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Secondary mathematics preservice teachers' perceptions of program (in)coherence

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Abstract

Teacher educators globally have argued that developing coherent programs can combat the fragmentation that often characterizes teacher education and better support teacher learning. Yet, there is little research on coherence in mathematics teacher education, especially from the perspectives of preservice teachers. To that end, in this article, we report how 13 secondary mathematics preservice teachers (PSTs) from one teacher education program perceived their program as coherent, specifically attending to the ideas and practices PSTs engaged with and the settings in which they engaged with those ideas/practices. Based on participatory diagramming interviews and network analysis, we found that PSTs experienced two main sources of incoherence. First, although PSTs had opportunities to learn about equity from multiple settings, they did not perceive that equity and other aspects of mathematics instruction together were coherently organized. Second, PSTs reported learning about two opposing instructional approaches—direct instruction and inquiry-based instruction. PSTs reported that opportunities to learn about inquiry-based instruction were primarily isolated to courses taught by the mathematics education program and were contradicted by learning and experiencing direct instruction in their special education courses, mathematics courses, and field and student teaching experiences. Findings highlight a need to attend to issues of equity, as well as connections among university coursework and between coursework and field. Based on our findings, we conclude with implications for how teacher education programs might respond to and engage with incoherence to support PST learning.

Keywords Program coherence · Mathematics teacher education · Equity · Field experiences · Network analysis · Participatory diagramming

Opportunities to learn in teacher education programs necessarily span a range of settings, which can vary in their promotion and representation of particular ideas and practices (Bain & Moje, 2012). For example, there is the well-established "two-worlds pitfall"

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(Feiman-Nemser & Buchmann, 1985, p. 54), in which new teachers experience differences in practices between university coursework and field placements. Even within the university, preservice teachers can experience inconsistency in the treatment of instructional ideas across their courses (Dack, 2019). To ensure that programs have an impact on teachers' practice, teacher educators globally have stressed the importance of program *coherence* so that preservice teachers (PSTs) have repeated experiences with a set of ideas and practices and opportunities to engage with such ideas/practices across multiple settings (e.g., Bamfield, 2014; Canrinus et al., 2019; Graber, 1996; Zeichner, 2010). For example, researchers in Canada (e.g., Bateman et al., 2008) and France (e.g., Gagné et al., 2013) have studied efforts to establish coherence across courses within a program. In the United States, Grossman et al.'s (2009) study of 15 teacher preparation programs highlighted the importance of connections between university coursework and field experiences. However, research including PSTs' own perceptions of coherence is limited (Canrinus et al., 2017), especially within mathematics education. Greater insights into their experiences could point to missed opportunities within and inform re-designs of teacher education programs. Therefore, this study investigates program (in)coherence reported by a group of secondary mathematics PSTs.

In the following sections, we describe the perspectives that oriented our attention to mathematics PSTs' perceptions of program coherence and review relevant literature. We first, however, provide some background on a vision for mathematics instruction in order to describe the potential goals and content around which mathematics teacher education programs might cohere.

Refining and enacting a vision for mathematics instruction

There is a longstanding "school mathematics tradition" (Cobb et al., 1992) of conventional direct instruction, in which teachers demonstrate—and ask students to replicate—procedures for solving types of problems. For decades, members of the international mathematics education community have been working to shift the teaching of mathematics toward more inquiry-based *pedagogies of investigation* (Nicol, 1999), where students have opportunities to author, explain, and justify their own ideas for solving problems (Boaler, 2008; Maaß & Artigue, 2013). After early progress in Japan, there have been other reform efforts with varying levels of success in other places, including China, Singapore, Australia, England, and the USA (Brown & Clarke, 2013). Situated in their own contexts, such efforts vary in their aims and orientations, but are, in general, informed by years of classroom-based research and the resultant arguments for approaching mathematics teaching and learning differently. Although the options are often over-simplified and reduced to a dichotomy (Munter et al., 2015), multiple illustrative classroom cases across multiple countries establish an array of practices that can be employed as alternatives to the school mathematics tradition. In the USA, such practices have been articulated and promoted by the National Council of Teachers of Mathematics (NCTM; 2014).

Although practices for moving beyond the school mathematics tradition may be increasingly clear, enacting them at a significant level of scale has been challenging (Gates, 2006; Hiebert, 2013). Change efforts have almost certainly been thwarted by the cultural inertia of "how we've always done it." But there have also been arguments from other communities of educators and mathematicians that have countered the reform efforts of mathematics educators (Brown & Clarke, 2013; Schoenfeld, 2004). Those communities also hope to replace conventional mathematics teaching, but with better models of direct instruction, which focus on conceptual understanding and are highly attuned to error correction and circumventing struggle (Munter et al., 2015). With appeals to disciplinary authority (e.g., Wu, 1997) or "evidence-based" practices (e.g., Hughes et al., 2016), such arguments can sound compelling to many.

As the challenge of scale has come into focus for the mathematics education community (Maaß & Doorman, 2014), so too has a shortcoming in the empirical basis for its new vision: attention to equity and diversity. Researchers have argued that pedagogies of investigation have the potential for more equitable classroom experiences (e.g., Boaler, 2002; Hiebert et al., 1997; Nasir et al., 2014). And in the U.S., the NCTM has, for some time, consistently named "equity" as an important principle for guiding reform (e.g., NCTM, 2000). But the research and discourse responding to that principle have been limited primarily to questions of *access* and *achievement* (and "achievement gaps"; Carey, 2014), with less attention to more "critical" dimensions of equity such as *identity* and *power* (Gutiérrez, 2012). Implications of attending to more critical dimensions have not been integrated into the field's characterizations of high-quality mathematics instruction (Barajas-López & Larnell, 2019; Martin, 2009). Consequently, as argued by Martin (2015), for African American, Latinx, Indigenous, and poor students, the benefits that NCTM's espoused instructional vision may afford "are metered by Whites and White design and are contingent on parallel benefits to Whites" (p. 21).

Thanks to the persistence of some scholars—typically scholars of color—(e.g., Aguirre et al., 2013; Frankenstein, 1990; Gholson & Martin, 2019; Ladson-Billings, 1997; Matthews, 1983; Nasir et al., 2008; Shah & Leonardo, 2016), the need for such an integration has become more widely acknowledged. In terms of official policy documents, U.S. progress on that front was well-captured by the Association of Mathematics Teacher Educators' (AMTE; 2017) *Standards for Preparing Teachers of Mathematics*. Authors of those standards specified the knowledge and skills that beginning teachers need, including a deep, integrated understanding of how "the social, historical, and institutional contexts of mathematics affect teaching and learning" (p. 21) and a recognition of the "the key roles identity and power play in mathematics education" (p. 35).

Such changes to teacher education, researchers have argued, are necessary if we hope to move past the school mathematics tradition (Leinwand & Burrill, 2001; Thames & Ball, 2013). At a minimum, teacher education programs need to (re)design curricula, courses, and practice-based learning opportunities for PSTs in line with this vision (Tatto et al., 2020). In that work, mathematics teacher educators likely must navigate competing commitments across an array of settings through which PSTs pass, including different perspectives on pedagogy and how equity is defined and integrated into a program. Such complexities raise questions of coherence, to which we next turn.

Program coherence

Reforming mathematics education through teacher education is challenging. Teacher education aligned with ambitious and equitable instruction must counter the "apprenticeship of observation," where new teachers, through years of observing as schoolchildren, come to know what teachers typically do, and despite years of preparation programs, teach in ways that align more with their own K-12 experiences (Lortie, 1975). Moreover, as noted previously, PSTs often encounter the two-worlds pitfall (Feiman-Nemser & Buchmann, 1985), where they observe instruction in their field placements that do not reflect what they are learning in university courses. In such cases, PSTs may reject the ideas and practices promoted by methods courses as they may not seem relevant or practical to classroom realities (Alsup, 2006; Vacc & Bright, 1999). Even within university settings, different perspectives on teaching and learning can manifest (Boyd & Bargerhuff, 2009; Sheppard & Wieman, 2020), resulting in misaligned experiences across coursework.

Some scholars argue that program coherence can combat these challenges (Bain & Moje, 2012; Darling-Hammond, 2000). Research has provided evidence that more coherent programs have a greater impact on teachers' learning and practice (e.g., Boyd et al., 2009; Graber, 1996; Tatto, 1996). In mathematics teacher education, researchers have found that providing PSTs opportunities to experience inquiry-based instruction, as learners, can support PSTs to adopt perspectives aligned with and enact inquiry-based mathematics instruction after becoming practicing teachers (Bahr et al., 2014; Jao, 2017).

In coherent programs, (1) faculty share common goals for the kind of teachers they want to graduate and (2) learning opportunities are arranged organizationally to achieve those common goals (Tatto, 1996). These two elements reflect the distinctions Feiman-Nemser (1990) made between conceptual and structural aspects of teacher education. Though not mutually exclusive, conceptual aspects of coherence include a shared instructional vision and unifying ideas across programmatic content, whereas structural aspects include program components (e.g., courses, field) sequentially aligned and organized around shared ideas (Hammerness, 2006). Therefore, the *what* of coherence might include a shared instructional vision or shared ideas and practices across settings. The *where* of coherence might include the settings of the university courses and field experiences, and the opportunities to make connections across such settings (Canrinus et al., 2017, 2019). Conceptual coherence, then, foregrounds how ideas and practices are connected (relations between "whats"), while structural coherence attends to how such ideas/practices are organized across settings (relations between "what" and "where").

Organizing learning opportunities around shared commitments, however, does not imply that all faculty "think alike" (Tatto, 1996, p. 176), as what such commitments look like in practice, and the means for achieving them, might vary. For Tatto, diversity in thought affords richer learning opportunities. This is not unlike Buchmann and Floden's (1991) caution against creating teacher education programs so structured that they "tie up all loose ends," as this might prevent PSTs from making connections to unexpected ideas (p. 71). Recent research provides evidence that some incoherence can indeed support learning (e.g., Hebard, 2016). For instance, Dack (2019) found that non-examples of differentiated instruction in the field prompted PSTs to reflect on their understandings and served as lessons of "what not to do."

