confirmed.

Here we analyse their ARPES results. In Fig. 4 of Ref. [6], the quasiparticle dispersion curves, $\text{Re } \Sigma(\varepsilon)$ and $\text{Im } \Sigma(\varepsilon)/2v_F$, v_F being the Fermi velocity, for normal and superconducting states of $(\text{Ba}, \text{K})\text{Fe}_2\text{As}_2$ are shown. We carefully read the experimental curves at 11 K of this figure, on the basis of Eq. (2.3), to obtain

$$\text{Re } \Sigma(\varepsilon) = -0.839 \varepsilon \left\{ 1 + \frac{\varepsilon}{200} + \left(\frac{\varepsilon}{71.2} \right)^2 \ln \frac{|\varepsilon|}{125} \right\} \text{ meV}, \quad (3.1)$$

$$\text{Im } \Sigma(\varepsilon) = 0.0063 \varepsilon^2 \left(1 + \frac{\varepsilon}{89} \right) \text{ meV}, \quad (3.2)$$

where we have used $v_F = 0.7 \text{ eV}\text{\AA}$ from Ref. [6]. In deriving Eq. (3.1), the zero point, $\text{Re } \Sigma(\varepsilon) = 0$, is shifted to $\varepsilon = -10.4 \text{ meV}$; this is caused possibly by experimental errors and an additional energy of breaking the Cooper pair. $\text{Re } \Sigma(\varepsilon)$, Eq. (3.1), is drawn in Fig. 1.

We should comment the accuracy of Eq. (3.1); the root-mean-square (rms) error of Eq. (3.1) to the original curve is found to be 0.39%. If the curve is expressed, without the logarithmic $\varepsilon^3 \ln |\varepsilon|$ term, by a power series up to ε^3 , ε^4 or ε^5 , the corresponding rms error is 1.93, 0.59 or 0.43%; this demonstrates the existence of the logarithmic term to be essential. The energy of the kink point is determined as 33.9 meV from Eq. (3.1). From Eqs. (3.1) and (3.2), α and α' in Eq. (2.1) are determined as

Fig. 1 Real part of the self-energy, $\text{Re } \Sigma(\varepsilon)$, for quasiparticles in $(\text{Ba}, \text{K})\text{Fe}_2\text{As}_2$ at 11 K is plotted as a function of ε . Circles show the experimental data [6], and the solid line is the theoretical curve, Eq. (3.1) in the text

$$\begin{aligned}\alpha &= -0.00026 \text{ (meV)}^{-2} \\ |\alpha'| &= 0.00019 \text{ (meV)}^{-2}.\end{aligned}\quad (3.3)$$

We have to note that, contrary to the general belief, the energy of the kink (34 meV) has no special meaning; if we assume that the $\varepsilon^3 \ln |\varepsilon|$ term is related to a collective or coherent motion with the characteristic energy σ , then we may have a term $a\varepsilon^3 \ln (|\varepsilon|/\sigma)$, to which we have to add ordinary $b\varepsilon^3$ term from Eq. (2.3), where a and b are constants; here a arises exclusively from the interactions, while b is the sum of an interaction-free (band) term and an interaction-dependent term. Since

$$a\varepsilon^3 \ln (|\varepsilon|/\sigma) + b\varepsilon^3 = a\varepsilon^3 \ln \left(|\varepsilon|/\sigma e^{-b/a} \right), \quad (3.4)$$

the kink energy is a fraction of $\sigma e^{-b/a}$, which is usually quite different from σ , since $e^{-b/a}$ ranges from 0 to ∞ , depending on the interaction strength.

4 The Electrical Resistivity

We have already stated the temperature dependence of the electrical resistivity, $\rho(T)$, for highly correlated electron system in Ref. [4]. Because of the importance of the problem, here we restate the essence of the problem.

For evaluating $\rho(T)$, we need the knowledge of the density-of-states function for 3D quasiparticles, $\nu(\varepsilon)$. The quasiparticle energy with momentum p for interacting

3D fermion systems is given, by Galitsky [24], in terms of the scattering length for 2 quasiparticles, a ,

$$\varepsilon(p) = \varepsilon(p_0) + O(s, s^2, s^3) - \lambda s^3 \ln |s|, \quad (4.1)$$

where $s = (p - p_0)/p_0$, and $O(s, s^2, s^3)$ denotes a sum of terms of order s , s^2 and s^3 ; $\lambda = (4/3\pi^2)(p_0^2/m)(k_F a)^2$ represents the interaction strength of the system, where m is the mass of a particle and $k_F = p_0/\hbar$ is the Fermi wave number. From this, $v(\varepsilon)$ for both spins is derived as

$$\begin{aligned} v(\varepsilon) &= \frac{8\pi p^2}{(2\pi\hbar)^3} \left(\frac{\partial \varepsilon}{\partial p} \right)^{-1} \\ &= \frac{mp_0}{\pi^2 \hbar^3} \left\{ 1 + O(\varepsilon, \varepsilon^2) + \left(\frac{k_F a}{\pi} \right)^2 \left(\frac{\varepsilon}{\varepsilon_0} \right)^2 \ln \frac{|\varepsilon|}{\varepsilon_0^*} \right\}, \end{aligned} \quad (4.2)$$

where $\varepsilon_0^* = p_0^2/2m^*$, $m^* = p_0/(\partial \varepsilon/\partial p)_{p_0}$ is the effective mass of a quasiparticle.

