



Enacting multilingual learner core practices: a PST's approximations of practice of mathematics language routines

Sarah A. Roberts¹ · T. Royce Olarte¹

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Abstract

This study considered a preservice teacher's (PST's) attention to multilingual learner core practices within her approximations of practice of mathematics language routines during a secondary mathematics methods course. We used a framework of approximations of practice to understand how a PST enacted and developed an understanding of multilingual learner core practices. To provide a vision for what multilingual learner core practices can look like in nontraditional instructional contexts, we qualitatively analyzed four approximations of practice of mathematics language routines from a single PST during a single semester of a methods course. We share how she navigated the remote teaching context and engaged sample students in all of the multilingual learner core practices despite the challenges of the COVID-19 pandemic, providing an example of what these practices may look like in remote instruction. We also discuss how these multilingual learner core practices interact and complement one another. We consider possible implications, limitations, and future research directions.

Keywords Preservice teachers · Multilingual learner core practices · Approximations of practice · Multilingual learners · Mathematics language routines

There has been a turn in teacher education toward the implementation *practice-based* pedagogies, where preservice teachers (PSTs) enact part of their teacher education work, or “practice” the work of teaching within classrooms (McDonald et al., 2013). Using *approximations of practice*, or “opportunities to enact practices in conditions similar to the authentic teaching practice” (Matsumoto-Royo & Ramirez-Montoya, 2021, p. 3), is a practice-based pedagogy. PSTs can hone their role as teacher, for instance, in mathematics methods courses. These are low-stakes opportunities, with conditions of limited complexity, where the consequences of failure are minimized (Matsumoto-Royo & Ramirez-Montoya, 2021).

Many teacher education programs embed *core practices* within such practice-based pedagogies (Matsumoto-Royo & Ramírez-Montoya, 2021). Here, practice within core practice

✉ Sarah A. Roberts
sroberts@education.ucsb.edu

¹ Gevirtz Graduation School of Education, University of California, Santa Barbara, CA 93106-9490, USA

relates to orchestrating understandings, skills, relationships, and issues of power and identity to accomplish activities with others (Core Practice Consortium, 2023). Core practices,¹ such as eliciting student thinking or leading a classroom discussion, are fundamental to teaching (Core Practice Consortium, 2023); can be used across disciplines (Matsumoto-Royo & Ramírez-Montoya, 2021); and allow a PST to develop professional identity, skills, and knowledge (Grossman et al., 2009). Grossman et al. note that core practices occur in high frequency, are present across content areas, are practices that novice teachers can begin to master, allow novices to develop their understanding of their students and own pedagogies, and have the capacity to improve student performance.

With a turn toward practice-based pedagogy has come a call to develop a shared understanding of core practices (Grossman & Dean, 2019). Grossman and Dean brought together key scholars around core practices, and while common elements were found across multiple definitions and practices, they identified that there is still space to define core practices as a field. A clearer equity focus is often cited (i.e., Kane, 2020; Philip et al., 2019) as missing in the common language of core practices. For example, Zeichner (2012) noted early on that PSTs needed to develop cultural competence, something that is currently still lacking in most core practices. There is a need for a broader notion of core practices, because, for instance, the current core practices lack attention to multilingual learners. There is space to foreground equity as the field continues to define these core practices.

Multilingual learners (also identified as English Learners) accounted for more than 10% of students enrolled in US classrooms in 2017 (National Center for Education Statistics, 2020). Language plays an intricate role in mathematics classrooms across the world, providing the medium for conveying and solving problems, as well as assessing this content (Essien, 2018). Globally, the language of teaching and learning often differs from a student's first language, with many students learning mathematics in their second or third language, frequently due to migration, colonialism, and/or the plurality of local languages (Salekhova, 2020). Multilingual learners' increasing classroom presence has prompted teacher education programs to focus more intently on preparing their preservice secondary mathematics teachers to effectively integrate content *and* language development (Johnson et al., 2016; Lyon et al., 2016).

With so many multilingual students learning the language of teaching and learning and mathematics, PSTs must understand how to provide ambitious and equitable pedagogy (Jackson & Cobb, 2010). Because multilingual learners have often been provided with less than ambitious pedagogy because of their status as “English learners” (Iddings, 2005; Planas & Gorgorió, 2004), it is important to provide PSTs with opportunities to learn about equity-based pedagogies. We define equity-based pedagogies as those practices that “center justice, with a recognition of and willingness to address historical and contemporary systems of oppression” (Philip et al., 2019, p. 260). Multilingual learner core practices for mathematics teaching, which attend to content and language, are a starting point for this work. We posit that supporting teachers' understanding of mathematics language routines (MLRs; Zwiers et al., 2017) and multilingual learner core practices is one way we can begin to combat systems that continue to push multilingual learners to the periphery and inhibit their access and participation within mathematics classrooms. MLRs are structured and adaptable routines that were developed to support simultaneous mathematics learning while acquiring English and “support *all* students learning mathematics” (Zwiers et al.,

¹ See Grosser-Clarkson and Neel (2020) for an exhaustive list of core practices.

2017, p. 3, emphasis in original). With multilingual learners experiencing historic patterns of exclusion, such as receiving less cognitively demanding work, MLRs and the multilingual learner core practices that teachers can embed within the MLRs are tools that teachers can develop to make their instruction more equitable for all students.

To confront historical systems of exclusion, we need a vision for what multilingual learner core practices look like in practice (Campbell & Bgrinderaldinger, 2022). In particular, we need to understand how PSTs enact these core practices. Adding equity-based core practices, like multilingual learner core practices, and capturing an instance during COVID-19 of a PST who attempted these core practices provides this vision through approximations of practice teaching multilingual learners. This study answers the following research question: How did a PST enact multilingual learner core practices through approximations of practice?

Framework

We draw on the conceptualizations of approximations of practice (Campbell & Baldinger, 2022) and core practices (e.g., McDonald et al., 2013) to frame this study. Specifically, we align our work with the notion that approximations of practice develop and support novice teachers' understanding of core practices. As PSTs engage in approximations of practice of the MLRs, they have opportunities to enact and develop a range of core practices, such as the multilingual learner core practices. Furthermore, we consider the examination of remote approximations of practice to provide us unique insight into how teachers engage in core practices in remote instructional contexts.

Approximations of practice

A key element of practice-based pedagogies is the integration of approximations of practice in teacher education programs. Our study focused on approximations of practice as a way for PSTs to enact, practice, and develop their understanding of multilingual learner core practices. Specifically, PSTs engaged in these approximations of practice, or "opportunities for enacting teaching in situations of reduced complexity" (Campbell & Baldinger, 2022, p. 508), as they asynchronously enacted MLRs. MLRs are scaffolded routines intended to lead to students' independent participation in the mathematics classroom through supporting sense-making, optimizing output, cultivating conversation, and maximizing linguistic and cognitive meta-awareness (Zwiers et al., 2017). MLRs include such routines as Three Reads or Stronger & Clearer and are described in further detail in the Method section. It is through these approximations of practice (i.e., the enactments of MLRs) that we observed PSTs' implementation of multilingual learner core practices for mathematics.