Cautioning against coherence as "an objective outcome where theory and practice (or standards and curricula and activities) are aligned," Richmond et al. (2019) instead characterized coherence as a process where various stakeholders negotiate questions such as "According to whom? To what end(s)?" (p. 188). As an answer to the latter question, the AMTE (2017) standards noted in the previous section might serve as one candidate, but arguments against the NCTM's (2014) vision of inquiry-based instruction and the limited integration of equity in that vision historically (Martin, 2015) suggest that answers to the question are contested. Regarding "according to whom," little research centering PSTs' perspectives has investigated coherence specific to mathematics teacher education. Understanding PSTs' perceptions is important because "the impact [on PSTs] is often different from what instructors or teaching supervisors may imagine or wish" (Clift & Brady, 2009, p. 331). Thus, this study is oriented by Grossman et al.'s (2008) argument that an important measure of coherence is the extent to which PSTs perceive they have coherent opportunities to learn. We, therefore, foreground PSTs' experiences and perceptions in characterizing the what and where of program (in)coherence.

Most research on program coherence has examined teacher education in general (e.g., Canrinus et al., 2017, 2019; Lamb & Jacobs, 2009). The little research that focuses specifically on mathematics teacher education, however, attends only to the settings of methods courses and field experiences (e.g., Bahr et al., 2014; Jao, 2017) or general aspects of coherence without consideration of the particularities of mathematics education (e.g., extent to which there are "clear links between courses;" Tatto et al., 2012). A notable exception is Mintos et al.'s (2019) study of opportunities to learn about equity in five U.S. secondary mathematics teacher education programs. Though not exclusive to the perspectives of PSTs, Mintos et al. (2019) found that PSTs had several opportunities to learn about equity. By contrast, in their study of mathematics teacher education programs across 17 countries, Tatto et al., (2012) found that, with exception of programs in the U.S., Botswana, and the Philippines, PSTs reported that they rarely or never had opportunities to learn about teaching mathematics to diverse groups of learners. Mintos et al. (2019) also found that, while PSTs' learning about access and achievement was often mathematics-specific, learning about identity and power considered more general issues. However, integrating issues of equity with subject-specific understandings may support PST learning. For example, in science teacher education, Rodriguez (1998) found that PSTs were more likely to make sense of equitable teaching when this learning was integrated into their science methods courses. In addition to integrating equity throughout coursework, Sleeter (2008) argued that programs that coherently address equity should also provide PSTs field experiences in classrooms with historically underserved students, as well as opportunities for cross-cultural community-based learning, where PSTs spend time in and learn about a community that is culturally different from their own.

While these few studies (Mintos et al., 2019; Tatto et al., 2012) provide some glimpses into settings in which PSTs are afforded opportunities to learn about equity in mathematics teacher education, more could be learned about how these learning opportunities are connected to learning about other issues of mathematics education, as well as how they are coherently structured across settings. Therefore, our study contributes to this research base by focusing specifically on secondary mathematics PSTs and how they experience aspects of mathematics teaching and learning as (in)coherent, the extent to which that is aligned with ambitious and equitable mathematics instruction (AMTE, 2017; NCTM, 2014), and how those learning opportunities are organized across programmatic components. That is, we investigate how PSTs perceive the ideas and practices (the what of coherence) promoted across the various settings (the where of coherence) of their teacher education program as coherent. Specifically, the research questions that guided our work were: (1) What ideas and practices do PSTs engage with and from what settings? (2) Across these ideas/practices and settings, where do PSTs perceive (in)coherence, and how do they characterize those instances of (in)coherence?

Methods

Context

This study recruited participants from one undergraduate secondary (grades 6–12) mathematics teacher education program at a university in the Midwest region of the USA during the second semesters of the 2019–2020 and 2020–2021 academic years. Organized to be completed in four years of undergraduate study, the program includes a variety of courses and field experiences (see Table 1). All secondary education majors complete a common set of courses from multiple departments. In addition to those, mathematics education majors complete mathematics courses in the mathematics department and four–five courses taught by mathematics education instructors, including one or two mathematics content courses for teachers (one focused on algebra and the other on geometry) and three pedagogy courses.

The typical sequence through the five mathematics education courses—and the one that most participants in our sample followed—is to complete one pedagogy and one mathematics content course in each semester of Year 3 and the third and final pedagogy course during the second semester of Year 4, following the student teaching internship in the first semester of that year. Those five courses are designed to align with AMTE (2017) standards (though they likely fall short of meeting all those standards). Year 3 courses are intended to support PSTs in becoming proficient in equitably providing opportunity for and attending to student reasoning by employing cognitively demanding tasks (Stein et al., 1996) and building lessons around students' inquiry (Smith & Stein, 2018; Stein et al., 2008). Following student teaching, the Year 4 pedagogy course emphasizes critical perspectives of school mathematics, with attention to refining instructional vision and practice, classroom assessment, curriculum, and institutional settings and change—with consistent attention to issues of equity throughout.

Beyond the mathematics education-specific components, the curriculum for the teacher education program is organized primarily according to the state's requirements for teacher certification, with coordinators and instructors across departments having considerable autonomy in course design. Thus, it is possible that PSTs are introduced to different, even contradictory, perspectives and practices across their program experience. And this extends beyond just university coursework. Across the many hours of "field experience" that PSTs spend in middle and high school mathematics classrooms, prior to and including student teaching, they likely encounter instructional practices that are different from what is promoted in the mathematics education-specific component of the program.

It is important to consider how our relation to the teacher education program and the PSTs may have influenced aspects of the study and the findings we report. The second author is an instructor in the program and contributed to the design of its curriculum and structure. Among the participants in this study, those interviewed following their student teaching internship would have, at the time, been enrolled in the Year 4 mathematics pedagogy course with the second author, who would have also served as their bi-weekly seminar leader during fall student teaching. Participants interviewed prior to student teaching would not yet have had any sustained interaction with the second author, but might have been aware of his role in the program. The first author interned in the second Year 3 pedagogy course with the second author, which included participants in the 2020 cohort.

Following Morris and Hiebert (2017), we view our subjectivity as both an asset and a potential pitfall to our analysis (Peshkin, 1988). The very origins of our inquiry into "coherence" were rooted in our intimate knowledge of the teacher education program (particularly the components specific to mathematics education), which also attuned our data collection and analysis to important nuances in participants' responses. But our familiarity and involvement with the program could also introduce bias into our study. The presence of the second author as a faculty member and co-designer of the program may lead to a respondent bias (Lincoln & Guba, 1985) if participants feel pressure to respond favorably about the program and their experiences in it. Likewise, although this was not a program evaluation, our data collection and analysis could be biased by a kind of "developer effect"

Table 1 Descriptions and labels of required courses, typical year completed, and host department

Course name	Description	Year	Dept	Label
School Health & Student Well-being	Explores and analyzes the critical role schools and teachers play to address students' physical, social, and emotional well-being. Focus areas include school safety, nutrition policy and health education	1	E	School Health
Experiencing Cultural Diversity	Examines cultural diversity in U.S. Society, to increase self-awareness related to worldviews and beliefs about diversity issues, and to increase understanding of the intersections of multiple group identities	1	E	Cultural Diversity
Empowering Learners with Technology	Examines models and strategies for integrating technology into the teaching and learning process, with a focus on transformative, meaningful learning instead of passive technology use	1	I	Tech
Schools, Community and Society I	Focuses on schooling in American society, the school community, the school culture and students' lives and identities. Studied are the political, cultural, and economic conditions of the schools	2	L	SCS
Inquiry into Learning	Focuses students on the central themes of learning and teaching. Emphasis will be placed on the interaction of theory, philosophy and practice as related to the field of education	2	E	Learning
Introduction to Teaching Mathematics in Middle & Secondary Schools	The nature of school mathematics; learning mathematics through sense-making and problem solving; middle and high school students' understanding of mathematics; influences on students' opportunities to learn mathematics; local, state, and national standards for the learning of mathematics; and local, state, and national organizations dedicated to K–12 mathematics education	3	L	Math Pedagogy
Teaching Mathematics in Middle & Secondary Schools 2	Planning and enacting sequences of lessons that (a) build from students' mathematical interests and sense-making, and (b) attend to issues of status and power in the classroom (e.g., constructing competence, composition of groups, "differentiation"); "judicious telling"; "classroom management"; and foundations of classroom assessment	3	L	Math Pedagogy
Reading and Writing in the Content Areas I	Explores specific ways teachers can help students improve reading and writing skills in content areas and ways they can be taught	3	L	RWICA

Table 1 (continued)

Course name	Description	Year	Dept	Label
Reading and Writing in the Content Areas II	Explores specific reading and writing content area strategies, with a focus on struggling students and new investigations in disciplinary literacy	3	L	RWICA
Behavioral Management for Exceptional Learners	Study of classroom management and applied behavior analysis strategies. Focus on teacher as decision-maker in the design, implementation, evaluation of individual and group management programs	3	S	SPED
Teaching the Exceptional Learner	Addresses topics in the foundations of pedagogy for students with disabilities in inclusive settings, including the roles and responsibilities of the general educator and related service personnel with respect to special education law and policy, behavior management, universal design for learning, and evidence-based practices	3	S	SPED
Teaching, Engaging and Assessing Middle-Level Students *	In this course students will learn about the specific and individual needs of middle-level students and develop the skills and understandings to meet these needs	3	L	TEAMS
Connecting Geometry to Middle and Secondary Schools	A detailed study of (Euclidean) geometry concepts and practices typical in the U.S. secondary curriculum, from an advanced perspective. Topics may include: Euclidean foundations; Pythagorean Theorem; transformations, congruence, invariance, symmetry and similarity; geometric measurement; Euler Characteristic; and proof	3	L	Math for Teachers
Connecting Algebra to Middle and Secondary Schools *	A detailed study of integer and rational arithmetic and algebra. Topics may include: Binomial Theorem, induction, division algorithm, Euclid's Algorithm, Fundamental Theorem of Arithmetic, Pythagorean triples, modular arithmetic andgeneralizations to polynomials, matrices and other axiomatic structures	3	L	Math for Teachers
Student Teaching		4	–	
Teaching Mathematics in Middle & Secondary Schools 3	Development of critical perspectives of school mathematics and various aspects of classrooms and institution, including foci on refining instructional vision and practice, classroom assessment, curriculum, and institutional settings and change—with consistent attention to issues of equity throughout	4	L	Math Pedagogy

Table 1 (continued)

Course name	Description	Year	Dept	Label
Teaching English to Speakers of Other Languages	Linguistic and pedagogical principles of teaching English to speakers of other languages	4	L	TESOL
Schools, Community, and Society II	Designed to transition students into the teaching internship through study of teacher roles, school organizations and cultures, and community contexts	4	P	SCS

Note. Courses with an asterisk denote those taken by only students in the Middle School Mathematics major. Host departments include Educational Leadership & Policy Analysis (P); Educational, School & Counseling Psychology (E); School of Information Science & Learning Technologies (I); Learning, Teaching, & Curriculum (L); and Special Education (S)

(Wolf et al., 2020) if our conclusions cast a too-positive light on a program in which we are invested.