The electrical resistivity arises mainly by the Coulomb scattering and impurity scattering. In terms of the corresponding conductivities σ_C and σ_{imp} , we have

$$\rho(T) = (\sigma_C)^{-1} + (\sigma_{\text{imp}})^{-1}. \quad (4.3)$$

These conductivities are given, in terms of the corresponding relaxation time $\tau(\varepsilon)$, as

$$\sigma = -\frac{2e^2}{3} \int d\varepsilon v^2 v(\varepsilon) \tau(\varepsilon) \frac{\partial f}{\partial \varepsilon}, \quad (4.4)$$

where e is the electronic charge, $v^2 = (\partial \varepsilon/\partial p)^2$ is the velocity squared of the quasiparticle, and f is the Fermi distribution function, $f = (e^{\beta \varepsilon} + 1)^{-1}$ with $\beta = 1/k_B T$, k_B being the Boltzmann constant.

Here v^2 in Eq. (4.4) contains, according to Eq. (4.1), a logarithmic term

$$v^2 = v_0^2 \left\{ 1 + O(s) - \frac{8}{\pi^2} (k_F a)^2 s^2 \ln \frac{|s|}{s_1} \right\}, \quad (4.5)$$

where $v_0 = p_0/m$, and s_1 is a constant. For more general Fermi liquids, since $s = (m^*/p_0^2)\varepsilon + O(\varepsilon^2)$, v^2 behaves as

$$v^2 = v_0^2 \left\{ 1 + \eta_1 \varepsilon - \eta_2 \varepsilon^2 \ln \frac{|\varepsilon|}{\eta^*} \right\}, \quad (4.6)$$

where η_1 , η_2 and η^* are constants. Concerning $v(\varepsilon)$, we can write, as a generalization of Eq. (4.2),

$$\nu(\varepsilon) = \nu(0) \left\{ 1 + \nu_1 \varepsilon + \nu_2 \varepsilon^2 \ln \frac{|\varepsilon|}{\nu^*} \right\}, \quad (4.7)$$

where ν_1 , ν_2 and ν^* are constants.

Concerning the Coulomb scattering, because of the Umklapp processes, $\tau(\varepsilon)$ is given by $\tau_0(\varepsilon)/T^2$, and the conductivity is given by $\sigma_C = (rT^2)^{-1}$, where $\tau_0(\varepsilon)$ is a function of ε and r is a constant [29]. If we consider the effect of logarithmic terms in $\nu^2 \nu(\varepsilon)$ of Eq. (4.4), there appears a term of the order $T^4 \ln T$ for σ_C , which will be ignored in this paper. Thus we have, for the Coulomb resistivity,

$$\rho_C = rT^2. \quad (4.8)$$

Concerning the impurity scattering, the form of the relaxation time $\tau(\varepsilon)$ varies with assumptions; whether the scattering of a quasiparticle with impurities is treated as the single particle scattering, the Born approximation or unitary approximation, $\tau(\varepsilon)$ becomes independent of, inversely proportional to, or proportional to $\nu(\varepsilon)$. Thus, we have

$$\nu^2 \nu(\varepsilon) \tau(\varepsilon) = \nu_0^2 \nu(0) \tau(0) \left\{ 1 + \phi_1 \varepsilon + \phi_2 \varepsilon^2 \ln \frac{|\varepsilon|}{\phi^*} \right\}, \quad (4.9)$$

where ϕ_1 and ϕ^* are constants, and ϕ_2 takes $-\eta_2 + \nu_2$, $-\eta_2$, or $-\eta_2 + 2\nu_2$ for above three assumptions.

From Eqs. (4.4) and (4.9), σ_{imp} is obtained as

$$\sigma_{\text{imp}}(T) = \frac{2e^2}{3} \nu_0^2 \nu(0) \tau(0) \left\{ 1 + \frac{\pi^2}{3} \phi_2 (k_B T)^2 \ln \frac{T}{T_{\text{imp}}} \right\}, \quad (4.10)$$

where the order T^2 term, $(\pi^2/3)/\phi_2'(k_B T)^2$, is considered to be included in $T^2 \ln(T/T_{\text{imp}})$ term, since

$$\phi_2'(k_B T)^2 + \phi_2(k_B T)^2 \ln \frac{T}{T^*} = \phi_2(k_B T)^2 \ln \frac{T}{T^* e^{-\phi_2'/\phi_2}},$$

and $T_{\text{imp}} = T^* e^{-\phi_2'/\phi_2}$; $\sigma_{\text{imp}}(T)$; Eq. (4.10) is rewritten simply as $\sigma_0 + \sigma_1 T^2 \ln(T/T_{\text{imp}})$.

The total resistivity, Eq. (4.3), is given by

$$\begin{aligned} \rho(\tau) &= rT^2 + \left\{ \sigma_0 + \sigma_1 T^2 \ln \frac{T}{T_{\text{imp}}} \right\}^{-1} \\ &= \frac{1}{\sigma_0} - \frac{\sigma_1}{\sigma_0^2} T^2 \ln \frac{T}{T_{\text{imp}}} + rT^2 + O\left(T^4 (\ln T)^2\right) \\ &= \rho_0 - \rho_1 T^2 \ln \frac{T}{T_\rho^*} + O\left(T^4 (\ln T)^2\right), \end{aligned} \quad (4.11)$$

up to order T^2 , where $\rho_0 = \sigma_0^{-1}$, $\rho_1 = \sigma_1/\sigma_0^2$ and $T_\rho^* = T_{\text{imp}} \exp(\sigma_0^2 r/\sigma_1)$. It should be noted that the rT^2 Coulomb resistivity is included in the second term of Eq. (4.11).

If we make the ratio ρ_1/ρ_0 , since $\rho_1/\rho_0 = \sigma_1/\sigma_0 = (\pi^2/3)k_B^2\phi_2$, it does not depend on the character of impurities, but reflects solely the nature of the electron system; as seen from Eq. (4.10), $\rho_1/\rho_0 \propto \phi_2$ represents the relative magnitude of the logarithmic term in $v^2\nu(\varepsilon)$, v^2 or $v^2\nu^2(\varepsilon)$.

