



Studying a mathematics teacher's documentational and identity trajectories over time

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Abstract

We study the interactions over 22 years between one mathematics teacher and his resources for teaching, especially digital ones, with a dual focus on the teacher's documentational and identity trajectories and professional development. We combined a theoretical framework on teachers' work with resources—documentational approach to didactics (DAD)—with a framework based on social practice theory—patterns of participation (PoP). The DAD analysis provided rich descriptions of which digital resources the teacher interacted with and of the transformative evolution of these interactions over time. The PoP analysis offered explanations of how and why the teacher transformed his interactions with the resources through foregrounding affective and contextual factors as significant for the teacher's formation of identity. We conclude that the combination of the two frameworks provided deeper and complementary insights into the teacher's long-term professional development with digital resources, and that such networking is needed to develop balanced understandings of teachers' long-term interactions with digital resources.

Keywords Digital resources · Documentational approach to didactics · Documentational trajectory · Patterns of participation · Identity trajectory

Introduction

Generally, research agrees that the teacher is the far most important factor when it comes to integrating digital resources (DRs) (e.g. e-textbooks, applets, Dynamic Geometry Systems—DGS) in mathematics classrooms (Clark-Wilson et al., 2020). However, an abundance of studies shows the difficulties teachers face when trying to exploit the affordances of DRs to support students' learning of mathematics in new and more profound ways (Chronaki & Matos, 2014; Clark-Wilson et al., 2020; Trouche et al., 2013). These studies stress several factors causing these difficulties: the fact that DRs are changing the nature of mathematics and the way that it is normally learned (Leung,

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2013); teachers' lack of competence and their beliefs towards DRs, mathematical knowledge and mathematics learning (Clark-Wilson et al., 2020); the time it takes to fully implement any change and the difficulty of making radical changes in school mathematics (Heid et al., 2013). Acknowledging such kinds of difficulties, Clark-Wilson et al., (2020) and Trouche (2019) have called for extended studies, arguing that teachers' deep professional development (PD) needs time. However, only a few studies have examined teachers' PD with DRs for a long time period. Multiple studies have investigated teachers' PD as an outcome of their participation in specific short-term PD programs, but not as a long-termed outcome of their daily activities (e.g. collaboration with colleagues) and participation in projects and activities outside their schools. Hence, teachers' long-term PD with DRs has not been sufficiently addressed. Furthermore, there is a research gap regarding how and why some teachers succeed in using DRs to support students' profound mathematics learning.

In this study, we examine over a time span of 22 years the professional career of a mathematics teacher in Greece. We refer to him here as 'Victor'. His case is an extreme one in two ways (Flyvbjerg, 2006). First, it is a success story, as Victor exceeds the high expectations of research and uses DRs to support students' profound learning of mathematics by engaging them in mathematical processes (e.g. problem-solving, reasoning). Second, Victor's case is extreme in the sense that he developed professionally from one extreme to another. His practice evolved from teaching mathematics traditionally without using DRs, to exploiting the affordances of DRs to teach under inquiry-based approaches. Also, Victor now designs materials both for his classrooms and at national level, and has achieved national recognition for this. By investigating Victor's case, we can address the existing literature gap regarding successful cases of teachers integrating DRs to support students' profound learning of mathematics and the more general question of how and why an initially traditional teacher, who resembled the average mathematics teacher in Greece, developed so tremendously over time in relation to DRs, and what motivated him to do so.

One focus of our study is to learn more about the phenomenon of teachers' long-term PD with DRs, while another more theoretical focus is to find lenses to analyse this phenomenon. Currently, only a few studies address the phenomenon, and they apply primarily the *Documentational Approach to Didactics (DAD)* (Trouche et al., 2018). This framework has been developed specifically to study teachers' interactions with resources, particularly DRs. Recently, Rocha (e.g. in Loisy et al., 2019) has developed theoretical and methodological constructs to study teachers' long-term interactions with DRs. However, our literature review reveals a need to take into account broader social constellations (e.g. teacher unions) and modes of reasoning beyond the classroom (e.g. teachers' obligations to use DRs in mathematics as conceived by leaderships or at the level of society) and adopt a more holistic, social, and participatory theoretical stance. Such theoretical approaches can be found in identity research. Thus, our study seeks to understand teachers' long-term PD with DRs by using an acknowledged theory within research on teachers' interactions with resources and a theory on teacher identity. More precisely, we combine the more cognitive-oriented DAD framework with a framework that adopts a participatory stance to teacher identities, called *Patterns of Participation (PoP)* (J. Skott, 2019).

First, we review how teacher PD with DRs has been studied within the two research fields: teachers' interaction with resources and identity research. Then, we outline the frameworks of DAD and PoP, formulate our research questions, describe our methodology approach, analyse Victor's trajectories using each framework and conclude by discussing our results.

Teacher professional development in relation to interaction with DRs

Mathematics teachers' interaction with resources has been a dominant theme of mathematics education research over the last two decades (e.g. Pepin et al., 2013; Remillard et al., 2009). The term *resources* has widened from conventional resources (e.g. textbooks and curriculum materials) to digital resources. A variety of theoretical perspectives and approaches have been developed to specifically model and study teachers' interactions with resources. These include theories around the notion of didactical design (Ruthven & Hofmann, 2013) such as didactical engineering (e.g. Artigue, 2014); theories of interpretation of, and participation with, a resource (Remillard et al., 2009); teaching as design (e.g. Brown, 2009) and DAD (e.g. Trouche et al., 2020).

In all these approaches, teacher PD is evidenced by evolutions of teachers' beliefs, knowledge or practice that shape and are shaped by their interaction with resources—especially DRs—in two-way processes of acting and being acted upon (Remillard et al., 2009). When engaging in designing for teaching, teachers need to interact with existing resources and through adaptations, to create something new that is interrelated with their professional knowledge and expertise. For instance, researchers, such as Remillard (2018), consider the teacher's design work with resources, especially digital curriculum resources, as a dynamic interaction between the affordances of the resources and the teacher's interpretation of what is possible in the classroom. Researchers within the 'teaching as design' approach consider that the teacher's interaction with resources is interrelated to improvements of their design capacity, meaning a sensitivity to mathematical and pedagogical aspects of designing resources and the flexibility to use them (Pepin et al., 2017). These researchers distinguish three main aspects of teacher design capacity: (1) the design goal(s); (2) a set of design principles ("robust", "flexible"); and (3) "reflection in action" (e.g. in instruction). In DAD, the experience acquired during the teacher's design and adaptations of resources provides the basis for their PD. Then, PD is indicated by analyses focusing on shifts in structures and evolutions of the resources a teacher uses so as to address complementary structuring and evolutions of their professional activity (Gueudet & Trouche, 2012).

Recently, increased research attention has been drawn to time as an essential component of teacher PD when attempting to capture the deep and slow changes in teaching practices. It has been indicated that teacher interaction with resources takes place over extended periods through facing multiple situations and callings for adaptations (Trouche, 2019). However, only a few studies have examined teachers' PD across more than four or five years (Loisy et al., 2019). As mentioned earlier, one exception is Rocha (2018, 2019) and her colleagues who focused on teachers' long-term interaction with resources, especially DRs, by modelling them as trajectories. In Rocha (2019) and Trouche et al., (2020), they, for instance, studied how a teacher's (Anna's) interactions with DRs benefitted her PD over more than 15 years. Their analysis uncovered Anna's existing and new resources in the collective-work context and allowed the authors to capture structuring elements of her PD. These included developing agency when adapting DRs and developing expertise to exploit Anna's experience of designing applications.

