Journal of Global Optimization
https://doi.org/10.1007/s10898-024-01368-2

Social equity in international environmental agreements

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Received: 23 June 2023 / Accepted: 15 January 2024

Abstract
The aim of this paper is to investigate the problem of designing and building International Environmental Agreements (IEAs) taking into account some normative properties. We consider n asymmetric countries of the world, each one generating a quantity of pollutant emissions from the production of goods and services. We assume that individual emissions yield private benefits and negative externalities affecting all countries. To determine its own level of pollution, each state conducts a cost-benefit analysis. The absence of a supranational entity imposing emissions reduction makes IEAs based on voluntary participation. Examining the standard static non-cooperative game-theoretical model of coalition formation, we discover that the resulting emissions allocations might not be equitable à la Foley. It means that there might exist at least one player preferring to implement some other agent's strategic plan instead of to play her own strategy. With the goal of studying whether equity, at least among coalesced countries, may be a criterion leading to social improvement, we introduce a new optimization rule. We require that members of an environmental coalition have to solve the maximization problem subject to the constraint imposing that they do not envy each other. Analyzing the particular case of two-player games, we get that, when countries are, in a sense, not too different from each other, our new mechanism endogenously induces social equity. By imposing a suitable total emission cap, the same results extend to all those games where our and standard solutions coexist and are different.

Keywords International environmental agreements · Social equity · Envy-freeness · Non-cooperative games

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Published online: 29 April 2024

1 Introduction

Garrett Hardin [1] in 1968 introduces for the first time the metaphor *tragedy of the commons*, to describe situations in which a finite natural resource can be over-exploited if a large number of individuals have uncontrolled access to it. This concept has been widely discussed throughout history, starting from ancient Greek philosophers to ecologists and economists in the modern age. The very nature of the problem suits with the tool provided by the game theory, that generates a large literature on common pool resources (see e.g., [2–4]). Enlarging the picture, most of the environmental problems, from the over-exploitation of common pool resources to global warming, are transnational and required an international agreement to be addressed. As is well known, a wide part of literature adopts a non-cooperative game-theoretical approach to investigate the building of International Environmental Agreements (IEAs) (see [5] for a vast survey on designing of IEAs using cooperative and non-cooperative game-theoretical models). The choice of this technique lies in two fundamental characteristics of IEAs. The former is the strategic nature of the problem: any emissions decision made by a country impacts on the well-being of all others states. The latter is the voluntariness with which countries have to decide whether or not to sign the deal: since there are no external constraints or supranational institutions imposing restrictions on emissions, states act on a voluntary basis.

Many papers explore the effect of different mechanisms on the building of an IEA. For instance, [6, 7] include punishments for non-member countries, [8–10] consider transfer schemes, [11, 12] take into account issue linkage, [13, 14] analyze the impact of farsightedness on environmental agreements and trans-boundary pollution, [15, 16] treat social externalities.

The aim of this paper is to examine IEAs taking into account some normative properties.

The huge literature on normative economics is rich of various equity notions proposed in decades of research. Many of these concepts require absence of envy among the primitive entities of the economy (see [17] for an extensive survey on fair allocation rules). The first criterion based on envy-freeness is due to Foley [18], according to which each agent prefers to consume her own bundle rather than receive the bundle of anybody else. As pointed out on [17], from a mathematical point of view, the operation that is carried out in this kind of evaluation of allocations allows to check "robustness under substitutions" or "under permutations." Indeed, according to Foley's notion, an agent is not envious if she is not better off after changing her bundle to someone else's. Similarly, an allocation is envy-free if no agent is better off after an arbitrary permutation of the components of the allocation.

Envy-freeness notion is widely studied and investigated in several areas. Among others [19, 20] examine existence, refinement and properties of envy-freeness in exchange economies and economies with production, while [21–23] extend the analysis on economies with uncertainty and asymmetric information, [24] implements in Nash equilibria envy-freeness solution to the problem of fair division, [8] introduces envy-freeness in IEA literature, while [25] considers a weaker notion of envy-freeness requiring absence of envy only among spatial neighbours. Interesting refinements of envy-freeness notion are those given in [26–28]. In [26] it is introduced *per-capita envy-freeness* notion, according to which each agent finds her bundle at least as desirable as the average bundle. In [27] it is defined *average envy-freeness* concept, following which each individual weakly prefers her own bundle to the average of what all the others receive. Both notions avoid "distortions" like legitimizing an agent to judge non-equitable an allocation, because she envies another individual, despite being much better than the large majority. A strengthened notion of average envy-freeness is due to [28], according to which, an allocation is said to be *strictly envy-free* if no agent envies the average bundle of any possible coalition she does not belong to.[1]

In this paper we consider n asymmetric countries of the world, each one engaged in economic activities to produce goods and services, generating pollution. As usual, we suppose the existence of a one to one increasing relationship between production and emission. Moreover, we assume that the level of emissions of each country brings private benefits and contributes to environmental damage affecting all states. As a consequence, individual benefits levels are supposed to be dependent uniquely on private emissions, while costs that each state has to bear are assumed to be function of the aggregate of all emissions. Each player makes her own decisions on the level of production, considering a cost-benefit analysis. She acts maximizing her own utility function, expressed by the difference between the benefit and the environmental damage cost functions. Adapting Foley's notion to our framework, an emissions allocation (and then the corresponding vector of level of production) is considered envy-free if each player weakly prefers to implement her own strategic plan instead of to play another country's strategy. The dependence of the costs on the total emission allows to check envy-freeness focus on benefits level.

As there is no higher entity imposing restriction on emissions or forcing cooperation, each player acts voluntarily and countries interested on environmental protection look for a stable collaboration. In other words, they try to conclude a self-enforcing agreement, meaning that no signatory country has incentive to withdraw and all non-signatories have no convenience to join the coalition. According to the standard rule, an environmental coalition acts as a single player with the aim of maximizing the joint welfare. Simultaneously, other players solve their own optimization problem. Analyzing the resulting allocations, we get that, not for all games, Nash equilibria pass envy-freeness test. We, then, characterize the class of games for which there exits at least an environmental coalition whose cooperation guarantees absence of envy. We conduct the same analysis requiring envy-freeness within environmental coalition, disregarding other countries, and we obtain some interesting properties.

It is clear that, from an individualistic point of view, requiring envy-freeness within environmental coalitions may be an ethic and righteous normative condition. Indeed, it avoids, for each signatory player, a negative feeling on what is established by the agreement. Moreover, it guarantees that each coalesced player, observing the vector of coalition emission, sees it coinciding with her idea.

With the aim of investigate whether equity, at least among signatory countries, may be a normative criterion inducing environmental, social and economic improvement, we introduce a new cooperation rule, according to which coalesced players maximize the joint utility function subject to the constraint imposing that they do not envy each other.

The analysis carried out in two-player games proves that, at the optimum, in several cases, our rule leads to social equity. Where environmental agreements non-avoiding envy and those imposing envy-freeness are both signable, using our mechanism, the most developed player is willing to give up a part of her levels of production and benefits to allow the least developed player to produce more and increase her benefits level. This balance of production, and therefore of emissions, is obtained endogenously in games where the agents are, in a sense, not too far apart in terms of benefit per unit of emissions and net marginal damage. By imposing a suitable total emission cap, we achieve the same results and considerations

[1] See [29] for an analysis on efficient, equitable and consistent solutions on atomless economies, [30] for a weaker notion of strict envy-freeness imposing the comparison only among spatial neighbours, [31] for an investigation in economies allowing the coexistence of negligible and non-negligible traders, [32] for a study on how certain limitations imposed on coalition formation may impact the set of strictly fair allocations, and [33] for a stronger version of strict fairness notion imposing absence of envy towards fuzzy coalitions.

for all games in which standard and envy-free environmental agreements may be signed and generate different emissions allocations. It means that, even for games where players are potentially distant in parameters, envy-free environmental agreements endogenously ensure social equity, giving the least developed country the opportunity to increase its benefits and, potentially, its resilience to possible environmental damage.

Moreover, we identify classes of games for which standard mechanism forces to non-cooperation, while envy-freeness rule conducts to stable cooperation, leading environmental, social and economic improvement.

It should be underlined that, as mentioned above, absence of envy is already introduced and analyzed in game theory framework, as well as in IEA literature. For instance, in [24] it is conducted a very interesting analysis on equity notions in game theory framework. More precisely, the author constructs games implementing in Nash equilibria several solutions to the problem of equitable division. He investigates, among other concepts, envy-freeness à la Foley, per-capita envy-freeness and average envy-freeness. Moreover, he proposes games implementing the intersection of the Pareto solution with each of the previously mentioned equitable solutions. The analysis carried out in our paper is different and the model investigated here can not be considered belonging to the class of games discussed in [24]. In [24] each agent is equipped with continuous, convex and strictly increasing preferences, while, as previously stated, we impose players making their decisions through a cost-benefit analysis. In [8], absence of envy à la Foley is introduced in IEA literature and a very interesting axiomatic analysis is carried out. Our investigation differs from that in [8] not only in the assumptions on the utility functions, but even for another fundamental aspect. In our games, monetary compensations are not considered. Part of our study is devoted to investigate whether a moral boost within the coalition can affect social growth and smooth out inequities by endogenously generating greater social justice, bringing "the stronger to help the weaker." So, we prevent the presence of external factors, as monetary compensations, that may force or create incentives for cooperation.

The paper is organized as follows. In Sect. 2 we introduce the model and the main definitions. In Sect. 3 we analyze global envy-freeness and absence of envy within environmental coalitions for IEAs resulting from the standard cooperation mechanism. In Sect. 4 two-stage envy-free game is introduced and the particular case of two-player envy-free games is deeply investigate. Section 5 contains some concluding remarks. All the proofs are collected in Appendix.

2 The model and the main definitions

We consider a finite number, n, of countries of the world (agents), indexed by i. Each country produces a level of goods and services to which correspond some benefits and a positive quantity of pollutant emission. As natural, we suppose, for each agent, the existence of a relationship between how much she can produce and the level of benefits she can enjoy. Furthermore, we assume that, for each country, there exists a one-to-one increasing correspondence between its level of productivity and the amount of pollution it releases into the environment. As a consequence, denoted, for each i, by y_i, e_i and b_i, respectively, the level of production, the amount of emissions and the degree of welfare brought about by the benefits, we can identify y_i with e_i and express b_i directly depending on e_i, i.e., $b_i = B_i(e_i)$, where B_i represents country i's *benefit function*. On the other hand, for each agent i, pollution resulting from production causes environmental damage and consequently some costs. As well as, for each i, the level of benefits b_i depends uniquely on the emissions produced by i, it is natural to assume that each agent's damage cost depends on all countries' emissions.

We define *group's emissions allocation* (or simply *emissions allocation*) a vector $\underline{e} = (e_1, ..., e_n)$ belonging to $\mathbb{R}^n_{++}$, in which each component represents the level of emissions of the corresponding country.[2] Given a group's emissions allocation $\underline{e}$, we define the related *aggregate* or *total emissions* as the positive real number $E = \sum_{i=1}^n e_i$. Finally, fixed a country i, a subset $C \subseteq I$ and a group's emissions allocation $\underline{e}$, with $\underline{e}_{-i}$ we identify the vector $(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n) \in \mathbb{R}^{n-1}_{++}$, while with $\underline{e}(C)$ we denote the vector $(e_i)_{i \in C} \in \mathbb{R}^{|C|}_{++}$.[3]

For each agent i, we identify by d_i the level of environmental damage cost that i has to bear. Hence, we can express d_i depending on $\underline{e}$, that is, $d_i = D_i(\underline{e})$, where D_i represents country i's *environmental damage cost function*.

We assume that, for every i,

– the law of the benefit function, $B_i : \mathbb{R}_{++} \to \mathbb{R}$, may be well described by a quadratic functional form

$$B_i(e_i) = \alpha_i e_i - \frac{1}{2} e_i^2,$$

where α_i is a positive parameter outlining the benefit per unit of emissions;

– the law of the environmental damage cost function, $D_i : \mathbb{R}^n_{++} \to \mathbb{R}$, may be well approximated by a linear function

$$D_i(\underline{e}) = \beta_i \sum_{i=1}^n e_i = \beta_i E,$$

where β_i is a positive parameter identifying the net marginal damage.[4]

The first assumption is standard in literature (see e.g., [34–38]), while the second is common (see e.g., [6, 7, 10, 37, 39–41]) and supported by empirical estimations (see e.g., [42]). Although it may be reductive to assume that the damage cost function is well approximated by a linear function, this requirement guarantees the orthogonality of the reaction function of countries with standard preferences. Indeed, the linearity allows to obtain the optimal level of emissions of a country independent of the choice of emissions of others.

Throughout the paper we make the following assumptions

(A1) for each i in I, $\alpha_i > \beta_i$;
(A2) for every i and j in I, if i is more developed than j, then $\alpha_i > \alpha_j$ and $\beta_i < \beta_j$.

Assumptions $(A1)$ and $(A2)$ are not only technical requirement. The former ensures that for each country the benefit per unit of emission level (which is equivalent to consider the benefit per unit of production) is greater than the net marginal cost. The latter means that the benefit per unit of production is the greater the more developed the country, while for the marginal net damage is worth the inverse inequality. The idea behind this requirement

[2] We denote by $\mathbb{R}^n_{++}$ the interior of the positive orthant of the Euclidean space $\mathbb{R}^n$. In the IEA literature, usually, emissions are assumed to be non-negative. Without loss of generality, we concentrate our analysis on the positive ones. Let us observe that our assumption is not really a theoretical or practical restriction. Indeed, in the literature, even if the emissions are allowed to be null, at the equilibrium, they are generally supposed to be interior solutions of maximization problems. Moreover, it is quite unrealistic to assume that a country might stop producing.

[3] Given a subset A of I, we denote by $|A|$ the cardinality of A.

[4] With abuse of notation, D_i may be considered as a function of E, i.e., $D_i : \mathbb{R}_{++} \to \mathbb{R}$, with the law $D_i(E) = \beta_i E$. Through the paper, when necessary and not confusing, we stress the dependence on E.

is that the more a country is developed, the greater its ability to build different skills and to grow technologies and procedures for adaptation to climate change. Having more advanced and specialized technology allows for a greater benefit from production, as well as a greater resilience to pollution. Assumption $(A2)$ is in line with the current literature (see e.g., [16, 39, 43–48]).

Each country makes its own decisions on the level of production, considering a cost-benefit analysis. More specifically, it acts maximizing its own utility function, u_i, expressed by the difference between benefit and environmental damage cost functions, i.e., $u_i : \mathbb{R}^n_{++} \to \mathbb{R}$ is defined by the law

$$u_i(\underline{e}) = B_i(e_i) - D_i(\underline{e}) = \alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E.$$

Formally, we consider a game $\Gamma = \{I = \{1, ..., n\}, \{S_i\}_{i\in I}, \{u_i\}_{i\in I}\}$ in which

- each player $i \in I$ represents a country of the world;
- for each $i \in I$, *player i's set of strategies of the game* Γ, S_i, coincides with $\mathbb{R}_{++}$ and each element of S_i represents a quantity of emissions e_i that agent i chooses to spread in the environment; hence a group's emissions allocation $\underline{e}$ is a *vector strategy of the game* Γ;
- for each i, *player i's utility*, $u_i : \mathbb{R}^n_{++} \to \mathbb{R}$, is given by the difference between benefits and costs, $u_i(\underline{e}) = B_i(e_i) - D_i(\underline{e}) = \alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E$.