To mitigate the threat of respondent bias, as explained shortly, all data were collected only by the first author, who had no prior relationship with most of the participants (i.e., the first author had not met most of the participants prior to recruiting them in the study). Moreover, all participants remained anonymous to the second author throughout. To minimize the potential bias stemming from our relation to the program, we chose not to conduct our own observations and assessment of coherence, but rather to invite PSTs' self-reporting of their experiences and examine those responses with a novel use of social network analysis, all of which we explain in the subsections that follow.

Participants

Participants included 13 secondary mathematics PSTs, who were mostly white, and mostly women. All members of 2020, 2021, and 2022 cohorts of the mathematics teacher education program were invited to participate. The sample included four (of 10), four (of nine), and five (of 18) participants from each cohort, respectively. Seven participated during the semester following their 16-week, full-time student teaching internship under the mentorship of an experienced secondary mathematics teacher. Six participated during the semester immediately prior to student teaching, which included at least 24 hours of field experiences in secondary mathematics classrooms. Eight PSTs were in the program during the 2020–2021 academic year. Those who had completed student teaching did so in a hybrid and/or virtual setting, in addition to in-person experiences. Those in the semester prior to student teaching did not have typical classroom field experiences and instead engaged in watching and reflecting on videos of teachers' practice, coordinated by instructors of the pedagogy courses. Five PSTs were interviewed in March and April of 2020, and eight were interviewed in March and April of 2021. Table 2 lists the participants, whether they had completed student teaching, their cohort, and year interviewed.

Data collection

The lead author (Phi) conducted data collection and analysis. Phi engaged PSTs in participatory diagramming interviews (Bagnoli, 2009; Bravington & King, 2019). In this approach, participants are invited to create diagrams that represent their experience, which entails deciding the elements to include, layering the elements in ways that reflect associations among them, and describing the visual. Participatory diagramming is useful for representing complex phenomena, as the process affords opportunities for participants' meaning-making and further interviewing as they interact with the diagram (Crilly et al., 2006). For this study, PSTs made decisions regarding which ideas/practices and settings to include and how to arrange and connect them, and were invited to share their rationales for doing so.

PSTs participated in one 60–90-min interview conducted through Zoom video conferencing and shared Google Slides. The interview protocol was piloted and refined with two PSTs not included in this sample, and not in the 2020–2022 cohorts (see Appendix 1 for interview protocol). First, to orient them to describing *ideas and practices* about teaching, PSTs were asked to describe what they would look for in a mathematics classroom to determine whether the instruction was high-quality (Munter, 2014). Beyond the ideas and practices those questions surfaced, PSTs were asked to describe additional ideas/practices

Table 2 Participant information

Name	Pre/Post student teaching	Cohort	Year interviewed
Ellie	Post	2020	2020
Gail	Post	2020	2020
Rosa	Post	2020	2020
Tori	Post	2020	2020
Bella	Post	2021	2021
Cady	Post	2021	2021
Walter	Post	2021	2021
Helen	Pre	2021	2020
Alan	Pre	2022	2021
Bennett	Pre	2022	2021
Fiona	Pre	2022	2021
Francisco	Pre	2022	2021
Lindsay	Pre	2022	2021

they engaged with in their teacher education program. This was then followed by questions regarding the *setting*s in which they engaged with those ideas/practices ("have you talked about this in any of your courses or field experiences?").

Then, PSTs used Google Slides to construct a representation of connections (where they perceived them to exist) across what/where they reported learning about teaching. PSTs were encouraged to (1) include ideas/practices and settings reported earlier in the interview, along with anything additional that emerged and (2) represent ideas/practices with pink ovals and settings with yellow ovals, and to connect those ovals with lines. Figure 1 displays an example of a diagram one PST (Gail) created during her interview. After creating their representation, Phi prompted PSTs to describe what they created ("can you walk me through your diagram?"), with follow-up questions about particular connections. For example, if PSTs connected an idea/practice to multiple settings, Phi asked questions such as "did all these courses talk about this idea/practice in similar ways, or differently?" Finally, PSTs were shown a list of the courses required for the program, and asked to share about what, if anything, they had learned about teaching in the courses they had not yet reported. If PSTs reported a new idea/practice that was not yet included in the diagram, Phi asked questions such as "You talked about this idea. How do you see this fitting or not fitting in your diagram?" to elicit PSTs' perceived connections to other ideas, as well as settings in which they engaged with that idea. Interviews were audio- and screen capture-recorded, and transcribed.

Analysis

Data analysis consisted of two complementary steps: (1) qualitative coding of interviews and (2) transforming interview data into network data to create and visually analyze network maps. Network visualization revealed structure among the ideas and practices PSTs engaged with from the various settings of the teacher education program that might have been overlooked with qualitative coding alone (Contandriopoulous et al., 2018; Koponen et al., 2019), while qualitative analysis gave meaning to the otherwise abstract ties and vertices that make up a network map (Bernhard, 2018; Crossley, 2010). In other words,

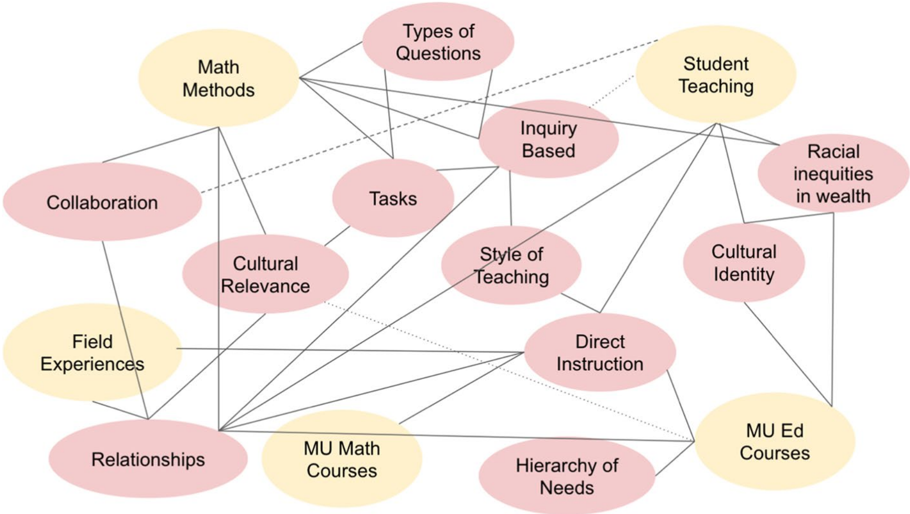

Fig. 1 Example of diagram created by PST during participatory diagramming interview

this mixed method approach allowed us to examine not only the structure among ideas/practices and settings, but also characterize how PSTs perceived that structure (Froehlich et al., 2020).

Qualitative interview coding

Phi began analysis of interview transcripts by sorting relevant excerpts into three categories: ideas/practices, settings, and coherence issues. These categories were informed by the research questions. By *excerpt*, we refer to "meaning unit," or "words, sentences or paragraphs containing aspects related to each other through their content and context" (Graneheim & Lundman, 2004, p. 106). For the first category, *ideas/practices*, Phi developed codes inductively to capture the content of participants' descriptions. For example, "inquiry-based instruction" and "direct instruction" were developed as two separate codes since PSTs described these two ideas differently, verbally in their interview and/or visually in their diagram (see Fig. 1 for an example). Phi drafted codes by first identifying elements reported among the five participants interviewed in 2020 and then applied the codes to the first four participants interviewed in 2021, adding and refining codes as needed. Phi then revisited the first set of interviews to see if the new codes applied, and, finally, analyzed the remaining interviews with the full set of codes.

Constituting the second category, *settings* were based on the required components of the teacher education program. As listed in Table 1, these included university courses, field experiences (which accompany particular courses), and the semester-long student teaching internship. Settings were collapsed and assigned the same abbreviated label when PSTs reported similar ideas from them. For example, all three of the mathematics pedagogy courses were collapsed into one setting, as were the two courses taught by the special education department.