The observed resistivity of the normal state of LSCO cuprates [30], Ce-based oxy sulfide [31] and iron-based superconductors [4] has been confirmed to follow clearly the $\Delta\rho \propto T^2 \ln T$ law. It should be remarked that extremely clear agreement between theory and experiment has been attained in a heavy fermion compound, CeCu₄Ga [28].

5 Nearly Ferromagnetic Ba(Fe_{1-x}Co_x)₂As₂

It is generally believed that, in the iron-pnictide materials, the antiferromagnetic fluctuations play an important role for the behaviours of this system. We here examine the type and degree of the fluctuations by taking the Ba(Fe_{1-x}Co_x)₂As₂ system as an example.

In order to define the magnetic properties in the normal state, we have to find the value of Landau parameter F_0^a given by

$$\frac{\chi(0)}{\gamma(0)} = \frac{3}{\pi^2} \frac{k_B^2}{\mu_B^2} \frac{1}{1 + F_0^a}, \quad (5.1)$$

where $\chi(0)$ and $\gamma(0) = (2\pi^2/3)k_B^2\nu(0)$ are the magnetic susceptibility and electronic specific heat coefficient at 0 K, and μ_B is the Bohr magneton. In Eq. (5.1) F_0^a , which ranges $-1 < F_0^a < \infty$, specifies whether the system is nearly ferromagnetic, paramagnetic or nearly antiferromagnetic.

Here we shall find the F_0^a value for $x = 0.085$ specimen whose T_C , 23 K, is near the highest in this series. The magnetic susceptibility for this specimen has been determined, on the basis of Wang et al.'s data [7], in our preceding paper [4];

$$\chi(T) = 5.88 \times 10^{-4} \left\{ 1 - \left(\frac{T}{731} \right)^2 \ln \frac{T}{1930} \right\} \text{emu/Oe} \cdot \text{mol}. \quad (5.2)$$

This $T^2 \ln(T/T^*)$ variation can be confirmed by comparing with the Knight shift, $K(T)$, experiment by Ning et al. [8]. From their data for $x = 0.09$ in Fig. 3, we find

$$K(T) = 0.231 \left\{ 1 - \left(\frac{T}{791} \right)^2 \ln \frac{T}{2230} \right\} \%. \quad (5.3)$$

It is remarkable that the relative temperature variations for $\chi(T)$ and $K(T)$ are almost the same in spite of data by different groups.

The corresponding $\gamma(0)$ value can be obtained from the electronic specific experiments by Yoshizawa et al. [9,32]. Their data have shown that, for the range $50 \text{ K} \lesssim T \lesssim 100 \text{ K}$, $\gamma(T)$ values do not change for changes of doping x , $0 \leq x \leq 0.245$. Thus, we may assume that, if the system stays non-superconducting, the $\gamma(T)$ values for $0 \text{ K} \leq T \lesssim 50 \text{ K}$ also should not change with x ; $\gamma(0)$ value for $x = 0.085$ is the same as that for the non-superconducting sample $x = 0.245$. For $x = 0.245$, their data of $\gamma(T)$ can be precisely fitted by

$$\gamma(T) = 12.5 \left\{ 1 + \left(\frac{T}{7.43} \right)^2 \ln \frac{T}{1.64} \right\} \text{ mJ/mol} \cdot \text{K}^2. \quad (5.4)$$

Thus, by putting $\chi(0) = 5.88 \times 10^{-4} \text{ emu/Oe} \cdot \text{mol}$ and $\gamma(0) = 12.5 \text{ mJ/mol} \cdot \text{K}^2$ in Eq. (5.1), F_0^a for $x = 0.085$ is obtained as

$$F_0^a = -0.71. \quad (5.5)$$

It is remarkable that the system is not nearly antiferromagnetic, but nearly ferromagnetic. The spin susceptibility is enhanced by $(1 + F_0^a)^{-1} = 3.4$ times compared with the value evaluated by the real (exact) density-of-states. The frequently mentioned statement that, because of the antiferromagnetic fluctuations, the system tends to be a non-Fermi liquid is not applicable to this system.

6 The 3D Fermi Liquid Description for the $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ System

Whether the system of quasiparticles in the normal state of iron-pnictide superconductors is a Fermi liquid or non-Fermi liquid has been a subject of principal concern for a long time. One of the most notorious iron-pnictides is the $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ system in the neighbourhood of $x = 0.3$.

Kasahara et al. [1], Nakai et al. [10] and Shishido et al. [11] have performed extensive experimental studies using high-quality single crystals of this substance; measurements of the resistivity, Knight shift, spin-lattice relaxation time, $1/T_1 T$, and de Haas–van Alphen effect have been taken. They have asserted that the 2-dimensional antiferromagnetic fluctuations play a central role for the non-Fermi liquid behaviour of this system; in the superconducting dome, the system has an antiferromagnetic critical point at $T = 0$ and $x = 0.33$; this is also a quantum critical point (QCP). For reasons of the non-Fermi liquid nature of the system, they have presented 3 points; outside and inside of the dome the effective mass of the quasiparticle is greatly enhanced near $x = 0.33$; the resistivity obeys the T -linear law and the Kohler's rule is violated.

On the contrary, we shall show that the system can be described in terms of the 3D normal Fermi liquid; outside of the superconducting dome in the T vs x phase diagram, the T -dependence of the resistivity, Hall coefficient and Knight shift follows the $T^2 \ln(T/T^*)$ variation. The reason why the Kohler's rule is not valid in this system is presented. Even inside of the dome, we can show that, from the analysis of the Knight shift experiment, the itinerant carriers behave as a 3D normal Fermi liquid.