We denote by $\mathcal{G}$ the family of games Γ satisfying $(A1)$ and $(A2)$.

The absence of external constraint or a supranational institution imposing restrictions on the level of emissions requires that states voluntarily agree to collaborate in order to safeguard the environment.

We define *coalition* a non-empty subset of I, while with the term *degenerate coalition* we mean a subset of I with cardinality 1.

For every non-degenerate coalition C, for every country i in C, we denote by C_{-i} the coalition $C \setminus \{i\}$. Moreover, if C is strictly included in I, for every i in $I \setminus C$, we denote by C_{+i} the coalition $C \cup \{i\}$.

We define *environmental coalition* a non-degenerate coalition composed by a set of countries deciding to cooperate in terms of "environmental protection."

Formally, an environmental coalition, $C \subseteq I$, is a non-degenerate coalition whose members play as a unique player with the aim to maximize the coalitional utility function $u_C : \mathbb{R}^n_{++} \to \mathbb{R}$, describing the joint welfare and defined by the law

$$u_C(\underline{e}) = \sum_{i\in C} u_i(\underline{e}).$$

Given an environmental coalition C, we denote by $\mathcal{P}(C)$ the partition of I composed by C and the degenerate coalitions $\{i\}$, with $i \in I \setminus C$, i.e., $\mathcal{P}(C) = \{C, \{i\}_{i\in I\setminus C}\}$. We say that an n-dimensional vector $\underline{a}$ is a $\mathcal{P}(C)$*-emissions allocation* if it is componentwise positive and

1. $\underline{a}(C)$ is the solution of

$$\arg\max_{\{e_i\}_{i\in C}} u_C(\underline{e}) = \arg\max_{\{e_i\}_{i\in C}} \left(\sum_{i\in C}\left(\alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E\right)\right); \quad (1)$$

2. for any $i \in I \setminus C$, a_i is the solution of

$$\arg\max_{e_i} u_i(\underline{e}) = \arg\max_{e_i} \left(\alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E\right). \quad (2)$$

We assume that players move simultaneously, namely, we look for Nash equilibria (a different analysis may be carried out investigating Stackelberg equilibria as, for instance, in [49]). We define $\mathcal{P}(C)$-*equilibrium problem* the problem identified by (1) and (2). We say that, if there exists a componentwise positive solution of the $\mathcal{P}(C)$-equilibrium problem, the cooperation among members of C is *admissible*, and then the environmental coalition C is *formable*. Otherwise, if the solution does not belong to the admissible set of strategies, the cooperation among agents in C is not possible, and, therefore, states have to look for other partitions of I. Indeed, allowing the solution to have some null components is equivalent to asking the corresponding countries to stop any production activity, which is not reasonable. Moreover, consenting to negative components is senseless, as it would require countries to have negative production levels.

Finally, we define *environmental agreement* a deal signed by an environmental coalition.[5]

From an individualistic point of view, each country in C computes its level of production with the aim of maximizing the joint welfare. Roughly speaking, it determines its contribution, in terms of emissions, to the environmental cause. In fact, by solving the joint optimization problem, each coalesced agent maximizes her own benefits level and minimizes her contribution to the damage costs, obtaining a reduction in global emissions. Notice that, by construction, even the states that do not participate in the environmental agreement benefit from the actions implemented by the signatory countries. In fact, reducing emissions by coalescing countries means lowering costs, even for non-signatory states. Consequently, for some countries, it could be convenient to let others commit to protecting the environment, thereby benefiting from their efforts.

To avoid this free-riding attitude, an agreement has to be self-enforcing. Following [50], we say that an environmental coalition is *stable* if it satisfies *internal* and *external stability*, meaning that no country (inside or outside the coalition) has an incentive to deviate. More specifically, every member of the coalition has no convenience in free-riding and will keep the agreements (internal stability) and no non-signatory player prefers to joint the coalition (external stability).

Let C be an environmental coalition and $\underline{e}$ a $\mathcal{P}(C)$-emissions allocation. The internal stability requires that for every signatory country i, if there exists a $\mathcal{P}(C_{-i})$-emissions allocation $\underline{\tilde{e}}$, then

$$u_i(\underline{e}) \geq u_i(\underline{\tilde{e}}). \tag{3}$$

Condition (3) means that each player i in the coalition C achieves a higher level of utility by not withdrawing from the agreement. Indeed, the increased level of benefits it would derive from playing alone is not enough to compensate for the increased damage costs deriving from its exit from the agreement, and the consequent formation of the coalition C_{-i}, i.e., $B_i(\tilde{e}_i) - B_i(e_i) < D_i(\underline{\tilde{e}}) - D_i(\underline{e})$.

The external stability requires that for every non-signatory country i, if there exists a $\mathcal{P}(C_{+i})$-emissions allocation $\underline{\tilde{e}}$, then

$$u_i(\underline{e}) \geq u_i(\underline{\tilde{e}}). \tag{4}$$

Condition (4) means that each player i outside C reaches a higher level of utility by not cooperating with members of C and playing alone. Indeed, the decreased damage costs it would derive from cooperation is not enough to compensate for the decreasing of the

[5] Through the paper, where necessary to avoid confusion, we refer to this cooperation calling it *standard cooperation*.

benefits of its entry into the agreement, and the consequent formation of the coalition C_{+i}, i.e., $D_i(\underline{e}) - D_i(\tilde{\underline{e}}) < B_i(e_i) - B_i(\tilde{e}_i)$.

3 The envy-freeness property

In this section we recall the envy-freeness notion introduced by Foley in [18] and we adapt it into our framework, with the aim to analyze if, for every game Γ in $\mathcal{G}$, for every formable coalition, the resulting Nash equilibrium satisfies envy-freeness property. In addition, we give and investigate a weaker version of equity.

Foley's concept of absence of envy states that an allocation is said to be envy-free (or equitable) if no agent prefers (or envies) the bundle of anybody else. In other words, each agent prefers to consume her own bundle instead of receiving that of someone else.

In our model, as a suitable adaptation of this kind of envy notion, we introduce the following definition.

Definition 3.1 Given a group's emissions allocation $\underline{e}$, a country i *envies at* $\underline{e}$ *a country* j if it prefers j's production level to its own.

Formally, at $\underline{e}$, i is envious of j, if and only if $\alpha_i e_j - \frac{1}{2}e_j^2 - \beta_i E > \alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E$. Hence, according to the above definition, an envious player i envies the possibility to have player j's level of production, since she would receive a higher level of benefits, leaving costs unchanged. In other words, a country i is envious of a country j at an emissions allocation $\underline{e}$, if and only if $B_i(e_i) < B_i(e_j)$. As a consequence, we can say that i is non-envious of j at $\underline{e}$, if it is impossible to find a vector strategy $\hat{\underline{e}}$, such that $\hat{E} = E$, $\hat{e}_i = e_j$ and $\hat{e}_j = e_i$, for which $B_i(e_i) < B_i(\hat{e}_i)$.[6] Equivalently, i does not envy j at $\underline{e}$, if she does not prefer the vector strategy $\tilde{\underline{e}}$ to $\underline{e}$, where $\tilde{e}_h = e_h$, if $h \notin \{i, j\}$, while $\tilde{e}_i = e_j$ and $\tilde{e}_j = e_i$. Hence, by virtue of these equivalences, to test absence of envy at $\underline{e}$, it is enough that each player focuses on the comparison between $\underline{e}$ and the allocation obtained by swapping her strategy with that of someone else.

As pointed out in [17], under Foley's notion, it could be checked "whether agent i is better off after her bundle has been *switched* with agent j's bundle, after her bundle has been *replaced* by a bundle identical to agent's j, after an arbitrary *permutation* of the components of the allocation." Notice that, in our framework, there is not a market for emissions, then players do not have the option to exchange them. The operation that each country does checking envy-freeness is to compare the level of utility she obtains with her own emission (production) level with what she would get with any other's level of emission (production). It means that, in a sense, each player's analysis reflects her view on how the emission levels, and equivalently the production levels, should be "assigned."

Given a vector $\underline{a} \in \mathbb{R}^n_{++}$, with $u_i(a_i, \underline{a}_{-i})$ and $u_i(a_j, \underline{a}_{-j})$ we denote the utility level that i obtains if it is played respectively the strategy $\underline{a}$ and the strategy defined starting by $\underline{a}$ and swapping between them a_i and a_j. Hence, Definition 3.1 may be reformulated as follows: a country i envies at $\underline{e}$ a country j, if and only if $u_i(e_i, \underline{e}_{-i}) < u_i(e_j, \underline{e}_{-j})$.

Definition 3.2 A group's emissions allocation $\underline{e}$ is *envy-free* if each country, assuming cost unchanged, prefers or is indifferent to implement its own strategic plan instead of to play an other country's strategy.

[6] As previously observed, for every i, D_i may be considered as a linear function of the total emissions, hence, the condition $\hat{E} = E$ implies $D_i(\hat{\underline{e}}) = D_i(\underline{e})$. Therefore, in the comparison, costs may be disregarded.

Formally, in the light of previous observations, $\underline{e}$ is *envy-free* if and only if for every i and j in I, $B_i(e_i) - D_i(E) \geq B_i(e_j) - D_i(E)$, that is equivalent to $B_i(e_i) \geq B_i(e_j)$ and to $u_i(e_i, \underline{e}_{-i}) \geq u_i(e_j, \underline{e}_{-j})$.

In many cases, it might be more reasonable to allow a player to focus her comparison only on those countries she would relate herself. For this reason, we consider a suitable modification of the idea of "local" envy-freeness introduced in [51]. In this weaker concept of absence of envy, we consider the set of countries partitionable into several, non-degenerate or degenerate, coalitions and impose that each player has to look only members of group she belongs to, disregarding the others.

Definition 3.3 Given a partition $\mathcal{P}$ of I, a group's emissions allocation is *$\mathcal{P}$-envy-free* if there is no envy within each coalition composing $\mathcal{P}$.

Notice that, by definition, given the partition $\mathcal{P} = \{\{i\}_{i \in I}\}$, the non-cooperative emissions allocation is $\mathcal{P}$-envy-free.

According to Definition 3.3 we say that, given an environmental coalition C, a $\mathcal{P}(C)$-emissions allocation is $\mathcal{P}(C)$-*envy-free* if agents in C do not envy each other; in this case we say that the cooperation among agents in C is envy-free.

As previously pointed out, if an emissions allocation is envy-free, each agent, observing it, sees it coinciding with her idea of how production levels should be distributed. Since this property is very interesting from economic and ethical point of view, we want to find the conditions under which an equilibrium is envy-free. Hence, the aim of the remaining part of this section is to consider emissions allocations resulting from agreements signed by environmental coalitions, C, investigate the conditions under which they satisfy $\mathcal{P}(C)$-envy-freeness and envy-freeness properties and deduce some interesting characteristics.

In the following two propositions, the proofs of which are collected in the Appendix, we analyze the existence and some properties of $\mathcal{P}(C)$-emissions allocations satisfying $\mathcal{P}(C)$-envy-freeness. More precisely, in Proposition 3.4 we determine the conditions on the parameters α_i and β_i under which cooperation ensures absence of envy among coalesced countries. It means that, we establish the conditions that a game has to satisfy for the existence of at least one $\mathcal{P}(C)$-envy-free emissions allocation. In other words, we characterize the subfamily $\mathcal{G}'$ of $\mathcal{G}$

$$\mathcal{G}' = \{\Gamma \in \mathcal{G} \text{ s.t. } \exists\, C \subseteq I \text{ for which the } \mathcal{P}(C)\text{−emissions allocation is } \mathcal{P}(C)\text{−envy − free}\}.$$

In Proposition 3.5 we show that $\mathcal{P}(C)$-envy-freeness solution is *consistent*, meaning that, if the cooperation for a given non-degenerate environmental coalition C is envy-free, then the cooperation for any subcoalition $C' \subseteq C$ is still envy-free. Formally, we prove that given a $\mathcal{P}(C)$-envy-free emissions allocation, for every $C' \subseteq C$, the solution of the $\mathcal{P}(C')$-equilibrium problem is a $\mathcal{P}(C')$-envy-free emissions allocation.

Proposition 3.4 *Given a game $\Gamma \in \mathcal{G}$ and a non-degenerate environmental coalition C, the solution of the $\mathcal{P}(C)$-equilibrium problem is a $\mathcal{P}(C)$-envy-free emissions allocation if and only if*

(a) *for every i in C, $\alpha_i > \sum_{h \in C} \beta_h$,*

(b) *for every i and j in C such that i is more developed than j, $\alpha_j \leq \alpha_i - 2 \sum_{h \in C} \beta_h$.*

Condition (a) ensures the existence of a componentwise positive solution of the $\mathcal{P}(C)$-equilibrium problem, and then the admissibility of the cooperation. As observed in the proof, for every game Γ in $\mathcal{G}$, each signatory player is non-envious, at $\underline{e}$, of any cooperator less developed than her. However, if condition (b) is not satisfied, there exists at least one coalesced player envying at $\underline{e}$ some members of the coalition more developed than her. In other words, Proposition 3.4 states that there might exist games for which Nash equilibria are not $\mathcal{P}$-envy-free and, a fortiori, envy-free.

Proposition 3.5 *Let Γ belong to $\mathcal{G}'$. Let C be a non-degenerate environmental coalition, for which the $\mathcal{P}(C)$-emissions allocation is $\mathcal{P}(C)$-envy-free. Then, for every environmental subcoalition $C' \subseteq C$ the solution of the $\mathcal{P}(C')$-equilibrium problem is a $\mathcal{P}(C')$-envy-free emissions allocation.*

Actually, the previous proposition allows the following characterization: for a game Γ, the full cooperation is envy-free if and only if for any environmental coalition the cooperation is envy-free. The proof of the following corollary is an immediate consequence of Proposition 3.5.

