Combining enactivism and systemic functional linguistics: a methodology for examining (mathematics teacher educator) language

Tracy Helliwell[1] · Andreas Ebbelind[2]

Abstract

As mathematics teacher educators (MTEs), we are motivated by the lack of research concerning the language that MTEs use in initial teacher education settings. In this paper, we turn our attention towards developing a methodological approach to studying the language-in-use during teacher education situations, with a specific focus on the language of the MTE in the form of a monologue. The methodological approach that we present draws upon two theoretical perspectives, specifically, the situated cognition theory of enactivism and the social semiotic perspective of systemic functional linguistics (SFL). To develop this methodology, we explore both of these theoretical perspectives, focussing on their respective conceptualisations of language, from which we derive a set of methodological principles and practices. A significant feature of the methodology presented in this paper, is that it takes into account the researchers' relationships with the subject of research. Thus we propose this methodology as being of particular significance to practitioner–researchers studying the language of other practitioners within the same field (e.g. MTEs studying the language of other MTEs), as well as to the study of one's own use of language in mathematics education settings. From our methodological perspective we explore the meaning of quality research, proposing relevant criteria. We exemplify the methodological principles and practices by analysing a transcript of a mathematics teacher education lecture for prospective primary teachers in Sweden.

✉ Tracy Helliwell
  tracy.helliwell@bristol.ac.uk

1 School of Education, University of Bristol, Bristol, UK

2 Faculty of Technology, Linnaeus University, Växjö, Sweden

 🖄 Springer

Introduction

Mathematics teacher education is recognised as a key area of research and scholarship that, as a field, has benefited from the vast and diverse research that takes place across the world. As a body of research, mathematics teacher education is concerned with issues relating to the education of mathematics teachers and mathematics teacher educators (MTEs) at all stages of their development. A vital (but sometimes overlooked) aspect within mathematics teacher education research is the MTE who supports the learning and development of mathematics teachers during any phase of their professional lives. A MTE can be described as "anyone engaged in the education or development of teachers of mathematics" (Beswick & Goos, 2018, p. 418). Thus, members of this group include MTEs involved in TE, for *prospective* teachers of mathematics, either school-based or university-based, as well as those involved in the facilitation of professional development (PD) of *in-service* mathematics teachers. In this paper, our focus relates to the former, that is, the knowledge and practices of MTEs in TE settings and specifically university-based MTEs and their role in educating prospective mathematics teachers (PMTs). In particular, we turn our attention to *how* we might develop meaningful ways of examining the language-in-use of MTEs within these settings. We see this as an important line of inquiry since, as a mathematics education community, we need to better understand how MTEs shape the learning of PMTs through their use of language. We currently know of no other studies with an explicit focus on the language used by MTEs in initial TE settings, yet we know that "language, talk, text and the production and interpretation of symbols are integral to the creation of learning" (Radford & Barwell, 2016, p. 275). Furthermore, as MTEs ourselves, we are compelled to study the language of other MTEs as a way of learning about our own use of language and to provide readers of this research (e.g. other practising teachers or teacher educators) with an opportunity to do the same.

With this paper, we contribute to the field of mathematics teacher education in different ways. Firstly, as we have already indicated, we extend the growing literature base relating to the knowledge and practices of MTEs and in particular the study of MTE language in TE settings. Secondly, we contribute methodologically by developing a novel methodology for studying language by "combining" (Gellert, 2008; Prediger et al., 2008) two theoretical perspectives, namely, the enactivist theory of cognition and the social semiotic perspective of systemic functional linguistics (SFL). According to Prediger et al. (2008), the networking strategy of combining is typical for developing *conceptual frameworks* (see Eisenhart, 1991), where the aim is not to articulate one coherent and complete theory, but to use "different analytical tools for the sake of a practical problem" (Prediger et al., 2008, p. 172). The practical problem we are concerned with is *how* we might meaningfully study and learn from the language of MTEs in TE settings specifically when that language is monologic (which we envisage to be a common feature of many TE situations where the MTE is addressing large groups of PMTs). This paper is fundamentally one concerning learning, that is, learning in relation to MTEs use of language in TE settings (in which we include our own learning as MTE researchers, the learning of mathematics teachers in TE settings, and the learning of readers of this research). Enactivism as a theory of learning that includes conceptions of language thus provides us with our overarching perspective for the study that can be framed in terms of learning. Although several mathematics education researchers have developed methodologies on the basis of enactivism (see sections "Enactivism and mathematics education" and "Establishing methodological principles" for examples), enactivist theory does not *specify* what to do when analysing text. Thus we use the

methodological tools of SFL and its associated categories to support our analysis of MTE language by enabling us to interpret text in enhanced ways. We claim that by combining enactivism and SFL, the methodology articulated in this paper is of particular relevance to practitioners seeking to study and learn from the language of other practitioners (e.g. MTEs studying the language of other MTEs). Thirdly, through articulating and enacting our methodology, we make a further contribution by identifying the kinds of insights relating to the language of MTEs that such a methodology can reveal. Thus, we are guided by the following interconnected research questions:

RQ1 What does a methodology for studying the language-in-use of MTEs consist of that combines the enactivist theory of cognition with the social semiotic perspective of SFL?

RQ2 What kinds of insights and new awarenesses can be revealed in relation to the language-in-use of MTEs from utilising such a methodology?

To address these questions, we first "compare" and to some extent "contrast" (Prediger et al., 2008, p. 171) each of the theoretical perspectives of enactivism and SFL that underpin our methodology for studying language before combining them to formulate our methodology. We then present our methodology which we exemplify by analysing the transcript of one mathematics teacher education situation (hereafter referred to as a 'lecture') at a university in Sweden, for prospective primary school teachers (for students aged 7–12 years). The lecture was delivered by an experienced MTE, and it is the language of this MTE in the form of a monologue that forms the basis of the transcript. We then present the insights and new awarenesses that emerged for us through the analysis process in relation to the language of the MTE and in relation to our own use of language based on such a methodology.

Theoretical foundations: enactivism and systemic functional linguistics

Prior to this study, we had separately become familiar with each of the two perspectives. Tracy had recently developed a narrative-enactivist methodology for researching how she was becoming a MTE (Helliwell, 2021) and Andreas had used SFL for researching the process of becoming mathematics teachers (Ebbelind, 2020). In this section we compare and contrast the different ways in which language is conceptualised from the two perspectives as a way of making their possible connections and their individual strengths more visible (Prediger et al., 2008) before combining them to develop a coherent methodology for studying MTE language. According to Prediger et al., "[e]ven theories with conflicting basic assumptions can be combined in order to get a multi-faceted insight into the empirical phenomenon in view" (Prediger et al., 2008, p. 173).

Enactivism

Enactivism offers a biological theory of cognition that is understood from an evolutionary standpoint. Enactivist theory provides a powerful alternative to cartesian and cognitivist views of mind where the cognising system is positioned purely within an internal mental environment (i.e. the brain of an individual) that is separable and distinct from both the body and the external world, and where learning is conceived as the acquisition of

knowledge and subsequent effects on inner mental structures. Enactivism as a situated cognition theory, rejects a view that the mind and body are distinct and separable and instead assumes the now familiar brain-body-world formulation where 'world' includes other individuals as well as "culturally constructed social and material settings" (Hutchins, 2010, p. 711). For enactivists, learning is not "the representation of a pregiven world by a pregiven mind", but rather "the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs" (Varela et al., 1991, p. 9). From this embodied view, cognition is not seen as a construction of an external reality, nor is it purely a function of the brain, rather, it is the continuous adaptive process in which individuals co-evolve with their environments.