Fig. 2 Temperature dependence of the electrical resistivity of $\text{BaFe}_2(\text{As}_{0.67}\text{P}_{0.33})_2$ from Ref. [1]. As described in the text, the T -dependence is analysed on the basis of 10 experimental points of the curve

First we shall analyse their resistivity data for $\text{BaFe}_2(\text{As}_{0.67}\text{P}_{0.33})_2$ in Fig. 3 of Ref. [1]. The authors of this Ref. have reported the results on the basis of the empirical law; $n = 1.0$ or 1.05 in the formula $\rho(T) = \rho_0 + AT^n$, where ρ_0 , A and n are constants. They have claimed that this system is a non-Fermi liquid, since n is highly deviated from 2. We reexamine their results; as shown in Fig. 2, we choose ten representative points from 40 K to 130 K on the curve, carefully read the points, and try to find the best fitting value of n . We have selected six n -values from $n = 1.00$ to 1.15 , and evaluate the rms error in %, δ , as a function of n to find, $\delta = 0.493 + 40.75(n - 1.069)^2$; the best fitting value of n is 1.07 , the error being 0.49% , and the resistivity is given by

$$\rho(T) = 32.2 + 0.801T^{1.07} \mu\Omega \cdot \text{cm} . \quad (6.1)$$

Correspondingly, the resistivity formula for the $T^2 \ln T$ variation is obtained as

$$\rho(T) = 49.3 \{1 - (T/85.1)^2 \ln (T/398)\} \mu\Omega \cdot \text{cm} , \quad (6.2)$$

and the error is 0.164% which is just $1/3$ of that of the $T^{1.07}$ law. The $T^2 \ln T$ law reproduces the experimental data the most accurately.

Generally stating, if one adopts the $\rho(T) = \rho_0 + AT^n$ law, $\partial^2 \rho / \partial T^2 \propto n(n - 1)T^{n-2}$ should be positive for $n > 1$. If one observes the experimental data for $\text{Ba}_2\text{Fe}_2(\text{As}_{1-x}\text{P}_x)$ compounds, Fig. 1a of Ref. [1], 6 curves from $x = 0.14$ to 0.41 are convex for the temperature range, $T_C \lesssim T \leq 300$ K, and $\partial^2 \rho / \partial T^2 < 0$. Thus, the $\Delta \rho \propto T^n$ law cannot describe the experimental data.

Concerning a difficulty concerning the Hall coefficient, R_H , which exceeds $1/ne$, n being the number of electrons (or holes), they have argued that quasiparticle scattering times are different on different parts of Fermi surfaces. It should be remarked that the simple two-band model is not applicable to this system. The difficulty has arisen from

Fig. 3 Temperature dependence of the Hall coefficient, $R_H(T)$, for $\text{BaFe}_2(\text{As}_{0.67}\text{P}_{0.33})_2$. Circles show the experimental data [1], and the solid line is the theoretical fit to the data from Eq. (6.4) in the text

their way of analysing the experimental data. They have fitted their data, Fig. 3 of Ref. [1], through the formula

$$R_H(T) = -0.0015 \left(1 + \frac{32}{T} \right) \text{ cm}^3/\text{C}, \quad (6.3)$$

for the temperature range $30 \text{ K} \lesssim T \lesssim 150 \text{ K}$. According to this, $R_H(T)$ would decrease indefinitely when T tends to zero.

Here we seek for the plausible form of $R_H(T)$ for multi-Fermi sphere systems. It is known, for two band metals with equal number of electrons and holes, R_H is proportional to $(\rho_e - \rho_h)/(\rho_e + \rho_h)$, where ρ_e and ρ_h are resistivity functions for electrons and holes. On the principle of analogy, for the multi-Fermi sphere system, R_H can be written as an algebraic function (functional) of plural resistivity functions arising from plural Fermi surfaces. Since each resistivity can be expanded in powers of T^2 and $T^2 \ln T$, the resultant R_H can be expressed in the $a - bT^2 \ln(T/T_H^*)$ form, a , b and T_H^* being constant; R_H can never be written in the form of Eq. (6.3). In fact, their data [1], Fig. 3, can be fitted by

$$R_H(T) = -0.00305 \left\{ 1 + \left(\frac{T}{149} \right)^2 \ln \frac{T}{217} \right\} \text{ cm}^3/\text{C}. \quad (6.4)$$

This curve is plotted in Fig. 3; Eq. (6.4) is seen to reproduce precisely the experimental data within experimental errors.

They have asserted that the apparent violation of the Kohler's rule is a direct proof of the non-Fermi liquid nature of the system. The Kohler's rule states that, in the presence of a magnetic field H , for the resistivity $\rho(T, H)$, $\Delta\rho(H)/\rho(T) = \{\rho(T, H) -$

$\rho(T, 0)/\rho(T)$ is given by a function of $\mu_B H/\rho(T)$ alone. Hereafter we define $\xi = (\mu_B H/\rho(T))^2$, $\eta = \Delta\rho(H)/\rho(T)$ and the Bohr magneton $\mu_B \equiv 1$. If the Kohler's rule is valid, $\eta(\xi)$ should be a universal function of ξ ; their experimental data, Fig. 3 of Ref. [1], however, have been shown not to satisfy this requirement; η depends on both ξ and T .