Corollary 3.6 *Let Γ belong to $\mathcal{G}$. The solution of the $\mathcal{P}(I)$-equilibrium problem is an envy-free emissions allocation if and only if for any non-degenerate environmental coalition C, the solution of the $\mathcal{P}(C)$-equilibrium problem is a $\mathcal{P}(C)$-envy-free emissions allocation.*

We can observe that in cases of supcoalitions it is not possible to obtain results similar to those of the Proposition 3.5, meaning that $\mathcal{P}(C)$-envy-freeness solution is not *converse consistent*. As shown in Example 3.7, given two environmental coalitions C and C', such that $C \subsetneqq C'$, the solution of the $\mathcal{P}(C')$-equilibrium problem, $\underline{e}'$, might be not an emissions allocation, even if the solution of the $\mathcal{P}(C)$-equilibrium problem, $\underline{e}$, is $\mathcal{P}(C)$-envy-free. Indeed, the positivity of the components of $\underline{e}'$ is not guaranteed, even if $\underline{e} \in \mathbb{R}^n_{++}$ and $\underline{e}$ is $\mathcal{P}(C)$-envy-free. Moreover, Example 3.8 proves that, even in cases where $\underline{e}'$ is an emissions allocation, it may be non $\mathcal{P}(C')$-envy-free. By adding agents to the coalition, individual countries' emissions decrease. As a consequence, some state i, non-envious at $\underline{e}$ of any j in C, might envy some j at $\underline{e}'$, since it could obtain a higher level of benefits through j's production level. As a consequence, it would prefer to $\underline{e}'$ the strategy where e'_i and e'_j are swapped.

Example 3.7 Let $\Gamma = \{I, \{S_i\}_{i\in I}, \{u_i\}_{i\in I}\}$, with $I = \{1, 2, 3\}$, $S_i = \mathbb{R}^3_{++}$, $u_i = \alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E$, where $\alpha_1 = 4, \alpha_2 = \frac{5}{2}, \alpha_3 = 2, \beta_1 = \frac{1}{4}, \beta_2 = \frac{1}{2}, \beta_3 = \frac{7}{4}$.

Considering $C = \{1, 2\}$, the solution of the $\mathcal{P}(C)$-equilibrium problem,

$$\underline{e} = (\alpha_1 - \beta_1 - \beta_2, \alpha_2 - \beta_1 - \beta_2, \alpha_3 - \beta_3) = \left(\frac{13}{4}, \frac{7}{4}, \frac{1}{4}\right),$$

is a $\mathcal{P}(C)$-envy-free emissions allocation, indeed, $u_1(e_1, e_2, e_3) > u_1(e_2, e_1, e_3)$ and $u_2(e_1, e_2, e_3) = u_2(e_2, e_1, e_3)$. However, the solution of the $\mathcal{P}(I)$-equilibrium problem,

$$\underline{e}' = (\alpha_1 - \beta_1 - \beta_2 - \beta_3, \alpha_2 - \beta_1 - \beta_2 - \beta_3, \alpha_3 - \beta_1 - \beta_2 - \beta_3) = \left(\frac{3}{2}, 0, -\frac{1}{2}\right),$$

is not an emissions allocation.

Example 3.8 Let $\Gamma = \{I, \{S_i\}_{i\in I}, \{u_i\}_{i\in I}\}$, with $I = \{1, 2, 3\}$, $S_i = \mathbb{R}^3_{++}$, $u_i = \alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E$, where $\alpha_1 = \frac{9}{2}, \alpha_2 = \frac{5}{2}, \alpha_3 = 2, \beta_1 = \frac{1}{4}, \beta_2 = \frac{1}{2}, \beta_3 = 1$.

Considering $C = \{1, 2\}$, the solution of the $\mathcal{P}(C)$-equilibrium problem,

$$e = (\alpha_1 - \beta_1 - \beta_2, \alpha_2 - \beta_1 - \beta_2, \alpha_3 - \beta_3) = \left(\frac{15}{4}, \frac{7}{4}, 1\right),$$

is a $\mathcal{P}(C)$-envy-free emissions allocation, since $u_1(e_1, e_2, e_3) > u_1(e_2, e_1, e_3)$ and $u_2(e_1, e_2, e_3) > u_2(e_2, e_1, e_3)$. However, the solution of the $\mathcal{P}(I)$-equilibrium problem,

$$\underline{e}' = (\alpha_1 - \beta_1 - \beta_2 - \beta_3, \alpha_2 - \beta_1 - \beta_2 - \beta_3, \alpha_3 - \beta_1 - \beta_2 - \beta_3) = \left(\frac{11}{4}, \frac{3}{4}, \frac{1}{4}\right),$$

is an emissions allocation non satisfying $\mathcal{P}(I)$-envy-freeness, since $u_2(e'_1, e'_2, e'_3) < u_2(e'_2, e'_1, e'_3)$, meaning that 2 envies 1 at $\underline{e}'$.

In the following we conduct the same analysis on envy-freeness of $\mathcal{P}(C)$-emissions allocations. More specifically, in Proposition 3.10 we determine the conditions on the parameter α_i and β_i under which, given an environmental coalition C, the $\mathcal{P}(C)$-equilibrium problem admits solution satisfying envy-freeness. In other words, we find a characterization of games Γ for which there exists at least one non-degenerate environmental coalition C such that the $\mathcal{P}(C)$-emissions allocation is envy-free. We denote by $\mathcal{G}''$ the subfamily of $\mathcal{G}$ composed by such games. Formally,

$$\mathcal{G}'' = \{\Gamma \in \mathcal{G} \text{ s.t. } \exists\, C \subseteq I \text{ for which the } \mathcal{P}(C)\text{−emissions allocation is envy−free}\}.$$

By construction $\mathcal{G}'' \subseteq \mathcal{G}'$. In Example 3.9 we show that the inclusion might be strict. After Proposition 3.10, in which we characterize games in $\mathcal{G}''$, we analyze the consistency of the envy-freeness property.

Example 3.9 Let us consider the same game Γ introduced in Example 3.7. As already proved, $\Gamma \in \mathcal{G}'$ since, for instance, considering $C = \{1, 2\}$, the $\mathcal{P}(C)$-emissions allocation is $\mathcal{P}(C)$-envy-free.

Let us show that $\Gamma \notin \mathcal{G}''$.

Considering $C = \{1, 2\}$, the solution of the $\mathcal{P}(C)$-equilibrium problem, $\underline{e} = \left(\frac{13}{4}, \frac{7}{4}, \frac{1}{4}\right)$, is not envy-free since, for instance, 3 envies 2 at $\underline{e}$, i.e., $u_3(e_1, e_2, e_3) < u_3(e_1, e_3, e_2)$.

Any other cooperation is not admissible, since for every non degenerated coalition $C' \neq C$, the solution of the $\mathcal{P}(C')$-equilibrium problem is not an emissions allocation.

Finally, if we consider the full non-cooperation, i.e., $\mathcal{P} = \{\{1\}, \{2\}, \{3\}\}$, the non cooperative emissions allocation $\underline{e}^{NC} = \left(\frac{15}{4}, 2, \frac{1}{4}\right)$ is not envy-free, since, for instance, 3 envies 2 at $\underline{e}^{NC}$, $u_3(e_1^{NC}, e_2^{NC}, e_3^{NC}) < u_3(e_1^{NC}, e_3^{NC}, e_2^{NC})$.

In the following proposition, which proof is shown in the Appendix, we determine the conditions for the existence of environmental agreement ensuring global absence of envy.

Proposition 3.10 *Given a game Γ in $\mathcal{G}$ and a non-degenerate environmental coalition C, the solution of the $\mathcal{P}(C)$-equilibrium problem is an envy-free emissions allocation if and only if*

(a) *for every i in C, $\alpha_i > \sum_{h \in C} \beta_h$,*

(b) *for every i and j such that i is more developed than j,*

(b1) *if i and j belong to C, $\alpha_j \leq \alpha_i - 2 \sum_{h \in C} \beta_h$,*

(b2) *if i and j belong to $I \setminus C$, $\alpha_j \leq \alpha_i - \beta_i - \beta_j$,*

(b3) *if $i \in I \setminus C$ and $j \in C$, $\alpha_j \leq \alpha_i - \beta_i - \sum_{h \in C} \beta_h$,*

(b4) *if $i \in C$ and $j \in I \setminus C$, (b4.1) $\alpha_j = \alpha_i + \beta_j - \sum_{h \in C} \beta_h$ or (b4.2) $\alpha_j \leq \alpha_i - \beta_j - \sum_{h \in C} \beta_h$.*

As well as for Proposition 3.4, condition (a) guarantees the admissibility of cooperation, that is the existence of an admissible solution of the $\mathcal{P}(C)$-equilibrium problem, while condition (b) ensures absence of envy among player.

Actually, Proposition 3.10 proves something more, allowing the following considerations. Given an environmental coalition C and a $\mathcal{P}(C)$-emissions allocation $\underline{e}$, each signatory (non-signatory) does not envy at $\underline{e}$ any less developed signatory (non-signatory), meaning that each country does not envy at $\underline{e}$ anyone less developed who made the same choice in terms of cooperation. Moreover, each non-signatory does not envy at $\underline{e}$ any less developed signatory. Therefore, we get that a non-signatory player does not envy at $\underline{e}$ any less developed player.

The following example shows that, in case of global absence of envy, consistency might fail. It means that even if the cooperation for a given environmental coalition C ensures global absence of envy, the cooperation for a subcoalition C' of C does not avoid envy. Indeed, given an envy-free $\mathcal{P}(C)$-emissions allocation, there might exist a subcoalition C' of C for which the $\mathcal{P}(C')$-emissions allocation is not envy-free.

Example 3.11 Let $\Gamma = \{I, \{S_i\}_{i\in I}, \{u_i\}_{i\in I}\}$, with $I = \{1, 2, 3, 4\}$, $S_i = \mathbb{R}^4_{++}$, $u_i = \alpha_i e_i - \frac{1}{2}e_i^2 - \beta_i E$, where $\alpha_1 = 11, \alpha_2 = 6, \alpha_3 = \frac{3}{2}, \alpha_4 = 1, \beta_1 = \frac{3}{14}, \beta_2 = \frac{5}{14}, \beta_3 = \frac{3}{7}, \beta_4 = \frac{1}{2}$.

Considering $C = \{1, 2, 3\}$, the solution of the $\mathcal{P}(C)$-equilibrium problem,

$$\begin{aligned} e &= (\alpha_1 - \beta_1 - \beta_2 - \beta_3, \alpha_2 - \beta_1 - \beta_2 - \beta_3, \alpha_3 - \beta_1 - \beta_2 - \beta_3, \alpha_4 - \beta_4) \\ &= \left(10, 5, \frac{1}{2}, \frac{1}{2}\right), \end{aligned}$$

is an envy-free emissions allocation, since, for every i and j in I, $u_i(e_i, \underline{e}_{-i}) \geq u_i(e_j, \underline{e}_{-j})$.

Consider, now, $C' = \{1, 2\}$. The solution of the $\mathcal{P}(C')$-equilibrium problem

$$\underline{e}' = (\alpha_1 - \beta_1 - \beta_2, \alpha_2 - \beta_1 - \beta_2, \alpha_3 - \beta_3, \alpha_4 - \beta_4) = \left(\frac{73}{7}, \frac{38}{7}, \frac{15}{14}, \frac{1}{2}\right),$$

is an emissions allocation non-satisfying envy-freeness, since $u_4(e'_4, \underline{e}'_{-4}) < u_4(e'_3, \underline{e}'_{-3})$, meaning that 4 envies 3 at $\underline{e}'$.

However, as we prove in the following proposition, given an environmental coalition C, if the $\mathcal{P}(C)$-equilibrium problem admits envy-free solution, then, for any subcoalition C' of C, the solution of the $\mathcal{P}(C')$-equilibrium problem, $\underline{e}'$, ensures absence of envy within C', $C \backslash C'$ and $I \backslash C$, meaning that $\underline{e}'$ is $\{C', C \setminus C', I \setminus C\}$-envy-free. Moreover, at $\underline{e}'$, any agent in C is non-envious at all. This result is quite interesting, since it allows to state the following sentence. If in a game there exists an environmental coalition whose agreement guarantees global absence of envy, if no external country chooses to cooperate, a member of that coalition, choosing to withdraw or remain even if others retreat, is still non-envious. The proof of the following proposition is given in the Appendix.

Proposition 3.12 *Given a game $\Gamma \in \mathcal{G}$ and a non-degenerate environmental coalition C, if the solution of the $\mathcal{P}(C)$-equilibrium problem is an envy-free emissions allocation, then, for every $C' \subsetneq C$, the solution of the $\mathcal{P}(C')$-equilibrium problem, $\underline{e}'$, is an emissions allocation such that*

(1) *$\underline{e}'$ is $\mathcal{P}$-envy-free, where $\mathcal{P} = \{C', C \setminus C', I \setminus C\}$;*
(2) *every i in C is not envious at $\underline{e}'$.*

Analyzing Example 3.11 and the proof of Proposition 3.12, we can remark that for envy-freeness solution the consistency might fail only if one of the cooperating countries, i, leaving the coalition envies a more developed player, j, who has not coalesced from the beginning. Moreover, we can observe that this could happen only if $\alpha_j = \alpha_i + \beta_j - \sum_{h \in C} \beta_h$. Hence, we can state that in games for which there exists an environmental coalition satisfying conditions (a), $(b1)$, $(b2)$, $(b3)$ and $(b4.2)$ of Proposition 3.12, envy-freeness solution is consistent. The proof of the following proposition is shown in the Appendix.

Proposition 3.13 *Let $\Gamma \in \mathcal{G}''$. Fixed a non-degenerate environmental coalition C, satisfying conditions (a), $(b1)$, $(b2)$, $(b3)$ and $(b4.2)$ of Proposition 3.12, for each $C' \subseteq C$, the solution of the $\mathcal{P}(C')$-equilibrium problem is envy-free.*

As predictable, as well as for the $\mathcal{P}(C)$-envy-freeness solution, the previous results can not be obtained considering supcoalitions. Example 3.14 shows that given two environmental coalitions C and C', such that $C \subsetneq C'$, the solution of the $\mathcal{P}(C')$-equilibrium problem, $\underline{e}'$, might be not an emissions allocation, even if the $\mathcal{P}(C)$-emissions allocation is envy-free. Example 3.15 proves that, even in cases where $\underline{e}'$ is an emissions allocation, it might be non-envy-free.

Example 3.14 Let $\Gamma = \{I, \{S_i\}_{i \in I}, \{u_i\}_{i \in I}\}$, with $I = \{1, 2, 3\}$, $S_i = \mathbb{R}^3_{++}$, $u_i = \alpha_i e_i - \frac{1}{2} e_i^2 - \beta_i E$, where $\alpha_1 = 6, \alpha_2 = 4, \alpha_3 = 2, \beta_1 = \frac{1}{3}, \beta_2 = \frac{2}{3}, \beta_3 = 1$.

Considering $C = \{1, 2\}$, the solution of the $\mathcal{P}(C)$-equilibrium problem,

$$\underline{e} = (\alpha_1 - \beta_1 - \beta_2, \alpha_2 - \beta_1 - \beta_2, \alpha_3 - \beta_3) = (5, 3, 1)\,,$$

is an envy-free emissions allocation, since for every i and j in I with $i \neq j$, $u_i(e_i, \underline{e}_{-i}) \geq u_i(e_j, \underline{e}_{-j})$.