Researchers have suggested that collective work in a variety of networks and groups around DRs' design and use likely contributes to teacher PD (Gueudet & Trouche, 2009). Numerous studies have been conducted within PD programmes—often under the lens of communities of practices (CoP) (Wenger, 1998). Such studies have evidenced that teachers' work with resources is an essential means of their PD. For instance, Gueudet and Trouche (2012) argued that the process of gathering, creating and sharing resources to

achieve a CoP's teaching goals results in a "shared repertoire of resources and shared associated knowledge" (p. 309). However, an open research concern is how to interrelate individual and collective work with resources. For instance, Sabra (2016) studied how design processes in a teacher CoP impacted individual teachers' practices. Results indicated that the integration of 'new' resources can cause a reconfiguration of community participation and resource reorganisation. Our study contributes to these current research strands by broadening the focus on teachers' long-term interaction with DRs occurring in different communities with a dual documentational and identity perspective.

Teacher professional development with DRs from an identity perspective

Since 2000, research on identity in mathematics education (Darragh, 2016) and particularly on mathematics teachers' identity (Lutovac & Kaasila, 2018b) has increased. One reason for this growth is that teachers' identity formation is perceived as closely connected with their PD, for instance by providing "the missing [theoretical] link" in the "complex dialectic between learning and its sociocultural context" (Sfard & Prusak, 2005, p. 15).

The concept of identity is approached in various theoretical (and sometimes non-theoretical) ways. Darragh's review (2016) identified five approaches:

1. Participatory: focuses on how professional identities are constructed through participation in social groups.
2. Narrative: uses stories people narrate (e.g. about themselves as teachers and learners of mathematics) to construct their identities.
3. Discursive: focuses on how discourses at micro and/or macro levels shape peoples' identities.
4. Psychoanalytic: focuses on how individual characteristics shape a person's identity.
5. Performative: focuses on how people construct their identities through performance (i.e. stylised repetition of acts over time).

Among these five, the participatory approach is dominant in teacher identity research and these approaches often build on social practice theory, in particular Wenger (1998) and Holland et al. (1998). The majority of studies on teacher identity focus on pre- and in-service teachers, but with a preference for pre-service and novice teachers (Lutovac & Kaasila, 2018b). Thus, the number of studies on in-service teachers' identity is few and only rarely focused on teacher identity development over time (Lutovac & Kaasila, 2018a). An exception is the study of Lutovac and Kaasila (2018a) who examined the identity work of one elementary teacher over 16 years by using a narrative approach but not focusing on the use of DRs.

The growth in identity research is not reflected in research on mathematics teachers' interactions with resources (e.g. Clark-Wilson et al., 2020; Darragh, 2016; Lutovac & Kaasila, 2018b). In our literature study, we found only three studies focused on teacher identities in relation to their use of DRs.

C. K. Skott et al. (2021) explored the complexity of factors influencing mathematics teachers' use of DRs by analysing the case of Sofia, an experienced teacher in Denmark highly committed to DRs. In the analysis, PoP was used to relate emerging classroom practices to significant aspects of Sofia's professional identities. Three identity markers to

describe Sofia's identities were identified: *a modern teacher*, *a digital spearhead* and *the traditions of mathematics teaching*. The analysis showed how these markers shifted in their orientations of Sofia's actions and meaning makings related to DRs during classroom interactions, which helped the authors to explain Sofia's reinforcement of her procedural contributions when using DRs. The analysis further shed light on the more general issues of how teachers' professional experiences are co-constituted by practices, characters and worlds in and beyond their schools and classrooms.

Chronaki and Matos (2014) examined the identity work of a group of Greek in-service teachers participating in an intensive PD program on the use of innovative DRs to promote students' mathematical reasoning and inquires. They applied a discursive approach to identity and analysed the teachers' articulation of meaning in relation to multiple, often competing discourses, and the teachers' identifications with positions offered at different discursive levels. For our purpose, two results are important. The groups' portrayal of being modern teachers on the "the cutting edge" (p. 115) and their prioritising of the pedagogical novelty of DRs by "considering its appeal to children, as well as, its potential to turn the mathematics classroom culture into a more talkative, collaborative, and active place" (p. 116). Hence, these teachers saw no potentials in the DRs to support students' more profound learning of mathematics.

Goos (2013) applied Valsiner's Zone Theory and a participatory approach to identity based on Lerman (2000) in which "participation develops identities as the practice becomes part of the individual" (Goos, 2013, p. 522). Goos analysed the long-term PD of one teacher in Australia, Brian, who struggled with using DRs to challenge his students mathematically, but then succeeded in it. For this, Brian sought (on his own) to overcome the tension between his students' unsatisfactory learning and his own underlying beliefs and traditional teaching approach. Goos used Brian's movements across zones to uncover the reasons and mechanisms behind his PD, which Goos then used to inform classroom teaching.

Our study resembles Goos' (2013) study, as both tell a success story of a teacher's use of DRs to support students' mathematical understanding and focus on long-term PD with DRs using a participatory identity approach. However, by using a different participatory approach and combining it with DAD, we expect to contribute more profound insights into the reasons and mechanisms behind teachers' long-term PD with DRs.

Two theoretical approaches

The documentational approach to didactics

DAD acknowledges the crucial role of *resources* (i.e. material and non-material elements including textbooks, discussions with colleagues) for teachers' work and PD. According to Trouche et al., (2018), the main aim of DAD "is to understand teachers' professional development by studying their interactions with the resources they use and design into/for their teaching" (p. 1). This individual or collective process that includes selecting, modifying and creating new resources is called teacher *documentation work*. During it, the teacher can transform resources gathered for facing a given class of situations in their regular activity (e.g. lesson preparation) to a *document* comprised of a resource and *scheme* of use (i.e. the beliefs and knowledge in action). A document is also considered a resource that can be further transformed to a new document over time.

The set of a teacher's resources defines their *resource system*. In this system, some resources have a distinct role. For instance, resources a teacher uses to develop a document are called *mother resources*. Resources resulting after the teacher has recombined and elaborated mother resources are called *daughter resources* (Hammoud, 2012). For Alturkmani (2015), a *structuring mother resource* is one around which a daughter resource devoted to teaching is elaborated and brings a new balance to the teacher's relation to the subject matter they teach. Resources that are central in the design of other resources are called *pivotal resources* (Trouche et al., 2018). The aforementioned authors claim that DRs can contribute to deep effects in teachers' documentation work, resource system and professional practice. Recently, Trgalova et al., (2019) proposed examining three aspects of the teacher's resource system—constitution, content and structure, evolution—in order to address if and how resources impact on their professional activity and PD.

Over the last years, there has been increasing interest in studying teachers' PD by focusing on evolutions of their resource system over extended periods. Rocha (in Loisy et al., 2019) introduced two concepts: *documentational experience* (i.e. a teacher's appropriation of professional events acquired during their interactions with resources, that were remarkable to their documentation work from their own perspective) and *documentational trajectory* (i.e. the path linking the events that took place along this experience) to study the long-term evolution of interactions between teachers and resources Rocha. The notion of *documentational dominant* is used to name a set of activity families orienting the teacher's documentation work during a given period (e.g. 'preparing and setting up lessons'). Trouche et al. (2020) denoted a *symbolic event of transition* a professional event that provokes a change in the teacher's hierarchy of activities (i.e. change in the documentational dominant) indicating a 'before' and an 'after' in their documentation work. For instance, a new collaboration with a colleague experienced with group work can influence the activity 'design and implement group work in the classroom.'