Core practices

Core practices are fundamental elements of teaching in that they help advance PSTs' skills and pedagogies (e.g., Ball & Forzani, 2009; Grossman et al., 2009); however, there is still limited work around equity-based core practices for multilingual learners. Additionally, although there is a developing knowledge base around effective instruction for multilingual learners (i.e., Moschkovich, 2013), there is limited scholarship focused on understanding

and implementing core practices that engage multilingual learners specifically, particularly in content area instruction, such as mathematics.

Grossman and Dean (2019) illustrated the importance of developing a shared understanding and shared language around core practices. We further argue that the field needs to develop a stronger shared knowledge around multilingual learner core practices in order for them to be more effectively integrated into mathematics methods courses in teacher education programs. By examining teachers' approximations of practice for these multilingual learner core practices, we enrich our understanding of what these core practices could look like in the classroom.

Literature review

We reviewed three bodies of the literature relevant to our study as related to multilingual learner core practices: equity-based core practices—multilingual learner core practices for mathematics teaching; equity-based approximations of practice with PSTs; and equity-based multilingual learner pedagogy in methods courses.

Equity-based core practices—multilingual learner core practices for mathematics teaching

We operationalized equity-based core practices as multilingual learner core practices for mathematics teaching (Roberts, 2017, 2019). Multilingual learner core practices for mathematics teaching draw on prior work of scholars in the field of mathematics education (e.g., Khisty & Chval, 2002; Moschkovich, 2002; Zahner, 2015), and they allow teachers to engage multilingual learners in mathematical work in purposeful ways. While these core practices are distinct, they are understood as reinforcing and overlapping with one another.

The first multilingual learner core practice for mathematics teaching is to *create a safe classroom*. A safe classroom allows for taking risks, asking questions, and engaging in reasoning and sensemaking—such that collaboration is fostered (Choike, 2000). The second multilingual learner core practice for mathematics teaching is to *build on and use multilingual learners' funds of knowledge and resources* (Lee et al., 2008; Moll et al., 1992; Moschkovich, 2002). Teachers identify, celebrate, and use the knowledge and skills students, their families, and their communities bring to the classroom during mathematics teaching. The third multilingual learner core practice for mathematics teaching is to *identify disciplinary language demands and supports* for multilingual learners (Aguirre & Bunch, 2012; Lyon et al., 2016). Teachers attend to the disciplinary language demands by providing appropriate supports, such as sentence frames, so that all students can share their ideas and reasoning. The fourth multilingual learner core practice for mathematics teaching is to provide multilingual learners *opportunities for rich language and literacy exposure and practice* (Khisty & Chval, 2002; Lee et al., 2013). Teachers create opportunities for students to receive comprehensible input through listening and reading, and teachers provide opportunities for multilingual students to produce comprehensible output through speaking and writing. The fifth multilingual learner core practice for mathematics teaching is to provide multilingual learners with *cognitively demanding work* (Stanford Graduate School of Education, 2013; Tekkumru-Kisa et al., 2015). Teachers focus on engaging students in the content standards and mathematical practices, while balancing conceptual understanding and procedural fluency (Moschkovich, 2013).

Equity-based approximations of practice with preservice teachers

Approximations of practice more generally are quite widespread in the USA and internationally (i.e., Anthony et al., 2015; Hammerness et al., 2020). We highlight the limited work on approximations of practice in secondary mathematics teacher education that focuses on equity-based pedagogies, especially outside of the USA.

Scholars have studied how approximations of practice afford opportunities for PSTs to enact and develop equity-based core practices (e.g., Polly & Colonnese, 2022; Strom & Martin, 2015). For example, Krause et al. (2020) conceptualized one-on-one interviews between students and bilingual (Spanish and English) PSTs as approximations of practice. PSTs met with students through an after-school program that engaged them in translanguaging while working on fraction problems with their students, which more equitably elicited their students' mathematical thinking.

Furthermore, a growing body of scholarship examines digital, web-based approximations of practice, which provide a safe space for ambitious teaching practices (Howell & Mikeska, 2021). This is especially relevant because we examined a PST's asynchronous approximations using web-based platforms (e.g., Desmos,² Desmos Studio, 2023). Howell and Mikeska used simulated classroom experiences (i.e., virtual students) to support PSTs in the core practice of leading mathematical discussions. An affordance of approximations of practice is that they can be designed to enact equity-based core practices, and even in web-based, digital approximations of practice, PSTs can attempt targeted practices in low-stakes settings.

Equity-based multilingual learner pedagogy in methods courses

We identified approaches to equipping PSTs to work with multilingual learners through methods courses. We acknowledge that most of this work has been done in the USA. A general approach for equipping PSTs to work with multilingual learners in a methods course is to familiarize them with the Sheltered Instruction Observation Protocol (SIOP; Echevarria et al., 2006). This protocol includes eight different instructional strategies for working with multilingual learners in any content area, such as using sentence frames and providing explicit vocabulary instruction (Echevarria et al., 2006), and is a general approach for PSTs to scaffold instruction for multilingual learners (e.g., Moore, 2018).

Several mathematics methods courses have used a content-specific focus for attending to multilingual learners, particularly at the elementary level (e.g., de Araujo et al., 2018, 2021; Fernandes, 2012). For example, de Araujo et al. (2021) analyzed PSTs' lesson plans and weekly reflections and conducted weekly interviews with PSTs to understand how PSTs' beliefs affected their use of mathematics tasks for multilingual learners. For instance, PSTs removed unfamiliar words and changed contexts. The noted that teacher educators need to disrupt notions about removing barriers for multilingual learners by providing instruction on appropriate scaffolds for multilingual learners. This study and others (i.e., I & Stanford, 2018) highlighted the importance of engaging mathematics PSTs in learning experiences explicitly focused on multilingual learners. While PSTs' pedagogies

² Desmos is a suite of online math tools, where teachers can develop or use community-developed lessons, a curriculum, or other tools, like a graphing calculator. Teachers can build a series of screens to walk students through a lesson.

developed during a methods course can be beneficial for all learners, these studies stand out for their specific attention to multilingual learners, a rarity in the mathematics education literature (de Araujo, 2018).

Several mathematics education scholars have suggested guiding principles for working with multilingual students in mathematics methods courses, similar to the multilingual learner core practices for mathematics teaching (Roberts, 2017, 2019) that helped direct this study. These common ideas include.

- Providing multilingual learners with challenging or cognitively demanding work and/or tasks (Lee et al., 2019; Moschkovich, 2012; Ramirez & Celedón-Pattichis, 2012);
- Using multilingual learners' cultural and linguistic differences as resources rather than obstacles (Moschkovich, 2012; Ramirez & Celedón-Pattichis, 2012); and
- Providing opportunities for students to engage with the complex language of the content area, while engaging in this discourse with others (i.e., multiple modes of communication, use of representations; Driscoll et al., 2016; Lee et al., 2019; Moschkovich, 2012).

There are common elements across many of these principles and the multilingual learner core practices for mathematics teaching that guide our own work. Importantly, it is possible to engage PSTs in creating classroom environments that directly engage multilingual students. This study used a content methods course that centered approximations of practice to develop the PSTs' practice around the multilingual learner core practices for mathematics teaching.