The enactivist view of cognition assumes an ontological position that Maturana calls "objectivity-in-parenthesis" (Maturana, 1988b, p. 27) in the sense that, for Maturana, objectivity is "constituted" (p. 3). Thus, enactivist theory neither belongs within a subjective discourse where things that exist, exist purely in the domain of experiences, as might be the case from a radical constructivist perspective, nor does it belong within an objective/realist discourse where things in the external world exist completely independently of the knower. Rather, enactivism takes a middle path within what Proulx (2008) describes as an "*interobjective* discourse" where "the distinction between objects and subjects collapses because both are co-determinations of one and the other" (p. 22). Through their interactions with one another, organisms and their environments co-adapt and in doing so experience mutual histories of structural changes through a process that Maturana and Varela (1998) call "structural coupling" (p. 75). The environment triggers changes in the organism and simultaneously the organism triggers changes in the environment in an ongoing, dynamic process of co-evolution. In this sense, individuals and world specify one another, knowledge (or knowing) is an active process that is brought forth (it is made ontological) through being in the world.

This sense of bringing forth a world is expressed by one of the key enactivist aphorisms, that "everything is said by an observer" (Maturana, 1987, p. 65), or more fully, "everything said is said by an observer to another observer that could be him or herself" (Maturana, 1988a, p. 5). What is possible to know depends on the observer:

> In the enactive approach reality is not a given: it is perceiver-dependent, not because the perceiver "constructs" it as he or she pleases, but because what *counts as a relevant world is inseparable from the structure of the perceiver* (Varela, 1999, p. 13, emphasis original)

From an enactivist perspective, "perception consists in perceptually guided action" (Varela et al., 1991, p. 173), that is, perception and action are fundamentally inseparable, and we can only perceive those phenomena that having a human body (with its various sensorimotor capacities) allows us to perceive. Cognition is embodied action (Reid & Mgombelo, 2015). Fundamentally, "*all doing is knowing, and all knowing is doing*" (Maturana & Varela, 1998, p. 26, emphasis original), knowledge does not result from an action, nor is it a source of action, knowledge (or knowing) "*is* the emergent action" (Proulx & Simmt, 2016, p. 102, emphasis original), emerging in the dynamics of interaction between subject and environment. The notion of emergence is a key concept in relation to the enactivist view of cognition in that learning can be described as a dynamic process that *emerges* in individuals' interactions with and in the environment, which includes their interactions with one another, through a process that has been described as co-emergence (Davis, 1996; Lozano, 2015). The word emergence suggests the coming into existence, or a being brought forth of something that up to that point, did not exist. Emergence can thus be described as "the

formation of a novel property or process out of the interaction of different existing processes or events" (Di Paolo et al., 2011, p. 40) and links to an enactivist view of learning described by Brown (2015) as "seeing more, seeing differently" (p. 190).

Furthermore, all living organisms are "structure-determined systems" (Maturana, 1987, p. 73) which means that it is the structure of an organism that determines how the organism responds given any stimulus from the environment (as opposed to the environment *causing* the nature of the response). In relation to a living organism, the word 'structure' is understood as its "biological constitution" which is "more than physical, as it is realised with/in experience, and through its histories of interactions" (Proulx & Simmt, 2016, p. 101). Thus, our structures as human beings are changed by our experiences, changes that are triggered by the interactions we have with and in our environments, but it is our structures that determine how any changes occur, i.e. how we respond to a situation. Two mathematics teachers, for example, could respond differently to the same stimuli, depending on their own history of interactions. It is not possible, therefore, for MTEs to determine *how* prospective teachers will respond in a particular situation or to a particular prompt although the MTE is certainly an active participant in the learning situation who can influence and *shape* the prospective teachers' learning.

Enactivism and language

When we perceive an object as separate from its background we are "making an *act of distinction*" (Maturana & Varela, 1998, p. 40, emphasis original). Learning requires a *change* to our ways of making distinctions, a change in the way we perceive and act in the world. Whenever we explain our experiences, we do so in relation to other experiences. We cannot claim to have access to a reality that is independent of our actions and our being in the world:

> Whatever we are able to say about that truth or reality is dependent on the availability of language. What is supposedly independent from us becomes describable only when language is available, emerges only through distinction by means of language (Maturana & Poerksen, 2004, p. 29).

Enactivism, along with its emphasis on action, rejects a 'code view' of language where language is conceived as the transmission of messages from one individual to another. It makes more sense from an enactivist perspective to consider language not as an object but in terms of a process of *languaging*. Languaging stems from our being active in the world with others, it arises through our recurrent interactions "in a flow of co-ordinations of co-ordinations of consensual behaviours" (Maturana & Verden-Zöller, 2008, p. 30). An enactivist view of language emphasises the relational dynamic of consensual coordination between individuals. According to Maturana (1988b), "the phenomenon of language takes place in the flow of consensual co-ordinations of consensual co-ordinations of actions between organisms" (p. 43) or in more simple terms, "languaging is the co-ordination of actions that are about other actions" (Reid & Mgombelo, 2015, p. 179). Maturana and Varela (1998) point out there is no similarity between a word and the object or situation designated by that word, e.g. the word 'chair' is not congruent with what we do to distinguish a chair, the word 'chair' is an arbitrary label. As Coles (2015) explains, objects (including social phenomena) are not out there in the world waiting to be labelled, objects come into existence through the process of languaging. It is the co-ordinations of actions in relation to the object, situation or phenomena that is relevant and not the form that the label takes.

Thus language is not a representation of the outside world, nor it is simply the communication of our inner thoughts or the process of operating with sets of symbols, rather, "symbols arise in language as distinctions of relations of distinctions" (Maturana, 1988b, p. 44). Maheux and Roth (2014) conceptualise language and communicative acts as "collective products of relations in and through which human beings constitute and articulate the lived life of [...] the worlds they inhabit" (p. 509). For these authors, language is not a means to share thinking or to negotiate ideas, rather, words and utterances are "the very fabric in which thinking takes places as a soci(et)al relation" (p. 504).

Enactivism and mathematics education

Enactivist theory has been used to advance a number of areas of research in mathematics education, pointing researchers towards the study of certain phenomena. In Reid and Mgombelo's (2015) survey of key enactivist concepts, they refer to some of the different notions used by mathematics education researchers to guide and inform their studies, including: cognition as an active process (e.g. Samson & Schäfer, 2011); research as perceptually guided action (e.g. Lozano, 2015); and knowing as mathematical doing (e.g. Maheux & Proulx, 2015). The use of enactivism in mathematics education can be traced back to the work of Tom Kieran (Reid, 2014) who initially used Maturana's ideas for the basis of what has come to be known as the Pirie-Kieran model of mathematical understanding (Pirie & Kieren, 1989). Kieren (1995) also used enactivist ideas to characterise mathematics teaching as "in-the-middle" of classroom activity, where the teacher and students are "bringing forth together a shared world of mathematical significance" (p. 10). According to Kieran, teachers who are "in the middle" can be observed doing various acts including "prompting explanatory reasoning"; "providing occasions [for learning]"; and "listening with students" (p. 15). More recently, an "enactivist mathematics pedagogy" (Abrahamson et al., 2022) has begun to be developed consisting of a set of principles for enactivist pedagogy design and facilitation.

