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Nonlinear Transport of Positive Ions Below a Free Surface of Topological Superfluid ^{3}He-B

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Abstract The B phase of the topological superfluid ^{3}He (^{3}He-B) shows many exotic phenomena at a surface as well as in bulk. In this article, we describe the first study of the nonlinear transport behavior of positive ions trapped below a free surface of ^{3}He-B measured at very low temperatures (below 300 μK), investigated for the purposes of observing the Landau critical velocity for pair breaking and finding a signature of the Majorana surface Andreev bound states. We observe that nonlinearity sets in at a velocity smaller than the Landau critical velocity for bulk pair breaking, and find that the nonlinear behavior does not depend on the depth of the trapped ions even though the depth is varied over the coherence length. The lack of depth dependence indicates that the nonlinear behavior does not reflect properties of the surface Andreev bound states. The onset of the nonlinearity at a velocity smaller than the bulk pair breaking velocity might be due to the inelastic scattering of bulk quasiparticles.

Keywords Transport of ion · Surface Andreev bound states · Pair breaking · Inelastic scattering

1 Introduction

Superfluid ^{3}He is a condensate of Cooper pairs with a relative orbital angular momentum $L = 1$ (p-wave) and a total spin $S = 1$ (spin-triplet) [1–3]. It offers an ideal

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platform for experimentally testing unusual properties expected in topological superfluids/superconductors because clear and conclusive results can be obtained thanks to its fully elucidated order parameter and its inherent cleanliness. One of the superfluid phases, ^{3}He-B, is recognized as a time-reversal invariant topological superfluid with broken relative spin-orbit symmetry [4–6]. Due to the bulk-surface correspondence, ^{3}He-B has surface Andreev bound states (SABSs) at a surface within the coherence length $\xi_0 \sim 80$ nm [6–8]. Surprisingly, the SABSs are predicted to show Majorana nature; their antiparticle is identical to their own particle [4–6,9]. The SABSs formed at a diffusive and a partially specular surfaces were observed recently [9–13] although there is no direct experimental demonstration of the Majorana nature.

In our previous experiments, we studied the mobility of ions trapped below the free surface of ^{3}He-B for the purpose of detecting the SABSs, and found that the mobility does not depend on the depth of the trapped ions even if the depth is changed over ξ_0 [14]. This indicates that the SABSs have little contribution to the scattering of the ion, which is possibly associated with the Majorana nature of the SABSs. In this article, we presents results of the nonlinear transport of positive ions trapped below the free surface of ^{3}He-B, which are performed with aims of observing the nonlinearity expected at around the Landau critical velocity for the pair breaking and finding a signature from SABSs formed at a free surface. A free surface is a microscopically smooth, specular surface, at which the SABSs are expected to have a linear dispersion called a Majorana cone, one of the important features of the Majorana fermion. By contrast, at a diffusive surface, the dispersion relation of SABSs is ill-defined [8]. Therefore, the free surface provides an ideal test bed for uncovering exotic properties of the Majorana SABSs. In this nonlinear investigation, we expect that an ion moving faster than the Landau critical velocity will cause breaking of Cooper pairs, which results in the creation of SABSs at the free surface as well as quasiparticles (QPs) in bulk. Past experiments of nonlinear transport of ions in bulk ^{3}He-B showed that excess dissipation sets in when the velocity of the ion approaches the Landau critical velocity for the pair breaking at Δ_B/p_F, when two bulk QPs are excited by breaking a Cooper pair, where Δ_B is the energy gap of ^{3}He-B and p_F is the Fermi momentum [15–17].

The nonlinear behavior associated with the Landau critical velocity has long been investigated for motion of a macroscopic object like a vibrating wire [18,19]. In the vibrating wire experiments, strong damping of motion has been found to set in at the velocity $\sim \Delta_B/3p_F$ [18,19]. Volovik recently pointed out that the onset of the damping at one third of Δ_B/p_F can be explained by the presence of zero-energy SABSs with $|\mathbf{p}_\parallel| = p_F$ formed at the diffusive surface of the vibrating wire ($\mathbf{p}_\parallel$ is the momentum parallel to the surface). He proposed that the damping is caused by either one of two processes; (i) by the escape of zero-energy SABSs from the surface of the vibrating wire or (ii) by the pair breaking which creates one SABS and one bulk QP [9].

Motivated by the vibrating wire experiments, we study here nonlinear transport of ions trapped below the surface. We expect that nonlinearity in the motion of the ion below the free surface arises from SABSs although it would be generated by a somewhat different process from that in the vibrating wire experiments; either by the (i) escape of SABSs formed *at the free surface* to the bulk or (ii) by pair breaking which creates one SABS *at the free surface* and one bulk QP. (We also expect that bound states formed around an ion [7,20] escape from the ion to the bulk at high ion velocities, which might induce other nonlinear behavior.) A preliminary study of nonlinear transport of positive ions trapped below a ^{3}He-B free surface was carried out by Shiino *et al.*, but data were taken only at 0.39 mK without investigating the depth dependence [21].