However, the solution of the $\mathcal{P}(I)$-equilibrium problem,

$$\underline{e}' = (\alpha_1 - \beta_1 - \beta_2 - \beta_3, \alpha_2 - \beta_1 - \beta_2 - \beta_3, \alpha_3 - \beta_1 - \beta_2 - \beta_3) = (4, 2, 0)\,,$$

is not an emissions allocation.

Example 3.15 Let $\Gamma = \{I, \{S_i\}_{i \in I}, \{u_i\}_{i \in I}\}$, with $I = \{1, 2, 3\}$, $S_i = \mathbb{R}^3_{++}$, $u_i = \alpha_i e_i - \frac{1}{2} e_i^2 - \beta_i E$, where $\alpha_1 = 6, \alpha_2 = 4, \alpha_3 = 3, \beta_1 = \frac{1}{8}, \beta_2 = \frac{1}{4}, \beta_3 = \frac{1}{2}$.

Considering $C = \{1, 3\}$, the solution of the $\mathcal{P}(C)$-equilibrium problem,

$$e = (\alpha_1 - \beta_1 - \beta_3, \alpha_2 - \beta_2, \alpha_3 - \beta_1 - \beta_3) = \left(\frac{43}{8}, \frac{15}{4}, \frac{19}{8}\right),$$

is an envy-free emissions allocation, since for every i and j in I with $i \neq j$, $u_i(e_i, \underline{e}_{-i}) > u_i(e_j, \underline{e}_{-j})$.

However, the solution of the $\mathcal{P}(I)$-equilibrium problem,

$$\underline{e}' = (\alpha_1 - \beta_1 - \beta_2 - \beta_3, \alpha_2 - \beta_1 - \beta_2 - \beta_3, \alpha_3 - \beta_1 - \beta_2 - \beta_3) = \left(\frac{41}{8}, \frac{25}{8}, \frac{17}{8}\right),$$

is not envy-free, since $u_3(e'_3, e'_{-3}) < u_3(e'_2, e'_{-2})$, meaning that 3 envies 2 at $\underline{e}'$.

For the sake of completeness we remind the conditions under which an environmental coalition is stable, that is it satisfies internal and external stability conditions (3) and (4).

Proposition 3.16 *Given a game Γ in $\mathcal{G}$, a formable environmental coalition C is stable if and only if*

(a) *for each i in C, $\beta_i \geq (2|C_{-i}|)^{-0.5} \sum_{h \in C_{-i}} \beta_h$;*

(b) *for each i in* $I \setminus C$, *for which the solution of the* $\mathcal{P}(C_{+i})$*-equilibrium problem is an emissions allocation,*[7] $0 < \beta_i \le (2|C|)^{-0.5} \sum_{h \in C} \beta_h$.

4 The two stage envy-free game

In the light of the results obtained in the previous section, we can state that not all environmental agreements ensure global absence of envy nor envy-freeness among coalesced countries. As observed, $\mathcal{P}(C)$-envy-freeness may be interpreted as a check whether the agreement reflects each coalesced player's idea on how much each member of the coalition should produce and therefore emit. Hence, it seems interesting to emphasize this criterion to ensure that each player feels properly recognized her effort within the coalition. Moreover, we believe that equity at least among cooperators may be a normative rule generating economic, social and human justice. For this reason, our investigation carries on by introducing a new type of game. We assume that, once decided to cooperate, agents have to solve a maximization problem subject to the constraint imposing envy-freeness among them. Therefore, we analyze how requiring absence of envy à la Foley inside the coalitional optimization problem may impact the agreement.

Definition 4.1 An *envy-free environmental coalition* is a set of countries, $C \subseteq I$, deciding to cooperate with the aim of maximizing the joint welfare $u_C(\underline{e}) = \sum_{i \in C} u_i(\underline{e})$ under the constraints imposing absence of envy inside the coalition: $u_i(e_i, \underline{e}_{-i}) \ge u_i(e_j, \underline{e}_{-j})$ for every i and j in $\in C$.[8]

Each country acts individually, voluntarily and without any kind of external constraint. Each one chooses whether to try to join the coalition and sign an envy-free environmental agreement. Assuming that agents want to try to consider a possible partition $\mathcal{P}(C)$ of I, members of C play as a unique player optimizing the joint utility function subject to the envy-freeness constraint. From an individualistic point of view, each player in C acts assuming that every cooperator maximizes the joint utility making sure not to envy each other. On the contrary, each country outside C maximizes its own utility function not caring to compare its level of utility with the one it would have if it emitted like someone else. If the coalitional maximization problem has an admissible solution, then the coalition should be formed, otherwise, agents have to look for other possible partitions of I, $\mathcal{P}(C)$, with C non degenerate or degenerate coalition. As usual in IEA literature, we refer to this stage of the game as *emissions stage*. If C is formable, players have to check if it is self-enforcing and consequently the agreement is signable. As usual in IEA literature, we call this step of the game *membership stage*.

Emissions stage
Assuming that agents agree to test the admissibility of the partition $\mathcal{P}(C)$ of I, in order to determine the level of emissions to produce, members of C solve the following optimization problem

[7] Note that if for every i in $I \setminus C$ the $\mathcal{P}(C_{+i})$-equilibrium problem has not admissible solution, no coalition C_{+i} is formable and then C is external stable.

[8] As previously observed the constraints may be written as $B_i(e_i) \ge B_i(e_j), \forall i, j \in C$.

$$\begin{cases} \max\limits_{\{e_i\}_{i\in C}} u_C(\underline{e}) = \max\limits_{\{e_i\}_{i\in C}} \left(\sum\limits_{i\in C} \left(\alpha_i e_i - \frac{1}{2} e_i^2 - \beta_i E \right) \right) \\ \text{subject to} \\ u_i(e_i, \underline{e}_{-i}) = \alpha_i e_i - \frac{1}{2} e_i^2 - \beta_i E \geq \alpha_i e_j - \frac{1}{2} e_j^2 - \beta_i E = u_i(e_j, \underline{e}_{-j}), \quad \forall i, j \in C \end{cases} ; (5)$$

simultaneously, any country outside the coalition, that is any $i \in I \setminus C$, solves its own optimization problem

$$\max_{e_i} u_i(\underline{e}) = \max_{e_i} \left(\alpha_i e_i - \frac{1}{2} e_i^2 - \beta_i E \right). \tag{6}$$

Given an envy-free environmental coalition C, we define

– $\mathcal{P}(C)$ *envy-free equilibrium problem*, or ef-$\mathcal{P}(C)$ *equilibrium problem*, the problem composed by (5) and (6);

– ef-$\mathcal{P}(C)$ *emissions allocation*, if there exists, a componentwise positive vector $\underline{e}$ such that in $\underline{e}(C)$ the maximum required in the problem (5) is achieved, while, for each i in $I \setminus C$, e_i is the point in which the solution of the maximization problem (6) is realized.

Notice that, for each envy-free environmental coalition $C \subseteq I$ for which the $\mathcal{P}(C)$-equilibrium problem has an admissible solution, the resulting ef-$\mathcal{P}(\mathcal{C})$ emissions allocation is $\mathcal{P}(C)$-envy-free. In particular, if there exists, the ef-emissions allocation is envy-free.

Membership stage

Assuming that the envy-free environmental coalition C is formable, in the membership stage it is tested its stability. In accordance with the standard case, an admissible envy-free environmental coalition C is stable if the ef-$\mathcal{P}(C)$ emissions allocation satisfies the envy-free internal and external stability conditions.

In line with the literature and the previous section, the envy-free internal stability condition requires that for every i in C for which there exists the ef-$\mathcal{P}(C_{-i})$ emissions allocation, $\underline{\tilde{e}}$,

$$u_i(\underline{e}) \geq u_i(\underline{\tilde{e}}); \tag{7}$$

while the envy-free external stability requires that for every i in $I \setminus C$ for which there exists the ef-$\mathcal{P}(C_{+i})$ emissions allocation, $\underline{\tilde{e}}$,

$$u_i(\underline{e}) \geq u_i(\underline{\tilde{e}}). \tag{8}$$

We define *envy-free environmental agreement* a deal signed by an admissible stable environmental coalition, that is an agreement in which the emissions allocation is a componentwise positive vector obtained solving (5) and (6), and satisfying (7) and (8).

4.1 The two-player envy-free game

In order to analyze how the absence of envy within the optimization problem impacts on the choice of emissions, let us consider the particular case in which $I = \{1, 2\}$. Without loss of generality, we assume that 1 is more developed than 2. Then, by assumption $(A2)$, $\alpha_1 > \alpha_2$ and $\beta_1 < \beta_2$. Hence,

$$\mathcal{G} = \{\Gamma = \{\{1, 2\}, \{S_i\}_{i=1,2}, \{u_i\}_{i=1,2}\} \; : \; \forall i = 1, 2, \; S_i = \mathbb{R}^2_{++}, \; u_i = \alpha_i e_i - \frac{1}{2} e_i^2 - \beta_i E,$$
$$\text{satisfying } (A1) \text{ and } (A2)\}.$$

In line with Proposition 3.4, solving the ef-$\mathcal{P}(I)$-equilibrium problem we get that adding the envy-freeness constraint in the maximization problem affects emissions decisions only if the set of parameters does not satisfy the conditions $\beta_1+\beta_2<\alpha_2\le\alpha_1-2\beta_1-2\beta_2$. In the following proposition we determine the functional form of ef-$\mathcal{P}(I)$-emissions allocation. The proof is shown in the Appendix.

Proposition 4.2 *Let Γ belong to $\mathcal{G}$. The solution, $\underline{e}^*$, of the ef-$\mathcal{P}(I)$-equilibrium problem is an emissions allocation defined as*

$$\underline{e}^*=\begin{cases}(\alpha_1-\beta_1-\beta_2,\alpha_2-\beta_1-\beta_2), \text{ if } \beta_1+\beta_2<\alpha_2\le\alpha_1-2\beta_1-2\beta_2,\\ \\ \frac{1}{2}(\alpha_1+\alpha_2,3\alpha_2-\alpha_1), \text{ if } \max\{\alpha_1-2\beta_1-2\beta_2,\frac{1}{3}\alpha_1,\beta_2\}<\alpha_2\le\alpha_1-\beta_1-\beta_2,\\ \\ \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2,\alpha_1+\alpha_2-2\beta_1-2\beta_2),\\ \text{if } \max\{2\beta_1+2\beta_2-\alpha_1,\alpha_1-\beta_1-\beta_2,\beta_2\}<\alpha_2<\alpha_1.\end{cases}$$

Let $\underline{e}^*$ and $\underline{e}^{NC}$ be respectively the solution of the $\mathcal{P}(I)$ envy-free equilibrium problem and the vector $(\alpha_1-\beta_1,\alpha_2-\beta_2)$, solution of the non-cooperative game.

Let us define the following subfamilies of $\mathcal{G}$

$$\mathcal{G}_1=\{\Gamma\in\mathcal{G}:\ \beta_1+\beta_2<\alpha_2\le\alpha_1-2\beta_1-2\beta_2\},$$
$$\mathcal{G}_2=\left\{\Gamma\in\mathcal{G}\ :\ \max\left\{\alpha_1-2\beta_1-2\beta_2,\frac{1}{3}\alpha_1,\beta_2\right\}<\alpha_2\le\alpha_1-\beta_1-\beta_2\right\},$$
$$\mathcal{G}_3=\{\Gamma\in\mathcal{G}\ :\ \max\{2\beta_1+2\beta_2-\alpha_1,\alpha_1-\beta_1-\beta_2,\beta_2\}<\alpha_2<\alpha_1\}.$$

Since, as already noted, for any $\Gamma\in\mathcal{G}_1$, $\underline{e}^*$ coincides with the standard cooperative emissions allocation, we focus in other cases.

In the following propositions, whose proofs are collected in the Appendix, we analyze the stability of the coalition $C=I$. Since $I\setminus C=\emptyset$, only condition (7) has to be checked. It is equivalent to requiring that no country prefers to act alone instead of cooperating. In other words, I is stable, and therefore players sign the agreement, if the level of utility that each one achieves if the coalition is formed is not lower than if everyone acted alone. Formally, the coalition I is stable if and only if, for $i=1,2$, $u_i(\underline{e}^*)\ge u_i(\underline{e}^{NC})$.

Proposition 4.3 *Given a game Γ in $\mathcal{G}_2$, the envy-free cooperation is stable if and only if the parameters satisfy the following conditions*

$$\beta_1>\frac{\sqrt{2}-1}{2}(\alpha_1-\alpha_2)\ \textit{and}\ \ \beta_2<\frac{-4\beta_1^2+8(\alpha_1-\alpha_2)\beta_1-(\alpha_1-\alpha_2)^2}{8\beta_1}. \tag{9}$$

Analyzing the proof of Proposition 4.3, we can observe that for any game in $\mathcal{G}_2$ the least developed player has no incentive to leave the coalition. The envy-free agreement, indeed, guarantees her a higher level of utility than non-cooperation. Therefore, the agreement is stable only for games in which the combination of parameters satisfies the stability condition of the most developed player.

Proposition 4.4 *Given a game Γ in $\mathcal{G}_3$, the envy-free cooperation is stable if and only if the parameters satisfy the following condition*

$$\frac{2+\sqrt{5}}{2}(\alpha_1-\alpha_2)<\beta_1<\beta_2\le\frac{1}{2}(\alpha_2-\alpha_1)+\sqrt{\beta_1(\alpha_2-\alpha_1+2\beta_1)}. \tag{10}$$

Looking at the computations made in the proof of Proposition 4.4 we can remark that for some game in $\mathcal{G}_3$ the least developed country has incentive to withdraw from the envy-free agreement. The level of utility she achieves by non-cooperating is, in fact, greater than she would receive if the coalition is formed. Moreover, in any game in which the least developed player would not cooperate, neither would the most developed. In addition, there are games in which the least developed player would agree to cooperate, while the most developed player would veto it. So, once again, in a sense, we could say that the most developed player has the power to determine cooperation.

Envy-free cooperation VS non-cooperation

Assuming as *status quo* the scenario that would occur without cooperation, the following considerations can be taken into account.

For any game $\Gamma \in \mathcal{G}$, the aggregate of the ef-cooperative emissions allocation is lower than equal to the aggregate of non-cooperative one, i.e., $E^* \leq E^{NC}$. This means that, even by adding the envy-freeness constraint to the optimization problem, cooperation ensures environmental protection. More specifically,[9] for every Γ in $\mathcal{G}_2$, $E^* = 2\alpha_2 \leq \alpha_1 + \alpha_2 - \beta_1 - \beta_2 = E^{NC}$, while for every Γ in $\mathcal{G}_3$, $E^* = \alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 < \alpha_1 + \alpha_2 - \beta_1 - \beta_2 = E^{NC}$.