Some DAD studies have highlighted the social dimension of teacher interaction with resources and their PD, and explored the relationships between the resource system a group develops collectively and personal schemes developed individually. Trouche (2019) aimed to orient a research direction and questioned to what extent it is possible to speak of a "shared repertoire", a "collective resource system", and "shared knowledge", etc. (p. 463) and indicated the need to consider the diversity of a teacher's collectives. Trouche's suggestions are very critical for studies such as ours that address teacher documentation work and PD over time, as this usually occurs at the interplay between individual and collective work. Such suggestions challenged us to look for further explanations of Victor's documentational trajectory and how his use of DRs evolved over time.

Patterns of participation

Recent developments in participatory approaches to identity that build on social practice theory influenced this study. In line with most such approaches, the PoP framework (Skott, 2019) draws on the notions of *practice* and *figured worlds*. Here, practice "connotes doing [but] doing in a social and historical context that gives structure and meaning to what we do" (Wenger, 1998, p. 47) and figured worlds are "socially and culturally constructed realm[s] of interpretation, in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p.52).

However, PoP differs from other participatory approaches using social practice theory by not focusing on how a teacher moves towards more extensive participation in a pre-established community or figured world, such as a world of inquiry-based teaching targeted by a PD program or teacher education. In contrast, PoP “focuses on how different social constellations and practices come to play a role for” a teacher’s shifting experiences of themselves as a teacher (Skott, 2019, p. 469). This is irrespective of whether the teacher is involved in PD programs or not. Therefore, PoP suited our purposes.

PoP understands a teacher’s participation in social interactions as being influenced by their interpretation of the immediate situation and their simultaneous meaning-making where the teacher “continuously interprets others’ actions symbolically, including their actual or possible reactions to one’s own behaviour” (Skott, 2019, p. 472). In terms of symbolic interactionism, the teacher takes the attitude of others to themselves. The others can be a colleague or a parent, but also a social group or community in which case the teacher takes the attitude to them of generalised others. In PoP, practices, characters and figured worlds are interpreted as possible generalised others, and we will refer to them as *identity markers*. By focusing on the teacher’s experience in social interactions and contexts, PoP seeks to re-centre the individual teacher as called for by Lutovac and Kaasila (2018b).

By using PoP, we define Victor’s professional identity “as his shifting experiences of being, becoming and belonging related to the profession” (Skott, 2019, p. 469) first at the auxiliary school he owned and later at North River School. This may include: experiences of being (in)competent (e.g. when collaborating with colleagues or researchers); of becoming better or worse (e.g. at using DRs) and of belonging (e.g. acknowledged as a competent teacher by colleagues, students or school leadership). Seeking to understand Victor’s identity formation over time, we focus on his *identity trajectory*, that is “the development of [his] professional experiences of being, becoming and belonging as a teacher” (Skott, 2019, p. 470).

The concept of experience is not yet well-defined in PoP, but it seems to relate to a teacher’s past experiences from previous situations and immediate experiences from current moments. Nevertheless, the concept is inspired by social interpretations of human functioning, to a greater extent than the concept of experience used in DAD.

Combining DAD and PoP

Emerging from a desire to exploit the richness of theories more systematically in mathematics education, a networking group developed a landscape of strategies for connecting theoretical approaches (Bikner-Ahsbabs & Prediger, 2014). For our purpose, the strategy of *combining* seems particularly relevant. Combining is usually used to gain deep insight into an empirical phenomenon by looking at it from different theoretical perspectives. These perspectives do not need to be completely compatible; even theories with conflicting basic assumptions can be combined to obtain multifaceted insight into the phenomenon (ibid). DAD and PoP approach teacher PD differently. From a DAD perspective, PD is described as cognitive changes resulting from teachers’ interactions with resources, whereas PoP conceives teacher PD as changes in their ways of participating in social interactions. As both frameworks have proven fruitful for studying teacher PD, it seems promising to combine them to study Victor’s long-term PD with DRs.

Research questions

Our first two questions address specifically Victor's case and seek to construct two of his trajectories by using each of the DAD and PoP frameworks. Our third question relates to the more general theoretical focus of our study.

- (1) What are the major professional events and corresponding DRs that structure Victor's resource system and how do his interactions with DRs influence his documentational trajectory?
- (2) How do Victor's experiences of being, becoming and belonging at his own auxiliary school and later at North River School influence his use of DRs and form his identity trajectory?
- (3) What are the benefits of combining the DAD and PoP frameworks when studying teachers' long-term PD with DRs?

Methodological approach

Context of the study

In Greece, the educational system is highly centralised. Innovations are introduced through a top-down approach by the Ministry of Education. The national textbook is the main resource teachers use to organise lessons. Auxiliary schooling (a coaching system undertaken by small-sized registered institutions called *para-education*) that builds on a traditional approach to mathematics teaching is widespread. Its purpose is to help students understand concepts and acquire skills needed for exam success. Most novice teachers work in para-education before entering public education. With respect to employment in secondary schools (years 7–12), teachers must hold a degree in mathematics and pass a competitive national written exam (the *ASEP exam*).

In 2010, the Ministry of Education launched the four-year, *New School* project aimed at modernising national mathematics curriculum from years 1–12 by drawing on research on learning trajectories and conceptualising teachers as active partners in designing teaching. A group of educational consultants, teachers (including Victor), researchers in mathematics and mathematics education (including the first author) collaborated in this project. In 2011, another project, *Digital School*, was initiated with the aim of producing DRs to be integrated within the online mathematics curriculum books.

Victor also participated in Digital School with others from New School (including the first author). In Digital School, two novel types of DRs were created. These were 1) *micro-experiments*: interactive applets developed through proliferating digital media (CAS, DGS, programmable software) which simulated experiments and provoked students to engage in inquiries around a set of concepts (e.g. inscribed angle, see Fig. 1) and 2) *synthetic activities*: generic scenarios designed to promote project-work based on modelling, inquiry and integrated representations through using micro-experiments (Kynigos, 2020). In both projects, a set of principles guided the work, including combining formal mathematical representations (e.g. graphs) with text and/or simulations, promoting students' dynamic manipulation of mathematical objects to explore and discuss their relationships/properties, and promoting students' conceptual understanding.

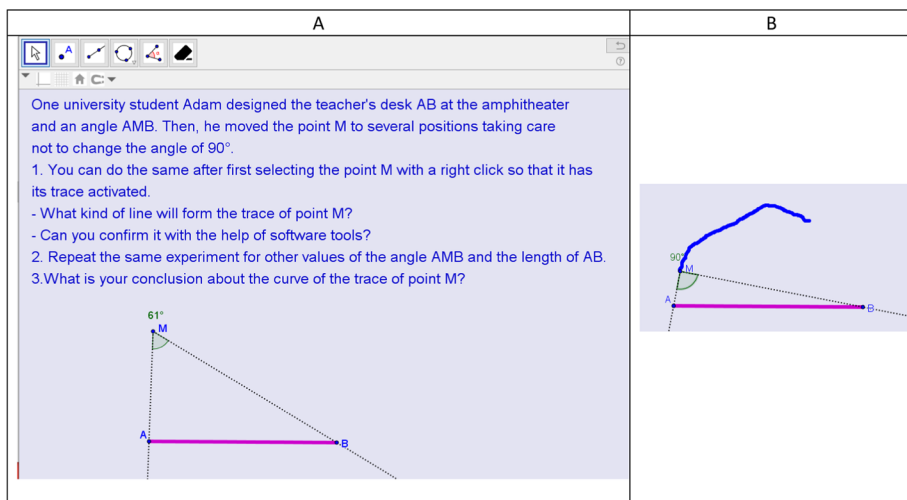


Fig. 1 'Angles in the amphitheatre': Micro-experiment for the property of the points of a semicircle that are vertices of inscribed angles from which a constant length is "seen" at a right angle (Grade 8, Topic: Inscribed angles, Tool: Geogebra). Students are expected to guess the type of line through dragging and then confirm with the use of other tools (e.g. construction of circle, measure) and generalise

Data generation and analysis

We found it challenging to generate data that would sufficiently cover the 22 years of Victor's career, that we focus on, and inform his two trajectories. We needed data on his personal history, important events and collectives, his design and use of resources, the context of North River School, social constellations and modes of reasoning within and beyond his school.