2 Experimental

We have investigated the properties of nonlinear transport of the positive ion called a snowball, which is a cluster of ^{3}He atoms with a radius R of 0.6 nm at zero pressure [22]. We used the same experimental cell as employed for the studies of ion mobility [14] and intrinsic Magnus force [23]. The cell had top, bottom, and guard electrodes (Fig. 1a). The top electrode, which was used for transport measurements, had three rectangular segments shown in Fig. 1b. The bottom electrode was a single rectangular electrode of size 18 mm ×18 mm. The top electrode was located 3 mm above the bottom electrode, and the free surface of liquid ^{3}He was placed halfway between the two electrodes (Fig. 1a). Positive ions were generated at ∼ 100 mK by field emission from carbon nanotubes with a high voltage (500–700 V) and were trapped below the surface by a positive dc voltage applied to the bottom electrode V_{dc}. The trapped ions formed a two-dimensional ion sheet below the surface at a depth d. We could tune d by changing V_{dc}, in the range $20 \le d \le 60$ nm, in our typical experimental conditions [14,24]. The trapped ion sheet was confined horizontally to rectangular shape (∼ 16 mm ×16 mm, estimated from our numerical calculation) by applying positive voltage to the guard electrode V_g.

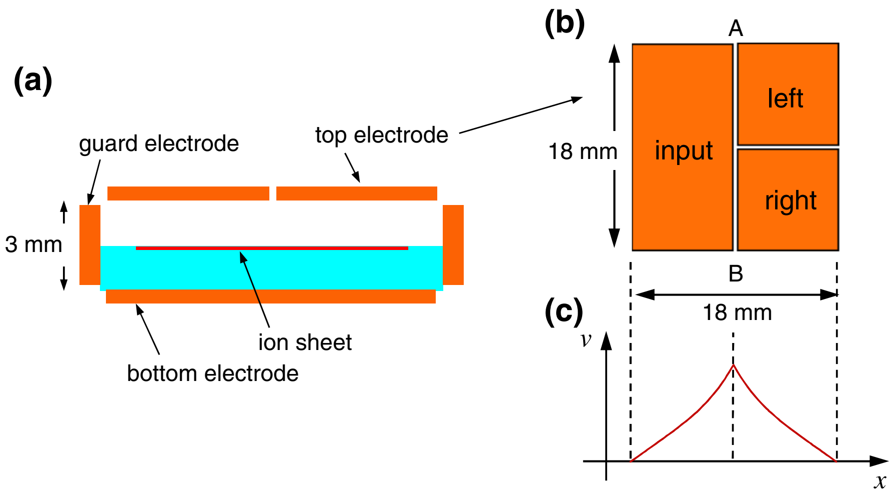

Fig. 1 **a** Configuration of the electrodes and the free surface of liquid ^{3}He. **b** *Top* electrode. In this study, the *right* and *left* electrodes are used as a single *output electrode*. **c** Distribution of the velocity of the *ions* along the *x-axis* (Color figure online)

In transport measurements, we used the right and left electrodes (Fig. 1b) as a single output electrode. An ac voltage $V_{\rm in}$ of frequency 61.1 Hz was applied to the input electrode, which produced an in-plane electric field $E_\parallel$ parallel to the x-axis in Fig. 1. The in-plane electric field then induced a current of the ions along the x-axis, which was detected by the output electrode (the output current is $I_{\rm out} = I_R + I_L$, where I_R and I_L are currents detected by the right and left electrodes, respectively). In the rectangular geometry, the velocity of the ions v has a distribution along the x-axis with a maximum at the line AB as shown in Fig. 1c. The maximum velocity is deduced from $I_{\rm out}$ with the relation of $v = \frac{C_U+C_L}{C_U}\frac{1}{enW}I_{\rm out}$, where W is the width of the ion sheet along the line AB and C_U (C_L) is the capacitance per unit area between the ion sheet and the top (bottom) electrode [14]. In the nonlinear measurements, we recorded $I_{\rm out}$ as a function of $V_{\rm in}$, and then converted them to v and $E_\parallel$. The temperature T of liquid ^{3}He was directly determined from the linewidth of a tuning fork immersed in the liquid, which was calibrated against a vibrating wire thermometer at a low magnetic field ($\sim$ 80 mT). Data presented here were taken at an ion density of 2.3×10^{11} m^{-2} at zero magnetic field. The mobility of the positive ion increases by about two orders of magnitude from T_c (= 0.93 mK) to 0.26 mK [14] mainly due to the reduction in the density of thermally excited bulk QPs. We note that, to realize $v > \Delta_B/p_F$ at a not so high $V_{\rm in}$, the mobility should be substantially large. ($V_{\rm in}$ should be sufficiently smaller than V_{dc} because the ions escape from the surface when $V_{\rm in}$ is comparable to V_{dc}. In our experiments, we used $V_{\rm in} \le 0.8$ V and $8 \le V_{dc} \le 30$ V.) Therefore, we could carry out the nonlinear measurement only at $T \le 0.33$ mK for the positive ion at our typical experimental conditions. For the negative ion, we could not perform the same experiment because its mobility is one order of magnitude smaller than that of the positive ion [14].