Method

Context

This research took place in a multilingual learner-focused secondary mathematics methods course within a small, 12-month, postbaccalaureate teacher education program at a research university in California. The teacher education program involves three sets of courses and experiences related to multilingual learners within their coursework (see Roberts, 2017 for further information). These include the three following areas: (1) Field Experiences, to try out and reflect on their practice; (2) General Multilingual Courses: three courses—one foundations of bilingual education and two English language development/Specially Designed Academic Instruction in English (SDAIE) courses; and (3) Mathematics Methods Courses: a total of three methods courses, with the course described in this paper focused specifically on methods to attend to multilingual learners in mathematics.

This methods course was the third mathematics methods course PSTs completed and was offered in their final semester of the program. PSTs also completed student teaching in a grade 7–12 mathematics classroom while in this course. The first nine months of student teaching were in person, while the last three months were via Zoom, as a result of the beginning of the COVID-19 pandemic. This methods course met weekly, for a total of nine 2-h meetings, over Zoom.

We aligned the methods course with an ongoing research project with in-service teachers on the use of multilingual learner core practices and MLRs (Zwiers et al., 2017). MLRs are meant to engage students, particularly multilingual learners, productively with content,

providing them with tools that they can familiarly return to so that they can solve mathematics tasks (Kelemanik et al., 2016). Zwiers et al. (2017) developed MLRs specifically for multilingual learners, and teachers can use these routines to amplify, assess, and develop multilingual students' language and content in mathematics classrooms. The four focal MLRs in the course were Three Reads; Clarify, Critique, & Correct; Co-Craft Questions; and Stronger & Clearer. Table 2 provides an overview of these MLRs.

PSTs learned about the five different multilingual learner core practices, which were aligned with these four focal MLRs (shown in Table 1). These alignments were purposeful; we felt the MLRs provided enactment opportunities for the aligned multilingual learner core practice. This did not mean that other core practices could not be aligned or would not be present; however, the alignments provided for more ample opportunities for PSTs to practice the multilingual learner core practice. For example, *cognitively demanding work* was aligned with Co-Craft Questions because of the cognitive demand of developing questions. We aligned a multilingual learner core practice with each MLR, with the others operating in the background.

We used a modified version of our studio days model (see Fig. 1; Von Esch & Kavanagh, 2018) that we used with in-service teachers in our research project to engage the PSTs with the MLRs, multilingual learner core practices, and the approximations of practice during two-week cycles. During the first week of the cycle ("Class Day 1" of Fig. 1), the instructor introduced PSTs, during a synchronous Zoom class, to a multilingual learner core practice and a MLR through a simulated lesson. PSTs engaged in activities as "students," such as through engaging in a Desmos lesson (Desmos Studio, 2023), in order to learn about the MLR and how it could be enacted in a class. PSTs then had time to work in breakout rooms to brainstorm their own implementation of the multilingual learner core practice and MLR for their approximation of practice.

Enactments of MLRs in the methods course were approximations of practice through which PSTs could execute and deepen their understanding of the multilingual learner core practices under conditions of reduced complexity (Campbell & Baldinger, 2022), because PSTs worked individually to implement the MLRs without actual students. We originally planned to have PSTs conduct these approximations of practice in person during the methods class; however, just as our spring course was about to begin in March 2020, the COVID-19 pandemic began in earnest. Our PSTs remotely completed both their student teaching takeover and this methods course remotely. Over the course of the next week (the second cell of Fig. 1), PSTs videotaped their approximation of practice and shared this in a Google Folder with a peer. These approximations of practice were completed without students (and without PST peers) present because of the pandemic. Most PSTs presented their lessons, as if there were students, which we called "sample" students. In the second week of the cycle ("Class Day 2" in Fig. 1), we discussed reflections and reviewed instructor-selected clips from the PSTs' videotaped approximations of practice.

Participants

Five mathematics methods students enrolled in this course during Spring 2020. One student, Ms. Severn, a pseudonym, served as a case study (Yin, 2018). We used a case study approach to investigate the phenomenon of multilingual learner core practices for mathematics teaching within the real-world, virtual context of Ms. Severn's enactments during the two-week cycles. Because the phenomenon of multilingual learner core practices, a type of equity-based pedagogies, is not currently well understood, particularly

Table 1 Course Alignment of MLRs and Multilingual Learner Core Practices for Mathematics Teaching Across Course

Weeks of course	Mathematics language routine	Aligned multilingual learner core practice for mathematics teaching	Description of why the core practice and MLR were aligned
Across All Weeks	[Attended to Across All Routines]	Create a safe classroom	In order to engage in any routine, students have to feel like they are part of a safe classroom, and teachers have to create these safe classrooms.
1	(No MLR) Introduction to Course	Overview of all multilingual learner core practices	(No MLR introduced.)
2–3	Three Reads	Identify disciplinary language demands and supports	Students unpack a mathematical reading passage, making sense of disciplinary language and using a disciplinary support (the Three Reads MLR) as they do so.
4–5	Clarify, Critique, & Correct	Opportunities for rich language and literacy exposure and practice	Students discuss and write (i.e., rich language practice and exposure) about a piece of mathematical work that is not their own, providing students opportunities to engage in multiple modes of communication (i.e., reading, writing, unpacking representations, discussing) around mathematics.
6–7	Co-Craft Questions	Cognitively demanding work	The development and refinement of mathematical questioning are usually left to teachers. Instead, students develop and refine mathematical questioning, which is cognitively demanding work.
8–9	Stronger & Clearer	Build on and use multilingual learners' funds of knowledge and resources	As students develop their Stronger & Clearer explanations, they not only use their own ideas, but they also use their peers' ideas, thereby drawing on their peers' strengths. This provides an opportunity for the teacher to celebrate these strengths.

As teachers enacted all of the MLRs, there were likely opportunities to use all the multilingual learner core practices for mathematics teaching. However, we chose to foreground particular multilingual learner core practices for mathematics teaching with specific MLRs, noting that other multilingual learner core practices for mathematics teaching would likely be backgrounded in these enactments