Finally, PSTs' descriptions were categorized as *coherence issues* if they included connections (or disconnections) that might contribute to coherence (or incoherence), where "connection" refers to a relation between two ideas/practices or between an idea/practice and setting. Informed by the literature on structural and conceptual aspects of coherence, Phi identified different types of (dis)connections by comparing the different ways PSTs related ideas/practices to settings or other ideas/practices. Regarding structural coherence, Phi coded for when PSTs reported engaging with an idea/practice in a particular setting (i.e., the idea/practice and setting were connected). Because some PSTs reported having few opportunities to enact a particular idea/practice in a setting, some connections were qualified as *limited*. Regarding conceptual coherence, Phi attended to when PSTs described an idea/practice in relation to another. Two ideas/practices were considered connected if PSTs' talk included both ideas/practices in the same excerpt. Sometimes, excerpts that included two ideas/practices explicitly described them as different or unrelated. In such instances, the ideas/practices were considered disconnected. In short, we found the following types of connections and disconnections: (1) learning or enactment of an idea/practice in a setting; (2) limited enactment of an idea/practice in a setting; (3) connection between ideas/practices; and (4) disconnection between ideas/practices. While an individual connection reveals PSTs' relation of a particular idea/practice to a particular setting or with another idea/practice, connections together collectively illustrate how PSTs perceive multiple ideas/practices as conceptually integrated, and structurally organized across program components. Similarly, disconnections, limited connections, and/or a lack of connections together reveal how PSTs perceive multiple ideas/practices as disparate or unconnected, with few opportunities to engage with such ideas/practices across multiple settings.

Network analysis

After completing the qualitative analysis of PSTs' interview responses, network analysis was then employed to map the (dis)connections (or ties) among ideas/practices and settings (or vertices). Network analysis allowed us to visually identify and represent structures and interconnections underlying PSTs' descriptions of their engagement with ideas/practices across settings, which facilitated an inductive interpretation of their perceptions of coherence.

We used the "igraph" package in R (Csardi & Nepusz, 2006). The first step—to transform interview data to network data—was accomplished by creating data matrices that detailed, for each participant, the different ties among ideas/practices and settings they reported. Then, we calculated degree centrality for each vertex (setting or idea/practice). Degree centrality captures the number of ties to a given vertex, and measures how central a vertex is in a network. With respect to PSTs' perceptions of coherence, an idea/practice with a high degree means that the PST connected it to several other ideas/practices (conceptual coherence) and/or described learning or enacting it in multiple settings (structural coherence). We also calculated frequencies for how many PSTs reported a particular tie or vertex.

Network maps were constructed for each participant, and for all 13 participants together. Vertex size was proportional to its degree centrality (number of ties to the vertex), and different types of (dis)connections were represented by different colored ties—(1) blue tie represents learning or enactment of an idea/practice in a setting; (2) green tie represents limited enactment of an idea/practice in a setting; (3) orange tie represents connection between ideas/practices; (4) red tie represents disconnection between ideas/practices. In addition, for network maps combining all participants, ties were given a weight to represent the number of PSTs who identified that particular tie (represented by tie thickness). The "igraph" package, by default, uses the Fruchterman–Reingold (1991) method to construct maps, where edges are similar in length and cross each other as little as possible. As a result, vertices that share more (weighted) ties are closer to each other, and those with fewer (weighted) ties are farther. Furthermore, vertices with greater degree centrality are pushed to the center and clusters of vertices are made visible (Contandriopoulos et al., 2018; Decuypere, 2020).

Findings

In this section, we first provide an overview of the findings by sharing network maps that illustrate the connections and disconnections PSTs reported among ideas/practices and settings. In doing so, we briefly outline the main sources of conceptual and structural coherence and incoherence perceived by PSTs. Following that, we further investigate instances of (in)coherence and describe how PSTs characterized them.

Table 3 presents the results of the inductive coding, defining the ideas/practices reported by at least half of PSTs, and the settings in which they engaged with these ideas/practices. Each idea/practice and setting is represented by a vertex in the network maps. PSTs related ideas/practices with settings as well as with other ideas/practices. Both connections—among ideas/practices, and between ideas/practices and settings—are represented in Fig. 2. The larger vertices for inquiry-based instruction (IBI), group work, meeting students' needs, and relationships indicate that PSTs connected them to several other ideas/practices and/or reported learning or enacting them in multiple settings.

Two clusters emerged from the analysis. In the lower left of Fig. 2, several PSTs connected IBI to rich tasks ($n=9$), conceptual understanding ($n=8$), math conversations ($n=8$), conceptual questions ($n=8$), and activity structure ($n=8$) (represented by orange ties). Toward the right is another cluster with equity-related ideas regarding meeting students' needs, bias and privilege, students' identities, relationships, and relevance. There are, however, few (orange) ties between the two clusters, and the ties that are present are of low weight (i.e., few PSTs reported these connections). This indicates something about conceptual incoherence: the ideas/practices PSTs associated with equity were not highly connected to other ideas/practices about mathematics teaching.

To further explore conceptual coherence, we share the network map with only ties between ideas/practices (Fig. 3). Of all ideas/practices, the vertex representing IBI is the largest, indicating that PSTs reported the most connections between IBI and other ideas/practices. PSTs also reported learning about another instructional approach, direct instruction, and that it was disconnected to IBI (represented by red tie). This incoherence is related to the idea of conceptual understanding, as IBI was connected (orange tie), but direct instruction was disconnected (red tie). While there are some ties among equity-related ideas, though not of high weight, there was also a disconnect between the ideas of meeting students' needs and bias and privilege. These conceptual incoherencies will be further discussed in subsections that follow.

To examine structural coherence, Fig. 4 displays the network map with only ties reporting the settings in which PSTs engaged with an idea/practice. The relatively large sizes of the vertices representing equity ideas indicate that PSTs reported learning about such ideas from multiple settings. While there was structural coherence regarding PSTs' learning

Table 3 Ideas and practices, their descriptions, the settings from which PSTs engaged with, and number of PSTs that reported idea/practice

Idea/Practice	Description (based on PST interviews)	Setting											Number of PSTs
		SPED	Math Ped	Math for teachers	RWICA	Math	SCS	Cultural diversity	Learning	School health	Field	Student teaching	
Meeting students' needs	Teachers providing students what they need to succeed	9	5		3					6			13
Direct instruction	Teacher-led instruction, typically in the form of lecture and demonstrations	7				9					8	6	12
Inquiry-based instruction	Instruction that provides opportunities for students to make sense of, explore, and discover important mathematics concepts		11	3							3 (L)	5 (L)	11
Conceptual questions	Questions that support students to think deeper about mathematics concepts		7		2								11
Rich tasks	Tasks that allow for multiple strategies; low floor, high ceiling tasks		10	2									11

Table 3 (continued)

Idea/Practice	Description (based on PST interviews)	Setting											Number of PSTs
		SPED	Math Ped	Math for teachers	RWICA	Math	SCS	Cultural diversity	Learning	School health	Field	Student teaching	
Conceptual understanding	Understanding mathematics concepts and big ideas, in addition to procedures										2 (L)		11
Group work	Opportunities for students to collaborate with peers	5	7		4								10
Math conversations	Talk between students about their mathematical thinking												10
Relationships	Relationships between students and teacher		7				4		3			5	10
Activity structure	"You, y'all, we;" launch-explore-summarize; Five Practices for Orchestrating Discussions		5										9
Relevance	Real-life connections; culturally relevant		7								2	2	9

Table 3 (continued)

Idea/Practice	Description (based on PST interviews)	Setting											Number of PSTs
		SPED	Math Ped	Math for teachers	RWICA	Math	SCS	Cultural diversity	Learning	School health	Field	Student teaching	
Bias and privilege	Teacher's positionality; biases about certain student groups; role of privilege in education		7				5	3					8
Students' identities	Students' gender, racial, mathematical, and other identities; students' home lives		2				2	3	2	2			7
Incorporating reading/writing	Incorporating reading and writing into mathematics teaching				7								7

Settings were limited to top four, and those at least two PSTs reported. "L" represents settings where PSTs reported limited engagement

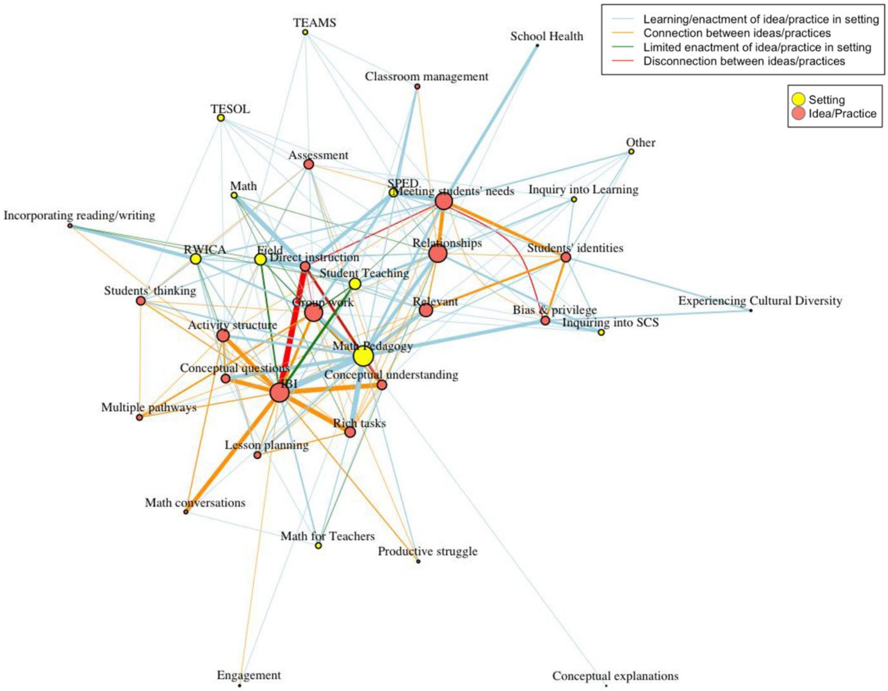

Fig. 2 Network map for all 13 participants, with all ties

about equity, opportunities to learn about IBI and direct instruction were not structurally coherent. From IBI is only one thick blue tie (to mathematics pedagogy courses), and two green ties (limited enactment in field and student teaching). From direct instruction, however, are four thick blue ties (to mathematics and SPED courses, field, student teaching). This indicates that, though several PSTs reported learning about or experiencing direct instruction in multiple settings, PSTs primarily engaged in learning about IBI in only one setting, and that these settings were mutually exclusive.