The enactivist view of cognition as embodied action considers gestures, body language and facial expressions as fundamental parts of the cognitive process (Towers & Martin, 2015), thus enactivism naturally invites researchers to examine these aspects of mathematics teaching and learning. Moreover, enactivism's emphasis on the coupling of individuals with their environments (which include other individuals) and the coordination of actions tends to point enactivist researchers to examine *collective* classroom activity, developing methodologies that enable this collective activity to be observed (e.g. McGarvey et al., 2022; Thom, et al., 2020; Towers & Martin, 2015). In relation to the study of language, Coles (2015) developed an enactivist-based methodology for studying classroom talk which he used to analyse two episodes from mathematics classrooms. Like Coles (2015), we also turn our attention towards analysing an episode of transcript but in our case we explore what enactivism has to offer the study of MTE language when the empirical material consists of a transcript in monologic form. From an enactivist perspective, the analysis of transcript in the form of a monologue presents a significant challenge (which we address in section "Methodology: combining enactivism and SFL") due to enactivism's emphasis on interactions/relations. Enactivist researchers in mathematics education therefore tend to analyse situations where interactions are observable. Maheux and Roth (2014), for example, analyse mathematics classroom conversations between teachers and students. In considering the fundamental unit of analysis as the student–teacher *relation* as opposed to the individual subjects, the researchers remind us that conversations are irreducible social

phenomena that cannot be reduced to the individuals' utterances. From this relational perspective, "utterances (a) are always a/in response to something else and as offerings and (b) find their actual signification (as a question, an answer, an explanation, a justification, and so on) in how it is responded to" (Maheux & Roth, 2014, p. 505). Conversations are thus manifestations of relations that are the higher order functions (such as mathematical concepts) subsequently attributed to individuals. Hence, in relation to monologic text, both text and reader/listener could be conceived as co-existing and co-emerging. The text, though easily regarded as static, can be seen as emerging as something else in/through the relationship with the reader/listener. The words in the text do not 'change', in that sense the text is static, yet the meaning that emerges in and through the interaction between the reader/listener and text is continuously in flux.

Within mathematics *teacher* education, more specifically, enactivist theory has been used by fewer researchers. Proulx (2008), for example, used the concept of structure determinism to explore implications for mathematics teacher learning, framing MTEs as triggers for teacher learning, with "the opportunity to open new possibilities […] new ways of making sense and of understanding" (p. 151). Other examples of ways in which enactivist ideas have been used within mathematics teacher education include: conceptualising teacher development (L. Brown & Coles, 2011); framing the process of reflection in learning to teach mathematics (L. Brown & Coles, 2012); exploring the expressed awarenesses of PMTs in relation to doing mathematics (Voutsina et al., 2022); analysing the role of the PD facilitator in using video (Coles, 2013); describing the design principles of a TE programme (J. Brown et al., 2021); comparing with professional noticing as a conceptual framework for teacher learning (L. Brown et al., 2019); and conceptualising the process of becoming a mathematics teacher (J. Brown et al., 2019) and a MTE (Helliwell, 2021, 2020).

A common feature across all of this research is an explicit focus on *learning*. Though our focus is not primarily one of PMT or MTE learning, as MTEs ourselves, the process of researching the language of another MTE is exactly a process of learning, not only about the language of the participating MTE, but about ourselves and our own use of language when working with PMTs.

Systemic functional linguistics

Systemic functional linguistics is situated within the frame of social semiotics. Social semiotics seeks to understand how people communicate by various means in particular social settings. Different settings have specific, socially, and culturally shared possibilities for meaning-making. Social semiotics recognises that meaning-making occurs in social contexts and that language use is functional (Morgan, 2006). SFL is a theory of language as "meaning potential" (Halliday, 1978), where meaning emerges through the process of "languaging" (Halliday, 1985) during different activities, situations, and social practices (Holmberg, 2012). The word 'potential' is used to highlight the range of choices (where 'choice' is meant in an abstract sense and does not correspond to actual choice) within different functions of language and is demonstrated through the distinction made by Halliday (1978) between what he refers to as text and what he refers to as context. For Halliday, the text is what the speaker says, which is always embedded within the context of what the speaker/author can do (i.e. the range of all potentialities) which is contingent on those listening to the speaker/reading the text as well as the situation at hand. Text is thus an instance of language use in activities, situations, and social practices (Halliday,

1978) which develops, in situ, through interaction with others. Languaging as a verb (as opposed to a noun) highlights SFL's conception of language as an activity or practice as opposed to language as an entity with pre-determined meaning. SFL concerns the relationship between language and its function since people use language as a resource to accomplish specific purposes (Halliday, 1978) although these functions are not determined by the speaker's words themselves but by all those engaged in the languaging process; the text itself only has the potential to make meaning. SFL investigates how the contextual demands of language impact the way in which a speaker uses language differently according to the intended function of their use, hence the use of the term Functional within SFL. To conclude, SFL emphasises the connections and relationships between language, context and culture.

According to SFL, experience determines how and what we perceive, forming the basis for the choices we make about how we proceed in our interactions with others. Halliday and Hasan (1989) point out that language is stratified, meaning that it is arranged (in strata) in a series of layers, levels, or gradations in an ordered system (hence the term Systematic in SFL). Stratification concerns the way present social practices connect historically to social practices from the past, meaning that our experiences are framed in both time and space. According to the theory of SFL, what happens when we use language is so complex that analytically it requires a process of uncovering multiple layers (Halliday, 1978; Thompson, 2013).

SFL and mathematics education

During the last twenty years, mathematics education research has brought increased attention to the study of language and the importance of language as a medium of teaching and learning in a move often referred to as the "linguistic turn" (e.g. Lerman, 2009). In this paper, we follow this line of research and, in line with Morgan (2006), propose that Halliday's social semiotic perspective, SFL (Halliday & Hasan, 1989), provides us with some useful tools for exploring the language of MTEs. Through using SFL to analyse language, researchers within mathematics education have investigated various social phenomena by attempting to identify and describe the speakers' arrangement of language and the potential consequences of that arrangement. For example, Herbel-Eisenmann and Otten (2011) used SFL to visualise semantic structures of mathematical content; Herbel-Eisenmann and Wagner (2010) identified patterns of speech in mathematics classrooms to examine interpersonal positioning as they relate to personal feelings, attitudes, and values; DeJarnette (2018) combined positioning theory with SFL to identify collaboration during students' paired work; Meaney et al. (2012) used SFL to identify different genres in students' written mathematical texts; and Segerby (2017) used SFL as a way of exploring how students develop their mathematical reasoning skills.

Typical for most of this work is that the analysis of language relates to the three different layers associated with an SFL perspective. These layers represent three dimensions of meaning potential, so-called *metafunctions*, which are understood as operating simultaneously during any utterance. The three metafunctions, *ideational*, *interpersonal*, and *textual*, link linguistic resources to a particular aspect of the context. Firstly, the ideational function relates to the content of the text, and is concerned with our perceptions of the world around us (e.g. objects, phenomena, activities). The interpersonal function enables speakers to enact their interpersonal relations thus concerns the relationship between those involved in any interaction. The textual function weaves the other two functions together in

a coherent way and concerns the mode and role of the communication being used (Halliday & Matthiessen, 2014). Morgan (2006) provides, in relation to these metafunctions, three overarching questions in researching within the field of mathematics education:

> What is the nature of mathematics and mathematical activity as it is constructed in a text? (*ideational* aspect)
> Who are the participants in the interaction (author and reader or speaker(s) and listener(s)) and what relationships do they have to each other and to the subject matter? (*interpersonal* aspect)
> What role does the text play within the context of situation? (*textual* aspect) (Morgan, 2006, p. 229, emphasis original)

The three metafunctions along with adapted versions of Morgan's questions form the basis for our set of analytical tools which we explain in more detail later (see sections "Establishing methodological practices" and "Exemplifying the methodology").

Responding to the issue of *context*

Halliday's ideas have the potential to be read in objectivist terms. For instance, both Coles (2015) from an enactivist perspective, and de Freitas (2010) from a postmodern perspective, have critiqued SFL theory for the distinction it makes between text and context, a distinction that is not recognised by enactivists or postmodernists. For enactivists, there is no context independent of text, rather, context arises and is brought forth through language. Put simply, without language there would be no context, context and text emerge simultaneously, through the process of structural coupling.