Now we shall prove that the Kohler's rule cannot be applied to the 3D Fermi liquid. Since, in the theory of magnetoresistance [33], the effect of melting of the Fermi surface by H^2 is the same as that by T^2 , $\rho(T, H)$ is given by

$$\rho(T, H) = \rho_0 + \rho_1 T^2 \ln(a_1 T^2 + b_1 H^2) + \rho_2 H^2 \ln(a_2 T^2 + b_2 H^2), \quad (6.5)$$

where ρ 's, a 's and b 's are constants. If we expand this in powers of H^2 up to H^4 , we have

$$\eta = A_1 \xi + A_2 \xi^2, \quad (6.6)$$

where

$$A_1 = \left\{ \frac{b_1 \rho_1}{a_1} + \rho_2 \ln(a_2 T^2) \right\} \rho(T), \quad (6.7)$$

and

$$A_2 = \left(\frac{b_2 \rho_2}{a_2} - \frac{b_1^2 \rho_1}{2a_1^2} \right) \frac{\rho(T)^3}{T^2}. \quad (6.8)$$

Here $\rho(T) = \rho(T, H = 0)$. The validity of Eq. (6.6) can be confirmed by examining whether or not $A_1/\rho(T)$ is a linear function of $\ln T$, and $A_2 T^2/\rho(T)^3$ is a constant or not. By analysing their experimental data of $x = 0.33$ sample for the range $40 \text{ K} \leq T \leq 100 \text{ K}$, we find that $A_1/\rho(T)$ is precisely fitted by

$$A_1/\rho(T) = (6.71 - 1.33 \ln T) \times 10^{-3} \mu\Omega \cdot \text{cm} \cdot \text{K}^{-4} \cdot \text{T}^{-2}, \quad (6.9)$$

and $A_2 T^2/\rho(T)^3$ is constant, $0.0018 \mu\Omega \cdot \text{cm} \cdot \text{K}^{-2} \cdot \text{T}^{-4}$ within 8% scattering. Thus, the violation of the Kohler's rule is not due to the non-Fermi liquid effect but represents the intrinsic nature of the 3D Fermi liquid.

Nakai et al. [10] have discussed the appearance of QCP in this compounds in the vicinity of $x = 0.3$. They have found experimentally that the spin-lattice relaxation, T_1 , follows $1/T_1 T = A + C/(T + \theta)$, A , C and θ being constants; θ changes sign at $x \sim 0.3$ where the antiferromagnetic transition occurs. This transition persists to absolute zero, thus the appearance of QCP. Shishido et al. [11] have determined the effective mass m^* through measuring the level spacing of the de Haas–van Alphen experiment, $e\hbar H/m^*c$, where e and c are the electronic charge and light velocity. Their results are reproduced in Fig. 4; m^*/m increases steeply from 1.9 to 3.2 for decreasing x from 0.64 to 0.41.

Fig. 4 Effective mass ratio m^*/m and the Knight shift ratio $\kappa(x) \equiv K_{\text{spin}}(0, x)/K_{\text{spin}}(0, x = 0.64)$ for $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ are plotted as functions of x . Solid circles show the experimental data, reproduction of Fig. 3b of Ref. [11], and open circles are deduced from the experimental data of Ref. [10]; solid lines are smoothed curves

Shibauchi et al. [12] have also examined the presence of QCP for the same substance. They have measured the London penetration depth λ_L near absolute zero by three different methods to find that λ_L shows a steep peak near $x = 0.3$; λ_L^2 is proportional to the mass of superconducting carriers. They have concluded that this effective mass enhancement is caused by the quantum fluctuations arising from the QCP transition at absolute zero. In relation to this, Yanase et al. [21] have shown theoretically that λ_L becomes longer if the vertex correction for the transport equation is rightly considered.

We reexamine the effective mass enhancement on the basis of Nakai et al.'s experiment for the Knight shift $K(T)$. $K(T)$ consists of the T -independent chemical shift, K_{chem} , and T -dependent $K_{\text{spin}}(T)$ which is proportional to the spin susceptibility $\chi(T)$; K_{chem} is estimated as $0.018 \pm 0.019\%$. We carefully read the experimental data and find that the data follow the $T^2 \ln T$ dependence; for instance, $x = 0.33$ specimen above T_C , we obtain

$$K(T) = 0.128 \left\{ 1 - \left(\frac{T}{888} \right)^2 \ln \frac{T}{382} \right\} \%, \quad (6.10)$$

where 0.128% is the extrapolated value to 0 K from the curve above T_C ; $K_{\text{spin}}(0) = 0.110\%$. It is known that the susceptibility $\chi(0)$ and hence $K_{\text{spin}}(0)$ is proportional to $(m^*/m)/(1 + F_0^a)$. In Fig. 4 we plot $\kappa(x) \equiv K_{\text{spin}}(0, x)/K_{\text{spin}}(0, x = 0.64)$ on the basis of 5 experimental points from $x = 0.27$ to 0.64 . It is seen that $\kappa(x)$ is almost constant. Thus, we conclude either that m^*/m and F_0^a simultaneously tend to infinity near $x = 0.33$ because of the antiferromagnetic transition, or that the both quantities hardly change with x . Our conclusion is that the effective mass enhancement is not confirmed here.

Fig. 5 Temperature dependence of the Knight shift experiment, $K(T)$, for $\text{Ba}_2\text{Fe}_2(\text{As}_{1-x}\text{P}_x)_2$ compounds, $x = 0.33$ and 0.41 . Circles are experimental data [34], and solid lines show theoretical fit to the data from Eqs. (6.11) and (6.12) in the text

It should be remarked that how the interactions affect the effective mass varies with experiments. In the de Haas–van Alphen experiment, m^* is determined by measuring the level spacing of Landau levels, while m^* obtained from the susceptibility is a quantity proportional to the density-of-states on the Fermi surface. These two effective masses are irrelevant.