The first author knows Victor well personally and professionally due to long-term collaboration in projects since 2010. This intimate knowledge enabled us to echo Victor's voice throughout our interpretations and ensure his authorship. Victor agreed to be involved in the study and showed confidence in sharing data with us, but he did not read all our analyses.

This paper draws on a huge amount of data collected by the first author from 2010 to 2018. The most important of these are as follows:

- Eight individual interviews on Victor's documentation work (e.g. old and new resources, his interaction with specific resources, classroom implementation, collective work),
- E-mails (about 80) with Victor about his documentation work on DRs,
- Victor's lesson materials (worksheets, lesson plans, DRs) for teaching specific units during the design and pilot implementation of the new curriculum (New School) (35 units in 2010–14) and other projects (2 units in 2015–16, 2 units in 2017–18),
- Activity templates, in which Victor described his teaching design and enactment of the lessons mentioned above.

The first author collected half of the data during their collaboration in New School and Digital School (2010–15) and two other projects (2015–18). The other half was collected

Regarding the PoP analysis, the idea of creating Victor's identity trajectory was fostered by our construction of the inferred/reflective mapping. We conducted a two-step analysis. The first step was both part of the methodology and of our results, as its resulting six identity markers served as an analytical lens in the second step. Therefore, we present the first step and its result before we describe how we conducted the second step.

Based on the inferred/reflective mapping, we initially conjectured what identity markers seemed to play significant roles for Victor's experiences at specific moments over his career. We developed our initial markers by analysing the follow-up interview line-by-line. Under a broadly grounded approach, we constructed codes, such as 'being aware of individual students' based on utterances in which Victor recounted his experiences of engaging in various practices, for example "make students work on their own so as to have time to help each student individually according to their needs". We compared these codes with our initial markers and constructed six identity markers by merging, deleting and consolidating our initial markers or adding new markers. Two of the six markers were characters that we identified from practices and figured worlds related to the teaching of mathematics and to broader political discourses of education and democracy. The six identity markers are as follows:

1. *Para-education*: In this world, educators (not necessarily certified to teach in primary or secondary education) prepared students for entry exams through traditional teaching.
2. *Reform mathematics teaching*: This world was primarily conveyed in the literature, teacher education and PD programs. Teachers in Greece generally found it difficult to teach in line with this world due to its focus on engaging students in mathematical processes, building classroom communications on student contributions and using a variety of representations to support their mathematical thinking.
3. *Collaborating with teachers, researchers, etc.*: Through this practice, Victor aimed to explore his own classroom teaching by collecting data and using these and research results to improve it.
4. *Designing teaching materials*: Through this practice, Victor aimed to produce teaching materials that could support him (and others) in solving the problems he faced in his classrooms.
5. *Embracing mathematics teacher*: This character reified Victor's deep preoccupation with both student-centred approaches to teaching and his political ideologies of empowering all citizens through education.
6. *Designer of DRs*: To Victor, this character aimed to design challenging mathematical tasks for (all) students by exploiting the affordances of DRs in a bold and creative way.

The second step zoomed in on the three critical periods identified by the DAD analysis using the six identity markers as an analytical lens. The analysis was primarily based on Victor's written reflections on the inferred/reflected mapping (coded as wr) and the follow-up interview (coded as int), while we used other data to confirm and nuance our interpretations. In contrast to other studies using PoP, we did not analyse Victor's participation in classroom interactions (or other interactions), but only his recounted experiences and especially those that we interpreted as significant for his interactions with DRs in classrooms. For each period, we analysed which identity markers played prominent roles for his experiences and how their roles shifted in dominance, and we drew corresponding landscapes hereof. Finally, we constructed Victor's identity trajectory by linking the three landscapes, thus connecting his past and present identities. This allowed us to uncover developments in

his identities throughout the period and to understand this in a holistic way that was sensitive to important contextual aspects. By using PoP, we were also able to re-centre Victor in his identity development. His identity trajectory is primarily our interpretation of his long-term identity development, but one that foregrounds his voice.

Victor's documentational trajectory

Global results

Based on our analysis of Victor's inferred/reflective mapping, we constructed four categories of events and corresponding representative resources (Fig. 3) that constitute his documentational trajectory: (a) taking part in professional exams (E2, E4); (b) teaching secondary students and in-service teachers (E1, E3, E6, E9); (c) engaging in research and development projects in mathematics education especially with the use of DRs (E5, E8, E10, E11, E12, E13, E14, E15, E16, E17); (d) participating in scientific and professional associations (E7).

In this trajectory, three periods turned out to be critical for Victor: 1) preparation for the ASEP exams (1998–2000), 2) collaboration with teachers and researchers (2004–2010), and 3) participation in teams designing DRs (2010–2015). At the beginning of each period are professional events that satisfy Trouche et al. and's (2020) criteria for a symbolic event of transition as they opened new opportunities for PD, led to a renewing resource system and happened in a year with other opportunities for PD (Fig. 4).

Each of the symbolic events is related to corresponding major documentational experiences underlying Victor's trajectory: *Preparing for the ASEP exams* foregrounded Victor's encounter with mathematics education research through personal reading; *Starting collaborations with teachers/researchers* foregrounded an extended period with collaboration on differentiated teaching; *Collaboration in New School and Digital School* foregrounded the longest period (to date) involving working together in teams framed by specific approaches and norms as regards the design of DRs. Three