Some readings of SFL, however, offer a view of language that coheres more closely with this enactivist view. For instance, in Halliday's (1989) conceptualisation of spoken language as *process*, we read language as having emergent properties:

> The spoken language presents a DYNAMIC view. It defines its universe primarily as process, encoding it not as a structure but as constructing–or demolishing. In the spoken language, phenomena do not exist; they **happen**. They are seen as coming into being, changing, moving in and out of focus, and as interacting in a continuous onward flow (Halliday, 1989, p. 97, capitalisation and emphasis original).

Similarly, in Halliday's (1993) conceptualisation of language as a dynamic open system, he considers language to persist only through constant change, where "this change takes place through interactive exchanges with [the] environment" in a state of "constant becoming" (p. 121). Thus language, from this view, is not seen as fixed, pre-determined, or unambiguous, but one that evolves in relation to the environment in which is arises. For language to be an open system, as in SFL, the relationship between an instance of language and the range of all potential instances (i.e. context, in SFL terms), is not conceived as being a simple process of selecting from a set of existing possibilities, rather, SFL "strives to account for the perturbing effect of novel utterances on the system and the cyclical relationship between the existing "structure" and the continuous processes of "constructing–or demolishing" – that structure" (Elorza et al., 2021, p. 2). Here we see affinities between how context is conceptualised by Halliday with the enactivist notion of structure, which we briefly explore here.

From an enactivist perspective, individuals bring their history of structural coupling, such that anything said is done so in relation to what is possible to say (i.e. dependent on

the individual's structure) and what has been said before. This does not mean we cannot say anything new, but that our use of language arises in relation to our own histories of interactions. From an SFL perspective, what is said (the text) is always said in relation to what is possible to say (i.e. the range of all possibilities and potential meanings) and is what is meant by context. Though SFL's context and enactivism's structure are positioned in different ways (with context being positioned primarily in the world beyond the speaker, and structure being positioned within the individuals, i.e. within the speaker or the readers/listeners), we see both of these concepts as relational (i.e. arising simultaneously with language) and that both concepts account (in slightly different ways) for the cultural–historical aspects of language. Thus, although we see SFL as making a linguistic distinction between text and context, and a difference in terms of how the relationship between text and context are conceived across the different perspectives of enactivism and SFL, we do not see these differences as problematic in terms of methodological implications.

Methodology: combining enactivism and SFL

Enactivism as a theory of learning does not come readily operationalised with a well-defined set of analytical tools. This is especially the case when the empirical material consists of monologic text as it does in this study. Having said that, methodological *principles* have been described coherently by Reid (1996) and several authors have explored enactivist methodology (see ZDM issue by Reid et al. (2015)) in relation to different mathematics education contexts, articulating a range of ways in which it is possible to do research from an enactivist perspective. In this section, we formulate our methodology as a set of *methodological principles* and *methodological practices* that include utilising the grammatical tools of SFL. This methodology is of particular relevance to practitioners seeking to study and learn from the language of other practitioners (e.g. MTEs) which we frame in relation to the analysis of monologic material.

Establishing methodological principles

Reid (1996) sets out two features of enactivist research, derived from key principles of enactivism, that are: "the creation of models and theories which are good-enough *for*, not definitively *of*" and "the importance of working from and with multiple perspectives" (p. 207). From an enactivist perspective, no model or theory used to describe or explain a phenomenon can ever be definitively of some external truth. This does not mean that models and theories are of no use, rather, models and theories are accepted as being good-enough descriptions and explanations *for* the phenomenon under study rather than mirrors of reality. According to Reid, "[t]heory and data coemerge in the medium of the researcher" (p. 206), that is, the process of analysis involves a dialogue between theory and data, with each one simultaneously transforming the other as well as the researcher. As two university-based MTE researchers from different countries and contexts, both interested in the language use of MTEs, we are shaped by our different histories which in turn shape our different ways of seeing the world of mathematics teacher education and the ways in which we each interact with theory and data. Utilising multiple perspectives is thus one way of expanding what is possible to grasp during the research process, a way of seeing more than is possible to see from any single (researcher or theoretical) perspective. A study of MTE language calls for analytical tools that trigger new awarenesses for us as MTE

researchers that our habitual ways of seeing would not allow us to see, and in the readers of our research, who are most likely to be other MTEs. Hence we use analytical tools that support us in seeing the data in new and different ways.

Goos and Beswick (2021) suggest that methodological challenges are amplified when those doing research with/on MTEs are often MTEs themselves, "likely to be involved in the milieus that they are researching as well as personally engaged with the same issues with which their research subjects […] are grappling" (p. 13). From an enactivist perspective, however, "it is desirable that the researcher has a history of being in the context, interacting and reflecting on what is observed not just occasionally but persistently" (Lozano, 2015, p. 231). As experienced MTEs, we have each developed increasingly specialised ways of observing TE situations. In enactivist terms, we have developed an increasingly refined set of distinctions/actions in relation to teaching PMTs based on our histories of structural-couplings. At the same time, we wish to challenge and potentially disrupt these specialised way of seeing by expanding our perspectives through the research process.

From certain perspectives, qualitative researchers look to reduce researcher bias by employing a variety of techniques such as "triangulation" or "member-checking". Employing these well-established techniques implies the existence of an external reality independent of the researcher and are designed to bring the researcher *closer* to that reality so that findings more closely reflect the actuality of the situation. These techniques are also associated with the development of criteria for quality research such as validity and reliability. From an enactivist perspective, where researcher and researched are inextricably linked, these techniques and criteria no longer seem appropriate. New criteria for quality research are needed when research is conceptualised as perceptually guided action. Lozano (2015) stresses the importance of ensuring that readers can engage fully with the research, which for Lozano means including a clear account "of how theories and ideas emerge in the process of doing research" (p. 230) and being clear about the research process itself. It is important, therefore, to provide a clear account of the research process at a detailed level. Moreover, at the heart of enactivist research, there is the need to generate novelty or newness, to develop ways of seeing beyond what we already see in relation to the research situation. Much research involves questions relating to 'what is (the current state of affairs)?' We propose that an enactivist framing supports questions more of the form 'what could be?', specifically, we see an enactivist methodology as one that supports the process of generating new possibilities. Proulx (2015), argues that a "fundamental aspect of a research study is its degree of generativity, that is, the ideas and distinctions that it generates" (p. 1). We suggest that a criteria of generativity is of particular relevance for practitioners studying the practices of other practitioners (within the same field), especially if one purpose of research is to enable critical reflection on one's own actions (e.g. as MTEs) that may lead to a change in practices.

Establishing methodological practices

With these methodological principles forming the basis of our approach to enactivist research, we now outline a set of methodological practices enabling us, as MTE researchers, to examine, in generative ways, the language in use of another MTE. To do this we draw on some of the enactivist informed "mechanisms" presented by Coles (2015) for the analysis of talk which we adapt in relation to our own research situation. In our case, the talk being analysed is in the form of a monologue, and those conducting the research are practitioners who themselves engage with PMTs as MTEs.

Practice 1: Considering and sharing context

Morgan (2006) reminds us that when adopting a social semiotic perspective such as SFL, interpretations of language must always be done in relation to the context of the situation in which that text was produced. Like Coles (2015), who discusses the importance of interpreting data within the context of the study, Morgan (2006), from the social semiotic perspective of SFL, endorses the importance of both considering and sharing the context (both situational and cultural) in which research is taking place. Morgan suggests that there are two important methodological issues to consider, namely, "how much of the context it is necessary to consider and what means to use to describe the context" (p. 239). Thus, it is necessary to account for both the context of the immediate research situation but also the context more broadly. In this study, it means not only that we share enough detail regarding the specific situation and context for the research to make sense to the reader, but that we consider our interpretations in relation to the context as we analyse the data. We also provide a description of our own backgrounds and context as MTEs and researchers.