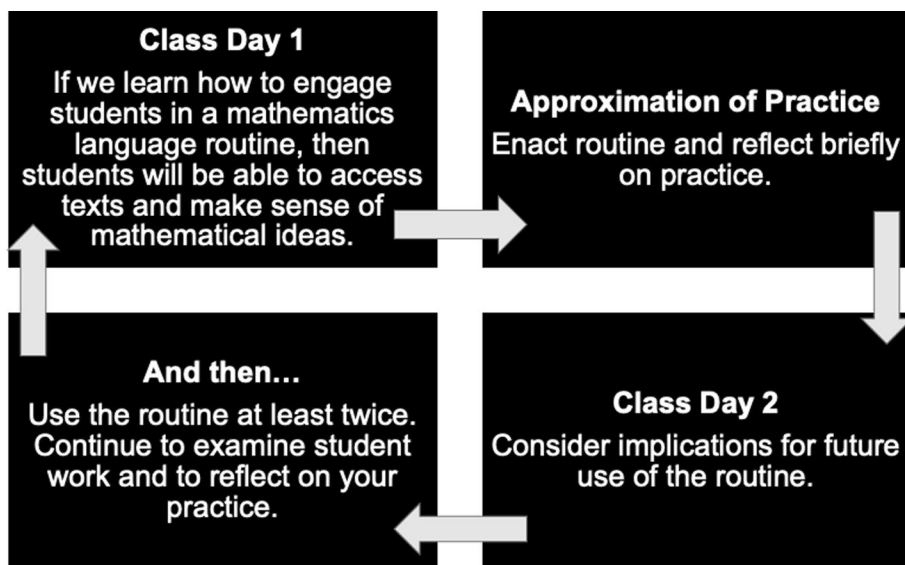


Fig. 1 Modified studio day model for mathematics methods course focused on MLR implementation

within the context of these mathematics methods courses, the use of a case study was particularly helpful (Yin, 2018). Yin noted that a case study benefits from the prior development of a theoretical framework to guide design and analysis, as enacted in this study through multilingual learner core practices for mathematics teaching; and a case study relies on multiple sources of data, as described in this study by both Ms. Severn's enactments and her reflections.

Our focal participant taught in an Integrated Math I placement during Spring 2020. We selected Ms. Severn because she was an unusual case in the spring methods class, which allowed us to maximize what we could learn from her regarding the enactments of multilingual learner core practices (Stake, 1995). Ms. Severn was unique because she used four different web-based technologies with each of the MLRs during the course, as shown in Table 2. Her peers usually videotaped their approximations of practice using a screenshare of a handout or slides on their computers, using the same format for every approximation of practice. Ms. Severn, instead, at her own discretion, planned each of her approximations of practice using a different type of technology, which provided both a revelatory and extreme case (Yin, 2018). No other PST chose to use technology in this way for each of their approximations, which allowed us to capture a distinct method for attending to multilingual learners, particularly at the onset of the COVID-19 pandemic, when different types of technology for asynchronous and synchronous instruction were just becoming more ubiquitous. While Ms. Severn was in a student teaching placement at the time of this methods course and was supporting remote instruction, she completed all approximations of MLRs without the live involvement of students. In some lessons, Ms. Severn later shared the same content with her students in her student teaching placement; however, students were not part of her approximations of practice. In her approximations of practice, she did pretend that students were present, as noted above, with "sample" students.

Table 2 Ms. Severn's Use of Technology with MLRs Across Course

MLR	Technology used	Overview of MLR and technology integration (drawn from Zwiers et al., 2017)
Three Reads	Edpuzzle	Three Reads supports students to access mathematical texts, prompts them to reflect on how mathematical questions are presented, and equips them to negotiate meaning. Ms. Severn used Edpuzzle (Edpuzzle Inc., 2023) to display slides. On each slide, she articulated what sample students should look for on each read, providing open-ended questions (e.g., What is the context of the problem?) to guide them through each read.
Clarify, Critique, & Correct	Zoom/notability	Clarify, Critique, & Correct tasks students to analyze, reflect on, and develop an incomplete or incorrect piece of mathematical writing that is not their own. Ms. Severn used Zoom (Zoom Video Communications, Inc., 2023) to share her Notability (Ginger Labs, 2023) screen, where she had uploaded the worksheet for Clarify, Critique, & Correct. This worksheet included an incorrect mathematical task and subsequent spaces for students to reflect on what they noticed, provide critiques, and offer corrections. Her accompanying Zoom video-guided sample students through the routine.
Co-Craft Questions	Desmos	Co-Craft Questions allows students to get inside of the context of a problem and produce the language of mathematical questions themselves. Ms. Severn created a Desmos (Desmos Studio, 2023) activity, which included a series of slides with videos of herself explaining each stage of the routine and space for sample students to submit their ideas, responses, and questions. Sample students needed to submit responses before clicking to the next slide.
Stronger & Clearer	Padlet	Stronger & Clearer allows students to revise and refine their ideas and verbal/written output both individually and collaboratively. Ms. Severn used Padlet (n.d.) to have sample students submit written and audio responses over the course of a week (Wednesday and Friday) after their initial response on a Monday. She tasked them to view and listen to their peers' responses before revising and strengthening their own response.

Data collection

Data collection was organized around Ms. Severn's four cycles of approximations of practice of multilingual learner core practices for mathematics teaching aligned with MLRs (see Table 1). The following served as data sources for the study: videos of Ms. Severn's enactments; Google Form reflections on her implementation of the MLRs; one- to two-paragraph reflections about MLR implementations; exit ticket reflections of 2–4 sentences; and a 100–200-word reflection at the end of the course, where students shared what they learned in the course. Ms. Severn recorded her own approximations of practice because of the pandemic. Google Form reflections asked PSTs to reflect on their experience implementing the multilingual learner core practice and MLR, asking such questions as: “What is one thing you learned from trying out this routine (related to Co-Craft Questions or *cognitively demanding work*) that you think is important to share with the larger group?” Using multiple data sources allowed for data and methodological triangulation to create a richer case study (Yin, 2018). Further, Campbell and Baldinger (2022) noted that “additional data, such as post-rehearsal reflections, can be designed to intentionally seek out information about components of [PSTs'] resources that are not apparent in the enactment itself” (p. 527).

Data analysis

We qualitatively analyzed Ms. Severn's assignments to understand how a PST enacted multilingual learner core practices for mathematics teaching through her approximations of practice. All approximations of practice, which were videos, were first transcribed verbatim. We then entered the transcriptions and the written reflections into the qualitative software NVivo, which allowed us to compile the data, code the data, and look for patterns within the data, such as through creating and examining matrices (Yin, 2016).

Our first round of coding used a priori codes (Saldaña, 2021) aligned with the multilingual learner core practices for mathematics teaching (see Table 3 for codebook) to examine Ms. Severn's approximations of practice and how she discussed these enactments in her reflections. A paragraph or complete response was our unit of analysis (e.g., within transcripts, Google Form responses). We reconciled our coding after we coded each 20% portion of the data and continued to refine our codebook. We achieved a Cohen's Kappa of between 44.2 and 68.5% (in our last effort), and, as a result, we pair coded all the data. This first round of coding allowed us to understand Ms. Severn's enactment of the multilingual learner core practices.

Our second round of coding followed a process inspired by Campbell and Baldinger (2022) to identify common features of the multilingual learner core practices for mathematics teaching during Ms. Severn's enactment. We first reviewed all the first-round coded data for each of the multilingual learner core practices individually. We each identified the common features, working to describe the key features of the multilingual learner core practices as enacted in Ms. Severn's approximations of practice. Then the co met to reconcile differences between each multilingual learner core practice and to develop a common set of features for each core practice. Table 4 provides an overview of the common features we identified, which we discuss further in the Findings section.