3 Nonlinear Transport of an Ion below the Free Surface of ^{3}He-B

Figure 2 shows v as a function of $E_\parallel$ measured at 0.26 and 0.29 mK for three different depths of the ions. At low $E_\parallel$, v is proportional to $E_\parallel$, and the slope corresponds to the mobility. At high $E_\parallel$, the v vs. $E_\parallel$ relation starts to deviate from the linear response at $v \sim 1.4$ cm/s ($\sim \Delta_B/2p_F$), indicating that excess dissipation sets in. The onset of the nonlinearity is not sharp, and the dissipation gradually increases as $E_\parallel$ becomes large. We note that our mobility data reported in Ref. [14] were taken at the velocity range 10^{-4} to 10^{-3} m/s, which is sufficiently smaller than the velocity at which the nonlinearity sets in. From Fig. 2, it is clear that the nonlinear behavior does not have depth dependence at $27 \le d \le 53$ nm although the local density of SABSs significantly changes as a function of depth in this depth range [6,8]. The lack of the depth dependence suggests that the nonlinear behavior does not reflect the properties of SABSs in the wide range of v ($0 \le v \le 1.8\Delta_B/p_F$).

As for the data in Fig. 2, we have to make a technical comment. To derive v from the relation $v = \frac{C_U+C_L}{C_U}\frac{1}{enW}I_{\rm out}$, we should know the actual width W of the horizontally confined ion sheet, which is determined by a confining potential produced by V_{dc} and V_g. When we changed d by varying V_{dc}, we also had to adjust V_g so that the actual width became the same at different depths (i.e., at different V_{dc}). We adjusted

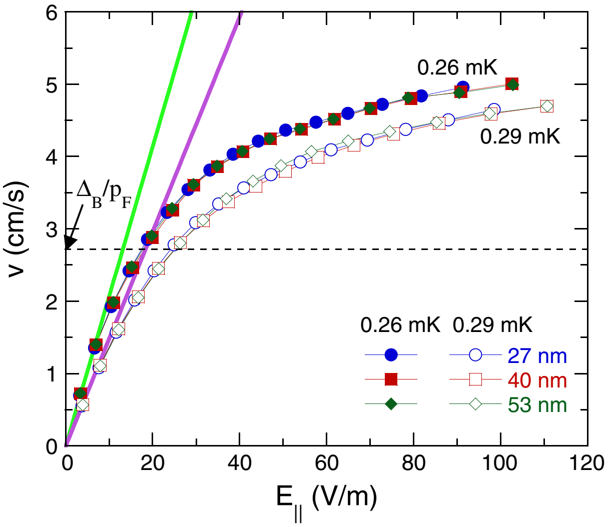

Fig. 2 The velocity as a function of $E_{\parallel}$ measured at 0.26 and 0.29 mK for the *positive ion* in ^{3}He-B at *zero* magnetic field. Data at three different depths are shown. The *slope* of the *solid lines* represents the mobility. The *arrow indicates* Δ_B/p_F (= 2.7 cm/s) (Color figure online)

V_g in such a way, but the actual width might be slightly different at different depths. The variation in the width caused an ambiguity in v after converting I_{out} to v (the variation is less than 3 %). In the data shown in Fig. 2, we had scaled the velocity by a multiplying factor A ($1 \leq A \leq 1.03$) so that the initial slope at low $E_{\parallel}$ gave the mobility obtained in our previous study [14]. Note that the mobility has no dependence on the depth [14], and therefore the initial slope should be the same at different depths.