The three network maps together reveal that PSTs did not perceive that opportunities to learn about equity and other aspects of mathematics instruction were coherently organized. In the next two subsections, we report findings according to the two themes of (1) equity and (2) instructional approaches.

Issues of equity

PSTs described learning about issues of equity in three different ways: meeting students' needs ($n=13$), students' identities ($n=7$), and bias and privilege ($n=8$). Relationships ($n=10$) and relevance ($n=9$) also intersected with ideas about equity. Meeting students' needs was often described in terms of equity versus equality. Lindsay, for example, explained that "equity was more about meeting the needs of your students… it's not about everybody having the same. It's about everybody having what they need." Regarding students' identities, PSTs reported learning about different groups of students and reflecting on their own identity. Finally, the ideas of bias and privilege pertained to

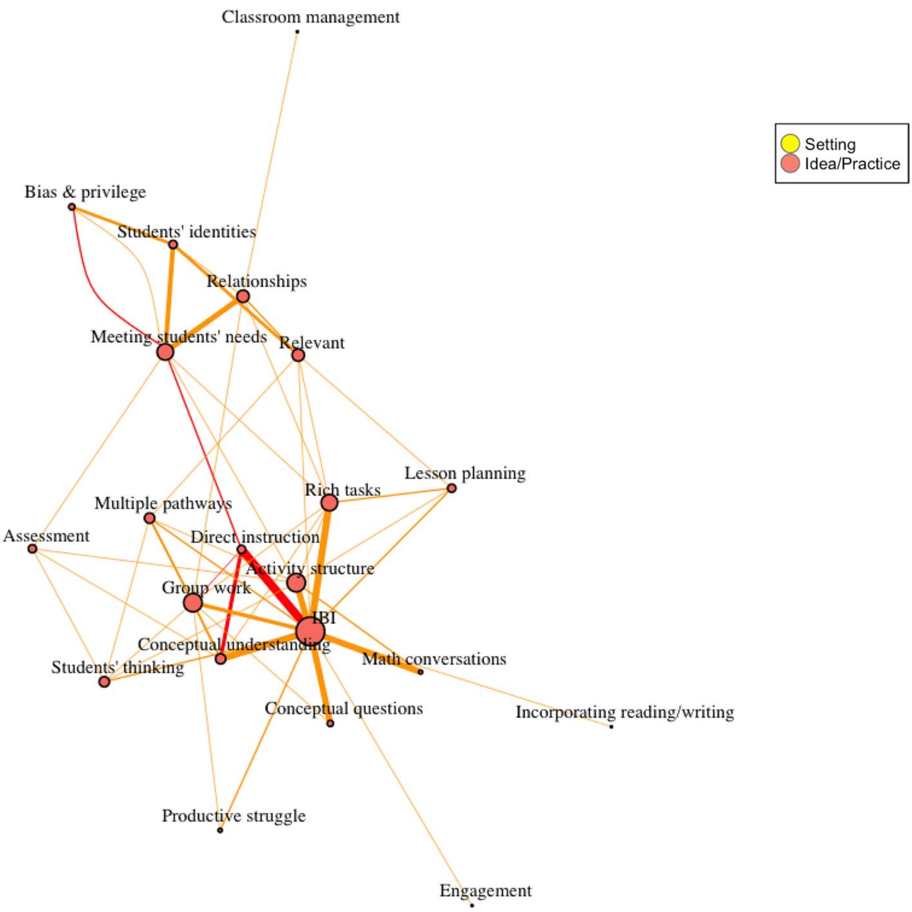

Fig. 3 Network map for all 13 participants, with ties between ideas/practices

reflecting on PSTs' own implicit biases and the structures and systems that may privilege or disadvantage certain student groups.

As noted previously, in general, PSTs perceived their learning about equity was structurally coherent as they experienced repeated opportunities to engage with such ideas in multiple settings. In fact, some PSTs expressed that multiple settings engaged them in thinking about issues of equity in very similar ways. For example, Ellie explained that "every education course, we learned about racism…I feel like they've all talked about it in the exact same way." Alan shared similar experiences, reporting that his mathematics pedagogy, School Health, and TEAMS courses all used the same "boxes" metaphor for equity (see Fig. 5): "They used the same, the same picture representation of like the three kids standing on the box." These quotes suggest that PSTs perceived some degree of structural coherence in their learning about equity.

Though PSTs perceived equity as a unifying theme across their coursework and were able to make connections between some aspects (PSTs connected students' identities to meeting students' needs and to bias and privilege), there were few connections between other aspects—and in fact a *disconnection* between meeting students' needs

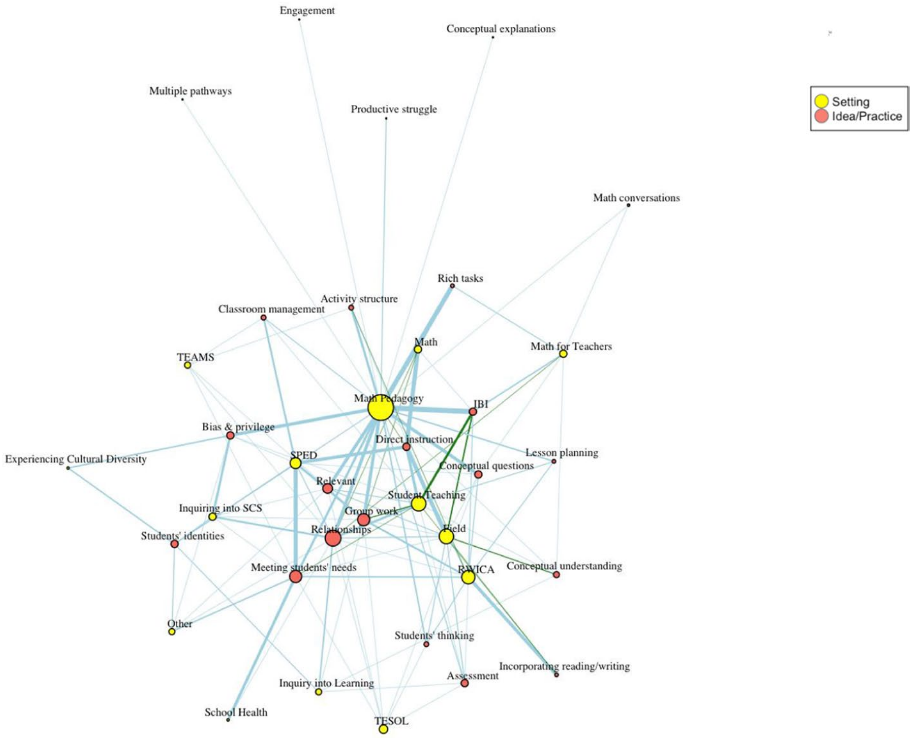

Fig. 4 Network map for all 13 participants, with ties between settings and ideas/practices

and bias and privilege. Furthermore, as we alluded to with the two clusters of vertices in Fig. 2, the ideas/practices PSTs associated with equity were not highly connected to other ideas/practices about mathematics teaching. We further describe how PSTs characterized these conceptual incoherencies in the following paragraphs, after first reporting how PSTs described the ideas of meeting students' needs and bias and privilege,

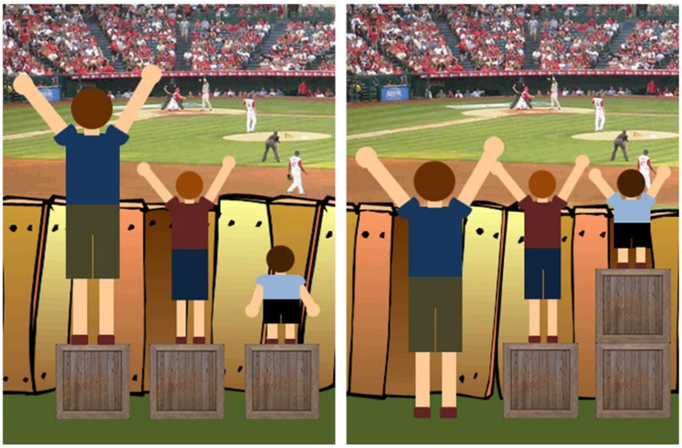

Fig. 5 Graphic commonly used to depict equality (on the left) and equity (on the right)

and how they connected to students' identities and, if applicable, relationships and relevance.

Meeting students' needs

PSTs reported learning about meeting students' needs in their special education courses ($n=9$), School Health course ($n=6$), mathematics pedagogy courses ($n=5$), and RWICA courses ($n=3$), represented in Fig. 4 with blue ties. These "needs" were often academic in nature. Especially in reference to special education courses, meeting students' needs took the form of adjusting instruction and providing interventions and accommodations. Helen, for example, explained "during the lesson or where I am instructing them, I can also adjust my instruction or my lesson to also include things that might help those students. So yes, SPED makes me think about also the students that could be struggling more. And what can I do for them."

Beyond academic needs, PSTs described that meeting students' needs incorporated learning about students and their lives outside of school. From their School Health course, PSTs often reported learning about Maslow's hierarchy of needs and adverse childhood experiences. Regarding the former, Gail described:

> Going back to what I was saying with hierarchy of needs, just that if they're hungry. If they haven't gotten sleep that they're not going to be able, or doing math is not going to be their main focus at the time. So I think being cognizant of if students are getting those needs. And why and how that can affect their experience in class…As the semester goes on, just like the more that you get to know them and build a relationship like they might become more vulnerable and be able to communicate those needs to you.

Here, Gail explained that students' experiences in mathematics classrooms are shaped by factors beyond teaching and learning. She also argued, as did six other PSTs, that learning about and meeting students' needs requires building relationships with students. In Fig. 3, this is represented by an orange tie between relationships and meeting students' needs ($n=7$).