We then examined the multilingual learner core practices for mathematics teaching, looking for similarities and differences to “further refine, group, and consolidate the set of

Table 3 First Round Codebook—Multilingual Learner Core Practices for Mathematics Teaching

Code	Description	Example From Ms. Severn
Create a safe classroom	Teacher employs a safe classroom by contributing to a culture of community learners, where collaboration, intellectual risk-taking, and diversity are not only valued but celebrated	[It] gave the students a good opportunity to be able to participate in the class without having to explicitly know the content perfectly, or the vocabulary for the content, because there were many opportunities to share noticings. [Clarify Critique Correct Post Google Form]
Build on and use multilingual learners' funds of knowledge and resources	Teacher employs students' funds of knowledge and resources by identifying, celebrating, and using the students', as well as their families' and communities', knowledge and skills in the classroom space	I know that's not quite possible right now, but if you want to pull in a sibling, or a grandparent, or a cousin, or a parent, or somebody to help you work on this, you can. You're not required to, but it can be fun to kind of bounce your ideas off of someone as you're coming up with those questions [Co-Craft Questions Transcript]
Identify disciplinary language demands and supports	Teacher employs or identifies disciplinary language supports for students by adequately scaffolding and producing language while attending to aspects of language that may be challenging for all students	I think that my implementation of the routine lacked in helping multilingual students that were not able to attend the Zoom meetings to communicate verbally, but it provided scaffolding and access to communicating with written language [Co-Craft Questions Post Google Form]
Opportunities for rich language and literacy exposure and practice	Teacher employs opportunities for rich language production when engaging in comprehensible input and output through listening, reading, speaking, representing, gesturing, and writing with their students	I think that giving students an opportunity to collaborate with their peers verbally is important, which is something that I missed but my peer did well [Three Reads Post Google Form]
Cognitively demanding work	Teacher employs cognitively demanding work when teachers focus on Common Core State Standards-Mathematics and provides the same opportunity to seize rich activities and assignments often reserved for English-only students	It's where I'm going to show you an image or a graph or something, in this case it is an image, and you're really going to be thinking like a teacher. Try to come up with "What could I ask about this if I were the teacher" [Co-Craft Questions Transcript]

Table 4 Second Round Coding—Common Features of Ms. Severn's Multilingual Learner Core Practices for Mathematics Teaching

Code	Description
Create a Safe Classroom	All students are able to participate equally Teacher shows students that their ideas are valuable Teacher has an expectation that no matter where students are, all students can participate Teacher encourages intellectual risk-taking (e.g., shows students their mistakes)
Build on and Use Multilingual Learners' Funds of Knowledge and Resources	Teacher draws on something from students' lives (e.g., picture of some job listings, pulling in a sibling, etc.) and uses it for instruction **This multilingual learner core practice might be harder to enact without live students
Identify Disciplinary Language Demands and Supports	Teacher clarifies expectations for students in ways that will help them Teacher is thoughtful about how to scaffold students' engagement with language and mathematics (e.g., scaffolding conversations within Stronger & Clearer; modeling and scaffolding conversations; using written instruments; using colors for Three Reads; and chunking problems) **This scaffolding could be used more because of the remote nature of the instruction
Opportunities for Rich Language and Literacy Exposure and Practice	Throughout activities, teacher gives students opportunity to read, write, and speak (i.e., multiple modes of communication) because she values these multiple modes of communication Teacher scaffolds in the form of sentence frames and guiding questions (i.e., how she's moving the routine forward) Teacher scaffolds (i.e., uses Scaffolded conversations) and provides access to communicate Teacher differentiates in ways that allow students to converse (e.g., through multimedia) Teacher reflects on how students can collaborate, even in asynchronous instruction **There is attention to these practices both in the moment and after she has simulated the enactment. There is attention to continuing to improve the multilingual learner core practice. In this case, there is attention to understanding that she may still be relying on these remote resources
Cognitively Demanding Work	Teacher prompts students to continually add evidence or clarify their ideas (e.g., within Stronger & Clearer and Clarify, Critique, & Correct) Teacher asks guiding questions that help students engage in the activity Teacher orients students to other students' work Teacher positions students to think like a teacher (e.g., with Co-Craft Questions) through creating questions Teacher uses a cognitively demanding task (i.e., how to find how many tiles in Fig. 100)

features evident” (Campbell & Baldinger, 2022, p. 515). We wrote analytic memos to continue to make sense of our ideas, to test our conjectures, and to track our research processes (Yin, 2016).

Findings

We share how a PST enacted multilingual learner core practices through her approximations of practice during her secondary mathematics methods course. Our findings draw on those common features outlined in Table 4. We further unpack these features using key examples from the data to illustrate the distinguishing characteristics of the core practices as enacted in Ms. Severn’s approximations of practice.

Create a safe classroom

Ms. Severn’s enactment of *create a safe classroom* began by considering how she would engage all students equally in the classroom. As she described her enactments of MLRs, she reflected on how multilingual learners would benefit from using the MLRs, saying, they allowed “students to enter into the routine wherever they are comfortable so that students with varying skill levels can all participate equally,” highlighting a key aspect of a safe classroom. We now illustrate two instances where Ms. Severn exemplified the core practice of *create a safe classroom* given her instructional context.

First, Ms. Severn reinforced the notion that her classroom was a safe place for students to share and discuss mistakes. In her enactment of Clarify, Critique, & Correct, an MLR focused on error analysis, using a problem of an incorrect setup of an inequality, she implemented a lesson in which she acted as if she were guiding sample students³ through discussions about analyzing errors in student work, while emphasizing that she did not expect perfectly corrected solutions (see Fig. 2). After facilitating various stages of the routine, where she asked students to analyze the example problem, to write down details they noticed, and to critique the example solution, Ms. Severn demonstrated the correct solution to her sample students by describing the error and helping her sample students make sense of the correct inequality. She said, “Awesome, so go ahead and make those corrections. If your solution wasn’t exactly correct, that’s OK. You’re still going to get the credit for that. I just ask that you try it. So, awesome, thanks for trying it.” Although she revealed the correct solution, Ms. Severn’s comments brought to light that she valued students’ engagement and participation over the correctness of the solution.

In our second example, Ms. Severn continued to explicitly tell her sample students that their ideas and contributions were valuable, for instance, in her enactment of the routine Co-Craft Questions, where students developed questions around two job listings within Desmos (Desmos Studio, 2023). Within her approximation, she described how a range of questions that the sample students offered were all important. Some of the questions included “How many miles away are each of the jobs?” or, “Which one is closer?” Ms. Severn pointed out that the sample class’s questions differed from her own questions in

³ There were no students present in the approximations of practice, as noted in the method. However, Ms. Severn acted as if students were present in the lessons. Therefore, notes of students in findings are approximations of students, called “sample students,” here.