It is important to clarify the nature of the superconducting state of this system. We pay attention to the Knight shift experiment shown in Fig. 5, the reproduction of Fig. 1 of Ref. [34]. It should be noted that, for $x = 0.33$ and 0.41 , $T < T_C$, the system does not show perfect diamagnetism but show T -dependent paramagnetism; the superconductivity is nodal, and appreciable numbers of itinerant carriers are present. For $x = 0.33$, $K(T)$ decreases almost linearly with T from 0.13% to nearly zero within the uncertainty of K_{chem} ; the system seems to be nearly antiferromagnetic. By carefully observing the experimental data for $x = 0.41$, we can find that the data follow the $T^2 \ln T$ variation. It is surely shown that $K(T)$ for $x = 0.33$ and 0.41 follows

$$K(T) = 0.0086 - \left(\frac{T}{96.2} \right)^2 \ln \frac{T}{87.5} \%, \quad (6.11)$$

and

$$K(T) = 0.0721 - \left(\frac{T}{128} \right)^2 \ln \frac{T}{84.1} \%. \quad (6.12)$$

Thus, the system of itinerant carriers in the superconducting dome is truly a 3D normal Fermi liquid.

From the above Knight shift data, we can estimate the number density of non-superconducting carriers, n^* , in the superconducting state. Since $\chi(0)$, and hence $K_{\text{spin}}(0)$, is proportional to $n^{1/3}$, on the crude assumption that $(m^*/m)/(1 + F_0^a)$ is near to 1, from Eqs. (6.10), (6.11) and (6.12), we obtain $n^*/n = (0.0086/0.128)^3 = 0.00030$ for $x = 0.33$ and $(0.0721/0.128)^3 = 0.176$ for $x = 0.41$. Thus, 0.03% ($x = 0.33$) and 18% ($x = 0.41$) of electrons (or holes) on the Fermi surfaces are free

Fig. 6 Temperature dependence of the thermoelectric power $Q(T)$ for $\text{LaFe}_{1-x}\text{Co}_x\text{AsO}_{0.89}\text{F}_{0.11}$ compounds, $x = 0.0125, 0.05$ and 0.1 , above the crystalline structural transition temperature. Circles show the representative points of the experimental data [17], and solid lines are the theoretical fit to the data from Eq. (7.5) in the text. The inset shows the $Q(T)$ curve for Pt which appeared in the Wilson's book [18]; the theory, Eq. (7.4), fits the curve precisely up to about 150 K

carriers; Nakai et al. [10] have reported that the analysis of the nuclear spin-lattice relaxation time leads to $n^*/n = 0.34$ for $x = 0.33$. In order to confirm these values, we should consult other experiments such as the specific heat measurement.

7 The Thermoelectric Power of $\text{LaFe}_{1-x}\text{Co}_x\text{AsO}_{0.89}\text{F}_{0.11}$

In the classic weighty book by A. H. Wilson, “The Theory of Metals”, published in 1953, he described precisely the thermoelectric power of metals [18]. He showed there experimental data on the thermoelectric power Q of three metals; the Q vs T curve of Pt is reproduced in the inset of Fig. 6. Being faced with these data, he wrote, “The behaviour at very low temperatures is still more complicated and cannot at the moment be reconciled with the theory”. The inverse- v shape, or peak, structure of the Q vs T curve at low temperatures was not able to be explained at that time.

The thermoelectric power is defined as an average of ε given by

$$Q = -(eT)^{-1} \frac{\int d\varepsilon \varepsilon v^2 v(\varepsilon) \tau(\varepsilon) \partial f / \partial \varepsilon}{\int d\varepsilon v^2 v(\varepsilon) \tau(\varepsilon) \partial f / \partial \varepsilon}. \quad (7.1)$$

Here we need to consider higher order term in Eq. (4.9);

$$v^2 v(\varepsilon) \tau(\varepsilon) = v_0^2 v_0 \tau_0 \left\{ 1 + \phi_1 \varepsilon + \phi_2 \varepsilon^2 \ln \frac{|\varepsilon|}{\phi^*} + \phi_3 \varepsilon^3 \ln \frac{|\varepsilon|}{\phi^{**}} \right\}, \quad (7.2)$$

where ϕ_3 and ϕ^{**} are constants. After taking the average, Q is obtained as

$$Q(T) = -\frac{\pi^2}{3} \frac{k_B^2}{e} \phi_1 T \left\{ 1 + \left(\frac{7\phi_3}{5\phi_1} - \frac{2}{3}\phi_2 \right) \pi^2 (k_B T)^2 \ln \frac{T}{T_Q^*} \right\}, \quad (7.3)$$

up to order T^3 , where T_Q^* is a constant which depends on ϕ^* , ϕ^{**} , ϕ_2 and ϕ_3 .

We apply Eq. (7.3) to reproduce Q of Pt, and obtain

$$Q(T) = 0.148T \left\{ 1 + \left(\frac{T}{121} \right)^2 \ln \frac{T}{276} \right\} \mu\text{V/K}. \quad (7.4)$$

This formula reproduces precisely the experimental data in the inset of Fig. 6 up to 150 K; thus, the appearance of the peak is explained by the Fermi liquid model. In order to get agreement at higher temperatures, we need to add the higher order term, $q_2 T^5 \ln(T/T_Q^{**})$, where q_2 and T_Q^{**} are constants. We have already shown that the thermoelectric power of a prototype heavy fermion compound, CeAl₃, can be explained by this law [27].