However, unlike the standard case, and then for games in $\mathcal{G}_1$, the lowering of the total emissions is not always dictated by the lowering of each individual player's emissions. Indeed, for any game in $\mathcal{G}_2 \cup \mathcal{G}_3$, under envy-free cooperation, the most developed country has to reduce its production level, but the same might not happen for the least developed player. Specifically, for each game Γ in $\mathcal{G}_2$, the level of coalitional emissions of the most developed player, $e_1^* = \frac{1}{2}(\alpha_1 + \alpha_2)$, is lower than what she would have chosen if she had not cooperated, $e_1^{NC} = \alpha_1 - \beta_1$. On the other hand, the level of coalitional emissions of the least developed player, $e_2^* = \frac{1}{2}(3\alpha_2 - \alpha_1)$, is lower than what she would have chosen if she had not cooperated, $e_2^* < e_2^{NC} = \alpha_2 - \beta_2$, if $\alpha_1 - 2\beta_2 < \alpha_2 < \alpha_1 - \beta_1 - \beta_2$, while is greater, $e_2^* > e_2^{NC}$, if $\alpha_1 - 2\beta_1 - 2\beta_2 < \alpha_2 < \alpha_1 - 2\beta_2$. Similarly, for each game Γ in $\mathcal{G}_3$, $e_1^* = \frac{1}{2}(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2) < \alpha_1 - \beta_1 = e_1^{NC}$, but $e_2^* = \frac{1}{2}(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2) < \alpha_2 - \beta_2 = e_2^{NC}$, if $\alpha_1 - 2\beta_1 < \alpha_2 < \alpha_1$, while $e_2^* > e_2^{NC}$, if $\alpha_1 - \beta_1 - \beta_2 < \alpha_2 < \alpha_1 - 2\beta_1$.

For any game Γ in $\mathcal{G}_2$, $\underline{e}^*$ and $\underline{e}^{NC}$ guarantee absence of envy among countries. Indeed, by construction, $\underline{e}^*$ is $\mathcal{P}(I)$-envy free, and then envy-free, while at $\underline{e}^{NC}$ no player envy the other. More precisely, $u_1(e_1^{NC}, e_2^{NC}) \geq u_1(e_2^{NC}, e_1^{NC})$ is satisfied for every Γ in $\mathcal{G}$; while $u_2(e_1^{NC}, e_2^{NC}) \geq u_2(e_2^{NC}, e_1^{NC})$ is satisfied for every Γ in $\mathcal{G}_1 \cup \mathcal{G}_2$.[10]

On the other hand, for any game Γ in $\mathcal{G}_3$, $\underline{e}^*$, by construction, ensures no envy among players, while $\underline{e}^{NC}$ is not envy-free. As previously observed, even in games belonging to $\mathcal{G}_3$, the most developed country does not envy the least developed one, but the vice versa is not true.

Envy-free cooperation VS standard cooperation

Let us make a comparison between envy-free cooperation - with emissions allocation $\underline{e}^*$ - and standard cooperation, according to which the emissions allocation is the vector $\underline{e}^C = (\alpha_1 - \beta_1 - \beta_2, \alpha_2 - \beta_1 - \beta_2)$, solution of the $\mathcal{P}(I)$-equilibrium problem.

[9] This result is already known in the case of standard cooperation and, therefore, it is not necessary to observe it for games in $\mathcal{G}_1$.

[10] Condition $u_1(e_1^{NC}, e_2^{NC}) \geq u_1(e_2^{NC}, e_1^{NC})$ is equivalent to $B_1(e_1^{NC}) = \alpha_1(\alpha_1 - \beta_1) - \frac{1}{2}(\alpha_1 - \beta_1)^2 \geq \alpha_1(\alpha_2 - \beta_2) - \frac{1}{2}(\alpha_2 - \beta_2)^2 = B_1(e_2^{NC})$, that can be reformulated as $(\alpha_1 - \alpha_2 - \beta_1 + \beta_2)(\alpha_1 - \alpha_2 + \beta_1 + \beta_2) \geq 0$. It is easy to note that it is satisfied for every Γ in $\mathcal{G}$. Condition $u_2(e_1^{NC}, e_2^{NC}) \geq u_2(e_2^{NC}, e_1^{NC})$ is equivalent to $B_2(e_2^{NC}) = \alpha_2(\alpha_2 - \beta_2) - \frac{1}{2}(\alpha_2 - \beta_2)^2 \geq \alpha_2(\alpha_1 - \beta_1) - \frac{1}{2}(\alpha_1 - \beta_1)^2 = B_2(e_1^{NC})$, that can be reformulated as $(\alpha_2 - \alpha_1 + \beta_1 - \beta_2)(\alpha_2 - \alpha_1 + \beta_1 + \beta_2) \geq 0$ that is satisfied only if Γ belongs to $\mathcal{G}_1 \cup \mathcal{G}_2$.

Let us start with the family $\mathcal{G}_2$ and let us partition it in $\mathcal{G}_2' \cup \mathcal{G}_2''$, where

$$\mathcal{G}_2' = \left\{ \Gamma \in \mathcal{G}_2 \ : \ \max\left\{ \alpha_1 - 2\beta_1 - 2\beta_2, \frac{1}{3}\alpha_1, \beta_2 \right\} < \alpha_2 \leq \beta_1 + \beta_2 \right\} \text{ and } \mathcal{G}_2'' = \mathcal{G}_2 \setminus \mathcal{G}_2'.$$

Notice that for every game Γ in $\mathcal{G}_2'$ standard cooperation is not admissible, since the second component of the solution of the $\mathcal{P}(I)$-equilibrium problem is non-positive, $e_2^C \leq 0$. Hence, imposing envy-freeness constraint within the optimization problem allows environmental cooperation that otherwise would not occur. Indeed, for any game Γ in $\mathcal{G}_2'$ satisfying (9), with the standard mechanism, countries would act alone, while with the rule imposing no envy between them, both have convenience to cooperate, and then the envy-free environmental agreement is signed.

Let us remark that, for every game Γ in $\mathcal{G}_2'$, our criterion leads to environmental and economic improvement. Indeed, with the standard mechanism the players are forced not to cooperate, generating a growing pollution and a reduction of individual and social well-being. In detail, for an environmental point of view, using our rule, the total emission is less than equal to the non-cooperative one, and therefore, greater environmental protection is guaranteed. On the other hand, from an economic and social point of view, each country is better off, receiving a level of utility greater than the one it obtained non-cooperating. Therefore, even joint welfare increases.

For each game Γ in $\mathcal{G}_2''$, the standard and envy-free cooperation are both admissible, since $\underline{e}^C$ and $\underline{e}^*$ belong to $\mathbb{R}^2_{++}$. Following Proposition 3.16, the standard environmental cooperation is stable if and only if

$$0 < \beta_1 < \beta_2 \leq \sqrt{2}\beta_1. \tag{11}$$

Then, comparing conditions (9) and (11) we get that for some games, both standard and envy-free cooperation are stable, while there exist games where stability is guaranteed only under one of the two mechanisms. In the following table we summarize the results obtained through algebraic computations.

Conditions on parameters	Standard EA	Envy-free EA
$\beta_1 \in \left(\frac{\sqrt{2}-1}{2}(\alpha_1 - \alpha_2), \frac{1}{2}(\alpha_1 - \alpha_2)\right)$ and $\beta_2 \in (a, b]$ where $a = \max\left\{\frac{1}{2}(\alpha_1 - \alpha_2 - 2\beta_1), \beta_1\right\}$, $b = \min\left\{\frac{-4\beta_1^2 + 8(\alpha_1-\alpha_2)\beta_1 - (\alpha_1-\alpha_2)^2}{8\beta_1}, \sqrt{2}\beta_1\right\}$	Stable	Stable
$\beta_1 \in \left(0, \frac{\sqrt{2}-1}{2}(\alpha_1 - \alpha_2)\right]$ and $\beta_2 \in (\beta_1, \sqrt{2}\beta_1)$ or $\beta_1 \in \left[\frac{3+\sqrt{2}}{14}(\alpha_1 - \alpha_2), \frac{1}{2}(\alpha_1 - \alpha_2)\right)$ and $\beta_2 \in \left(\frac{-4\beta_1^2 + 8(\alpha_1-\alpha_2)\beta_1 - (\alpha_1-\alpha_2)^2}{8\beta_1}, \sqrt{2}\beta_1\right)$	Stable	Non-stable
$\beta_1 \in \left(\frac{\sqrt{2}-1}{2}(\alpha_1 - \alpha_2), \frac{3+\sqrt{2}}{14}(\alpha_1 - \alpha_2)\right)$ and $\beta_2 \in \left(\sqrt{2}\beta_1, \frac{-4\beta_1^2 + 8(\alpha_1-\alpha_2)\beta_1 - (\alpha_1-\alpha_2)^2}{8\beta_1}\right)$	Non-stable	Stable

Let us denote by $\tilde{\mathcal{G}}_2''$ the subfamily of $\mathcal{G}_2''$ composed by games for which standard and envy-free cooperation are both stable.

The aggregate of the ef-cooperation emissions allocation is greater than the aggregate of standard cooperative one, i.e., $E^C = \alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2 < 2\alpha_2 = E^*$. Moreover, for each player i, the relationship $e_i^C < e_i^*$ holds,[11] indeed, $e_1^C = \alpha_1 - \beta_1 - \beta_2 < \frac{1}{2}(\alpha_1 + \alpha_2) = e_1^*$ and $e_2^C = \alpha_2 - \beta_1 - \beta_2 < \frac{1}{2}(3\alpha_2 - \alpha_1) = e_2^*$. It means that by signing an envy-free environmental agreement, each country emits a higher level of pollution than it would by subscribing an environmental agreement that allows for envy between cooperators. The central point is that, by not cooperating, players do not envy each other, while with a standard environmental agreement the least developed country envies the most developed one. In the standard cooperation, in fact, each player reduces her level of production and the least developed country lowers it to such an extent that she ends up envying the more developed one. Under no-envy constraint, therefore, the least developed player has to be able to produce more than e_2^C. As a consequence, the production level of the least developed country goes up to the point of forcing the most developed player to produce more than e_1^C, otherwise she would envy the least developed country. However, these production increases cause higher costs for both players. The second player, to reach the level of production required by the envy-free environmental agreement, faces a cost increase, $D_2(\underline{e}^*) - D_2(\underline{e}^C)$, that is not offset by the benefit increase, $B_2(e_2^*) - B_2(e_2^C)$. Therefore $u_2(\underline{e}^*) < u_2(\underline{e}^C)$. On the other hand, for the most developed player $B_1(e_1^*) - B_1(e_1^C)$ does not compensate $D_1(\underline{e}^*) - D_1(\underline{e}^C)$, and then $u_1(\underline{e}^*) < u_1(\underline{e}^C)$, if and only if $\alpha_2 < \alpha_1 - 6\beta_1 + \beta_2$. Hence $\underline{e}^*$ is Pareto improved by $\underline{e}^C$ for any Γ in $\tilde{\mathcal{G}}_2''$ for which $\alpha_2 < \alpha_1 - 6\beta_1 + \beta_2$. See Remark 5.1 for computational details.

Summing up the results obtained from previous comparisons, we get the following proposition.

Proposition 4.5 (i) *For any game Γ in $\mathcal{G}_2'$, the $\mathcal{P}(I)$-equilibrium problem has no admissible solution, while the ef-$\mathcal{P}(I)$-equilibrium problem has admissible solution. Moreover, for games satisfying (9) the solution of the ef-$\mathcal{P}(I)$ equilibrium problem is stable.*

(ii) *For any game Γ in $\mathcal{G}_2''$ both equilibrium problems have admissible solution, moreover, denoting by $\underline{e}^C$ and $\underline{e}^*$ respectively the solution of the $\mathcal{P}(I)$-equilibrium problem and the solution of the ef-$\mathcal{P}(I)$ equilibrium problem*

(a) *the following relationships hold*

 (a1) $E^C < E^*$,
 (a2) *for any* $i = 1, 2$, $\underline{e}_i^C < \underline{e}_i^*$,
 (a3) $u_2(\underline{e}^*) < u_2(\underline{e}^C)$,
 (a4) $u_1(\underline{e}^*) < u_1(\underline{e}^C)$ *if and only if* $\alpha_2 < \alpha_1 - 6\beta_1 + \beta_2$;

(b) *there exist games for which I is stable under $\underline{e}^C$, while it is not stable under $\underline{e}^*$;*
(c) *there exist games for which I is stable under $\underline{e}^*$, while it is not stable under $\underline{e}^C$.*

Let us now consider the family $\mathcal{G}_3$ and let us partition it in $\mathcal{G}_3' \cup \mathcal{G}_3''$, where

$$\mathcal{G}_3' = \{\Gamma \in \mathcal{G}_3 \ : \ \max\{2\beta_1 + 2\beta_2 - \alpha_1, \alpha_1 - \beta_1 - \beta_2, \beta_2\} < \alpha_2 \leq \beta_1 + \beta_2\} \text{ and } \mathcal{G}_3'' = \mathcal{G}_3 \setminus \mathcal{G}_3'.$$

Notice that for every game Γ in $\mathcal{G}_3'$ standard cooperation is not admissible, since $e_2^C \leq 0$. Hence, even for game in $\mathcal{G}_3$ imposing envy-freeness constraint within the optimization problem might allow environmental cooperations that otherwise would not occur. Indeed,

[11] Notice that $e_1^C < e_1^*$, $e_2^C < e_2^*$, and then $E^C < E^*$, are true for any game in $\mathcal{G}_2$. We focus on games Γ in $\tilde{\mathcal{G}}_2''$, since it is not interesting to compare between them agreements if it is known that one of these would not be signed.

for any game Γ in $\mathcal{G}_3'$ satisfying (10), with the standard rule countries are forced to non-cooperation, while with envy-free mechanism both have interest to cooperate, signing and maintaining the envy-free environmental agreement. Once again, we identify a family of games for which our criterion generates environmental and economic-social improvement.

For each game Γ in $\mathcal{G}_3''$, the standard and envy-free cooperation are both admissible, since $\underline{e}^C$ and $\underline{e}^*$ belong to $\mathbb{R}^2_{++}$. Comparing conditions (10) and (11), we get that for any game in which envy-free cooperation is stable, so is standard cooperation, while the reverse is not true.

The aggregate of the ef-cooperation emissions allocation is equal to the aggregate of standard cooperative one, i.e., $E^C = E^*$. Moreover, the relationships $e_1^* < e_1^C$ and $e_2^C < e_2^*$ hold,[12] indeed, $e_1^* = \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2) < \alpha_1-\beta_1-\beta_2 = e_1^C$ and $e_2^C = \alpha_2-\beta_1-\beta_2 < \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2) = e_2^*$. It means that by signing an envy-free environmental agreement, the most developed country agrees to emit less, letting the least developed country emit more. In other words, the first player is willing to decrease her production level allowing the second player to produce more.