We now discuss the expected critical velocity associated with the SABSs formed at the free surface although they are not detected in Fig. 2. The SABSs formed at a specular surface have the linear dispersion $E_S = c\left|\mathbf{p}_{\parallel}\right|$ with $c = \Delta_B/p_F$ [6,8]. The energy in the frame of the ion moving at a velocity $\mathbf{v}$ is shifted due to the Doppler effect: $E'_S = E_S - \mathbf{v}\cdot\mathbf{p}_{\parallel}$. The energy of bulk QPs E_B is also shifted as $E'_B = E_B - \mathbf{v}\cdot\mathbf{p_F}$. The SABSs can escape from the surface to bulk when $\min(E'_S) \geq \min(E'_B)$, which corresponds to $v \geq \Delta_B/p_F$. (Note that $\min(E'_S) = 0$ at $\mathbf{p}_{\parallel} = \mathbf{0}$ when $v \leq c$.) Instead of this process, a second different process associated with the pair breaking can also generate nonlinear transport of the ion. In this process, the breaking of a Cooper pair creates one SABS and one bulk QP. This process occurs when $\min(E'_S) + \min(E'_B) \leq 0$, which also gives $v \geq \Delta_B/p_F$. Therefore, the critical velocities for these two processes are the same as that for the pair breaking velocity in the bulk (Δ_B/p_F), suggesting that the presence of the SABSs does not change the critical velocity from the bulk pair breaking velocity. Note that the critical velocity Δ_B/p_F for the SABSs is a direct consequence from the linear dispersion of the SABSs with the minimum in energy at $\mathbf{p}_{\parallel} = \mathbf{0}$. If there exist zero-energy SABSs at $\mathbf{p}_{\parallel} \neq \mathbf{0}$ as similar to the diffusive surface [8], the above critical velocities can be smaller than Δ_B/p_F, following from a similar argumentation as given in Ref. [9]. In addition, the superflow induced around the ion would make the critical velocities smaller although it is not clear what flow field is generated around the ion whose size is much smaller than ξ_0.

From the above argument, the SABSs are expected to give excess dissipation at $v \geq \Delta_B/p_F$. If the above processes are relevant, the nonlinear behavior at $v \geq \Delta_B/p_F$ should be more affected by the SABSs when the ions are trapped closer to the surface, because of a larger density of states of the SABSs. Then the nonlinear behavior in Fig. 2 should have a depth dependence at $v \geq \Delta_B/p_F$. However, no depth dependence is observed at $v \geq \Delta_B/p_F$ in Fig. 2. This indicates that the nonlinear processes associated with the SABSs occur at a rate much lower than the bulk nonlinear processes.

In the above argument, it is implicitly assumed that the nonlinear transport reflects the local properties of ^{3}He-B at the position of the ion. However, it is not trivial whether it really reflects local properties of the region of the size of an ion because SABSs and bulk QPS are meaningful concepts in a region larger than ξ_0.

The lack of the depth dependence suggests that the observed behavior has to be considered as that of the ion in bulk ^{3}He. Indeed, the behavior in Fig. 2 is qualitatively similar to that found in the bulk ^{3}He-B [16,17] although the nonlinearity in the bulk was found to set in at $v \sim \Delta_B/p_F$ at high temperatures ($T/T_c > 0.62$). (The data in bulk ^{3}He-B have been taken at only $T/T_c > 0.62$.) In the bulk, Fetter and Kurkijärvi theoretically pointed out that, in addition to the nonlinearity associated with the pair breaking at $v \geq \Delta_B/p_F$, the transport becomes nonlinear as v approaches $k_B T/p_F$ because of the inelastic scattering of a QP [25]. This inelastic scattering has not been observed in the previous experiments in the bulk superfluid at high temperatures ($T/T_c > 0.62$) [16,17] because $k_B T/p_F$ is close to the critical velocity for pair breaking (Δ_B/p_F). (In normal phase, the nonlinear effect which possibly arises from the inelastic scattering was observed [16].) In our experiments at 0.26 mK, $k_B T/p_F = 0.43$ cm/s, thus it is well separated from $\Delta_B/p_F = 2.7$ cm/s. The onset of the nonlinearity at $v \sim 1.5$ cm/s, which is well smaller than Δ_B/p_F, is possibly due to the inelastic scattering. To further understand the effect of the inelastic scattering, a measurement of the nonlinearity in a wide temperature range would be important.

4 Summary

We reported properties of nonlinear transport of positive ions trapped below a free surface of ^{3}He-B. This is the first study of the nonlinearity in ^{3}He-B at very low temperatures ($T/T_c \lesssim 0.3$) and the first systematic investigation of the nonlinearity as a function of the ion depth. We found that the ion velocity deviates from the linear behavior at a velocity of about a half of Δ_B/p_F. We furthermore found that the v vs. $E_\parallel$ relation does not have a dependence on the depth of the ions from the surface even if the depth is changed over a distance comparable to the superfluid coherence length ξ_0. The lack of depth dependence indicates that no excess dissipation due to the presence of the SABSs is generated although our theoretical analysis suggests that the nonlinear processes associated with the SABSs are expected at velocities higher than Δ_B/p_F. The lack of depth dependence also suggests that the observed nonlinear behavior is completely dominated by the bulk nonlinear processes. The nonlinear

behavior at velocities higher than Δ_B/p_F is caused by the bulk pair breaking, and the onset of the nonlinearity observed at a half of Δ_B/p_F might be due to the inelastic scattering of bulk QPs.

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