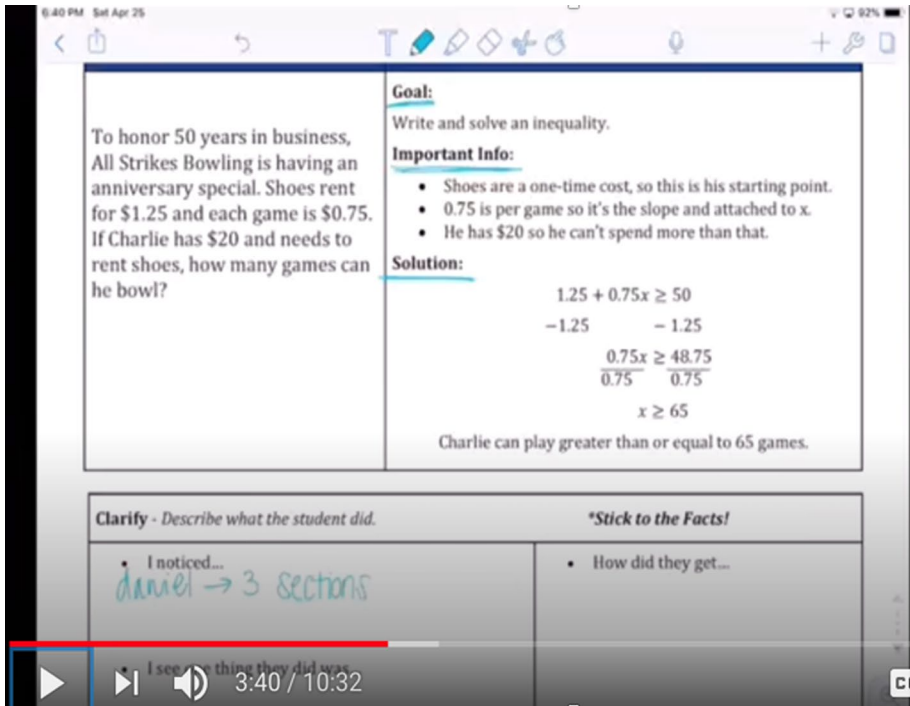


Fig. 2 Ms. Severn's focal problem and solution for Clarify, Critique, & Correct and Scaffolds Therein

that some were more open-ended or required further research. Ms. Severn's reflection on her enactment mentioned, "I think that it is a great idea to follow up the routine by having the student complete the questions that they came up with. It shows them that their contributions are valued." Within Desmos, Ms. Severn made space for students to do this work, as part of her approximation of practice. One Desmos slide had a "teacher move" (built into the technology) that noted, "Here I would pace the activity so that students cannot move past this page, forcing them to put ample thought into the comparison," using the technology to create space for students to engage with the questions and each other. By orienting sample students to each other's questions, choosing to answer preprepared sample student-generated questions, and using embedded technology, Ms. Severn created a safe classroom that demonstrated that student ideas and contributions were valued in the classroom.

Build on and use multilingual learners' funds of knowledge and resources

Building on and using multilingual learners' funds of knowledge and resources meant that a PST attempted to draw on something from students' lives and used that lived experience toward her mathematics instruction. This multilingual learner core practice was the least common of the core practices within Ms. Severn's approximations of practice, in part, because it was likely hard to know and understand students' lived realities without having actual students. Ms. Severn likely drew on extrapolated funds of knowledge from her prior student teaching experiences, without the opportunity to member check with

students during the COVID-19 shutdown, when few PSTs were allowed to be involved in instruction.

We provide two examples of the multilingual learner core practice of *building on and using multilingual learners' funds of knowledge and resources* as enacted. The first was Ms. Severn's use of local job listings, in the MLR Co-Craft Questions, which were relevant to her sample students, as noted above. Ms. Severn asked her sample students to create three questions based on the image of the two local stores that they might have visited, while also providing their local city. These stores connected to *building on and using multilingual learners' funds of knowledge and resources* in the following ways: (1) her student teaching placement students or their families might have previously shopped at these stores; (2) her student teaching placement students' families might have worked at these stores; and/or (3) her student teaching placement students were at an age to start looking for jobs in retail.

Ms. Severn drew on sample students' familial expertise, a second example of *building on and using multilingual learners' funds of knowledge and resources*, during Co-Craft Questions. She suggested:

If you want to pull in a sibling, or a grandparent, or a cousin, or a parent, or somebody to help you work on this, you can. You're not required to, but it can be fun to kind of bounce your ideas off of someone as you're coming up with those questions.

Here, Ms. Severn drew on the funds of knowledge of her sample students' family members, acknowledging that families were a resource (Civil, 2016).

Identify disciplinary language demands and supports

Identifying disciplinary language demands and supports focused largely on the disciplinary language supports, which Ms. Severn provided to her sample students within her approximations of practice. These were both language and mathematics scaffolds that included modeling and scaffolding mathematical conversations, scaffolding written tasks, and using different colors to highlight passages to make sense of the reading within the MLR Three Reads. We highlight Ms. Severn's scaffolding in two different ways: in action within the MLR Clarify, Critique, & Correct and within her reflections after the fact.

Scaffolding Clarify, Critique, & Correct

Figure 2 offered an example from Zoom (Zoom Video Communications, Inc., 2023) and Notability (Ginger Labs, 2023) of Ms. Severn scaffolding writing and reflection for sample students for the MLR Clarify, Critique, & Correct. She provided a document with the written instructions, as one scaffold; sentence frames, both verbally and in writing, to begin their work, as a second scaffold; and reviewed the directions verbally with them, as a third scaffold. She explained:

So, you're just saying, "I noticed that they did this." "I see one thing they did was..." And, in a little bit we're going to ask some questions. So, right now, go ahead and jot down two things that you noticed in the solutions [section within the Clarify, Critique, & Correct.]

These scaffolds would begin to provide students access to the disciplinary content of the task, helping them to consider something that they noticed within the mathematics, while

also supporting students with the language that they might need to share their thinking. Having students begin to deconstruct the given task, as part of the error analysis of Clarify, Critique, & Correct (again, see Fig. 2) using written and verbal directions to support their sensemaking, would help students enter the task. Then students could use sentence frames to begin to reflect on the mathematics provided as part of the MLR.

Reflecting on providing disciplinary language supports (Scaffolds)

Ms. Severn reflected that she felt the scaffolds would be useful in future instruction, but she wanted to continue to develop her use of scaffolds and models. For example, with the above implementation of Clarify, Critique, & Correct, Ms. Severn shared that the scaffolds would give students a “good opportunity to be able to participate in the class without having to explicitly know the content perfectly, or the vocabulary for the content, because there were many opportunities to share noticings.” However, in the MLR, Stronger & Clearer, Ms. Severn noted, “I do wish I had scaffolded the conversation a bit more,” which she explained that she might do in the future through creating “more of a structured written instruction page beforehand,” such as an instruction sheet aligned with the one seen in Fig. 2.

Opportunities for rich language and literacy exposure and practice

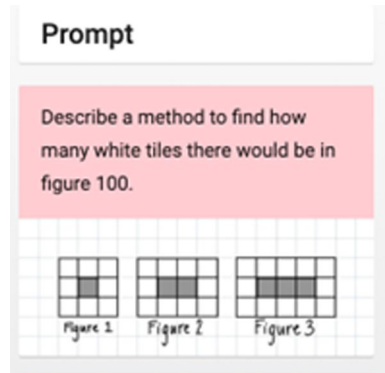
In ways similar to her work *identifying disciplinary language demands and supports*, Ms. Severn provided a number of scaffolds for sample students to participate in these *opportunities for rich language and literacy exposure and practice*. We use Ms. Severn’s approximation of practice of Stronger & Clearer to illuminate *opportunities for rich language and literacy exposure and practice*. We then discuss how Ms. Severn reflected on her work to continue to improve this multilingual learner core practice.