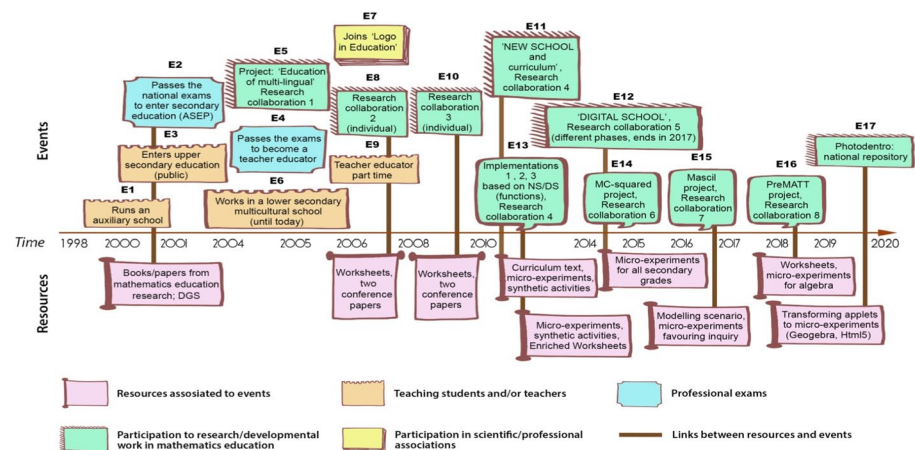


Fig. 3 Victor's inferred/reflective mapping

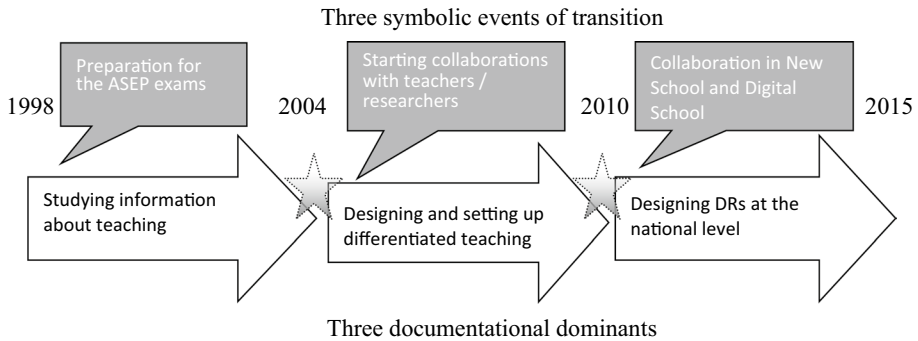


Fig. 4 A structure of Victor's documentational trajectory

common features of these experiences—evident in Victor's trajectory and reflections—allowed us to relate them to the major resources he used at the time, the corresponding documentational dominants and the evolutions they promoted in subsequent parts of the documentational trajectory:

- 1998–2000: Victor's major documentational experience trait concerned his readings about topics including philosophy of mathematics and mathematics education research, and his discussions with a knowledgeable colleague about digital technologies. Therefore, we called the corresponding documentational dominant *studying information about teaching*.
- 2004–2010: Victor's collaboration with teachers and researchers promoted his sensitivity to designing worksheets (one of Victor's main resources), considering issues addressed by mathematics education research. For instance, Victor started to modify tasks to address students' difficulties as described in research literature in collaboration with Elen (researcher in mathematics education). We called the documentational dominant *designing and setting up differentiated teaching*.
- 2010–2015: Victor's collaboration with teams in New School and Digital School was characterised by designing DRs and targeting pedagogical innovation at the national level. The key DR in these teams—micro-experiments—seemingly played a formative role in Victor's documentation work at the time and in his subsequent steps as a resource designer and user in and for teaching. Thus, Victor takes a step further from designing DRs for his teaching to that with pedagogical potential for public availability/others. We called the documentational dominant *designing DRs at the national level*.

Victor's resource system

Table 1 shows part of Victor's resource system that we constructed based on his documentational trajectory (Fig. 2).

Below, we provide the results of our analysis of Victor's resource system (Table 1) according to three aspects during each critical period: constitution, content and structure, and evolution.

Table 1 Part of Victor's resource system

Critical period 1 (1998–2000): preparation for the ASEP exams	Critical period 2 (2004–2010): collaboration with teachers and researchers	Critical period 3 (2010–2015): participation in teams designing DRs
Books/papers addressing mathematics teaching (e.g. mathematics education research, philosophy of mathematics): <i>reading about theory-informed approaches to teaching</i>	Researchers / colleagues with a PhD in educational research: <i>discussing teaching and its features under different perspectives (e.g. differentiation, group work); co-designing lessons</i>	Design team of Digital School: <i>designing DRs (i.e. micro-experiments, synthetic activities) for challenging students epistemologically in mathematics; sharing DRs with team members and discussing their educational value; modifying micro-experiments through cycles of design-reflection/feedback-redesign</i>
A friend with extensive knowledge of DRs for mathematics: <i>selecting information about the use/functionalities of digital environments for mathematics teaching</i>	Worksheets developed in collaboration with other teachers based on textbooks; DGS; Student questionnaires; <i>promoting autonomous group work and differentiated teaching; distributing questionnaires to students for feedback</i>	Design team of New School: <i>developing curriculum units according to learning trajectories; designing and sharing new kinds of DRs (micro-experiments, synthetic activities)</i>
Textbooks—official and other ones: <i>preparing the daily lesson</i>	Worksheets developed in collaboration with a colleague (Peter, PhD in science education): <i>designing and using resources to support students' collaborative work and learning</i>	Enriched Worksheets: <i>developing new kinds /recombining resources involving the use of DRs (i.e. micro-experiments, synthetic activities); organising classroom teaching in relation to learning trajectories</i>
Worksheets: <i>starting to create worksheets for first time including traditional questions in textbook-like style</i>	Worksheets developed in collaboration with a researcher (Elen, PhD in mathematics education): <i>designing and implementing tasks taking into account findings from mathematics education research (students' difficulties)</i>	Micro-experiments in everyday teaching: <i>designing for challenging students epistemologically in mathematics; posing learning aims in each lesson; using micro-experiments to bring mathematical challenge into the classroom; modifying micro-experiments through their actual use in teaching</i>
Digital tools such as DGS, Graphmatica: <i>starting to create geometrical figures and graphs with digital tools</i>	Digital tools such as DGS, Geogebra: <i>designing tasks/DRs involving the use of digital tools</i>	Teacher education materials for mathematics teaching with DRs: <i>organising his teaching around micro-experiments targeting teachers' development of criteria for 'good' DRs; cultivating joint reflection on ways to challenge students epistemologically in mathematics</i>
—	Teacher education materials with DRs (e.g. scenarios): <i>educating teachers on the functionalities of digital tools for mathematics teaching</i>	—

Constitution

In period 1, Victor prepared himself for the ASEP exams. Apart from the widely used national textbook, books and papers including theory (philosophy, pedagogy) and mathematics education, research was central to his preparation. His subsequent documentational work dealt with designing prototypic worksheets he considered innovative at the time—although the tasks were rather traditional and the use of DRs at the surface level. Victor was not yet ready to take advantage of using DRs in his teaching and make them fit his existing resources designed to serve para-education.

In period 2, Victor started to collaborate at an individual level with researchers or colleagues with a PhD. His main resource was again worksheets designed around textbook-like exercises. However, compared to the first period, Victor now attempted (a) to adopt differentiated teaching through collaborative group work, considering students' difficulties from the literature, thus his worksheets had a different format providing scaffolding (e.g. repetitions, notes) and used more inclusive language (e.g. 'we' rather than orders) and (b) to design questionnaires for his students to get their reactions to the new kind of lesson. As regards DRs, Victor used DGS to design tasks for his classroom teaching—a step further from period 1. However, they seemingly did not play a special role in his resource system following the exercise-oriented approach of the worksheets. Even in his teacher education classes, Victor's teaching was mainly focused on teachers' learning of the technologies *per se* (e.g. Geogebra).

In period 3, Victor's design of DRs in New School and Digital School marked a shift in his trajectory: from selecting and designing resources for his own lessons to designing DRs for curriculum use at the national level (i.e. micro-experiments). Victor's collaboration with the two teams deeply affected his interactions with DRs and promoted the alignment of his resource system with DRs available for others. We highlight two further points related to a renewing constitution of this system. First, Victor developed micro-experiments under shared design principles concerning the pedagogical uses of DRs to challenge students epistemologically (i.e. mathematical challenge). Second, he integrated micro-experiments into his daily teaching. As a result, this new resource gave birth to a new resource system that included other novel resources including enriched worksheets consisting of theory (e.g. definitions), different tasks and scenarios (i.e. synthetic activities), multiple representations and micro-experiments. Furthermore, Victor integrated micro-experiments into his teacher education materials to target more reflective views by teachers about the quality of DRs for mathematics teaching.