In addition to relationships, meeting students' needs was connected to the ideas of bias and privilege ($n=1$) and students' identities ($n=6$), represented with orange lines in Fig. 3. Francisco, for example, explained that he went to a school that was "pretty diverse," but that the majority of teachers were white women:

> And so being able to connect or like have some sort of understanding of like the different cultures that students are coming from especially if we're trying to make education accessible for all students and not just teaching in a general way that really only benefits certain groups. So it's really important to know about the students, where they're coming from and for, like, students with disabilities, how we can adjust to make sure we are adequately teaching them. So, students that are from different racial backgrounds, how can we adjust material that will connect to them as well, make them feel included and that works with a lot of their identities just to make students feel a part of the curriculum.

Here, Francisco described that teachers, particularly those from dominant backgrounds, should be culturally competent so that their instruction is “accessible for all students.” To mitigate instruction that “only benefits certain groups,” teachers should learn about students’ identities in order to, for example, adjust instruction to meet the needs of students with disabilities, and ensure that students of color feel included—one strategy for which is to create relevant curricula. This connection between relevance and students’ identities was reported by four PSTs.

Bias and privilege

PSTs reported learning about issues related to bias and privilege in their mathematics pedagogy courses ($n = 7$), SCS courses ($n = 5$), and Cultural Diversity course ($n = 3$). The idea of bias and privilege pertained to one’s own implicit biases, and systems and structures. With respect to the former, PSTs made connections to students’ identities ($n = 4$), and the implicit biases teachers might have about certain student groups. For example, Fiona explained that:

> This class [Cultural Diversity] actually does connect a lot with the SPED class right now, because when we’re talking about identifying learning disabilities and intellectual disabilities, there is so many studies on like boys are diagnosed with ADHD at like a 66% rate but it’s really, there’s no, you know, like gender doesn’t matter. It should be 50–50, so you’re thinking a lot about like biases in teaching too. Like am I weighing toward somebody that I think is going to be smarter, I think that is going to struggle, because of the way that they look or the way they were raised.

Here, Fiona shared that she made connections between conversations in different courses, and this connection centered on the biases teachers might have about different student groups, and how those biases might affect the learning opportunities afforded to students.

While considerations of implicit biases extended to all facets of social identity, PSTs reported that their learning about systems and structures centered race. PSTs reported learning about redlining and the history of segregation, both nationally and in local contexts. For example, Gail shared that “both of my classes this semester [mathematics pedagogy and SCS] actually talked about how income and wealth of Black communities is lower because of the redlining and the home loans that were and were not given a long time ago, to give white people more power.”

Characterizing incoherence with issues of equity

Though some PSTs were able to connect students’ identities to meeting students’ needs and bias and privilege, there was only one connection between the latter two (Francisco’s quote). Moreover, two PSTs described these two ideas as separate and unrelated (represented in Fig. 3 by red tie). For example, Tori explained “the contrast is math education and [SCS] are focusing on your positioning and how you are influencing students, whereas special education and [TESOL] is like ‘Here’s the student, how are you going to change your practice?’” Here, Tori described that different courses engaged her in thinking about different ideas about equity: while her math pedagogy and SCS courses focused on teachers' biases, her special education and TESOL courses focused on meeting students' needs through adjusting instruction, and that her learning about these "contrast[ing]" ideas were unconnected. This quote, in addition to there being only one reported tie, indicates that PSTs did not perceive strong connections between the equity-oriented ideas of meeting students' needs and bias and privilege, nor opportunities for sensemaking across settings.

Though PSTs reported some connections among equity-related ideas, and had opportunities to engage with these ideas across various settings, there were few connections to other aspects of mathematics instruction (represented by two clusters in Fig. 2). Not only were these connections infrequently reported (represented by low-weighted orange ties in Fig. 3), some PSTs actually suggested that these ideas were unrelated. Gail, for example, explained that "my math methods class, the one with [second author], I'd say types of questions, tasks, inquiry-based, cultural relevance, those are the main things we've talked about and they don't really connect with the conversations that I've experienced in my other classes." As a reminder, in a previous quote, Gail shared that both her mathematics pedagogy and SCS courses provided opportunities to learn about race and racism. Together, these quotes reveal that, though she engaged in learning about equity in multiple settings, including her mathematics pedagogy courses, she did not perceive that those conversations—about "redlining" and "racial inequities in wealth"—were connected to the conversations about pedagogy, including those about IBI and relevant instruction. This quote, in addition to there being two clusters in Fig. 2, suggests that PSTs did not perceive conceptual coherence between equity-related ideas and other aspects of instruction, and that this incoherence was experienced within their mathematics pedagogy courses, and also between other courses.

Instructional approaches

PSTs reported learning about and experiencing two main instructional approaches: direct instruction and inquiry-based instruction (IBI). Of the 13 PSTs, 12 reported engaging with direct instruction, while 11 reported learning about IBI. Regarding the former, Alan explained "when I think about direct instruction, I think about like the teacher being the sole educator to the students." By contrast, PSTs described IBI as providing opportunities for students to make sense of, explore, and discover important mathematical concepts. Rosa, for example, shared that "inquiry-based allows students to create their own understanding. Like, they're able to figure it out by themselves with the assistance of a teacher." In what follows, after reporting how PSTs described IBI and direct instruction, and the settings in which they engaged with these ideas, we characterize how PSTs perceived the incoherence between the two instructional approaches.

Inquiry-based instruction

While PSTs reported sustained and iterative opportunities to learn IBI from their three mathematics pedagogy courses ($n=11$), and some from their mathematics for teachers courses ($n=3$), their engagement with these ideas was disconnected from other courses. Furthermore, PSTs had only limited opportunities to enact IBI in field ($n=3$) and student teaching ($n=5$) (represented by green lines in Fig. 4). In other words, PSTs reported opportunities to learn about IBI came primarily from courses specific to the mathematics education program.

PSTs defined IBI through a collection of instructional ideas and practices, including rich tasks ($n=9$), conceptual understanding ($n=9$), math conversations ($n=8$), conceptual questions ($n=8$), activity structure ($n=8$), and, to a lesser extent, group work ($n=5$) (represented by orange ties in Fig. 3). This robust definition of IBI is exemplified by Ellie's description of an inquiry-based lesson:

> So inquiry-based, like the first part would be the launch. Like how are you going to launch this task with the kids, how are you going to catch their attention and keep them interested and wanting to do it? And then throughout the lesson you should be questioning kids, like what questions are you going to ask the students that are stuck and get them to the place without giving away the answers… And at the end they kind of present their information, right and wrong.

Here, Ellie described that IBI involves a certain activity structure where, after launching a task, students work collaboratively in small groups while the teacher monitors the classroom and asks questions that support students in making sense of mathematical concepts. The lesson then culminates with a teacher-facilitated discussion (math conversation) around the lesson's central mathematical ideas as students share their thinking and make connections across strategies and representations.

Central to Ellie's description of IBI are the ideas of rich tasks and activity structure. Regarding the former, Bella explained that "a good math task doesn't have a clear path like how to get there. It's going to produce multiple different responses." And, engaging students in such tasks, according to PSTs, involves a certain activity structure. PSTs described this sequence as "launch-explore-summarize," "you, yall, we," (Green, 2014) and/or in reference to the *Five Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2018).

Direct instruction

While PSTs reported learning about IBI primarily in their mathematics pedagogy courses, PSTs learned about (as future teachers) or experienced (as students themselves) direct instruction from several settings. They described that they experienced direct instruction in their university mathematics courses ($n=9$) and special education courses ($n=7$), and had opportunities to observe and enact it in their field experiences ($n=8$) and student teaching ($n=6$).

Unlike IBI, direct instruction was not connected to many other ideas or practices, even though it was experienced in many settings. Some PSTs, however, described that direct instruction was disconnected to, or did not support, conceptual understanding ($n=5$, represented by red line in Fig. 3). This disconnect, for all five PSTs, was derived from their own experiences as a student in direct instruction-oriented mathematics classrooms. Four PSTs referenced their experiences in the university mathematics courses. Fiona, for example, explained that the goal of "calculus and finite [mathematics] and all of that… is probably to make me better at math but they're doing direct teaching, which means I'm not really learning anything. I'm just trying to get an A in the class and reproduce what the teacher directly taught." For Fiona, experiencing direct instruction as a mathematics learner revealed its ineffectiveness for supporting conceptual understanding.

Characterizing incoherence with instructional approaches

In learning about and experiencing two different instructional approaches from different settings in the teacher education program, eleven PSTs described these two approaches as opposing and contradictory (represented by red line in Fig. 3). Several PSTs, like Gail, used direct instruction as a contrast to IBI: "So inquiry-based is just kind of the structure of the class where they presented the task and then students are working to find their own strategies, rather than direct instruction, where they're given the strategy and have to replicate it." Here, Gail explained that IBI engages students in rich tasks that allow them to "find their own strategies" and make sense of important ideas, while direct instruction expects students to replicate a procedure the teacher presented. As we alluded to earlier, conceptual understanding, rich tasks, and activity structure seemed to be important ideas that distinguished these two instructional approaches for PSTs.

Viewing IBI and direct instruction as contradictory instructional approaches prompted PSTs to wrestle with and make sense of what effective mathematics teaching looks like. This was most apparent in how PSTs experienced direct instruction in their university mathematics courses. All eight PSTs who reported their mathematics courses were taught in a direct instruction approach explained that these experiences served as "what not to do" (Dack, 2019) in their future teaching. Bella, for example, shared that "most of the math classes I've taken I feel like show me what I don't want to do as a math teacher. I can't even see what they're writing on the board, and I don't know what they're talking about because they don't turn around and talk to us." Here, experiencing direct instruction as a mathematics learner reinforced to Bella that direct instruction was not effective, which supported her in making sense of how she does (and does not) want to teach in the future.

By contrast, experiencing direct instruction in field and student teaching experiences, with little opportunities to enact IBI, seemed to limit the extent to which PSTs perceived IBI as applicable and relevant. In thinking about her future teaching, Tori explained "I would like to be working toward inquiry-based lessons and away from direct instruction, but I also think starting like that would be unrealistic based on what I have done, what I have seen." For Tori, not seeing many examples of IBI raised skepticism about whether IBI would be realistic in actual mathematics classrooms, at least as a beginning teacher. Cady similarly questioned whether IBI is well-suited for covering a broad curriculum: "I've noticed this through my four years here that teachers have so much content that they have to get through."