At the equilibrium $\underline{e}^*$, the most developed country gets a lower level of benefits than it would get in standard cooperation, $B_1(e_1^*) < B_1(e_1^C)$, while the least developed country sees its benefits level grows, $B_2(e_2^*) > B_2(e_2^C)$. Therefore, since $\underline{e}^*$ and $\underline{e}^C$ generate the same costs, i.e., $D_i(\underline{e}^*) = D_i(\underline{e}^C)$ for $i = 1, 2$, then $u_1(\underline{e}^*) < u_1(\underline{e}^C)$ and $u_2(\underline{e}^*) > u_2(\underline{e}^C)$. Hence, imposing envy-freeness constraint in the maximization problem ensures a kind of social equity. The most developed country is willing to lower its utility level and to allow the least developed country to increase its production and benefits levels. See Remark 5.2 in the Appendix for computational details.

The substantial difference between games in $\mathcal{G}_2$ and in $\mathcal{G}_3$ lies in the absence/presence of envy in non-cooperation. For any game in $\mathcal{G}_3$ where standard cooperation is allowed, it is not envy-free, nor is non-cooperation. In both cases, the least developed country envies the most developed. Therefore, in order not to be envied and, at the same time, not to become envious, the most developed player is willing to balance the production growth of the least developed player with a decrease in her own level of production.

We can observe that, for each game belonging to $\mathcal{G}_3$ and satisfying (10), our approach ensures environmental preservation and social gain. In particular, for games in $\mathcal{G}_3'$, compared with our mechanism, the standard rule causes an increasing in pollution and a lowering of the level of individual and social utility. For games in $\mathcal{G}_3''$, our criterion, compared with the standard one, ensures the same environmental safety and generates social equity, allowing a balance of the level of production and a consequent growth, in terms of welfare, of the least developed country.

Summarizing the results obtained from previous comparisons we have the following proposition.

Proposition 4.6 (i) *For any game Γ in $\mathcal{G}_3'$, the $\mathcal{P}(I)$-equilibrium problem has no admissible solution, while the ef-$\mathcal{P}(I)$-equilibrium problem has admissible solution. Moreover, for games satisfying (10) the solution of the ef-$\mathcal{P}(I)$ equilibrium problem is stable.*

(ii) *For any game Γ in $\mathcal{G}_3''$ both equilibrium problems have admissible solution, moreover, denoting by $\underline{e}^C$ and $\underline{e}^*$ respectively the solution of the $\mathcal{P}(I)$-equilibrium problem and the solution of the ef-$\mathcal{P}(I)$ equilibrium problem*

(a) $E^C = E^*$, $\underline{e}_1^C < \underline{e}_1^*$, $\underline{e}_2^* < \underline{e}_2^C$, $u_1(\underline{e}^*) < u_1(\underline{e}^C)$, *and* $u_2(\underline{e}^C) < u_2(\underline{e}^*)$;

[12] Notice that $e_1^* < e_1^C$ and $e_2^C < e_2^*$ are satisfied for any game in $\mathcal{G}_3$, but, once again, we focus our comparison only on games Γ where both agreements would be signed.

(b) *if I is stable under $\underline{e}^*$, then I is stable under $\underline{e}^C$.*

Focusing on the aggregate emissions allocations, considering envy-freeness constraint in the maximization problem leads to solutions whose aggregate, E^*, satisfies the inequalities $E^C \leq E^* \leq E^{NC}$. More precisely, for any game Γ in $\mathcal{G}_1 \cup \mathcal{G}_3$, $E^C = E^* < E^{NC}$, while, for every Γ in $\mathcal{G}_2$, $E^C < E^* \leq E^{NC}$. In order to avoid the inequality $E^C < E^*$, we may impose E^C as an envy-free cooperative total emission cap. In a sense, we assume that members of an envy-free environmental coalition dictate a constraint requiring a threshold beyond which the level of emissions is judged unacceptable. Requiring the existence of a cap is in line with a branch of IEA literature (see among others [52]).

Formally, we ask that in the emission stage the signatories solve the following optimization problem

$$\begin{cases} \max\limits_{(e_1,e_2)} u_C(\underline{e}) \\ \text{subject to} \\ u_i(e_i, e_{-i}) \geq u_i(e_j, e_{-j}), \quad \forall i, j \in \{1, 2\} \\ e_1 + e_2 \leq E^C \end{cases} \tag{12}$$

In line with Propositions 3.10 and 4.2, we get that adding the boundary condition $e_1+e_2 \leq E^C$ affects emissions decisions only for games in $\mathcal{G}_2$. Indeed, the solution of the problem (12) has the following functional form

$$\underline{e}^* = \begin{cases} (\alpha_1 - \beta_1 - \beta_2, \alpha_2 - \beta_1 - \beta_2), \text{ if } \beta_1 + \beta_2 < \alpha_2 \leq \alpha_1 - 2\beta_1 - 2\beta_2, \\ \\ \frac{1}{2}(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2, \alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2), \\ \text{if } \max\{\alpha_1 - 2\beta_1 - 2\beta_2, 2\beta_1 + 2\beta_2 - \alpha_1, \beta_2\} < \alpha_2 < \alpha_1. \end{cases}$$

Defining $\tilde{\mathcal{G}} := \{\Gamma \in \mathcal{G} : \max\{\alpha_1 - 2\beta_1 - 2\beta_2, 2\beta_1 + 2\beta_2 - \alpha_1, \beta_2\} < \alpha_2 < \alpha_1\}$, it is easy to show that, fixed a game Γ in $\tilde{\mathcal{G}}$, the cooperation under $\underline{e}^*$ is stable if and only if (10) holds. Moreover, any game in which envy-free cooperation is stable, so is standard cooperation, while the reverse is not true. With algebraic computation, we can show that $u_1(\underline{e}^*) < u_1(\underline{e}^C)$ and $u_2(\underline{e}^*) > u_2(\underline{e}^C)$. Then, by signing the agreement, countries protect the environment and guarantee social equity.

As previously observed, indeed, for any game Γ in $\tilde{\mathcal{G}}$ for which conditions $\alpha_2 < \beta_1 + \beta_2$ and (10) hold, the standard criterion forces to non-cooperation, while the rule imposing to solve (12) leads players to an equal sharing of E^C. This enables better protection of the environment and an increase in individual and joint welfare. For other games in $\tilde{\mathcal{G}}$ satisfying (10), the new rule ensures the same environmental safety guaranteed by standard cooperation and generates social equity, allowing the least developed country a growth in terms of level of production, benefits and welfare.

5 Conclusion

In this paper we analyze envy-freeness notions in international environmental agreements framework. In particular, we focus on deal in which no country envies à la Foley another state, and on those in which absence of envy is satisfied at least among players belonging to the same coalition. We show that not all agreements resulting from the standard environmental coalition formation rule satisfy these equity conditions. We, then, characterize games guaranteeing the existence of at least one formable environmental coalition generating an emissions allocation ensuring global absence of envy or at least among coalesced players.

From an individual point of view envy-freeness among players is relevant for a double motivation. On the one hand, it is equivalent to excluding, for each country, a negative feeling on the emission level established by the agreement. On the other hand, it ensures that each member of the coalition sees her own idea of emissions allocation reflected in the deal. With the spirit of testing if equity at least among coalesced countries may be a normative criterion inducing environmental, social and economic improvement, we introduce a new mechanism for cooperation. We require that signatory countries maximize the joint utility function subject to the constraint imposing that they do not envy each other.

The analysis carried out in two-player games shows that imposing envy-freeness as a constraint in the optimization problem provides advantages from both the individual and the social point of view. Considering the social aspect, at the optimum, in several cases, our rule leads to social equity. Comparing the results of environmental agreements non-avoiding envy with those imposing envy-freeness, we get that, in order to be neither envious nor envied, the most developed player is willing to give up a part of her levels of production and benefits to allow the least developed player to produce more and increase her benefits level. This balance of production, and therefore of emissions, is obtained endogenously in games where the agents are, in a sense, not too far apart in terms of benefit per unit of emissions and net marginal damage. In these games, players agree to share standard coalitional aggregate, producing the same quantity of emissions. This result is in line with the well known equal treatment property satisfied by envy-free allocation, i.e., identical agents are treated equally. Indeed, it is easy to note that, when the parameters pair (α_2, β_2) converges to (α_1, β_1), and then players are similar in characteristics and tend to be of the same type, the envy-free allocation $\underline{e}^* = \frac{1}{2}(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2, \alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2)$ collapses to the standard cooperative allocation in games with homogeneous players, i.e., $\underline{e}^C = (\alpha_1 - 2\beta_1, \alpha_1 - 2\beta_1)$.

By requiring that total emissions should not exceed the standard cooperative aggregate, we achieve the same results and considerations for all games in which standard and envy-free environmental agreements are signable and comparable, although different. It means that, even for games where players are potentially distant in parameters, envy-free environmental agreements endogenously ensure social equity, causing agents to share emissions, balancing production level. This equal sharing of standard cooperative emissions gives the least developed country the opportunity to increase its level of production, its benefits and, in the future, its resilience to possible environmental damage.

It would be interesting to analyze n-player games (with $n > 2$) and investigate how the envy-freeness constraint inside the rule impacts on the size of stable coalitions, discovering if it can help to solve the small coalitions puzzle in IEA.

Moreover, it would be interesting to conduct an analysis in line with [24], that is investigate on various equity notions, compare them with each other and analyze their impact in terms of social equity and size of stable coalition. For example, we can extend *per-capita envy-freeness* notion, due to Pazner [26], and *average envy-freness* notion introduced by Thomson [27]. Following Pazner's idea, a per-capita envy-free environmental agreement is a deal for which each signatory weakly prefers her production level to the average level of the entire coalition. On the other hand, according to Thomson's idea, an average envy-free environmental agreement is a deal for which each coalesced player does not prefer the average level of all other members of coalition to her own.

Funding Open access funding provided by Università Parthenope di Napoli within the CRUI-CARE Agreement.

Declarations

Competing interests The authors have no competing interests to declare that are relevant to the content of this article. The manuscript has no associated data.

Appendix

Proof of Proposition 3.4 Let C be an environmental coalition. Let $\underline{e}$ be a solution of the $\mathcal{P}(C)$-equilibrium problem. By definition, $\underline{e}(C)$ solves (1), while for every $i \in I \setminus C$, e_i solves (2). Therefore, by the first order equilibrium conditions, $e_i = \alpha_i - \sum_{h\in C} \beta_h$, for each $i \in C$, and $e_i = \alpha_i - \beta_i$, for each $i \in I \setminus C$.

Notice that by $(A1)$ and (a), $\underline{e} \in \mathbb{R}^n_{++}$, then $\underline{e}$ is an emissions allocation.[13]

Let us prove that $\underline{e}$ is $\mathcal{P}(C)$-envy-free.

By Definition 3.3, for every i and j in C, the envy-freeness condition, $u_i(e_i, \underline{e}_{-i}) \geq u_i(e_j, \underline{e}_{-j})$, that is equivalent to $B_i(e_i) \geq B_i(e_j)$, has to be satisfied, i.e.,

$$\alpha_i\left(\alpha_i - \sum_{h\in C}\beta_h\right) - \frac{1}{2}\left(\alpha_i - \sum_{h\in C}\beta_h\right)^2 \geq \alpha_i\left(\alpha_j - \sum_{h\in C}\beta_h\right) - \frac{1}{2}\left(\alpha_j - \sum_{h\in C}\beta_h\right)^2.$$

By algebraic computation it leads

$$\frac{1}{2}(\alpha_i - \alpha_j)\left(\alpha_i - \alpha_j + 2\sum_{h\in C}\beta_h\right) \geq 0. \tag{13}$$

By $(A2)$, if i is more developed than j, (13) is always satisfied, since $\alpha_i > \alpha_j$ and then $\alpha_i - \alpha_j + 2\sum_{h\in C}\beta_h > 0$. On the other hand, if i is less developed than j, since $\alpha_j > \alpha_i$, (13) holds if and only if $\alpha_i \leq \alpha_j - 2\sum_{h\in C}\beta_h$, i.e., condition (b), appropriately rewritten, is satisfied. □

Proof of Proposition 3.5 Let C be an environmental coalition for which the $\mathcal{P}(C)$-emissions allocation, $\underline{e}$, is $\mathcal{P}(C)$-envy-free. Let $C' \subseteq C$ and $\underline{e}'$ be the solution of the $\mathcal{P}(C')$-equilibrium problem. As already proved, for any i and j in C, conditions (a) and (b) of Proposition 3.4 hold and

$$e_i = \begin{cases} \alpha_i - \sum_{h\in C}\beta_h, & \text{if } i \in C \\ \alpha_i - \beta_i, & \text{if } i \in I \setminus C \end{cases} \qquad e'_i = \begin{cases} \alpha_i - \sum_{h\in C'}\beta_h, & \text{if } i \in C' \\ \alpha_i - \beta_i, & \text{if } i \in I \setminus C' \end{cases}.$$

By $(A1)$ and (a), $\underline{e}' \in \mathbb{R}^n_{++}$ and therefore is a $\mathcal{P}(C')$-emissions allocation.

[13] If (a) is not satisfied, coalition C can not formed. Indeed, it is impossible for agent in C to sign an agreement, since it would ask someone not to produce (which is not reasonable) or to emit a negative amount of pollution (which is impossible).

It remains to prove that each country i in C' does not envy any j in C'. As observed in the previous proposition, it is equivalent to require that

$$\frac{1}{2}(\alpha_i - \alpha_j)\left(\alpha_i - \alpha_j + 2\sum_{h\in C'}\beta_h\right) \geq 0. \tag{14}$$

By $(A2)$, if i is more developed than j, (14) is satisfied, while if i is less developed than j, condition (b) of Proposition 3.4 implies $\alpha_i < \alpha_j - 2\sum_{h\in C'}\beta_h$ and then (14), that concludes the proof. □

Proof of Proposition 3.10 Let C be an environmental coalition. By Proposition 3.4, under $(A1)$, $(A2)$, (a) and $(b1)$, the solution of the $\mathcal{P}(C)$-equilibrium problem, $\underline{e}$, is a $\mathcal{P}(C)$-envy-free emissions allocation. Let us prove that there is no envy among countries outside C and between i and j with $i \in C$ and $j \in I\setminus C$.

We remind that, for every i in C, $e_i = \alpha_i - \sum_{h\in C}\beta_h$, while, for every i in $I \setminus C$, $e_i = \alpha_i - \beta_i$.

Let i and j belong to $I \setminus C$. Envy-freeness condition $B_i(e_i) \geq B_i(e_j)$ is equivalent to

$$\alpha_i(\alpha_i - \beta_i) - \frac{1}{2}(\alpha_i - \beta_i)^2 \geq \alpha_i(\alpha_j - \beta_j) - \frac{1}{2}(\alpha_j - \beta_j)^2.$$

By algebraic computation it leads

$$\frac{1}{2}(\alpha_i - \beta_i - \alpha_j + \beta_j)(\alpha_i + \beta_i - \alpha_j + \beta_j) \geq 0. \tag{15}$$

By $(A2)$, if i is more developed than j, (15) is always satisfied, since $\alpha_i - \beta_i - \alpha_j + \beta_j$ and $\alpha_i + \beta_i - \alpha_j + \beta_j$ are both positive. On the other hand, if i is less developed than j, (15) is satisfied if and only if $\alpha_i \leq \alpha_j - \beta_i - \beta_j$, i.e., $(b2)$, appropriately rewritten, holds.