Practice 2: Systematically searching for patterns

Enactivist researchers focus on what is observable. In the case of text (including monologue), it is possible to observe and agree upon the occurrence of patterns within the speech. The systematic search for pattern involves splitting or segmenting data in a systematic way so that an observable pattern can be identified (Coles, 2015). According to Coles, "[a] pattern could comprise as few as two instances" (p. 240). Those parts of the data that are significant to multiple researchers are then "privileged and might be interrogated further" (p. 240). SFL provides tools for highlighting linguistic patterns so that we can examine the MTE's use of language. In our own systematic search for pattern, we use the grammatical tools of SFL as a way of identifying patterns within the MTE's language that point us to particularly significant moments within the transcript that merit further attention.

Practice 3: Conducting a micro-analysis

Once significant moments from the transcript have been identified, there is a need to for a more detailed analysis. According to Coles (2015), micro-analysis involves approaching "small sections of transcript with a slow and repeated reading, keeping some questions in mind" (Coles, 2015, p. 241). Since we are utilising the tools of SFL as a way of analysing the text, we are interested in what the MTE chooses says in relation to the range of possible choices that as MTEs we imagine could have been said. Morgan (2006) suggests that that text can be examined by, "[f]ocussing on the choices provided by the functional system […] identifying how the text might be different and considering the effects of the choices that are realised" (p. 229). Thus, in our slow and repeated reading, we asked ourselves the following questions having marked the text using the processes and categories associated with each of SFL's three metafunctions:

  -From the options available in each moment, which was selected?
  -What else might have been said in that moment?

-What are the potential consequences of the choices made?

All three questions are asked in relation to each aspect of the text (i.e. the ideational, interpersonal and textual aspects). Having considered a section of transcript with a slow and repeated reading keeping these questions in mind, we frame our findings around the three questions provided by Morgan (2006) for researching within mathematics education using the tools of SFL (see section "SFL and mathematics education") for which we have derived our own versions for the particular research situation:

> *What is the nature of mathematics and mathematics teaching depicted in the teacher education lecture? (Ideational aspect)*
> *Who are the participants and what relationships do they have to each other and to mathematics and mathematics teaching? (Interpersonal aspect)*
> *What role does the text play within the mathematics teacher education situation? (Textual aspect)*

Practice 4: Reflecting explicitly on our learning as mathematics teacher educators

Coles (2015) warns us that intentions cannot easily be interpreted from the words of others. In the case of analysing monologic text, this challenge is amplified. As two MTE researchers from different contexts (e.g. UK/Sweden, Secondary/Primary) we do not have access to the MTE's intentions, nor to the responses of the PMTs where the MTE's words become significant (and observable). Yet, through the process of joint analysis and reflection, possibilities for meaning emerge *for us*, both in relation to the specific research situation as well as in relation to our own practices as MTEs. This meaning can be made explicit as part of the research process both through the way in which the analysis is reported (i.e. as interpretations rather than objective truths) but also through reflecting explicitly on our learning as MTEs. The grammatical tools of SFL provide the means to make new distinctions that become available to us to consider in light of our own practices. In this study we present our reflections under the same questions used during the process of micro-analysis, but this time in terms of our own learning.

Exemplifying the methodology

Considering and sharing context

The authors of this paper are both MTE researchers. Tracy is a university-based MTE from the UK where she teaches prospective mathematics teachers on a one-year postgraduate course for secondary (aged 11–18 years) mathematics teachers. Before moving to teach at university, Tracy taught mathematics in secondary schools for thirteen years. Andreas is a university-based MTE from Sweden where he teaches prospective pre-school (aged 1–6 years) and prospective primary school (aged 7–12 years) mathematics teachers. Before moving to teach at university, Andreas taught mathematics in pre-school (age 6 years) and lower primary school (aged 7–9 years) for ten years. The context in Sweden where the TE lecture took place, a transcript of which comprises the corpus of data, is the reform mathematics movement where school mathematics is promoted as "students' creative engagement in exploratory and problem-solving activities as they develop their understandings of significant mathematical concepts and procedures" (Skott et al., 2018, p. 164). In Sweden, prospective primary teachers learn to become generalists (as opposed

to subject specialists). Consequently, they will teach a range of different subjects and their level of education in each of the school subjects is varied. Their professional background is often linked less to the teaching of specific subjects than to the profession as a whole.

Systematically searching for patterns

The entire lecture was transcribed in Swedish by Andreas, who then translated the text into English. Having marked and classified the entire transcript of the lecture using the grammatical tools of SFL, we looked for patterns that we both recognised as being significant. The following extract exemplifies an observable pattern found in the lecture where the MTE, on several occasions, uses the words "have to " and "must" (marked in bold in the extract below). We were intrigued by this pattern and decided to conduct a micro-analysis of this extract keeping our three questions in mind to address the broader questions (detailed in "Practice 3") relating to each of the three aspects of the language used by the MTE. [Note that … denotes a short pause]:

> At the same time, **you have to** work with their language in the future … **You have to** look at the national exams for grades three and six… there are a lot of tasks where **they have to** calculate, write and above all explain what they have done and so on… preferably in different ways. **We must**… **you must**, in the future, be able to write mathematically yourself... **we have to** give students these tools to pass the national tests. It is no longer enough just to calculate and calculate.

Conducting a micro-analysis

We started with the *ideational metafunction* (Table 1) and the transitivity system. *Transitivity* is a linguistic concept that directs us to consider the relationship between the participants being referred to and the process or action being described. By asking the question, "what is going on?", we marked (in Table 1) the process verbs (in bold) and classified them (in brackets). Process verbs are the actions that the MTE refers to, such as calculate, think or explain. Processes used in the analysis are: Material processes that involve physical actions, like competing; calculating; and teaching where there is an actor (doer) that does something. Mental processes like understanding; thinking; wanting; and knowing where there is a sensor sensing something. Relational processes which emphasise relations between objects and how entities are related to each other. For example, "are" in the transcript below highlight the relation between 'national exams' and 'a lot of tasks'. Verbal processes express something being said. However, since transitivity concerns the

Table 1 Example of SFL analysis (*ideational metafunction*)

Ideational metafunction

Processes (material, mental, relational, verbal), participants and objects

*At the same time, <u>you</u> have to **work** (Material + actor) with t<u>heir language</u> in the future … <u>You</u> have to **look** (Material + actor) at <u>the national exams</u> for grades three and six… there **are** (Relational "national exam and a lot of tasks") a lot of tasks where <u>they</u> have to **calculate** (Material + actor), **write** (Material + actor) and above all **explain** (Verbal + Sayer) what <u>they</u> have done and so on… preferably in different ways. We must… you must, in the future, be able to **write** (Material + actor) mathematically yourself… <u>we</u> have to **give** (Material + actor) students these <u>tools</u> to pass the national tests. <u>It</u> is no longer just **to calculate** (Material + actor) and calculate*

---

relationship between the participants and the process, we also marked (underlined) the participants and relevant objects.

Through the ideational metafunction we can ask: *What is the nature of mathematics and mathematics teaching depicted in the lecture?* Our reading of the overall ideational meaning of this extract is that it is focussed on the general goals and objectives of teaching mathematics in a way that prepares students for academic success by passing the national examination. The material process *work* (the process of carrying out an activity) seems to introduce that process which is followed by more specific material processes such as *calculate* (the process of performing mathematical operations) and *write* (the process of producing written text). The MTE also uses the verbal process *explain* in relation to information about the oral aspects of mathematics. We interpret these processes us being used to imply the importance of teaching mathematics in a way that develops students' oracy and mathematical reasoning. The MTE appears to be emphasising the need for students to both be able to perform calculations and communicate their processes and reasoning in a variety of ways.