Concerning the experimental study of iron-pnictide-compounds, we here take Q of La Fe_{1-x}Co_xAsO_{0.89}F_{0.11} examined by Sato et al. [17]. On the basis of extensive experimental data including Q and R_H , they have described these substances as an “anomalous metal”. Here we shall show that the substances are not anomalous; the T -dependence of Q for this system is analysed by the form

$$Q(T) = -q_1 T \left\{ 1 + \left(\frac{T}{T_1} \right)^2 \ln \frac{T}{T_Q^*} \right\}. \quad (7.5)$$

In Fig. 6, their data for $x = 0.0125, 0.05$ and 0.1 in Fig. 4a [17] and our analysis are presented. As shown in the figure, the data for the temperature range above the crystalline structural transition temperature, $75 \text{ K} \lesssim T \lesssim 250 \text{ K}$, can be precisely fitted by Eq. (7.5) with $q_1 = 1.28, 1.03$ and $0.631 \mu\text{V/K}^2$, $T_1 = 223, 233$ and 263 K and $T_Q^* = 479, 491$ and 504 K ; these q_1 values (the absolute value) are from 4 to 9 times larger than that of Pt, but about 1/30 of that of CeAl₃.

We have also analysed the experimental data of the Hall coefficient of this system and find that the data follow the $a - bT^2 \ln(T/T^*)$ law. Thus, our conclusion is that the quasiparticles of this system are not “anomalous” but form a real 3D Fermi liquid.

8 The ρ_1/ρ_0 and T_C vs x Diagrams and Superconductivity Types

As mentioned in Sec. 4, the ρ_1/ρ_0 values for $\rho(T)$ represent the inherent nature of electrons; from the ρ_1/ρ_0 values we may obtain precious information concerning the electron system. As described in the preceding paper [4], we have found that, for Ba(Fe_{1-x}Co_x)₂As₂, the ρ_1/ρ_0 values have a close correlation with the superconducting transition temperature T_C as a function of the doping content x . In the original BCS theory, the Cooper pair is formed between two quasiparticles whose momenta differ

by $2p_0$, while the logarithmic quasiparticle energy, $\epsilon^3 \ln \epsilon$, arises mainly from the interactions of two quasiparticles whose momenta are almost equal or differ nearly by $2p_0$; thus, the strength and the nature of this interaction is closely connected with T_C -values. It is frequently stated in the literature that the spin density fluctuations produce s_{+-} -wave superconductivity [35,36], while the charge density (or orbital) fluctuations produce s_{++} -wave superconductivity [37,38]. It is also stated that nodeless s_{++} superconductors have relatively high T_C values while nodal s_{++} superconductors have low T_C values.

When our analyses proceed to other compounds, however, we shall find that the correlation between ρ_1/ρ_0 and T_C is not always positive; each compound has a proper characteristic diagram in which the regions of positive, negative and no correlation are variously intermingled. The variety of the ρ_1/ρ_0 and T_C vs x diagrams reflects the variety of the electron correlation; the point of correlation change may suggest the crossover of the superconductivity types.

In order to confirm above views, we shall present to analyse the ρ_1/ρ_0 and T_C vs x diagrams of various superconductors.

First, in Fig. 7a, we reproduce the T_C and ρ_1/ρ_0 vs x diagram of $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ for the range $0 \leq x \lesssim 0.18$ deduced from Chu et al.'s experiment [39]; Wang et al.'s experiment [7] yields essentially the same figure. Here we see a clear positive correlation between ρ_1/ρ_0 and T_C ; the superconductivity seems to arise mainly via intra-band parings. Since the single particle excitations are not observed in the specific heat experiment, the superconductivity seems to be nodeless s_{+-} -wave state. As has been discussed in 5., however, because this system is nearly ferromagnetic, we should carefully examine the possibility of the p -wave superconductivity similar to liquid He^3 and UPt_3 .

Next we take up $\text{PrFeP}_{1-x}\text{As}_x\text{O}_{0.9}\text{F}_{0.1}$ system for which Takemori et al. [40] and Miyasaka et al. [3] have measured $\rho(T)$; in Fig. 7b, the ρ_1/ρ_0 and T_C vs x diagram is shown from the $\rho(T)$ data by Takemori et al. for the range $100 \text{ K} \leq T \leq 300 \text{ K}$; Miyasaka et al.'s data give almost the same diagram. The figure shows that the system can be classified to 2 regions of x ; for $0 \leq x \lesssim 0.6$, T_C is relatively low and ρ_1/ρ_0 and T_C show a negative correlation, while for $0.8 \lesssim x \leq 1.0$, T_C is high and they show a positive correlation. Miyasaka et al. have mentioned anomalies of this system for the range $0.6 \lesssim x \leq 0.8$; they have stated that, although the antiferromagnetic fluctuations seem to play the principal role for this superconductivity, the spin fluctuation theory cannot explain observed anomalies. By observing Fig. 6b, we see that, just in this range, the system seems to exhibit the transition; the crossover of states from s_{++} to s_{+-} -wave occurs.

In Fig. 7c, the ρ_1/ρ_0 and T_C vs x diagram for $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ system from Kasahara et al.'s data [1] is presented. At a glance, T_C and ρ_1/ρ_0 have an intimate correlation as functions of x . By a close look, however, although ρ_1/ρ_0 increases by 5 times for the change of x from 0.27 to 0.33, T_C keeps constant at about 30 K; there is no correlation for the range of x up to 0.40. Kontani and Onari [37] have described, with consideration of the electron-phonon interaction, the superconducting state showing a crossover from s_{+-} -wave to s_{++} -wave state as the impurity concentration increases. Here it should be remarked that, as shown in 6., the Knight shift inside the super-