All PSTs who had completed their student teaching provided a similar rationale to Cady's, explaining the various obstacles to their own, or their cooperating teacher's, enactment of IBI. That is, from the perspective of a learner of mathematics teaching, PSTs perceived direct instruction in their field experiences and student teaching as an answer to some of the classroom challenges they perceived as inhibiting teaching mathematics in an inquiry approach.

Other ideas and practices

Ten PSTs reported learning and engaging with the idea of relationships in the teacher education program, specifically from their mathematics pedagogy courses ($n = 7$), student teaching ($n = 5$), SCS courses ($n = 4$), and Inquiry into Learning course ($n = 3$), represented in Fig. 4 with blue ties. Beyond relationships being an avenue for learning about and meeting students' needs, as reported earlier, PSTs also described relationships as important for creating a safe and welcoming classroom community. Additionally, seven PSTs reported learning about methods for incorporating reading and writing into the mathematics classroom, though this was limited to their RWICA course sequence. PSTs, however, did not connect this idea to other ideas/practices. It is also worthwhile to note the ideas/practices and ties that are absent. For example, mathematics courses were mostly reported in reference to direct instruction, and rarely as sources of learning content knowledge, pedagogical or otherwise.

Discussion

This study sought to describe how PSTs in one secondary mathematics teacher education program perceived and characterized program coherence. Unlike the research on coherence in teacher education in general, by focusing on mathematics, our findings reveal the specific ideas and practices related to mathematics teaching and learning that PSTs experience as (in)coherent, and how these ideas/practices are organized across programmatic settings. In particular, our findings provide insights into PSTs' experiences with two longstanding challenges within mathematics education—getting beyond the school mathematics tradition (Cobb et al., 1992) to enact more pedagogies of investigation (Nicol, 1999), and integrating attention to equity into a discipline that is typically perceived as an "ethics-free domain of thought" (Ernest, 2018, p. 194). In the following, we foreground incoherence, though, throughout our discussion, we also attend to coherence.

Our findings suggest PSTs perceived two main sources of incoherence. First, although PSTs reported equity as a unifying theme across the various settings of the program, they did not perceive that equity and other aspects of mathematics instruction were conceptually integrated. Second, PSTs perceived both conceptual and structural incoherence between IBI and direct instruction. Specifically, PSTs described these two approaches as opposing and contradictory, often characterizing direct instruction as a contrast to IBI and what effective mathematics teaching looks like. Structurally, while PSTs experienced direct instruction in several settings, including field experiences and special education and mathematics courses, they had opportunities to engage with IBI only in their mathematics pedagogy courses, and to some extent, their mathematics for teachers courses and field and student teaching.

These two sources of incoherence varied not just with respect to *what*—equitable teaching or instructional approach—but also with respect to *where* and *how* PSTs characterized incoherence. Regarding how, though PSTs did not often connect or make sense of equity in relation to other aspects of mathematics teaching, they explicitly described IBI and direct instruction as contradictory approaches to teaching mathematics. That is, the conceptual incoherence between equity and other aspects of instruction was a lack of connection, while the conceptual incoherence between instructional approaches was a contradiction—one that was likely exacerbated by the structural incoherence (the where) noted in the previous paragraph. These differences have implications for how teacher educators and teacher education programs might respond to and engage with incoherence to support learning. We discuss these implications after first considering the potential reasons why PSTs in our study perceived each source of incoherence.

Opportunities to learn about equitable mathematics teaching

Though PSTs in our study reported that equity was integrated throughout university coursework, the program seemed to lack two other components that Sleeter (2008) argued teacher education programs that coherently addresss equity should provide: opportunities for cross-cultural community-based learning and field experiences in classrooms with students from historically underserved communities. In addition, few PSTs reported connections between equity-related ideas and other aspects of mathematics instruction. One potential explanation is that PSTs often learned about equity from courses outside the mathematics education sequence, which is consistent with Mintos et al.'s (2019) finding that mathematics teacher education programs often treat equity, especially the more critical dimensions, in subject-general terms. It is possible that such treatment of equity did not sufficiently support PSTs to make connections, as some research has found that PSTs are more likely to make sense of equitable teaching when their learning is integrated into content-specific methods courses (e.g., Rodriguez, 1998). This explanation, however, does not account for the lack of connection PSTs perceived even within the mathematics education courses.

An alternative explanation is that multiple settings of the teacher education program engaged PSTs in thinking about equity "in the exact same way," in the words of Ellie. That is, while PSTs perceived to have repeated opportunities to engage with equity across multiple settings, these learning opportunities might not have been sequentially organized to support progressively robust understandings. Arguably, teacher education programs should support PSTs to build understanding of important and complex ideas, like those related to equity, over time, with increasing depth (Canrinus et al., 2017, 2019).

It is possible that PSTs in this teacher education program were not afforded such opportunities, in their mathematics education courses or across coursework from multiple departments. For example, were students, as Alan suggested, repeatedly introduced to the picture in Fig. 5, which is commonly used in professional development settings to distinguish "equity" from "equality" (Froehle, 2016), with few other definitions or representations? If so, how might that have shaped their conceptions of "meeting students' needs?" Similarly, was the practice of redlining, as Gail described, the only example of structural racism offered? If so, what implications would PSTs have drawn for their practice as mathematics teachers? Raising such questions leads us to consider how much and "what kind of coherence is desirable" (Hammerness, 2006, p. 1262). To the extent that the program engaged PSTs with a limited repertoire of equity-related ideas or representations, it is possible that the coherence achieved was merely *redundance* with concepts, which failed to support PSTs in bridging to their practice.

Opportunities to learn about mathematics instructional approaches

Although equity was perceived to be a unifying theme across the teacher education program, PSTs reported learning about two opposing and contradictory instructional approaches—IBI and direct instruction. That PSTs perceived such an instructional dichotomy potentially reflects a broader societal tendency to polarize mathematics teaching (e.g., the "math wars"). It is also possible, however, that the teacher education program contributed to such a perception. For example, as practices that support IBI are introduced, are they consistently defined and framed as an alternative to conventional direct teaching? It could even be that instructors' framing is rooted in compassion—acknowledging that for some, IBI is significantly different from their own school mathematics experiences and therefore may require patience and persistence. But even when affirming PSTs' experiences and reactions, it is possible that such framing may contribute to constructing and reinforcing a dichotomy of disparate models of instruction, rather than a nuanced understanding of various pedagogies for mathematics and the ways in which they are similar and different (Munter et al., 2015). While the program does focus on ways that practices often associated with different instructional models should be integrated (e.g., supporting productive struggle with "judicious telling";Freeburn & Arbaugh, 2017; Lobato et al., 2005), it is possible that message is drowned out by the program's "over-correcting" (Brownell, 1956).

This conceptual incoherence was also reinforced by structural incoherence: PSTs reported that opportunities to learn about IBI were primarily isolated to courses taught by the mathematics education program and were contradicted by learning and experiencing direct instruction in their special education courses, mathematics courses, and field and student teaching experiences. That PSTs experienced incoherence between their methods courses and the field is not surprising; this fragmentation has been well-documented in the literature (e.g., Canrinus et al., 2017, 2019; Grossman et al., 2008). Our findings extend this research base by revealing that this incoherence exists within the university as well, and that the "two-worlds pitfall" (Feiman-Nemser & Buchmann, 1985) is exacerbated by pitfalls within just one world, namely the "fault lines" between university departments (Bain & Moje, 2012).

Given that stark structural divide, it would not be unreasonable to question whether the mathematics education courses were the source of pedagogical incoherence. However, given that, as noted previously, those courses are generally aligned with the field's professional standards (i.e., AMTE, 2017; NCTM, 2014), what we observed is likely a manifestation of the broader struggle in recent decades to disrupt the "school mathematics tradition" (Cobb et al., 1992) and the durability of the direct instruction status quo. It is likely that the differing pedagogical perspectives between the mathematics education program and the special education and mathematics departments in this teacher education program reflect wider epistemological differences between these fields (Boyd & Bargerhuff, 2009; Sheppard & Wieman, 2020). These differences, however, pose obstacles to developing shared goals and commitments to mathematics teacher education, a necessary prerequisite for program coherence (Tatto, 1996). In the next section, we discuss possible implications of our findings for teacher education.

Implications for teacher education

Having identified possible sources of incoherence, a first, practical implication is for our program to attempt to address those sources. As we do so, it is important to consider the "where" of incoherence, as PSTs' perceptions of incoherence varied by setting and their role in that setting. Specifically, experiencing direct instruction in the university mathematics courses, as learners of mathematics, offered an image of "what not to do" in future teaching (Dack, 2019), while experiencing direct instruction in field and student teaching, as learners of mathematics *teaching*, revealed to PSTs challenges to, and rationales for not enacting IBI (Alsup, 2006; Vacc & Bright, 1999). Therefore, addressing incoherence between these different settings, over time, likely necessitates different approaches. We can imagine three, which we alliteratively refer to as *preempt*, *protect*, and *partner*.

In teacher education programs such as the one that is the context of our study, where the curriculum is distributed across departments, instructors are acting with academic freedom and may engage students with ideas misaligned with the vision at the foundation of our mathematics education curriculum. In such cases, we may wish to "preempt" those ideas, either by directly critiquing them (e.g., teaching against behaviorist approaches to "classroom management") or by framing them as an opportunity to learn "what not to do." Though scholars (e.g., Dack, 2019; Hebard, 2016) have found that some incoherence can support teacher learning, we raise concerns about the diminishing returns of such learning, particularly in learning to enact IBI. The potential learning from experiencing "what not to do" is likely achieved after seeing such non-examples one time, especially given that PSTs have had a lifetime of experiencing direct instruction as schoolchildren (Lortie, 1975). Moreover, there is, arguably, potential for more learning in experiencing quality examples of IBI than in seeing non-examples, especially in field placements as these were the settings that raised PSTs' skepticism about whether IBI would be realistic in actual mathematics classrooms.