Let $i \in C$ and $j \in I \setminus C$.

Country i does not envy country j if and only if $B_i(e_i) \geq B_i(e_j)$, that is equivalent to

$$\alpha_i\left(\alpha_i - \sum_{h\in C}\beta_h\right) - \frac{1}{2}\left(\alpha_i - \sum_{h\in C}\beta_h\right)^2 \geq \alpha_i(\alpha_j - \beta_j) - \frac{1}{2}(\alpha_j - \beta_j)^2.$$

By algebraic computation it gives

$$\frac{1}{2}\left(\alpha_i - \alpha_j + \beta_j - \sum_{h\in C}\beta_h\right)\left(\alpha_i - \alpha_j + \beta_j + \sum_{h\in C}\beta_h\right) \geq 0. \tag{16}$$

The absence of envy of j against i, $B_j(e_j) \geq B_j(e_i)$, is equivalent to

$$\alpha_j(\alpha_j - \beta_j) - \frac{1}{2}(\alpha_j - \beta_j)^2 \geq \alpha_j\left(\alpha_i - \sum_{h\in C}\beta_h\right) - \frac{1}{2}\left(\alpha_i - \sum_{h\in C}\beta_h\right)^2$$

and leads

$$\frac{1}{2}\left(\alpha_j - \alpha_i - \beta_j + \sum_{h\in C}\beta_h\right)\left(\alpha_j - \alpha_i + \beta_j + \sum_{h\in C}\beta_h\right) \geq 0. \tag{17}$$

Therefore, if i is more developed than j, since $\alpha_i - \alpha_j + \beta_j + \sum_{h\in C}\beta_h > 0$, i does not envy j if and only if $\alpha_j \leq \alpha_i + \beta_j - \sum_{h\in C}\beta_h$, while j does not envy i if and only if $\alpha_j \leq \alpha_i - \beta_j - \sum_{h\in C}\beta_h$ or $\alpha_j \geq \alpha_i + \beta_j - \sum_{h\in C}\beta_h$.

Hence, there is no envy between i and j, if and only if $(b4)$ is satisfied.

If j is more developed than i, meaning that the more developed country belongs to $I \setminus C$, since both $\alpha_j - \alpha_i - \beta_j + \sum_{h\in C} \beta_h$ and $\alpha_j - \alpha_i + \beta_j + \sum_{h\in C} \beta_h$ are positive, j does not envy i.

On the other hand, since $\alpha_i - \alpha_j + \beta_j - \sum_{h\in C} \beta_h$ is negative, i does not envy j if and only if $\alpha_i - \alpha_j + \beta_j + \sum_{h\in C} \beta_h$ is non-positive, that is if and only if $(b3)$, appropriately rewritten, holds. □

As observed in the proof, condition (13) is always satisfied if i is more developed than j. It allows to say that, under cooperation, each player in the coalition is not envious at $\underline{e}$ of any other cooperating player less developed than her. On the other hand, condition (15) always holds if i is more developed than j, as well as condition (17) holds if j is more developed than i. Therefore, a non-signatory country does not envy at $\underline{e}$ any less developed state.

Proof of Proposition 3.12 Let C be a non-degenerate coalition for which the solution of the $\mathcal{P}(C)$-equilibrium problem, $\underline{e}$, is an envy-free emissions allocation.

Let $C' \subsetneqq C$ and $\underline{e}'$ be the solution of the $\mathcal{P}(C')$-equilibrium problem.

By Proposition 3.12, $\underline{e}'$ is $\mathcal{P}(C')$-envy-free, hence there is no envy within C'.

Let us start proving the statement 1).

For every i in $I \setminus C$, since i does not belong to C nor to C', then $e_i' = \alpha_i - \beta_i = e_i$. By assumption $\underline{e}$ is envy-free, hence, for every $j \in I \setminus C$, $B_i(e_i) \geq B_i(e_j)$ and then $B_i(e_i') \geq B_i(e_j')$, meaning that i does not envy j at $\underline{e}'$. Therefore, there is no envy within $I \setminus C$.

Let i, j belong to $C \setminus C'$. Since they do not belong to C', $e_i' = \alpha_i - \beta_i$ and $e_j' = \alpha_j - \beta_j$. Hence the envy-freeness condition $B_i(e_i') \geq B_i(e_j')$ is equivalent to

$$(\alpha_i - \beta_i - \alpha_j + \beta_j)(\alpha_i + \beta_i - \alpha_j + \beta_j) \geq 0. \tag{18}$$

As previously observed, if i is more developed than j, (18) is always satisfied. On the other hand, if i is less developed than j, since $\underline{e}$ is envy-free, condition $(b1)$ of Proposition 3.10 is satisfied, that implies (18). Therefore, there is no envy within $C \setminus C'$, that concludes the proof of the statement (1).

Let us now prove the statement (2).

By (1), each country in C' does not envy at $\underline{e}'$ any other in C', while each agent in $C \setminus C'$ does not envy at $\underline{e}'$ any other in $C \setminus C'$. Hence, in order to prove that they are not envious at $\underline{e}'$, we have to check that

(a) every country in C' does not envy at $\underline{e}'$ any $j \in I \setminus C$;
(b) every country in $C \setminus C'$ does not envy at $\underline{e}'$ any $j \in (I \setminus C) \cup C'$.

Let us prove (a).

Mimicking the computations of the previous proposition, a country i in C' does not envy at $\underline{e}'$ a country j in $I \setminus C'$ if and only if

$$\left(\alpha_i - \alpha_j + \beta_j - \sum_{h\in C'} \beta_h\right)\left(\alpha_i - \alpha_j + \beta_j + \sum_{h\in C'} \beta_h\right) \geq 0. \tag{19}$$

If i is more developed than j, (19) is equivalent to

$$\alpha_j \leq \alpha_i + \beta_j - \sum_{h\in C'} \beta_h. \tag{20}$$

Since $\underline{e}$ is envy-free, if $j \in I \setminus C, \alpha_j \leq \alpha_i + \beta_j - \sum_{h \in C} \beta_h$ implying (20), while if $j \in C \setminus C'$, $\alpha_j \leq \alpha_i - 2 \sum_{h \in C} \beta_h$ from which (20) derives.

If i is less developed than j, (19) is equivalent to

$$\alpha_i \leq \alpha_j - \beta_j - \sum_{h \in C'} \beta_h. \tag{21}$$

Since $\underline{e}$ is envy-free, if $j \in I \setminus C$, $\alpha_j \leq \alpha_i - \beta_i - \sum_{h \in C} \beta_h$, and hence (21), while if $j \in C \setminus C'$, $\alpha_i \leq \alpha_j - 2 \sum_{h \in C} \beta_h$, and then (21).

Therefore, (a) is proved.

Let us prove (b).

Consider $i \in C \setminus C'$ and $j \in I \setminus C$, hence $e'_i = \alpha_i - \beta_i$ and $e'_j = \alpha_j - \beta_j$. As previously observed i does not envy j if and only if (18) is satisfied. One more time, if i is more developed than j, (18) is always satisfied. On the other hand, if i is less developed than j, envy-freeness of $\underline{e}$ implies (18).

Finally, let $i \in C \setminus C'$ and $j \in C'$. Then, i does not envy j at $\underline{e}'$ if and only if

$$\left(\alpha_i - \alpha_j - \beta_i + \sum_{h \in C'} \beta_h \right) \left(\alpha_i - \alpha_j + \beta_i + \sum_{h \in C'} \beta_h \right) \geq 0. \tag{22}$$

Condition (22) is always satisfied if i is more developed than j.

If i is less developed than j, by envy-freeness of $\underline{e}$, $\alpha_i \leq \alpha_j - 2 \sum_{h \in C} \beta_h$ that implies $\alpha_i \leq \alpha_j - \beta_i - \sum_{h \in C'} \beta_h$, under which (22) holds. That concludes the proof.

Proof of Proposition 3.13 Let us denote by $\underline{e}$ the solution of the $\mathcal{P}(C)$-equilibrium problem. By Proposition 3.12 $\underline{e}$ is envy-free. Let us consider $C' \subsetneq C$ and let us denote $\underline{e}'$ the solution of the $\mathcal{P}(C')$-equilibrium problem. In Proposition 3.12 we have shown that, at $\underline{e}'$, any country in C is non-envious at all, and that any i in $I \setminus C$ does not envy any j in $I \setminus C$. It remains to prove that i does not envy at $\underline{e}'$ any country j in C.

If j belongs to $C \setminus C'$, i does not envy j at $\underline{e}'$ if and only if (18) is verified. Notice that if i is more developed than j, (18) is always true, while, if i is less developed than j, condition $(b3)$ implies (18).

If j belongs to C', i does not envy j at $\underline{e}'$ if and only if

$$\left(\alpha_i - \alpha_j - \beta_j + \sum_{h \in C'} \beta_h \right) \left(\alpha_i - \alpha_j + \beta_j + \sum_{h \in C'} \beta_h \right) \geq 0. \tag{23}$$

Notice that (23) is always satisfied if i is more developed than j, while it derives by $(b4.2)$ if i is less developed than j.

Proof of Proposition 4.2 The ef-$\mathcal{P}(I)$-equilibrium problem assumes the following form

$$\begin{cases} \max\limits_{(e_1,e_2)} u_C(\underline{e}) = \max\limits_{(e_1,e_2)} \left(\alpha_1 e_1 + \alpha_2 e_2 - \dfrac{1}{2}(e_1^2 + e_2^2) - (\beta_1 + \beta_2)(e_1 + e_2) \right) \\ \text{subject to} \\ B_1(e_1) = \alpha_1 e_1 - \frac{1}{2} e_1^2 \geq \alpha_1 e_2 - \frac{1}{2} e_2^2 = B_1(e_2) \\ B_2(e_2) = \alpha_2 e_2 - \frac{1}{2} e_2^2 \geq \alpha_2 e_1 - \frac{1}{2} e_1^2 = B_2(e_1) \end{cases}$$

Let us rewrite the constraints as $-\alpha_1(e_1-e_2)+\frac{1}{2}(e_1^2-e_2^2) \le 0$ and $-\alpha_2(e_2-e_1)+\frac{1}{2}(e_2^2-e_1^2) \le 0$ and consider the Lagrangian function

$$\begin{aligned}&\mathcal{L}(e_1, e_2, \lambda_1, \lambda_2)\\ &= \alpha_1 e_1 + \alpha_2 e_2 - \tfrac{1}{2}(e_1^2+e_2^2) - (\beta_1+\beta_2)(e_1+e_2) - \lambda_1\left(-\alpha_1(e_1-e_2)+\tfrac{1}{2}(e_1^2-e_2^2)\right)\\ &\quad - \lambda_2\left(-\alpha_2(e_2-e_1)+\tfrac{1}{2}(e_2^2-e_1^2)\right).\end{aligned}$$

Then, the first-order conditions generate the following system

$$\begin{cases}\alpha_1 - e_1 - \beta_1 - \beta_2 + \lambda_1\alpha_1 - \lambda_1 e_1 - \lambda_2\alpha_2 + \lambda_2 e_1 = 0\\ \alpha_2 - e_2 - \beta_1 - \beta_2 - \lambda_1\alpha_1 + \lambda_1 e_2 + \lambda_2\alpha_2 - \lambda_2 e_2 = 0\\ \lambda_1\left(-\alpha_1(e_1-e_2)+\frac{1}{2}(e_1^2-e_2^2)\right) = 0\\ \lambda_2\left(-\alpha_2(e_2-e_1)+\frac{1}{2}(e_2^2-e_1^2)\right) = 0\\ \lambda_1 \ge 0\\ \lambda_2 \ge 0\\ \alpha_1(e_1-e_2) - \frac{1}{2}(e_1^2-e_2^2) \ge 0\\ \alpha_2(e_2-e_1) - \frac{1}{2}(e_2^2-e_1^2) \ge 0\end{cases}$$

Under $(A2)$, by algebraic computation the solution is given by

$$\underline{e}^* = \begin{cases}(\alpha_1-\beta_1-\beta_2, \alpha_2-\beta_1-\beta_2), & \text{if } 0 < \alpha_2 \le \alpha_1 - 2\beta_1 - 2\beta_2,\\ \frac{1}{2}(\alpha_1+\alpha_2, 3\alpha_2-\alpha_1), & \text{if } \alpha_1 - 2\beta_1 - 2\beta_2 < \alpha_2 \le \alpha_1 - \beta_1 - \beta_2,\\ \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2, \alpha_1+\alpha_2-2\beta_1-2\beta_2), & \text{if } \alpha_1 - \beta_1 - \beta_2 < \alpha_2 < \alpha_1.\end{cases}$$

For $\underline{e}^*$ be an emissions allocation, the condition $\underline{e}^* \in \mathbb{R}^2_{++}$ has to be satisfied. Therefore, under $(A1)$, we get

$$\underline{e}^* = \begin{cases}(\alpha_1-\beta_1-\beta_2, \alpha_2-\beta_1-\beta_2), \ \text{if } \beta_1+\beta_2 < \alpha_2 \le \alpha_1 - 2\beta_1 - 2\beta_2,\\ \\ \frac{1}{2}(\alpha_1+\alpha_2, 3\alpha_2-\alpha_1), \ \text{if } \max\{\alpha_1 - 2\beta_1 - 2\beta_2, \frac{1}{3}\alpha_1, \beta_2\} < \alpha_2 \le \alpha_1 - \beta_1 - \beta_2,\\ \\ \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2, \alpha_1+\alpha_2-2\beta_1-2\beta_2),\\ \text{if } \max\{2\beta_1+2\beta_2-\alpha_1, \alpha_1-\beta_1-\beta_2, \beta_2\} < \alpha_2 < \alpha_1\end{cases}$$

that is the thesis.

Proof of Proposition 4.3 Let Γ belong to $\mathcal{G}_2$.

As previously observed, in order to study the stability of I, only the internal stability condition has to be checked.

Let us start analyzing condition (7) for the more developed player.

By definition, $u_1(\underline{e}^*) \ge u_1(\underline{e}^{NC})$ can be written as $B_1(e_1^*) - B_1(e_1^{NC}) - D_1(\underline{e}^*) + D_1(\underline{e}^{NC}) \ge 0$, that is $\frac{1}{2}\alpha_1(\alpha_1+\alpha_2) - \frac{1}{2}\left[\frac{1}{2}(\alpha_1+\alpha_2)\right]^2 - \alpha_1(\alpha_1-\beta_1) + \frac{1}{2}(\alpha_1-\beta_1)^2 - \beta_1(\alpha_2-\alpha_1+\beta_1+\beta_2) \ge 0$.