In the next stage of the analytical process, we looked at the interpersonal metafunction. The interpersonal metafunction is concerned with social relationships and interactions between people through language. It concerns the roles and relationships between speakers and their listeners. In this part of the analysis, we first focussed on the mood system. *Mood* is a grammatical system that direct us towards the purpose of what MTE is saying (e.g. statement, question, offer or command), reflecting a stance towards the content of the message. At the same time, we marked *modality*, which suggests the level of certainty we might associate with particular forms (Herbal-Eisenmann, 2007), expressing a position that lies somewhere between a definite "yes" and a definite "no". The distinction we have used (in Table 2) is high, medium and low modality. The participants and relevant objects are still underlined in this part of the analysis. In the second step, we marked (in bold) the

Table 2 Example of SFL analysis (*interpersonal metafunction*)	*Interpersonal metafunction*
	Mood and modality
	*At the same time, <u>you</u> must (**obligation**) work with <u>their language in the future</u> (**command with high modality**) … <u>You</u> must (**obligation**) look at the <u>national exams</u> for grades three and six (**command with high modality**)… there are a lot of <u>tasks</u> (**statement with high modality**). <u>They</u> must (**obligation**) calculate, write and above all explain what <u>they</u> have done and so on (**statement with high modality**)… preferably in different ways (**offer with medium modality**). <u>We</u> must (**obligation**) … <u>you</u> must (**obligation**), in the future, be able to write mathematically yourself (**command with high modality**)… <u>we</u> must (**obligation**) give students these tools to pass the national tests (**command – high modality**). <u>It</u> is no longer enough just to calculate and calculate (**statement with high modality**)*
	Tense and polarity
	*At the same time, **you have to work** (future–- positive) with their language **in the future** (future–- positive)… **You have to look** (present–- positive) at the national exams for grades three and six… there are a lot of tasks where **they have to** (present–- positive) calculate, write and above all explain what they have done and so on… preferably in different ways. We must… **you must in the future** (future–- positive), be able to write mathematically yourself… **we have to** (future–- positive) give students these tools to pass the national tests. It is no longer enough just to calculate and calculate*

tense to classify (in brackets) where we view the clause as being situated in time. We do this to consider the temporal relationships between the different clauses brought into the lecture. Thereafter, we classified the *polarity* to stress whether we viewed the propositions as appearing in positive or negative form (note, there are no examples of negative polarity in this piece of transcript).

Through the interpersonal metafunction we can ask: *Who are the participants and what relationships do they have to each other and to mathematics and mathematics teaching?* We viewed the *mood* of this extract as imperative, the MTE seems to be giving instructions and commands to the prospective teachers, emphasising the need for action and the importance of teaching mathematics in a certain way. The *modality* we viewed as primarily expressed through the use of "have to" and "must". To us, these verbs suggest a sense of urgency and importance in teaching mathematics in a way that prepares students for national examinations. The statement "It is no longer enough just to calculate and calculate" suggests a shift in expectations emphasising the importance of preparing students for something that is not generally done today. Overall, we read the modality in this passage as expressing a strong sense of obligation (emphasising that this obligation is not located in the words "you must" but in our reading of those words). The use of imperatives, such as "you have to" and "we have to" as opposed to "you could" or "we might", suggest to us that the MTE could be attempting to influence the PMTs and persuade them to take specific future actions.

Finally, we looked at the *textual metafunction*. The textual metafunction relates to how language organises and structures a text. Through this metafunction, we can gain insight into how the language is constructed and the way in which it has the potential to convey meaning. Firstly, in Table 3, we looked at the thematic structure and the way the message seems to us to be conveyed. Secondly, we looked at the information structure, for example, how the first clause establishes a theme in the text and how the repetitions in the text lead to a summary and conclusion that for us reinforces the overall message.

Through the textual metafunction we can ask: *What role does the text play within this mathematics teacher education situation?* The MTE seems to connect and create a sense of progression through the use of "in the future". Additionally, the MTE uses cohesive devices such as repetition ("calculate" and "must") creating coherence and linking the ideas together. Through these repetitions we also experienced a sense of urgency for something to be done by the PMTs in relation to their teaching. For us, the last clause in the extract seems to reinforce the overall message about the PMTs' responsibilities as future mathematics teachers.

Table 3 Example of SFL analysis (*textual metafunction*)	*Textual metafunction*
	Different aspects of structure – cohesive relations
	At the same time, **you have to** *(repetition – urging)* work with their language in the future … **You have to** *(repetition – urging) look at the national exams for grades three and six… there are a lot of tasks where they have to calculate (repetition), write* **and** *(linking) above all explain what they have done and so on…* **preferably** *(linking) in different ways.* **We must** *(repetition – urging) …* **you must** *(repetition – urging), in the future, be able to write mathematically yourself…* **we have to** *give students these tools to pass the national tests.* It is no longer enough just to calculate *(repetition)* and calculate *(repetition–- reinforcing the message)*

Concluding thoughts

Throughout this paper we have turned our attention specifically towards developing a novel methodology for studying the language-in-use of MTEs and in doing so we believe we have answered our first research question (*RQ1*). In formulating our methodology we began to address our second research question (*RQ2*) which relates to the *kinds* of insights and new awarenesses that can be revealed in relation to the language-in-use of MTEs from utilising such a methodology, by specifying the three overarching questions (section "Establishing methodological practices") that guided and framed our analysis. Having interrogated the data using the grammatical tools of SFL, we were able to examine aspects of the language used by the MTE during the TE lecture in Sweden. From the perspective developed in this paper, we acknowledge that our 'findings' were based on our own interpretations of what was being said, as opposed to the reality of the situation as distinct from us as observers. The analytical tools provided us the means to make new distinctions that we have previously not made, distinctions that are now available for us to consider in light of our own practices as MTEs. Most significant for us are therefore the insights and new awarenesses that we have developed in relation to our own use of language in our respective TE settings, hence we finish this paper by enacting our fourth methodological practice and presenting some of these insights as a set of reflections under each of the questions that framed our analysis. We see our process of reflecting as part of what it means for research to be generative.

Reflecting explicitly on our learning as mathematics teacher educators

What is the nature of mathematics and mathematics teaching depicted in our own language when working with prospective teachers of mathematics?

> Tracy: I know I have views on mathematics and mathematics teaching, which I mainly developed from being in the classroom. I grapple with how much of this I want to communicate with prospective teachers. It is important to me that they develop their own sense of mathematics and mathematics teaching, to become the teachers they want to become. I do not want to set them up to fail by suggesting there is only one way to teach mathematics, so I try to avoid offering one model of teaching. I question, however, how possible it might be to entirely avoid communicating my views. By examining another teacher educator's language, their choice of phrasing, the words they emphasise, I have become more conscious of my own use of language. I wonder what views on mathematics teaching I might be communicating that I am not yet aware of. From a prospective teacher's perspective, how might I respond if I view my own experiences of mathematics or mathematics teaching as being in contrast to those portrayed by the teacher educator? Might I become more or less open to reconsidering my own experiences in light of these new ideas?
>
> Andreas: I have found our research process overwhelming at times. What I might consider problematic as a teacher educator is common sense for another. By interpreting the language of other teacher educators, I find myself thinking about what it might mean in the end for our prospective teachers. I also question myself. Do I share a deficit story of mathematics teaching with my colleagues or, worse, do I share it with the prospective teachers? Do I have a specific prospective teacher in mind when

I am teaching, without really knowing them? Sometimes I envy other researchers that seem sure about things. I question if my teaching affords or constrains prospective teachers' development. I ask myself, how does my use of language enable prospective teachers in developing a sense of agency? How can I support them in finding their own need to develop their view of mathematics and mathematics teaching?

Who are the participants and what relationships do they have to each other and to mathematics and mathematics teaching when working with prospective teachers of mathematics?

Andreas: In my early days as a MTE, my objective differed from that of the prospective teachers. While I wanted to guide and support them in their learning and social development as teachers-to-be, they seemed to want other things. They had other priorities that I did not cohere with. I struggle because our course content is so specific. It feels impossible to engage with the prospective teachers' concerns when there is this content to be covered. Ultimately, they need to align with the content to pass the course. This focus on content is not, however, allowing the prospective teachers to explore their own relationships to mathematics and mathematics teaching, and this feels like it should be a priority.