Opportunities for rich language and literacy exposure and practice through multiple modes of communication

Ms. Severn created opportunities for her sample students to engage in discourse. For example, before enacting Stronger & Clearer, Ms. Severn suggested, “I am thinking about ways to do this routine only asynchronously. Maybe there is a way to do this through a forum or a place that students could post audio recordings of their explanations that would change each time.”

Ms. Severn then adapted Stronger & Clearer to do just this. In her enactment, as noted in Table 2, students were tasked with collaborating with their classmates to build on the ideas and reasoning of others both verbally and in writing. Using the online software Padlet (n.d.), which is similar to online cork boards, users start with a blank slate, or Padlet, to which items can be posted. Ms. Severn posted prompts, for which her sample students needed to respond in two different ways, in writing and verbally, and included a video with instructions for how to complete the assignment. She provided sample students a problem and figure, found in Fig. 3 (we have removed her video instructions to anonymize our participant). Ms. Severn provided a visual of the problem, allowed students to read the problem, and provided an audio file for students to hear directions of the problem, enabling students to work with the problem in multiple ways as they got started.

Fig. 3 Tile task for stronger & clearer



She also provided detailed directions for the sample students about how to engage with the Padlet (n.d.) platform, the mathematics, and with each other:

First, you're going to be giving your initial written response in the first column and your initial audio response in the second column. If you look at the prompt column, which I'm calling column zero, you will see the prompt there. It says, "Describe a method to find out how many white tiles there would be in figure 100." ... So, what you're going to be doing today, is, you're going to be, is, you're going to think about that prompt for three to five minutes, and click the plus to add a post, and you're going to write a post. So, that's just a written description for how to find how many tiles will be in Figure 100. And then you are also going to click the plus here, and you are going to give an audio response here. So, that's something where you are describing your method to your classmates. So, you're going to click on these three dots here, and you can do a voice recording or a film recording, whichever you are more comfortable with. You could do those two.

These directions provided sample students with the opportunity to write and talk about the mathematics, which would be shared with a sample peer in the coming days, via audio or video. This would provide sample students the opportunity to use multiple modes of communication, on their own terms, timewise and technology-wise; would create agency for the students; and would provide space for students, particularly multilingual learners, to use multiple resources. In the following days, if used in an actual classroom, students would provide a second, revised audio response for peers and a final, revised verbal, and written response. Ms. Severn encouraged her sample students, "And, at the end, you should go back, look at your initial response, and at your Friday response, see how much that they've changed over the course of adding in ideas from your classmates." This Stronger & Clearer was a clear example of providing multiple opportunities for speaking, listening, writing, and working with representations.

Reflecting on how to provide opportunities for rich language and literacy exposure and practice

Across Ms. Severn's reflections, she noted the importance of providing *opportunities for rich language and literacy exposure and practice*, the need for scaffolds for these

opportunities, and her desire to continue to use what she had learned during her methods course. She shared that she.

had been looking for a way for students to practice their verbal language skills if they cannot make it to the meetings, and so, I think that audio or video responses like these [in Stronger & Clearer] might be the answer.

The use of a technology tool, like Padlet (n.d.), seemed not only advantageous to provide *opportunities for rich language and literacy exposure and practice* but also important during the COVID-19 pandemic, when students were more likely to miss synchronous sessions. Additionally, Ms. Severn explained she would.

definitely use these routines in my class as ways to scaffold conversations, because I have found it extremely helpful to guide students on how they should have conversations and tell them the goals of conversations, rather than just giving them unstructured collaboration time.

She found that the scaffolds of the MLRs supported students to understand how to engage in *opportunities for rich language and literacy exposure and practice*, helping students understand what rich language mathematical conversations might entail.

Cognitively demanding work

Cognitively demanding work primarily centered around Ms. Severn's tasks and teacher actions, such as prompting her sample students to continually add evidence to their responses or asking students to clarify their ideas (e.g., within Stronger & Clearer and within Clarify, Critique, & Correct), asking guiding questions to help sample students engage in the activity, asking sample students to consider and build on the ideas of sample classmates, and positioning sample students to think like a teacher (e.g., within Co-Craft Questions) to develop questions. Two key illustrations of Ms. Severn's enactment of the multilingual learner core practice of *cognitively demanding work* were her approximations of practice of the routines Stronger & Clearer and Co-Craft Questions.

Build on other's ideas and provide more evidence

First, in Stronger & Clearer, Ms. Severn asked her sample students to provide richer evidence and to clarify their arguments within her approximation of practice of the MLR. This task, as shown in Fig. 3, centered around problem-solving, including constructing and critiquing others' ideas and strategies and providing one's own arguments and evidence, which are known to be cognitively demanding mathematical activities for students (Stein et al., 1996). Prior scholars have also illustrated students' difficulties in writing arguments from evidence (e.g., Choi & Hand, 2020; Yamamoto et al., 2022), and Ms. Severn's task required her sample students not only to reason with evidence, but to consider and to build on the ideas of their classmates. Furthermore, despite the asynchronous nature of her implementation, through her technology choice of Padlet (n.d.), she provided multiple avenues for her sample students to engage in *cognitively demanding work* by giving students opportunities to write responses, to read the responses of their classmates, and even to audio record new responses that built on previous ideas. We begin to see connections between multilingual learner core practices, specifically, how providing students *opportunities for rich language*

and literacy exposure and practice and multiple modes of communication can equip students to engage in *cognitively demanding work*. Through conversing, writing, and using representations around mathematical thinking, students have opportunities to construct and critique others' ideas (as well as their own thinking) to move their own and others' thinking forward.

On a proposed midweek lesson, using Padlet (n.d.), during the Stronger & Clearer approximation of practice, Ms. Severn displayed a prompt to her sample students asking them to contribute audio responses that built on others' ideas. She said,

You add in something that you heard from your partner, something that makes your idea stronger or clearer. So, you want to add extra evidence. You can add a new idea to it. Anything that would help your response be stronger or clearer.

Here, Ms. Severn attended to both *cognitively demanding work* and *opportunities for rich language and literacy exposure and practice* by asking her sample students to engage with their peer's arguments and add to that mathematical work, while engaging with the language in multiple ways.

Think like a teacher and develop questions

Another example of Ms. Severn's attention to *cognitively demanding work* was her positioning of her sample students to think like teachers and develop questions through the MLR Co-Craft Questions. Existing literature has revealed developing mathematical questions and formulating problems is rigorous for students (e.g., Rosenshine et al., 1996; Silver, 1994). In Co-Craft Questions, as shown in Table 2, Ms. Severn tasked her sample students to work through a series of small-group and whole-class discussions using Desmos (Desmos Studio, 2023) to develop questions about a hook. She prompted her sample students to think like teachers as they developed these questions, saying,

I'm going to show you an image or a graph or something, in this case it is an image, and you're really going to be thinking like a teacher. Try to come up with "What could I ask about this if I were the teacher" [questions].