Content and structure

In period 1, challenged by his readings and his knowledgeable friend (mother resources), Victor started to design worksheets (daughter resources) with a trivial use of DRs (see Table 1). Period 2 was characterised by Victor's interaction with researchers, who took the role of structuring mother resources as they contributed to the development of his didactical design and new daughter resources for teaching. In period 3, the Digital School and New School design teams and their resources (especially micro-experiments) seemingly took the role of structuring mother resources in Victor's renewed resource system. These contributed further to structuring his documentation work in two ways. First, micro-experiments were flexible and could be integrated in design of more complex daughter resources

like enriched worksheets and Victor relied essentially on this type of resources to organise his teaching. Second, they brought a new balance to Victor's mathematics teaching as his activities in relation to the corresponding daughter resources involved: designing DRs to challenge students epistemologically; reflecting on DRs' educational value; planning and enacting lessons based on students' learning trajectories; modifying DRs after actual use in the classroom and challenging teachers' reflection on the DRs' quality in the teacher education context. Hence, micro-experiments were involved in several of Victor's activities (e.g. lesson planning, actual teaching, designing in teams/collectives) and apparently helped to restructure and reorganise his resource system. Therefore, they stand out also as pivotal resources.

Evolution

The evolution of Victor's resource system brings to the fore the importance of the role of teams and researchers; as in Victor's case, they constituted important levers for a deep transformation of his system. Initially, Victor relied on official textbooks, read literature about mathematics and mathematics teaching and used DRs for constructing illustrations. Later, influenced by his individual collaboration with teachers and researchers, he started to develop resources (worksheets, DRs) and use them in teaching while taking into account research-informed perspectives (e.g. differentiated teaching). Finally, Victor developed DRs at the national level based on specific design principles. These new resources aimed to introduce teachers to new design approaches regarding the functionalities and use of available DRs, and open discussions and sharing ideas for future use. Further, developing such resources was a demanding process that required associated documentation work with high technical demands (e.g. design in Geogebra) and pedagogical demands (e.g. synthetic activities). The design experience gained through micro-experiments has been evident in multiple activities Victor enacted in all subsequent projects (until today) concerning: collaboratively designing DRs aimed to link authentic situations and mathematics and science teaching (scenario on modelling the design of a car park through micro-experiments, project Mascil, 2014–2016); designing c-books, a new genre of authorable e-books, c for creativity (project MC-squared, 2015–2017); developing authoring tools for designing DRs (from Flash to Geogebra, Digital School phase 2017–2020). We note that micro-experiments occupy dominant positions in all these subsequent forms of Victor's resource system.

Victor's resource system and documentational trajectory shaped by his long-term interaction with DRs

We now answer our first research question about which major professional events and corresponding DRs structured Victor's resource system and how his interactions with DRs influenced his documentational trajectory. We summarise the three documentational experiences and corresponding documentational dominants of Victor's documentational trajectory and comment on the evolutions of his resource system and documentational work.

1. *Victor's encounter with broader views of mathematics teaching when he prepared for the ASEP exams.* Through this experience, Victor encountered 'theoretically' the potential of using DRs to improve classroom teaching. Although his use of DRs was trivial at

that time, Victor started to consider resources and potentially DRs as a 'theme' in his professional life.

2. *Victor's collaboration with researchers through which he started considering DRs as part of his means to improve teaching.* Although the worksheets were the dominant resources, he took two critical steps as regards DRs: (a) designing DRs to support all students' understanding in his classroom and (b) attempting to address the impact of his didactical choices (including the use of DRs) on students' learning.
3. *Victor's collaboration with teams designing DRs.* This experience highlighted a new branch of Victor's documentational trajectory characterised by designing DRs framed by specific pedagogically-informed principles targeting students' conceptual understanding. Micro-experiments that appear as pivotal and structuring mother resources have led to a metamorphosis of Victor's resource system and activity. We retain here four main results.
 - New DRs (micro-experiments) appear. These
 - play a structuring role in all subsequent forms of his resource system, e.g. recombining them with other resources to develop new kinds of daughter resources (e.g. enriched worksheets) and
 - contribute to the appearance of a new family of activity, i.e. designing DRs to challenge students epistemologically in mathematics.
 - Sharing and discussing/reflecting appear to be interrelated to modifying/redesigning leading to a signifying role of collective work in individual documentation work as regards DRs.
 - Designing and using DRs in classroom teaching is based on reform-oriented principles of mathematics education (e.g. inquiry, learning trajectories).
 - Most of the resources are DRs shared with others via specific portals (e.g. e-textbooks, the Greek national portal Photodentro).

The different forms of Victor's resource system in each period relate mostly to changes caused by his interaction with DRs. The resource system structure seems closely linked to the structure of Victor's documentation work: from simple design of figures/graphs in the first period, to DRs to improve his teaching in the second period and to DRs designed and used under pedagogically-informed perspectives to challenge all students mathematically in the third period. Also, these evolutions indicated profound changes in Victor's relation to teaching mathematics as he: (a) relied essentially on this type of resource to organise his teaching in school and teacher education; (b) considered epistemological challenge as a fruitful way to teach and learn mathematics; (c) planned his lessons according to learning trajectories and (d) brought to the fore the quality of DRs as a central issue in his teacher education classes. Finally, the documentational dominant of *designing DRs at the national level* was progressively enriched by other activities concerning sharing, discussing and redesigning DRs.

Victor's identity trajectory

We now present the results of our PoP analysis. We show, first, Victor's identity development during each of the three critical periods and then his identity trajectory across the whole period.

Preparation for the ASEP exams 1998–2000

During the first period, Victor worked in the auxiliary school he owned (1997–2001) and experienced belonging to the traditional world of para-education. He characterised this world as exhibiting “stereotyped perceptions of mathematics” (wr) and understanding student learning as “copying” formal “pieces of knowledge from the board” (wr).

By reading mathematics education literature in preparation for the ASEP exams, Victor “discovered another unknown world of mathematics teaching” (int) and realised that “something was wrong [with his teaching] as it did not help all the students” (wr). These experiences marked a critical shift in his identity development: He started to draw on the world of reform mathematics teaching, wanting to close “the gap in my education” (int). The student-centeredness of this reform-world combined well with his strong political ideologies of education for all. In our interpretation, Victor reified those ambitions in the character of the embracing teacher that emerged during this period.

Drawing on the reform-world, Victor engaged in designing worksheets to “make students work on their own” (wr) so he would have time “to help each student individually according to their needs” (wr). Worksheets were a novelty in para-education at the time. Equipped with digital graphics of mathematics, they became even more innovative, as DRs was in general very rare in Greece. So from early in his career, Victor experienced being at the forefront of using DRs in mathematics teaching, even though his steps were initial. Figure 5 shows Victor’s first landscape of significant identity markers (a dark blue arrow shows strong dominance).

Collaboration with teachers and researchers 2004–2010

In 2002, Victor started teaching at North River School a multicultural public school with significant cultural and language challenges. This strongly impacted his experiences during the second period. Above all, he experienced that the issue of equity became still more important for him, which strengthened his devotion to become the embracing teacher and engage with the reform-world. Wanting to break “the norm in the traditional mathematics classroom of copying from the blackboard” (int), Victor engaged further in designing worksheets where “the theory emerged” (int) and all students were active “in the lesson and with their learning” (int). Specifically, Victor developed design principles to foster productive group work among his diverse body of students and used DRs (e.g. DGS) to engage them in exploring mathematics. Hence, Victor gradually drew less on para-education when he faced challenges in his classroom, and this world faded into the background during this period (see Fig. 6).