Fig. 7 ρ_1/ρ_0 and T_C vs x diagrams for, **a** $\text{Ba}(\text{Fe}_{1-x}\text{Co}_x)_2\text{As}_2$ compounds from Chu et al.'s experiment [39], **b** $\text{PrFeP}_{1-x}\text{As}_x\text{O}_{0.9}\text{F}_{0.1}$ compounds from Miyasaka et al.'s experiment [3], **c** $\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$ compounds from Kasahara et al.'s experiment, [1] **d** $\text{LaFeP}_{1-x}\text{As}_x\text{O}_{0.9}\text{F}_{0.1}$ compounds from Miyasaka et al.'s experiment [3], **e** $\text{NdFe}_{1-x}\text{Ru}_x\text{AsO}_{0.89}\text{F}_{0.11}$ compounds from Lee et al.'s experiment [41], and **f** $\text{LaFe}_{1-x}\text{Co}_x\text{AsO}_{0.89}\text{Fe}_{0.11}$ compounds from Sato et al.'s experiment [17]; solid and open circles show T_C and ρ_1/ρ_0 values, and solid lines are smoothed curves

conducting dome exhibits partly the normal state values; this may be the proof of the p -wave superconductivity for which we need more careful observation.

In Fig. 7d, the ρ_1/ρ_0 and T_C vs x diagram for $\text{LaFeP}_{1-x}\text{As}_x\text{O}_{0.9}\text{F}_{0.1}$, from Miyasaka et al.'s experiment [3], is shown. Here clear correlation between T_C and ρ_1/ρ_0 is not

Fig. 8 A schematic diagram for obtaining higher- T_C superconductors on the basis of the ρ_1/ρ_0 and T_C vs x diagrams

seen; low T_C and high T_C regions are clearly separated in the vicinity of $x = 0.3$ where the crossover from s_{++} to s_{+-} -wave state seems to occur. In view of high- T_C and low ρ_1/ρ_0 values for $x \gtrsim 0.4$, here the interband pairing seems to be dominant.

The diagram for $\text{NdFe}_{1-x}\text{Ru}_x\text{AsO}_{0.89}\text{F}_{0.11}$ is presented in Fig. 7e from Lee et al.'s experiment [41]. Remarkably, here is found no correlation between T_C and ρ_1/ρ_0 ; although, for a change of doping x from 0 to 0.1, ρ_1/ρ_0 decreases to $1/60$, T_C decreases only by 10 K (2%); in spite of that the small Ru-doping affects tremendously the electronic states, it hardly affects the electrons (or holes) which are associated with superconductivity. Lee et al. have stated that, in the small x -region, the lowering of T_C is too small to be explained by the s_{+-} -wave state. We cannot judge, by simply observing Fig. 7e, whether, in the vicinity of $x = 0.3$, the crossover from s_{+-} to s_{++} -wave state really occurs or not; Eliashberg equation for 3D system may give the definite answer.

Finally, the diagram for $\text{LaFe}_{1-x}\text{Co}_x\text{AsO}_{0.89}\text{F}_{0.11}$ is presented in Fig. 7f from Sato et al.'s experiment [17]. Here we find very clear correlation between ρ_1/ρ_0 and T_C ; when the ρ_1/ρ_0 value becomes negative, the superconductivity vanishes. From the diagram, it is clearly seen that the interaction of two quasiparticles whose momenta differ by $2p_0$ is attractive for $0 \leq x \leq 0.075$.

9 A Program for Obtaining Higher- T_C Superconductors

By observing diagrams presented in the preceding section, we can pursue the possibility of obtaining higher- T_C superconductors. In Fig. 7b for $\text{PrFeP}_{1-x}\text{As}_x\text{O}_{0.9}\text{F}_{0.1}$, if we can find a substance whose electronic structure (Fermi surfaces) is situated on a virtual system $x_A (> 1)$ as the continuation of the ρ_1/ρ_0 curve for $0.8 \lesssim x \leq 1$, as shown in Fig. 8, then we expect the corresponding T_C value, T_A , may be much larger than T_C at $x = 1$, T_M ; thus, the problem is whether or not we may find compounds

situating on this line by using As, Sb or/and Bi. The same argument can be applied to the compound $\text{NdFeP}_{1-x}\text{As}_x\text{O}_{0.9}\text{F}_{0.1}$.

Furthermore, on the basis of Fig. 7e for $\text{NdFe}_{1-x}\text{Ru}_x\text{AsO}_{0.89}\text{F}_{0.11}$, if we can find the virtual system $x < 0$, as the smooth continuation of the ρ_1/ρ_0 curve to $x < 0$, then we may get higher T_C substances; the problem is whether or not we can find such transition elements in place of Ru in $\text{NdFe}_{1-x}\text{Ru}_x\text{AsO}_{0.89}\text{F}_{0.11}$.

10 Conclusions

We have shown that the itinerant carriers (electrons or holes) in the normal state of all iron-based superconductors form strictly a 3D Fermi liquid. Because of interactions and the presence of sharp Fermi surfaces, the thermodynamic and transport properties obey the logarithmic formula with respect to T^2 , H^2 or M^2 , M being the magnetization; the electronic specific heat coefficient, magnetic susceptibility, electrical resistivity, Hall coefficient and thermoelectric power divided by temperature follow the formula $a - bT^2 \ln(T/T^*)$. This formula explains observations for the normal state of all iron-based superconductors. Remarkably, in certain cases, even the itinerant carriers inside the superconducting dome behave as a 3D normal Fermi liquid.

In most papers, in this field the authors describe the resistivity through the formula $\rho = \rho_0 + AT^n$, ρ_0 , A and n being constant; when n deviates from 2, they call the system a non-Fermi liquid. In view of the present paper, the notion of non-Fermi liquids or anomalous metals is not needed.

Finally, in order to determine theoretically the superconductivity type and transition temperature, we have to solve the Eliashberg equation on the basis of the 3D system.

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