As she facilitated the routine, she asked guiding questions to facilitate sample students to compare and contrast their questions with those of their "peers," so the sample class could work together to refine their questions. Ms. Severn's questions provided a starting place for considering and evaluating questions, supporting *cognitively demanding work*.

Discussion

We address five components: (1) approximations of practice in the midst of COVID-19, (2) technology use within approximations of practice, (3) multilingual learner core practices within approximations of practice, (4) MLRs as a medium for completing approximations of practice, and (5) secondary mathematics methods courses focused on multilingual learners.

Approximations of practice provided a context within which PSTs could learn about and execute the multilingual learner core practices, allowing authentic teaching practices, without being in an actual classroom (Matsumoto-Royo & Ramirez-Montoya, 2021). This was particularly important during the start of the pandemic, to provide PSTs with a platform

to continue to develop their pedagogical practice while not in classrooms. However, the enactments were both likely constrained and expanded. Ms. Severn did not actually have access to students (using “sample” students, instead), which constrained getting to know students’ funds of knowledge and students’ lived realities. However, there were other possibilities that were realized as a result of the pandemic. For example, Ms. Severn was able to consider “students” in this moment with approximations of practice to move her own teaching practice forward using multiple technologies. Our study supports the use of approximations of practice during moments of challenge, like the pandemic, and explores how to support PSTs during these times.

Because approximations of practice are of lower complexity (Campbell & Baldinger, 2022) than a traditional in-class lesson, Ms. Severn seemed to take advantage of the opportunity to try out the core practices with multiple technology platforms. This could have proved especially useful going into a first-year teaching placement during Fall 2020. Ms. Severn turned to these technology tools immediately to find scaffolds and ways to engage her multilingual learners, through the multilingual learner core practices and her asynchronous approximations of practice. In her enactments, Ms. Severn used various web-based platforms to provide multilingual learners with access to multiple modes of communication (Moschkovich, 2002) and cognitively demanding tasks. These online platforms possibly provided Ms. Severn flexibility with her approximations of practice, as Howell and Mikeska (2021) noted, because of her options to control parameters and design aspects within the technology. Supporting PSTs to engage with technology while completing approximations of practice can be useful, particularly during times that require asynchronous enactments.

Approximations of practice also provided an opportunity for Ms. Severn to engage with all of our multilingual learner core practices. In our examination, we compared our core practices to the set of characteristics Grossman et al. (2009) provided for core practices more generally. Some of these characteristics were present in Ms. Severn’s enactments of the multilingual learner core practices (i.e., occur in high frequency, can begin to master). Others are more difficult to identify (particularly within the COVID-19 setting) and to capture and so would require more research (i.e., develop an understanding of students, have capacity to improve student performance). This highlighted the need for ongoing research of core practices.

We found that Ms. Severn enacted all five of the multilingual learner core practices for mathematics teaching through her approximations of practice and enactments of MLRs, providing evidence of multilingual learner core practices as equity-based pedagogies. For example, Ms. Severn provided *opportunities for rich language and literacy exposure and practice* through using the technology platform Padlet (n.d.), having her sample students write responses, listen to peers’ responses, write revised responses, provide verbal responses, and have access to representations, a vast opportunity for multiple modes of communication (Moschkovich, 2002). Through writing and revising, sample students also had opportunities to engage with rich *cognitively demanding work* through completing nonalgorithmic work and self-monitoring their own cognitive processes (Stein et al., 1996), which is often less available to multilingual learners (Iddings, 2005; Planas & Gorgorió, 2004). The approximations of practice add to our understanding of equity-based pedagogies, particularly in mathematics, and they provide a starting point for multilingual learner core practices and their possible classroom enactments.

Ms. Severn’s approximations of practice also allowed us to see how the multilingual learner core practices interacted with one another in complementary ways, such as the above example. There were also a number of ways that the two language-specific core

practices of *opportunities for rich language and literacy exposure and practice* and *identify disciplinary language demands and supports* were related, even if this was not all that unexpected. For example, there was a clear focus on scaffolding in these two core practices. The multilingual learner core practices did not occur in isolation and benefited from being used with other practices, again, adding to our understanding of the practices.

The multilingual learner core practices provide a tangible vision for what these practices look like in the classroom for pre- and in-service teachers, as well as for mathematics teacher educators. Instructors of mathematics methods courses globally can consider multilingual learner core practices because of the important role language plays in the mathematics classroom (Essien, 2018) and because of our need to attend to multilingual learners in these methods courses, regardless of location.

MLRs provided a medium for completing the approximations of practice and learning the multilingual learner core practices. Ms. Severn noted that she would use the MLRs again in her class, particularly to scaffold conversations for students. Lampert et al. (2010) noted that teaching routines are not practiced within a vacuum. In the case of our methods course, the multilingual learner core practices were practiced within the MLRs. Research on MLRs is currently a burgeoning field (i.e., Zahner et al., 2021a, 2021b), and their implementation is argued to lead to more authentic language use in mathematics classrooms (Dimas, 2020). Our study provides insights into a PST's use of MLRs to understand core practices currently missing in the field.

The use of MLRs, approximations of practice, and multilingual learner core practices occurred within a secondary mathematics methods course organized specifically around multilingual learners, currently a rarity. With little current work being done around mathematics methods courses focused specifically on multilingual learners (e.g., de Araujo et al., 2018), this study adds to our knowledge of these methods courses and the practice-based approaches within them.

Limitations and future research

We acknowledge that this study was limited by its focus on a single PST and its focus on a single term of a methods course in the USA. However, with our singular focus, we were able to delve deeply into Ms. Severn's practice and to understand more thoroughly how she enacted the multilingual learner core practices. We also used a variety of data sources across each of her approximations of practice to triangulate the data. Furthermore, we recognize that the present study was situated within a particular national context and within a specific teacher education program that has its own policies, norms, and history. In this setting, a focus on supporting multilingual learners was emphasized within the program structure and coursework for preservice teachers. This afforded a pointed examination of MLRs and multilingual learner core practices, which can be revised and applied across other national contexts and teacher education programs throughout the world. We see this work as a starting point to consider equity-based pedagogies around multilingual learner core practices in mathematics methods courses. To expand this work in the future, we suggest a larger sample size and an in-person context, perhaps across multiple methods courses that have a focus on multilingual learner core practices.

Future research in this area should attend to the further use of multilingual learner core practices in methods courses with PSTs. Expanding our notion and development of

equity-based core practices is important, whether those focus on multilingual learners, as here, or consider equity and social justice more broadly. We must continue to examine what is missing from our core practices, as Zeichner (2012) noted over 10 years ago.

Conclusion

We are in an exciting moment, where there is space to contribute to an ongoing conversation about how the field will continue to define core practices. We provide one such suggestion for equity-based core practices through multilingual learner core practices. Such equity-based core practices have the potential to engage PSTs in the work of equity-based teaching more readily from the outset of their teaching careers.

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Declarations

Conflict of Interest All certify that they have no affiliations with or involvement in any organization or entity with any financial interest or nonfinancial interest in the subject matter or materials discussed in this manuscript.

Informed consent This research was conducted under an approved UCSB Institutional Research Board protocol using informed consent.

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