Tracy: It is important to me to position the prospective teachers as both teachers and learners from the outset of the course. Teachers of mathematics and mathematics students and learners of mathematics and mathematics teaching. I try to make this explicit in my use of language, although I will certainly pay more attention to this from now on. Perhaps working in secondary mathematics teacher education, I have more time to engage with prospective teachers' concerns. I know I am listening for when a prospective teacher might be expressing something about the nature or mathematics or mathematics teaching. I would hope to encourage them to bring that view into explicit awareness, so there it an opportunity for it to be unpacked with other prospective teachers. I suppose I am listening for implicit assumptions, so those assumptions can be held up for question. Having considered the relationships of the participants in the language of another teacher educator, I would like to pay attention to the ways in which I refer to both students and teachers of mathematics.

What role does our language play within our own mathematics teacher education situations?

Andreas: Most prospective teachers have experienced 12 years of schooling at the point they enter teacher education. Some of them seem to want to question their view of teaching, whilst others do not. If teachers teach who they are, my language needs to allow the prospective teachers to develop a sense of who they are. I try to create these opportunities. I have been told that sometimes I do not "stand up for the mathematics", I am not sure that is true. I think mathematics is essential, but teaching as telling what to know and how to teach could be counterproductive. I do not view my role as using language in that way. I argue that prospective teachers' prior experiences need to be used whilst encouraging them to reflect on their future to help them become more aware of themselves. This cannot be done on the surface if we take the role of language seriously and believe that we teach who we are.

Tracy: I wonder how my words could be reinforcing certain ideas and discouraging others, intentionally or unintentionally. I try to avoid communicating a sense of certainty in relation to teaching mathematics, but do my words imply something else? When I tell stories of teaching, I try to avoid telling stories from my own time as a

teacher. I like to tell stories of others, other prospective teachers maybe, or teachers with whom I have worked. I want to expose the prospective teachers to a range of ways of becoming a teacher, ways that might resonate with them so they tell stories too. I wonder what phrases I repeat, and what this repeating might be emphasising. How much of what I say opens up possibilities and how much closes them down. For me, one role of language is to expand the prospective teacher's perspectives, to offer them alternatives to what might feel quite certain to them. Studying the language of another teacher educator has certainly done this for me.

References

Abrahamson, D., Dutton, E., & Bakker, A. (2022). Towards an enactivist mathematics pedagogy. In S. A. Stolz (Ed.), *The body, embodiment, and education: An interdisciplinary approach* (pp. 156–182). New York: Routledge. https://doi.org/10.4324/9781003142010-9

Brown, L., Fernández, C., Helliwell, T., & Llinares, S. (2019). Prospective mathematics teachers as learners in university and school contexts: From university-based activities to classroom practice. In G. M. Lloyd & O. Chapman (Eds.) *International handbook of mathematics teacher education (second edition)* (vol. 3, pp. 343–366). Brill. https://doi.org/10.1163/9789004419230_014

Brown, J., Brown, L., Coles, A., & Helliwell, T. (2019). Learning to teach mathematics: The lesson de-brief conversation. In S. Llinares & O. Chapman (Eds.) *International handbook of mathematics teacher education (second edition)* (vol. 2, pp. 85–108). Brill. https://doi.org/10.1163/9789004418967_004

Brown, J., Brown, L., Coles, A., & Helliwell, T. (2021). Working with awareness as mathematics teacher educators: Experiences to issues to actions. In M. Goos & K. Beswick (Eds.) *The learning and development of mathematics teacher educators* (pp. 187–204). Springer. https://doi.org/10.1007/978-3-030-62408-8_10

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *Journal of Mathematics Teacher Education, 21*(5), 417–427. https://doi.org/10.1007/s10857-018-9416-4

Brown, L. (2015). Researching as an enactivist mathematics education researcher. *ZDM, 47*(2), 185–196. https://doi.org/10.1007/s11858-015-0686-3

Brown, L., & Coles, A. (2011). Developing expertise: How enactivism re-frames mathematics teacher development. *ZDM, 43*(6), 861–873. https://doi.org/10.1007/s11858-011-0343-4

Brown, L., & Coles, A. (2012). Developing "deliberate analysis" for learning mathematics and for mathematics teacher education: How the enactive approach to cognition frames reflection. *Educational Studies in Mathematics, 80*(1), 217–231. https://doi.org/10.1007/s10649-012-9389-7

Coles, A. (2013). Using video for professional development: The role of the discussion facilitator. *Journal of Mathematics Teacher Education, 16*(3), 165–184. https://doi.org/10.1007/s10857-012-9225-0

Coles, A. (2015). On enactivism and language: Towards a methodology for studying talk in mathematics classrooms. *ZDM, 47*(2), 235–246. https://doi.org/10.1007/s11858-014-0630-y

Davis, B. (1996). *Teaching mathematics: Towards a sound alternative*. Garland.

de Freitas, L. (2010). Regulating mathematics classroom discourse: Text, context, and intersubjectivity. In M. Walshaw (Ed.), *Unpacking pedagogy: New perspectives for mathematics* (pp. 129–151). Information Age Publishing.

DeJarnette, A. (2018). Using student positioning to identify collaboration during pair work at the computer in mathematics. *Linguistics and Education, 46*(1), 43–55. https://doi.org/10.1016/j.linged.2018.05.005

Di Paolo, E., Rohde, M., & De Jaegher, H. (2011). Horizons for the enactive mind: Values, social interaction, and play. In J. Stewart, O. Gapenne, & E. Di Paolo (Eds.), *Enaction: toward a new paradigm for cognitive science* (pp. 33–87). Cambridge: MIT Press. https://doi.org/10.7551/mitpress/9780262014601.003.0003

Ebbelind, A. (2020). *Becoming recognised as mathematically proficient: The role of a primary school teacher education programme* [Doctoral thesis, Linnaeus University]. http://lnu.diva-portal.org/smash/record.jsf?pid=diva2%3A1379999&dswid=-6091

Eisenhart, M. (1991). Conceptual frameworks for research. Ideas from a cultural anthropologist; implications for mathematics education researchers. In R. Underhill (Ed.), *Proceedings of the 13th Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. (pp. 202–219). Virginia, USA.

Elorza, I., Arus-Hita, J., & Bartlett, T. (2021). SFL approaches to language dynamics and contrast. *Lingua, 261*, 103098. https://doi.org/10.1016/j.lingua.2021.103098

Gellert, U. (2008). Validity and relevance: Comparing and combining two sociological perspectives on mathematics classroom practice. *ZDM, 40*(2), 215–224. https://doi.org/10.1007/s11858-008-0074-3

Goos, M., & Beswick, K. (2021). Introduction: The learning and development of mathematics teacher educators. In M. Goos & K. Beswick (Eds.), *The learning and development of mathematics teacher educators* (pp. 1–20). Springer. https://doi.org/10.1007/978-3-030-62408-8_1

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Edward Arnold.

Halliday, M. A. K. (1985). Systemic background. In J. Benson & W. Greaves (Eds.), *Systemic perspectives on discourse* (Vol. 1, pp. 1–15). London: Ablex Press.

Halliday, M. A. K. (1989). *Spoken and written language* (2nd ed.). Oxford University Press.

Halliday, M. A. K. (1993). Language and the order of nature. In M. A. K. Halliday & J. R. Martin (Eds.), *Writing science: Literacy and discursive power* (pp. 106–123). Falmer Press.

Halliday, M. A. K., & Hasan, R. (1989). *Language, context, and text: Aspects of language in a social-semiotic perspective* (2nd ed.). Oxford University Press.

