



Understanding the complex work of mathematics teaching

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The work of mathematics teaching is a complex activity during which teachers face a variety of decisions and challenges that influence the course of instruction, and ultimately, students' opportunities to learn. Adding further complexity is that teaching is a highly situated activity that requires adaptation to the in-the-moment interactions of teachers and students (Borko, 2004; Greeno et al., 1996). As such, teachers need to flexibly apply their knowledge while not compromising on the underlying principles that support the development of students' understanding. Teachers need to understand what to do, how to do it and why to do it (e.g., Bereiter, 2014; Darling-Hammond & Bransford, 2005).

Over the past two decades, considerable progress has been made to understand the complex work of mathematics teaching, as evidenced by the number of articles in JMTE focused on teaching, teacher learning and teacher decision-making. The articles in this issue are no exception. One common thread that cuts across many of the articles in this issue is a focus on understanding the work entailed in one phase of teaching, namely teachers' planning. Planning is an important and often underappreciated phase of teaching, during which teachers make decisions that ultimately impact students' opportunities to learn (Clark & Peterson, 1986; Darling-Hammond & Bransford, 2005). Sometimes referred to as the hidden side of teaching, planning commonly refers to the time teachers spend preparing and designing activities for students. From problems and activities used during instruction to engage students to instructional practices and strategies to employ during a lesson, teachers consider a variety of aspects of their teaching practice before students even enter the classroom (Fernandez & Cannon, 2005).

In this issue, Melville and Corey describe a component of the Japanese lesson study infrastructure, namely *Kyouzaikenkyuu*, a part of the lesson study process that is often neglected by teachers outside of Japan. *Kyouzaikenkyuu* is the planning phase of lesson study where teachers first study the instructional materials from a teaching perspective and then study the same materials to understand them from students' perspective. Drawing on a larger ethnographic study of Japanese teachers' lesson study practices, the authors make visible the process of *Kyouzaikenkyuu*. Through observations and interviews with two teachers, Melville and Corey (this issue) identify two different methods: a textbook adaptation approach (i.e., existing materials are adapted to the particular needs of students), and a curriculum-development approach (i.e., identify learning goals and draw on various resources to develop unit/lesson plans). They conclude by describing how the primary goal

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of *Kyouzaikenkyuu* is ultimately to support teacher learning with a lens toward developing and preparing lesson plans.

Leavy and Hourigan (this issue) similarly focus on the planning phase of teaching, with a particular focus on problem posing. They define problem posing as the development of new mathematics problems and the modification of existing textbook problems. In problem posing, teachers need to consider problem contexts that are mathematical meaningful and relevant, as well as the structural features of the problem. Many teachers often rely solely on textbook problems to use during lessons, with little if any attention to modifying problems that are cognitively challenging and best suited to the needs of students. Using a letter writing intervention, Leavy and Hourigan (this issue) engage a group of prospective teachers in an iterative process of problem (re)design. Prospective teachers posed mathematics problems that were delivered to student “pen pals” as part of the letter writing activity. The letters functioned as a form of communication between students and prospective teachers and informed the design and modification of the problems. Findings from their letter writing intervention show that while it is challenging for prospective teachers to pose cognitively demanding problems that required multiple steps, they did enhance their problem posing skills over time.

Stein and colleagues (this issue) similarly focus on the planning phase of teaching in their study of mathematics coaches’ learning. In their article, they describe a model for the development of mathematics instructional coaches that is focused on enhancing coaches’ capacity to engage in one-on-one pre-lesson co-planning of mathematics lessons. Using data from five different coaching cycles over two years, Stein and colleagues (this issue) show how participating coaches improved their capacity to discuss mathematics learning goals with teachers at increasing levels of depth and increased the depth of their discussion of the use of advancing questions over time. In doing so, they demonstrate that mathematics coaching focused on teachers’ lesson planning is a productive approach for the professional learning of mathematics coaches. Their study not only illustrates the complexities of lesson planning, but also how mathematics coaches can learn to support teachers in understanding what to do during planning and why to do it.

The other articles in this issue zoom out from teacher planning and focus more broadly on the work of teaching. In their article, Nachlieli and Heyd-Metzuyanim (this issue) study teachers’ learning of explorative teaching practices through a discourse lens. Taking a commognitive approach to learning (Sfard, 2008), they conceptualize learning as changes in teachers’ pedagogical discourses in a professional learning experience, discourses of which are located on a continuum between Delivery Pedagogical Discourse (DPD) (i.e., teachers’ delivering knowledge to students who acquire it) and Exploration Pedagogical Discourse (EPD) (i.e., students’ agentic exploration of their thinking, externalization of thinking processes and group discussions). In particular, Nachlieli and Heyd-Metzuyanim (this issue) analyzed the ways in which commognitive conflicts between teachers’ and facilitators’ discourses changed over time, and how those conflicts elicited teachers’ thinking and ultimately contributed to teacher learning. Their study offers a methodological approach to the study of teacher learning.

Finally, Copur-Gencturk, Jacobson and Rasiej (this issue) analyze teacher knowledge assessment instruments to provide users of these instruments with a stronger foundation on which to make claims about teachers’ mathematics learning. In their article, Copur-Gencturk and colleagues (this issue) analyze the extent to which two widely used teacher knowledge assessments align to a set of mathematics content standards currently prevalent in the USA. Specifically, they developed content maps using the Common Core State Standards for Mathematics to analyze the LMT and DTAMS assessment instruments for

elementary school teachers. Copur-Gencturk and colleagues found that the content alignment with the standards and the assessment items on both instruments varied depending on which assessment form was used, and that different forms did not include the same content with the same frequency. Mostly notably, in the case of both instruments, they found few items that measured teachers' pedagogical content knowledge. This is particularly important to consider as both instruments are often used to measure teacher knowledge and make claims about teachers' learning in professional development activities.

Together, the articles in this issue provide us with insight into different aspects of the work of teaching and offer new designs for and approaches to studying teacher learning. At the same time, I want to continue to encourage the JMTE community to study the work of teaching with an attention to the larger organizational systems within which teachers are embedded. We need to continue to consider how to account for the broader contexts and multiple communities within which teachers reside and in which teacher learning occurs. Indeed, teachers operate within complex ecosystems, wherein classrooms are embedded in schools located in communities within larger geographical regions, states, nation states and regions of the world (e.g., Bronfenbrenner, 1994; Cohen et al., 1993). What happens at any one level impacts and is impacted by what happens at other levels of these ecosystems, further contributing to the complexity of mathematics teaching.

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