By algebraic computation, it is easy to show that the previous inequality can be reformulated as $\frac{1}{8}(\alpha_2-\alpha_1+2\beta_2)(\alpha_1-\alpha_2+2\beta_1) - \beta_1(\alpha_2-\alpha_1+\beta_1+\beta_2) \ge 0$, that is $\frac{1}{8}[-4\beta_1^2 + 8(\alpha_1-\alpha_2-\beta_2)\beta_1 - (\alpha_1-\alpha_2)^2] \ge 0$.

Hence, condition $u_1(\underline{e}^*) \ge u_1(\underline{e}^{NC})$ is satisfied if and only if

$$\begin{aligned}&\frac{\sqrt{2}-1}{2}(\alpha_1-\alpha_2) < \beta_1 < \frac{1}{2}(\alpha_1-\alpha_2)\\ &\text{and } \max\left\{\frac{1}{2}(\alpha_1-\alpha_2-2\beta_1), \beta_1\right\} < \beta_2 < \frac{-4\beta_1^2 + 8(\alpha_1-\alpha_2)\beta_1 - (\alpha_1-\alpha_2)^2}{8\beta_1}.\end{aligned} \tag{24}$$

Since Γ belongs to $\mathcal{G}_2$, conditions $\alpha_2 < \alpha_1 - \beta_1 - \beta_2$ and $\beta_1 < \beta_2$ hold and imply $\beta_1 < \frac{1}{2}(\alpha_1 - \alpha_2)$ and $\beta_2 > \max\left\{\frac{1}{2}(\alpha_1 - \alpha_2 - 2\beta_1), \beta_1\right\}$. Then, in $\mathcal{G}_2$, (24) is equivalent to (9). It means that, in $\mathcal{G}_2$ the most developed player prefers envy-free cooperation than acting alone, only in games for which (9) holds.

Let us now consider condition (7) for the less developed player.

By definition, $u_2(\underline{e}^*) \geq u_2(\underline{e}^{NC})$ can be written as $B_2(e_2^*) - B_2(e_2^{NC}) - D_2(\underline{e}^*) + D_2(\underline{e}^{NC}) \geq 0$, that is $\frac{1}{2}\alpha_2(3\alpha_2 - \alpha_1) - \frac{1}{2}\left[\frac{1}{2}(3\alpha_2 - \alpha_1)\right]^2 - \alpha_2(\alpha_2 - \beta_2) + \frac{1}{2}(\alpha_2 - \beta_2)^2 - \beta_2(\alpha_2 - \alpha_1 + \beta_1 + \beta_2) \geq 0$.

By algebraic computation, it is easy to obtain that the previous inequality can be reformulated as $\frac{1}{8}(\alpha_2 - \alpha_1 + 2\beta_2)(\alpha_1 - \alpha_2 + 2\beta_2) - \beta_2(\alpha_2 - \alpha_1 + \beta_1 + \beta_2) \geq 0$, that is $\frac{1}{8}[-4\beta_2^2 + 8(\alpha_1 - \alpha_2 - \beta_1)\beta_2 - (\alpha_1 - \alpha_2)^2] \geq 0$.

Hence, condition $u_2(\underline{e}^*) \geq u_2(\underline{e}^{NC})$ is satisfied if and only if

$$0 < \beta_1 < \frac{1}{2}(\alpha_1 - \alpha_2) \text{ and } \max\left\{\frac{1}{2}(\alpha_1 - \alpha_2 - 2\beta_1), \beta_1\right\} < \beta_2 < \alpha_1 - \alpha_2 - \beta_1.$$

The previous conditions hold, since Γ belongs to $\mathcal{G}_2$. Therefore, $u_2(\underline{e}^*) \geq u_2(\underline{e}^{NC})$ is fulfilled, meaning that, in each game $\Gamma \in \mathcal{G}_2$, the less developed player prefers envy-free cooperation than acting alone.

Hence, we can conclude that, given a game Γ in $\mathcal{G}_2$, I is stable if and only if the parameters of Γ meet condition (9). □

Proof of Proposition 4.4 Let Γ belong to $\mathcal{G}_3$.

In order to investigate condition (7), let us analyze the differences $B_i(e_i^*) - B_i(e_i^{NC})$ and $D_i(\underline{e}^*) - D_i(\underline{e}^{NC})$, for $i = 1, 2$.

Notice that, since $E^* < E^{NC}$, envy-free cooperation leads to lower costs. Hence, we can focus on the differences between the benefit levels generated by $\underline{e}^*$ and $\underline{e}^{NC}$.

$B_1(e_1^*) - B_1(e_1^{NC}) = \frac{1}{2}\alpha_1(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2) - \frac{1}{2}\left[\frac{1}{2}(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2)\right]^2 - \alpha_1(\alpha_1 - \beta_1) + \frac{1}{2}(\alpha_1 - \beta_1)^2 < 0$ is equivalent $\frac{1}{8}(\alpha_2 - \alpha_1 - 2\beta_2)(\alpha_1 - \alpha_2 + 4\beta_1 + 2\beta_2) < 0$, that is satisfied for each game in $\mathcal{G}_2$. Hence, envy-free cooperation induces, for the more developed player, a decreasing of benefit level.

The inequality $u_1(\underline{e}^*) - u_1(\underline{e}^{NC}) = B_1(e_1^*) - B_1(e_1^{NC}) - D_1(\underline{e}^*) + D_1(\underline{e}^{NC}) \geq 0$ is equivalent to $\frac{1}{8}[-4\beta_2^2 + 8\beta_1^2 + 4(\alpha_2 - \alpha_1)(\beta_1 + \beta_2) - (\alpha_1 - \alpha_2)^2] \geq 0$, that is satisfied if and only if the set of parameters of the game Γ meet (10). Hence, the most developed country prefers to cooperate if and only if (10) holds.

$B_2(e_2^*) - B_2(e_2^{NC}) = \frac{1}{2}\alpha_2(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2) - \frac{1}{2}\left[\frac{1}{2}(\alpha_1 + \alpha_2 - 2\beta_1 - 2\beta_2)\right]^2 - \alpha_2(\alpha_2 - \beta_2) + \frac{1}{2}(\alpha_2 - \beta_2)^2$ is equivalent $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1)(\alpha_2 - \alpha_1 + 2\beta_1 + 4\beta_2)$.

Since for every game in $\mathcal{G}_3$, $\alpha_2 - \alpha_1 + 2\beta_1 + 4\beta_2 > 0$, then $B_2(e_2^*) - B_2(e_2^{NC}) > 0$ if and only if $\alpha_1 - \alpha_2 - 2\beta_1 > 0$. Hence, for the least developed player, envy-free cooperation, compared with the status quo $\underline{e}^{NC}$, induces an increasing of benefit level if the parameters satisfy the conditions $\alpha_1 - \beta_1 - \beta_2 < \alpha_2 < \alpha_1 - 2\beta_2$, while generates a decreasing of benefit level if $\alpha_1 - 2\beta_1 < \alpha_2 < \alpha_1$.

Investigating the inequality $u_2(\underline{e}^*) - u_2(\underline{e}^{NC}) = B_2(e_2^*) - B_2(e_2^{NC}) - D_2(\underline{e}^*) + D_2(\underline{e}^{NC}) \geq 0$, we get that it is equivalent to $\frac{1}{8}[-4\beta_1^2 + 8\beta_2^2 + 4(\alpha_1 - \alpha_2)(\beta_1 + \beta_2)^2 + (\alpha_1 - \alpha_2)^2] \geq 0$. By algebraic computation, we show that the previous inequality is satisfied if and only if

$$0 < \beta_1 < \frac{\sqrt{5}-2}{2}(\alpha_1 - \alpha_2)$$
$$\text{and } \frac{1}{2}(\alpha_1 - \alpha_2) - \sqrt{\beta_1(\alpha_1 - \alpha_2 + 2\beta_1)} < \beta_2 < \frac{1}{2}(\alpha_1 - \alpha_2) + \sqrt{\beta_1(\alpha_1 - \alpha_2 + 2\beta_1)} \tag{25}$$

or

$$\frac{\sqrt{5}-2}{2}(\alpha_1 - \alpha_2) < \beta_1 < \beta_2 < \frac{1}{2}(\alpha_1 - \alpha_2) + \sqrt{\beta_1(\alpha_1 - \alpha_2 + 2\beta_1)}. \tag{26}$$

Hence, the less developed country prefers to cooperate if and only if for the set of parameters of the game Γ one between (25) and (26) holds.

We can conclude that, given a game Γ in $\mathcal{G}_3$, I is stable if and only if the parameters of Γ meet condition (10). □

Remark 5.1 Let us consider a game Γ in $\tilde{\mathcal{G}}_2''$ and compare the utility levels generated by emissions allocations $\underline{e}^*$ and $\underline{e}^C$.

Notice that, since $E^C < E^*$, envy-free cooperation, compared to standard cooperation, leads to greater costs.

Let us start investigating the benefit levels and the utility levels generated by $\underline{e}^*$ and $\underline{e}^{NC}$ for the most developed player.

$B_1(e_1^C) - B_1(e_1^*) = \alpha_1(\alpha_1 - \beta_1 - \beta_2) - \frac{1}{2}(\alpha_1 - \beta_1 - \beta_2)^2 - \frac{1}{2}\alpha_1(\alpha_1 + \alpha_2) + \frac{1}{2}\left[\frac{1}{2}(\alpha_1 + \alpha_2)\right]^2$ is equivalent to $\frac{1}{8}\left[(\alpha_1 - \alpha_2)^2 - 4(\beta_1 + \beta_2)^2\right]$ that can be reformulated as $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)(\alpha_1 - \alpha_2 + 2\beta_1 + 2\beta_2)$.

Since for every Γ in $\mathcal{G}_2$, $\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2 < 0$, while $\alpha_1 - \alpha_2 + 2\beta_1 + 2\beta_2 > 0$, we get $B_1(e_1^*) - B_1(e_1^C) < 0$, meaning that, to the most developed player, envy-free cooperation brings a benefit level lower than that guaranteed from the standard cooperation.

Comparing the utility levels $u_1(\underline{e}^C)$ and $u_1(\underline{e}^*)$, by the previous computation, we get that the difference $u_1(\underline{e}^C) - u_1(\underline{e}^*)$ may be written as $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)(\alpha_1 - \alpha_2 + 2\beta_1 + 2\beta_2) - \beta_1(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)$, that can be reformulated as $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)(\alpha_1 - \alpha_2 - 6\beta_1 + 2\beta_2)$. Notice that it is positive if and only if $\alpha_2 > \alpha_1 - 6\beta_1 + 2\beta_2$. It means that envy-free cooperation brings, to the most developed player, a utility level greater than that guaranteed from the standard cooperation if and only if $\alpha_2 > \alpha_1 - 6\beta_1 + 2\beta_2$.

Let us now consider the least developed player.

$B_2(e_2^C) - B_2(e_2^*) = \alpha_2(\alpha_2 - \beta_1 - \beta_2) - \frac{1}{2}(\alpha_2 - \beta_1 - \beta_2)^2 - \frac{1}{2}\alpha_1(3\alpha_2 - \alpha_1) + \frac{1}{2}\left[\frac{1}{2}(3\alpha_2 - \alpha_1)\right]^2$ is equivalent to $\frac{1}{8}\left[(\alpha_1 - \alpha_2)^2 - 4(\beta_1 + \beta_2)^2\right]$, that can be reformulated as $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)(\alpha_1 - \alpha_2 + 2\beta_1 + 2\beta_2)$.

Since for every Γ in $\mathcal{G}_2$, $\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2 < 0$, while $\alpha_1 - \alpha_2 + 2\beta_1 + 2\beta_2 > 0$, we get $B_1(e_1^*) - B_1(e_1^C) < 0$, meaning that envy-free cooperation brings, to the least developed player, a benefit level lower than that guaranteed from the standard cooperation.

Comparing the utility levels $u_2(\underline{e}^C)$ and $u_2(\underline{e}^*)$, by the previous computation, we get that the difference $u_2(\underline{e}^C) - u_2(\underline{e}^*)$ can be written as $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)(\alpha_1 - \alpha_2 + 2\beta_1 + 2\beta_2) - \beta_2(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)$ that can be reformulated as $\frac{1}{8}(\alpha_1 - \alpha_2 - 2\beta_1 - 2\beta_2)(\alpha_1 - \alpha_2 + 2\beta_1 - 6\beta_2)$. It is easy to see that this product is negative for any game in $\mathcal{G}_2$. This points out that, compared with standard cooperation, envy-free cooperation brings, to the least developed player, a lower utility level.

As a consequence, envy-free cooperation is Pareto dominated by standard cooperation in every game Γ in $\tilde{\mathcal{G}}_2''$ for which $\alpha_2 < \alpha_1 - 6\beta_1 + 2\beta_2$.

Remark 5.2 Let us consider a game Γ in $\mathcal{G}_3''$ and compare the utility levels generated by emissions allocations $\underline{e}^*$ and $\underline{e}^C$.

Notice that, since $E^C = E^*$, the analysis attempts to compare the levels of benefits induced by $\underline{e}^*$ and $\underline{e}^{NC}$.

By construction, for the most developed player, the difference between $B_1(e_1^*)$ and $B_1(e_1^C)$ is explicable as $\alpha_1 \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2) - \frac{1}{2}\left[\frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2)\right]^2 - \alpha_1(\alpha_1+\alpha_2-\beta_1-\beta_2) + \frac{1}{2}(\alpha_1+\alpha_2-\beta_1-\beta_2)^2$ that is equivalent to $\frac{1}{8}(\alpha_2-\alpha_1)(\alpha_1-\alpha_2+4\beta_1+4\beta_2)$.

Since for every Γ in $\mathcal{G}_3$, $\alpha_1-\alpha_2+4\beta_1+4\beta_2>0$, while $\alpha_2-\alpha_1<0$, we get $B_1(e_1^*) - B_1(e_1^C) < 0$, meaning that envy-free cooperation brings, to the most developed player, a benefit level (and then utility level) lower than that guaranteed from the standard cooperation.

By construction, for the least developed player, the difference between $B_2(e_2^*)$ and $B_2(e_2^C)$ is explicable as $\alpha_2 \frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2) - \frac{1}{2}\left(\frac{1}{2}(\alpha_1+\alpha_2-2\beta_1-2\beta_2)\right)^2 - \alpha_2(\alpha_2-\beta_1-\beta_2) + \frac{1}{2}[\alpha_2-\beta_1-\beta_2]^2$ that is equivalent to $\frac{1}{8}(\alpha_1-\alpha_2)(\alpha_2-\alpha_1+4\beta_1+4\beta_2)$.

Since for every Γ in $\mathcal{G}_3$, $\alpha_1-\alpha_2+4\beta_1+4\beta_2$ and $\alpha_1-\alpha_2$ are both positive, we get $B_2(e_2^*) - B_2(e_2^C) > 0$, meaning that envy-free cooperation brings, to the least developed player, a benefit level (and then utility level) greater than that guaranteed from the standard cooperation.

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