Halliday, M. A. K., & Matthiessen, C. (2014). *Halliday's introduction to functional grammar* (4th ed.). Routledge.

Helliwell, T. (2021). *Developing a narrative-enactivist methodology for becoming a mathematics teacher educator* [Doctoral thesis, University of Bristol]. https://research-information.bris.ac.uk/en/studentTheses/developing-a-narrative-enactivist-methodology-for-becoming-a-math

Helliwell, T., & Brown, J. (2020). Seeing more, seeing differently as mathematics teacher educator-researchers. *For the Learning of Mathematics, 40*(0), 9–14.

Herbel-Eisenmann, B., & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education, 42*, 451–485. https://doi.org/10.5951/jresematheduc.42.5.0451

Herbel-Eisenmann, B., & Wagner, D. (2010). Appraising lexical bundles in mathematics classroom discourse: Obligation and choice. *Educational Studies in Mathematics, 75*(1), 43–63. https://doi.org/10.1007/s10649-010-9240-y

Holmberg, P. (2012). Kontext som aktivitet, situationstyp och praktik: En kritisk analys av kontextbegreppet i systemisk-funktionell teori. *Språk Och Stil, 22*, 67–86.

Hutchins, E. (2010). Cognitive ecology. Topics in Cognitive. *Science, 2*(4), 705–715. https://doi.org/10.1111/j.1756-8765.2010.01089.x

Kieren, T. (1995). *Teaching mathematics (in-the-middle): Enactivist view on learning and teaching mathematics*. Paper presented at the Queens/Gage Canadian National Mathematics Leadership Conference, Queens University, Kingston, Canada.

Lerman, S. (2009). Pedagogy, discourse, and identity. In L. Black, H. Mendick, & Y. Solomon (Eds.), *Mathematical relationships in education: Identities and participation* (pp. 147–156). London: Routledge. https://doi.org/10.4324/9780203876114-22

Lozano, M. D. (2015). Using enactivism as a methodology to characterise algebraic learning. *ZDM, 47*(2), 223–234. https://doi.org/10.1007/s11858-015-0678-3

Maheux, J. F., & Proulx, J. (2015). Doinglmathematics: Analysing data with/in an enactivist-inspired approach. *ZDM, 47*(2), 211–221. https://doi.org/10.1007/s11858-014-0642-7

Maheux, J. F., & Roth, W. M. (2014). The relationality in/of teacher–student communication. *Mathematics Education Research Journal, 26*(3), 503–529. https://doi.org/10.1007/s13394-013-0096-1

Maturana, H. (1988a). Ontology of observing: The biological foundations of self-consciousness and the physical domain of existence. In R. E. Donaldson (Ed.) *Texts in cybernetic theory: An in-depth exploration of the thought of Humberto Maturana, William T. Powers, and Ernst von Glasersfeld*. American Society for Cybernetics. https://cepa.info/597

Maturana, H., & Varela, F. (1998). *The tree of knowledge: The biological roots of human understanding* (original work published 1987). Shambhala.

Maturana, H. (1987). Everything is said by an observer. In W. Thompson (Ed.), *Gaia, a way of knowing: Political implications of the new biology* (pp. 65–82). Lindisfarne Press.

Maturana, H. (1988b). Reality: The search for objectivity or the quest for a compelling argument. *The Irish Journal of Psychology, 9*(1), 25–82. https://doi.org/10.1080/03033910.1988.10557705

Maturana, H., & Poerksen, B. (2004). *From being to doing: The origins of the biology of cognition*. Carl-Auer Verlag.

Maturana, H., & Verden-Zöller, G. (2008). *The origin of humanness in the biology of love*. Imprint Academic.

McGarvey, L., Glanfield, F., Mgombelo, J., Thom, J., Towers, J., Simmt, E., Markle, J., Davis, B., Martin, L., Proulx, J. (2022). Layering methodological tools to represent classroom activity. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 177–201). PME. https://www.igpme.org/wp-content/uploads/2022/04/Volume-1_final.pdf

Meaney, T., Trinick, T., & Fairhall, U. (2012). Collaborating to meet language challenges in indigenous mathematics classrooms. *Springer*. https://doi.org/10.1007/978-94-007-1994-1

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*(1–2), 219–245. https://doi.org/10.1007/s10649-006-5477-x

Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics, 9*(3), 7–11.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: First steps towards a conceptual framework. *ZDM, 40*(2), 165–178. https://doi.org/10.1007/s11858-008-0086-z

Proulx, J. (2015). Going beyond validity criteria in mathematics education research: towards the generativity of a research study. *Chroniques–Fondement et épistémologie de l'activité mathématique*, 1–6.

Proulx, J., & Simmt, E. (2016). Distinguishing enactivism from constructivism: engaging with new possibilities. In C. Csíkos, A. Rausch & J. Szitányi, (Eds.) *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (vol. 4, pp. 99–106). https://www.igpme.org/publications/current-proceedings/

Proulx, J. (2008). Some differences between Maturana and Varela's theory of cognition and constructivism. *Complicity: an International Journal of Complexity and Education, 5*(1), 11–26. https://doi.org/10.9173/cmplct8778

Radford, L., & Barwell, R. (2016). Language in mathematics education research. In Á. Gutiérrez, G. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 275–313). Brill. https://doi.org/10.1007/978-94-6300-561-6_8

Reid, D. (1996). Enactivism as a methodology. In L. Puig & A. Gutierrez (Eds.) *Proceedings of the 20th Annual Conference of the International Group for the Psychology of Mathematics Education* (vol. 4, pp. 203–209). Valencia, Spain: PME.

Reid, D., Brown, L., Coles, A., Lozano, M. D. (Eds.) (2015) Enactivist methodology in mathematics education research. *ZDM Mathematics Education, 47*(2).

Reid, D. (2014). The coherence of enactivism and mathematics education research: A case study. *Avant, V*(2), 137–172. https://doi.org/10.26913/50202014.0109.0007

Reid, D., & Mgombelo, J. (2015). Survey of key concepts in enactivist theory and methodology. *ZDM, 47*(2), 171–183. https://doi.org/10.1007/s11858-014-0634-7

Samson, D., & Schäfer, M. (2011). Enactivism, figural apprehension and knowledge objectification: An exploration of figural pattern generalisation. *For the Learning of Mathematics, 31*(1), 37–43.

Segerby, C. (2017). *Supporting mathematical reasoning through reading and writing in mathematics: Making the implicit explicit* [Doctoral thesis, Malmö University] https://doi.org/10.24834/2043/21479

Skott, J., Mosvold, R., & Sakonidis, C. (2018). Classroom practice and teachers' knowledge, beliefs, and identity. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education* (pp. 162–180). Routledge.

Thom, J., Glanfield, F., Mgombelo, J. Proulx, J., McGarvey, L. & Towers, J. (2020). Research tools for collectivity: tracking mathematics classes. In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.), *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 159–161). Mexico. Cinvestav/AMIUTEM/PME-NA. https://doi.org/10.51272/pmena.42.2020

Thompson, G. (2013). *Introducing functional grammar* (3rd ed.). Routledge. https://doi.org/10.4324/9780203431474

Towers, J., & Martin, L. (2015). Enactivism and the study of collectivity. *ZDM, 47*(2), 247–256. https://doi.org/10.1007/s11858-014-0643-6

Varela, F. (1999). *Ethical know-how: Action, wisdom and cognition*. Stanford University Press.

Varela, F., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. The MIT Press.

Voutsina, C., Alderton, J., Wilson, K., Ineson, G., Donaldson, G., & Rowland, T. (2022). Preservice teachers' expressed awarenesses: Emerging threads of retro-spection of learning and pro-spection of teaching. *Journal of Mathematics Teacher Education, 25*(2), 191–215. https://doi.org/10.1007/s10857-020-09484-y

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.