Thus, it is likely insufficient for mathematics education instructors to continually position PSTs' incoherent experiences as opportunities to learn "what not to do." A second, more proactive response may therefore be to "protect" our PSTs from incoherence by more creatively curating their experiences while still meeting programmatic and state certification requirements. For required field hours, this could include more purposeful selection of practicing mentor teachers, placing multiple PSTs in one classroom, or looking beyond traditional classrooms altogether. Within the university, we might expand the curriculum where we have direct control (e.g., rather than delegating PSTs' learning about state-required "classroom management" to special education courses, integrate it into mathematics education courses).

Although preemptive and protective approaches may help in the short-term, a likely more sustainable and expansive approach is to "partner" with stakeholders in increasing program coherence. Ideally, such partnership is built on a shared vision. For example, mathematics teacher educators could offer professional development and other forms of support to establish and grow a cohort of secondary mathematics teachers who are interested in enacting more equitable and inquiry-based instructional approaches. Within the university, faculty could establish a common framework for important cross-cutting ideas like inclusion, diversity, and equity and work to enact a learning progression as PSTs progress through the program.

These three approaches—preempt, protect, and partner—represent efforts to address potential incoherence beyond the mathematics education curriculum. However, we should not ignore possible sources *within* that curriculum, over which we do have some direct control. In particular, our findings reveal a need to better support PSTs in translating ideas about equity into their practice. To do so, we should more clearly identify areas in the "equity curriculum" across the entire program that we can extend into practice. For example, if PSTs are introduced to the history of racial segregation and discrimination in housing and schooling elsewhere, rather than re-hashing those ideas, the mathematics education courses can focus on how to respond as teachers, including mathematizing social problems (e.g., Wager & Stinson, 2012) and/or becoming agents of change who challenge policies and perspectives that fail to account for those histories (Martin, 2007).

All of these responses to incoherence raise questions about whose "job" it is to oversee the pursuit of coherence. While it may be easy to conclude that it is the job of anyone who works with PSTs, we suspect that achieving a shared vision might require coordination by individuals in formal positions of leadership in teacher education. Even then, however, it is likely that some epistemological divides may be too wide to achieve a shared vision. In the program in our study, this may be the case when it comes to differences between perspectives of mathematics and special educators, which, in our view, are revealed in our findings about both sources of incoherence.

With respect to equity, PSTs' characterizations of what they had learned about "meeting students' needs" and the disconnect they perceived between that idea and bias and privilege are emblematic of a possibly significant divide. In some cases, PSTs used potentially problematic language in talking about and making sense of issues of equity. For example, some PSTs, including Fiona and Francisco, related race and disability. While we interpret this as PSTs' understanding that both people of color and people with disabilities have been excluded based on social constructs (Annamma et al., 2013; Ferri & Connor, 2005), it is possible that PSTs problematically view these student groups as needing "accommodation" because of something inherently related to their identity, rather than due to limitations imposed by the constraints of an ableist society predicated on Whiteness. And messages about "meeting students' needs"—whether as depicted by people on boxes at a baseball game (Froehle, 2016) or as being met by "multi-tiered systems of support" (Stoiber & Gettinger, 2016)—may lack sufficient nuance to help PSTs draw such important distinctions.

Addressing possible sources of incoherence such as these, which may originate from academic traditions with different epistemologies, may be difficult. In those cases, perhaps a different kind of "partnering" may be necessary. For example, in the spirit of "adversarial collaboration" (Gilovich et al., 1998; Mellers et al., 2001), we could more directly work with stakeholders to come to agreements on exactly how we *disagree*, so that we at least achieve "coherence" in how we each talk about the varying perspectives and practices across the program. Here too, coordination by those in leadership positions would likely be helpful.

Limitations and future research

Before concluding, we wish to acknowledge limitations of our study and related possibilities for future work. First, it may be viewed as including a small sample of PSTs. This is partly because of the necessary focus on a single program: because coherence is something that is achieved (or not) with a given program. Had more PSTs chosen to participate, that would have increased the sample size. Our sample, however, included representatives from three cohorts, and the convergence of their responses give us confidence that we captured important themes. That is, by recruiting from three different cohorts, we were able to develop meaningful understandings about PSTs' perceptions of coherence that are valid and relevant to PSTs across the teacher education program and not unique to its configuration in a single year. In future, it would be helpful to conduct similar studies with PSTs in other secondary mathematics teacher education programs. Comparing the results and aspects of their respective programs and contexts could help to expand the field's understanding of PSTs' perceptions of coherence.

A second possible limitation is the scope of the claims warranted by our data. We do not offer our own conclusion about program coherence based on an independent analysis of course designs and implementation. As stated previously, however, we purposefully investigated PSTs' perceptions as an important indicator of coherence that often goes unaccounted. This approach also helped to mitigate the potential effects of our biases, given our relationships to the program and some of the PSTs. In other words, we minimized the "developer effect" (Wolf et al., 2020) by eliciting and examining PSTs' self-reporting of their experiences, rather than conducting our own of assessment of coherence. Relatedly, we cannot make claims about what were "actually" PSTs' experiences or how they contributed to PSTs' learning, only what they reported as their perceptions. Our findings, however, point to potentially fruitful areas for future work to more closely examine how (in)coherence contributes to PSTs' learning. With a broader scope, subsequent studies could even examine relations between program coherence and "impacts" on graduates' practice during the first few years of teaching.

Finally, we wish to discuss the value of participatory diagramming and network analysis for data collection and analysis. In our study, inviting PSTs to create visual diagrams elicited a multitude of ideas/practices and settings, and even more connections among them. Moreover, interacting with the diagram supported PSTs and the interviewer in communicating about the different components, as off-loading all the ideas/practices and settings on the board served as a memory-aid and facilitated thinking about a large volume of information without having to remember it all (Bagnoli, 2009; Bravington & King, 2019). We also found that network analysis helped us to uncover and represent underlying structures that were implicit in the interviews. For example, as we described in the findings, we noticed two clusters of vertices in the network, which oriented us to attending to how our interviewees described these clusters and how a lack of connection between them contributed to incoherence. These methods helped us to investigate PSTs' perceptions of program coherence, though we imagine they can also reveal deep insights about other measures of coherence, such as those based on course implementation. These methods might also be useful for investigating other phenomena important to mathematics teacher educators, such as the links between domains of mathematical knowledge for teaching (e.g., Koponen et al., 2019).

Conclusion

Teacher education is often fragmented and disconnected, with different settings (e.g., courses, field experiences) promoting different ideas and practices. Therefore, to better support mathematics PSTs to teach in equitable and inquiry-oriented ways, teacher education programs will likely look for opportunities to build coherence, so that PSTs are afforded repeated experiences with a set of ideas and practices, and opportunities to engage with such ideas/practices across multiple settings. There is, however, little research on coherence in mathematics teacher education, especially from the perspectives of PSTs. Attending to PSTs' perceptions of (in)coherence can point to possible spaces for collaboration between the different settings of a teacher education program and inform future redesign efforts. In our study, we found that PSTs perceived two main sources of incoherence: equity and other aspects of teaching are not conceptually integrated; and opportunities to learn about inquiry-based instruction in mathematics methods courses are undermined by limited opportunities to experience it in the field, and by learning and experiencing direct instruction in special education and mathematics courses. Our findings highlight a need for mathematics teacher educators to support PSTs in connecting equity to other aspects of mathematics teaching, and to develop greater coherence among university courses as well as between coursework and field with respect to mathematics instructional approaches.

Appendix

Interview Protocol

Introductory Question

1. First, I'd like to ask you to share about your current status in the teacher education program. What is your major, year, and are you currently taking classes or student teaching?

    a. (If taking classes) What classes are you currently taking?
    b. (If student teaching) Where are you student teaching and what class(es)?

Vision of high-quality mathematics instruction

2. If you were asked to observe a teacher's math classroom for one or more lessons, what would you look for to decide whether the math instruction is high quality?

    a. Why do you think ___ is important?
    b. What are some of the things you would expect to find the teacher actually doing in the classroom for instruction to be of high quality?
    c. Is there anything else you would look for? If so, what? Why?
    d. Have you talked about this in any of your courses or field experiences?

Ideas and Settings

3. Tell me some things about teaching you've talked about, and where you've talked about it. This could be from your coursework, instructors, peers, and field placements. I will be taking some notes on our shared Slides.

    a. *(For any words/ideas that are not defined)* Can you tell me what that means? Have you heard others using that term or was it specific to [source]?
    b. How is this related to your role as a math teacher?
    c. Have you had opportunities to apply these practices in the field? Have you had opportunities to discuss/connect this to other courses or field experiences?

4. Is there anything else about teaching you've talked about? Where did you talk about that?

Participatory Diagramming

5. For the remainder of the interview, I'm going to ask you to share your screen of our shared Slides. From what we've talked about, can you draw me a representation of how you see it related or connected, or not connected? The list I wrote based on what you talked about might be helpful. Feel free to add anything else that you think of.

    a. You can use pink ovals for ideas/concepts and yellow ovals for places.
    b. If you see things you've talked about as related, you can connect them. If you don't see them as related, don't connect them.
    c. You can use different lines for different types of relationships.

6. Can you walk me through your diagram?
7. *(For connections)* You connected these things together. Can you tell me more about that?

    a. Did all these courses talk about this idea/practice in similar ways, or differently?

8. *(For misalignments/contradictions)* You described ____. Can you tell me more about that?

    a. How do you see them as different or similar?

9. *(For those lacking connections)* You described ____. Can you tell me more about that?

    a. Why do you see this as not connected to the rest?

10. You've talked about these experiences and these courses, but not these other courses or field experiences. (Share list- note the sources PST have not talked about.) Can you talk to me about what you learned there?

    a. How do you see what you've learned as fitting or not fitting in this diagram?

Closing Question

11. Is there anything that I have not asked that would help me better understand your experiences here as a preservice teacher?

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