Victor began to collaborate with a few colleagues and in particular, two mathematics education researchers (e.g. Elen). He gained experiences with a research world of

Fig. 5 Victor’s first landscape of significant identity markers

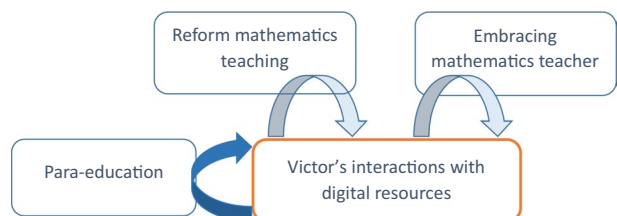
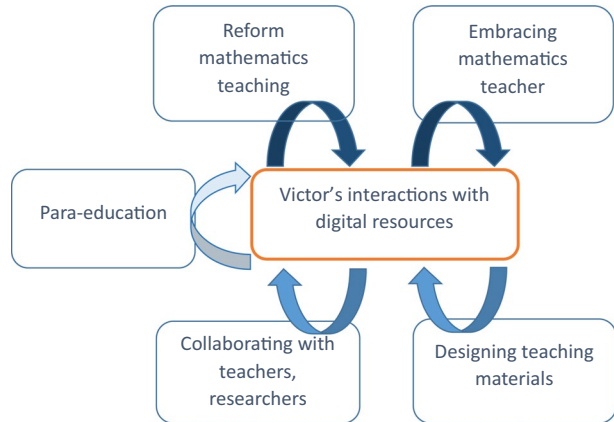


Fig. 6 Victor's second landscape of significant identity markers



mathematics education including research methods and theories, “Our research focus was on the collaborative classroom under a socio-cultural approach ... but you need to be close to the students, to “hear” what happens” (int). Victor also published research papers, but as the quote suggests, he was primarily concerned with students and classroom interactions. Thus, he orientated himself more towards the reform-world and the embracing-character, than towards being part of a research community—so such a world did not become significant to him.

Importantly, Victor experienced that his participation in the collaborating-practice supported him in designing materials (e.g. by providing research findings on student difficulties) that helped him deal with classroom problems. He developed an inquiry teaching stance towards his own teaching approach and gradually experienced belonging more to the reform-world and becoming more like the embracing teacher. For this, Victor was increasingly recognised by researchers, experiencing that, “[Elen] always pushed me” (wr) to participate in conferences on mathematics education and to be invited into national projects (e.g. New School, Digital School). He valued this confidence as it “has always made me try to do my best” (wr). Therefore, and due to his political commitment (e.g. the teacher union), Victor experienced having a central position in both practices—collaborating with colleagues and designing materials—and being generally recognised by the Greek mathematics education community as a teacher striving to develop new and innovative teaching approaches and promoting better education for all.

Participation in teams designing DRs 2010–2015

Victor characterised this period as “the most productive period in my professional life ... [New School and Digital School] was a big step, a very big step ... one of the most important things I have done so far” (int). In this period, Victor transformed his ways of collaborating and designing, which in our interpretation contributed to the emergence of his identity as a designer of DRs (see Fig. 7).

Victor experienced being involved in new and more complex collaborative work. This regards the number of participants (up to 30 people), their diversity (e.g. researchers, teachers, educational consultants) and work-scope (designing at a national level). Though challenged by collaborating in these new ways, especially “to experiment and try to improve our designs” (int), Victor recounted that:

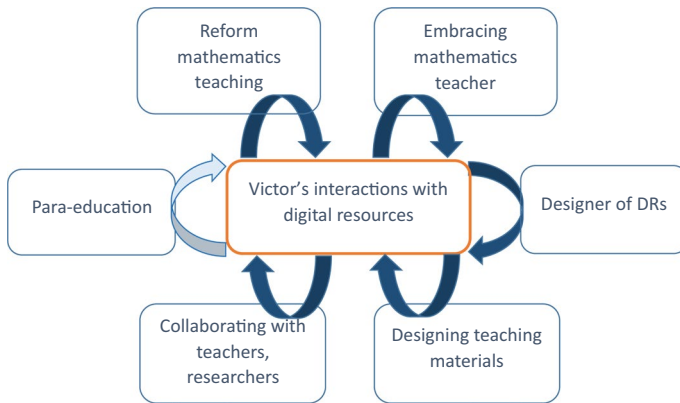


Fig. 7 Victor's third landscape of significant identity markers

I felt like a member of a team. I didn't feel like a member of a committee that considers teachers as being at a "lower" level. I was free to express my opinion and I felt that people could hear me and understand me (int).

He also experienced that the use of research helped him become a more competent teacher, "I knew a lot, but it was first when I considered the curriculum [in terms of learning trajectories] that I understood it deeper" (int). Thus, Victor's experiences of being recognised as a valued collaborator and becoming a better teacher were strengthened considerably during this period.

Victor also participated in designing novel DRs in new ways, especially micro-experiments. Victor characterised micro-experiments as "an autonomous application" (int) with integrated text, questions and digital elements that "starts with a problem. You need to design the problem to create space for the next step, the mathematics" (int). Victor experienced it as challenging to exploit the affordances of the DRs to facilitate the targeted reform-intentions, but recounted that he, thereby "understood more deeply what using technology means ... the role of representations, the idea of exploration, the dynamism of dragging a slider and observing what happens" (int). Victor tried the micro-experiments in his own classrooms "I implemented it ... I am happy: to some extent it worked for all students" (int). By experiencing such success in his classrooms, Victor found himself becoming more like the embracing teacher.

In interviews, Victor began to tell stories of himself as a designer of DRs, "through this process [Digital School] I recognised new possibilities for me as a designer ... I feel that I became mature digitally" (wr). Victor's experiences of becoming this new character and, furthermore, belonging to an elite group of designers of DRs marked a significant shift in his identity trajectory. Victor recounted that by collaborating with experts from different fields, "I learned a lot ... I observed what you [among others the first author] designed, and I took ideas at the technical level but also at the didactical one" (int). Even at present, Victor says that he re-engages in these ways of designing DRs, "I still turn to these materials to see how you [ditto] solved design problems in a specific micro-experiment. So, in this way I take what I need, and I connect it to other things. I put it in my own design" (int). As a designer of DRs, Victor was bold and creative, "these projects provoked me to dare trying innovative ideas" (int). He also intended to design mathematically challenging tasks for all students "Students can do a lot of

things ... I speak of relevant tasks according to their level of knowledge ... but demanding” (int).

In this third period, we see the most significant shifts in Victor’s identity development. We interpret that his new experience with collaborating and designing led to the culmination: he became a designer of DRs. However, Victor’s ambition was not to achieve this professional status. He aimed to become the embracing teacher. As he recently emphasised, “the basic thing for me is to be able to say after leaving the job 12 years from now: “I had a very good time with my students. All of them learnt as much as possible” ... [as a teacher] you never reach a final solution ... you learn from your experiences” (int).

Victor’s identity trajectory over the course of his professional career

We now answer our second research question of how Victor’s experiences of being, becoming and belonging at his own auxiliary school and later at North River School influenced his use of DRs and formed his identity trajectory. By connecting the three landscapes of Victor’s significant identity markers, we show how his experiences of being, becoming and belonging shifted during his career as he drew on a multiplicity of past and present practices and figured worlds—the most significant being the six identity markers—and how the roles of the markers shifted in dominance for his experiences across the three critical periods. The markers and the connected three landscapes constitute his identity trajectory (see Fig. 8).

The development of Victor’s identities is tremendous. His identity changed from being a traditional mathematics teacher belonging to the world of para-education with almost no use of DRs, to becoming a reform-oriented teacher and a national designer of DRs who was well-known and recognised as such by the Greek mathematics education community at large. We highlight our interpretation of the three most important experiences for his identity development.

First, Victor’s experiences of becoming and being the embracing teacher were a significant and constant driving force throughout his career. This character reified for him both the student-centeredness of the reform and his political ideologies. Early in his career, Victor experienced that using DRs in designing teaching materials helped him become this character. Our analysis showed how this motivated him to continuously seek and engage with new opportunities for agency and development that evolved at North River School and

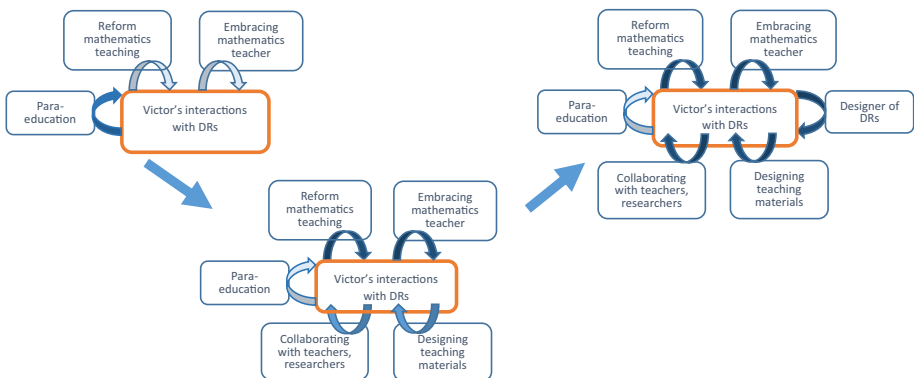


Fig. 8 Victor’s identity trajectory

beyond, such as collaborating with colleagues and researchers, and participating in projects targeted designing materials including DRs.

Second, during his career, Victor increasingly experienced being recognised as a competent reform-oriented teacher and a talented designer of DRs by among others, colleagues and researchers. Our analysis showed that his ways of participating in the two practices—collaborating and designing materials—evolved closely intertwined over his career and that DRs played an increasingly significant role in these practices. New School and Digital School had the strongest impact on his participation due to the spirit and expertise under which the DRs were designed as malleable artefacts open to change and adaptation, and oriented towards the reform-world. By participating still more sophisticatedly in collaborating and designing, Victor increasingly experienced being recognised by others.

Third, especially during the last period, Victor experienced becoming a designer of DRs and belonging to an elite group of such designers. We interpret that the development of this new identity was a result of Victor's increasingly sophisticated participation in other practices and worlds, most importantly collaborating, designing materials and the reform-world. Importantly, Victor learned to design tasks that exploited the affordances of DRs. This seemed to enable him to engage fully in the reform-world in his classroom and become the embracing teacher. Victor's identity as a designer of DRs was very significant for his identity development.

The three experiences are still crucial for Victor's present identity: he is being recognised as a reform-oriented teacher by the Greek community of mathematics education, he belongs to an elite group of designers of DRs, and yet his underlying motivation to develop further is to become the embracing teacher.

Discussion and concluding remarks

Having investigated the two trajectories of Victor's professional life, we now answer our third research question about the benefits of combining DAD and PoP when studying the phenomenon of teachers' long-term PD with DRs. Next, we address our study's contribution to understanding the phenomenon.

Methodologically, the two analyses were combined in two ways. First, the idea of creating Victor's identity trajectory was fostered by our construction of the inferred/reflective mapping, and this mapping comprised the basis for both analyses. Second, we structured the PoP analysis in relation to the three critical periods identified by the DAD analysis. Theoretically, the frameworks were based on conflicting basic assumptions, but we will argue below that their combination contributed a multifaceted insight into teachers' long-term PD with DRs by complementing and supplementing each other.

The analyses complemented each other in two interrelated ways. First, the DAD analysis provided a descriptive account of Victor's interaction with DRs and its evolution, while the PoP analysis provided an explanation of how and why Victor developed his interactions with DRs. Second, the DAD analysis focused on the resources that Victor used inside and outside of school, while PoP foregrounded affective and contextual aspects of Victor's PD with DRs within and beyond his school. Although the DAD analysis contributed rich descriptions of Victor's documentary trajectory and resource system, it did not satisfactorily address the reasons and mechanisms behind his long-term PD. Affective and contextual aspects such as social recognition, the sense of one-self as a professional and motivational forces were not considered, but, in Victor's case, they turned out to be decisive for

his successful use of DRs to support students' learning. Several authors, also from within the DAD community, point to the need to address such aspects and provide more explanatory accounts (Haspekian, 2020; Trouche, 2019). The ways the two frameworks complemented each other contributed to filling in this gap in research on teachers' interactions with DRs.

Each analysis supplemented the other in important ways. Victor's identity trajectory was enriched by his documentational trajectory as it specified the evolution of his interaction with DRs. For example, the PoP analysis showed that Victor experienced becoming a designer of DRs, but the analysis did not provide further details about how Victor was as a designer of DRs. The DAD analysis supplemented such details. Also, Victor's documentational trajectory was enriched by his identity trajectory. It identified shifts in his participation in different collectives and showed how these influenced his design and interactions with DRs. For instance, the PoP analysis offered motives and explanations of why Victor engaged in more complicated interactions with DRs and how this transformed and promoted his long-term development with DRs.

Next, we highlight two contributions of our study to understanding the phenomenon of teachers' long-term DP with DRs in general and successful cases of using DRs to support students' profound learning in particular. The first contribution is that affective and contextual aspects influence teachers' long-term PD with DRs, including social recognition, interactions and relationships with students, teachers' past experiences, and broader modes of reasoning. This is also the case for teachers' long-term PD without DRs (Lutovac & Kaasili, 2018a). Such aspects can explain the ineffectiveness of several PD programs with/without DRs. To this end, Victor's successful case is informative as it identified an individual adventure lasting about 10 years (i.e. critical periods 1 and 2), when he strived by himself to identify and create opportunities to learn more about designing and teaching with DRs. By looking globally at his trajectories, it becomes clear how Victor created and engaged persistently with opportunities for PD outside official channels. This kind of agency seems important for successful integration of DRs, as also suggested by Goos (2013).

The second contribution is that our study highlights the need for richer insights into the phenomenon and for more explanatory power than hitherto provided by DAD studies (e.g. Rocha, 2019; Trouche et al., 2020) or identity studies (e.g. Chronaki & Matos, 2014). As regards the DAD studies, our use of new analytical tools (e.g. constitution, content and structure, evolution) allowed a more complete picture of Victor's interactions with DRs, which was richer in terms of evidence of his activities, their plurality in relation to specific DRs in different contexts and critical periods, and the evolution of his resource system. For instance, Trouche et al., (2020) also described changes in Anna's resource system and elements of her professional learning, but without describing and elaborating on changes in her classroom teaching or specific use of DRs to support student learning. In some studies on teacher identity and use of DRs, teachers experienced themselves as being at "the cutting edge" (Chronaki & Matos, 2014, p. 115), which significantly contributed to their identity formation and their strong position. However, the teachers only challenged traditional teaching at a surface level by prioritising the pedagogical novelty of DRs. Our study shows what it may take to use DRs to support students' mathematics learning more profoundly and explain how Victor's experiences from contexts beyond his school and other social factors were reflexively related to and co-determined his use of DRs in mathematics classrooms.

We conclude that by combining the two frameworks, we gained deeper insights into Victor's PD with DRs over the 22 years. Moreover, these insights extend beyond the single

successful case of Victor. They highlight the complexity of how teachers' participation in social constellations co-relates with their professional experiences regarding their long-term use of DRs. Thus, our approach provides a new theoretical lens for studying teachers' long-term DP with DRs in general.

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