Journal of Mathematics Teacher Education (2020) 23:1–4
https://doi.org/10.1007/s10857-020-09455-3

Promoting explicit connections in mathematics teaching: scopes for the teachers learning in context

Salvador Llinares[1]

Published online: 30 January 2020

One of the key issues in initial mathematics teacher education and professional development is to help preservice and practice teachers to connect specific knowledge with the teaching practices, and also to become aware of these connections. Both processes can be seen from different perspectives and be supported with a wide range of tools. One claim behind this key issue is the claim that improvement in the practice of mathematics teaching is related to the teacher's awareness of how the rationality of practice can work. This claim opens the possibility of considering the teaching of mathematics as a space for the learning in the context of preservice and practice teachers. Promoting the awareness of connections in mathematics teaching is a contextual process which is related to mathematics teachers' knowledge and the epistemological positions.

This idea raises issues about our understanding of the teaching complexity, how we can frame the preservice and practice teachers' learning and the perspectives through which we try to understand the recontextualization processes of knowledge from mathematics methods course to instruction. For instance, questions about how teachers' knowledge shape their instruction, what theoretical frameworks can help us to better understand the preservice and practice teachers' learning and what types of personal and contextual factors determine how teachers use their knowledge in teaching.

The four papers of this issue underline connections among different aspects of mathematics teaching practice, teachers' learning and the contexts in which teachers teach. Furthermore, the papers point out more or less explicitly that teacher must be aware of these connections. These connections are studied from different perspectives and in different contexts. The papers address how practice teachers in different educational levels struggle to connect different aspects into teaching. The studied connections range from children mathematics thinking to children's linguistic, cultural and family funds of knowledge (Kinser-Traut and Turner); teachers' noticing of reasoning forms to the features of instruction (Melhuish, Thanheiser and Guyot); mathematical ways of thinking as a component of teachers' professional knowledge base to the quality and coherence of teachers' instruction (Tallman and Frank); and the use of learning trajectories to how and what teachers notice taking into account what is noticed to thinking about the instruction (Wickstrom and Langrall).

✉ Salvador Llinares
sllinares@ua.es

[1] Universidad de Alicante, Alicante, Spain

The way in which some of these connections shape the instruction and the teacher's learning is shown using case studies of teachers at various stages of their teaching professional career and teaching different topics in different educational levels. Specifically, Kinser-Traut and Turner's paper focuses on a teacher in transition from a mathematics methods course (initial primary teacher education) to her early career teaching years. Melhuish and colleagues' paper considers the recontextualization of the practices of justifying and generalizing from a professional development to the instruction, and the similarities and differences of how primary teachers and teacher educators notice the instruction. Tallman and Frank's paper focuses on the connections between a secondary teacher's knowledge of quantitative reasoning and the way in which the instruction is developed. These authors also provide empirical support to the claim 'habitual forms of reasoning that govern teachers' concept-specific understanding affect the quality of their instruction. Finally, Wickstrom and Langrall paper describes the efforts of a primary teacher in recontextualizing the knowledge from a learning trajectory to design and enact lessons.

Kinser-Traut and Turner focus on supporting effective mathematics teaching by connecting it to children's mathematical thinking (CMT) and children's linguistic, cultural and family funds of knowledge (CFoK). Changes in how a teacher learns to integrate CMT and contextual knowledge into instruction are identified considering the recontextualization of the knowledge into practice and how the teacher makes sense to the notion of authority. Particularly, they consider how teaching practices and ideas introduced in method courses are enacted by the teacher during the first years of teaching. Recontextualizing knowledge from method course to early career teaching years is evidenced by the changes in the recognition and realization rules. These changes show the connection between talking about and implementing specific practices. The teacher's awareness of the transition from talking about to implementing, and from the recognition rules to realization rules, is an example of the teacher's contextual learning. The Kinser-Traut and Turner's paper reports two different trajectories in the development of practices for eliciting and connecting CMT and CFoKs. These trajectories highlight the difficulty that teachers face in connecting some aspects in teaching. This difficulty is evidenced on how the teacher positions herself as the authority and reflects a lack of sharing authority that hinders her capacity to connect CMT and CFoK. This study shows the sharing authority as a generative practice which is a scope for teacher learning in context. One of the tools in this scope is the curriculum of the methods course and student teaching. Kinser-Traut and Turner suggest as an implication of this study that 'explicitly offering recognition and realization rules around sharing authority could support recontextualization as it may offer a generative approach to connecting to and eliciting CMT and CFoK'.

The paper by Melhuish, Thanheiser and Guyot focuses on the relationships between what primary teachers' notice regarding the practices of justifying—reasoning with meaning of ideas, showing/refuting the validity of an idea—and generalizing—seeing the general in the specific—and what an external observer can notice. Justification and generalization are both considered as mathematical reasoning forms. In this study, the connection is about noticing and promoting specific practices in teaching. The hypothesis behind this connection is that primary teachers can only promote this type of practices if they are able to notice them. The analysis of the effect of a professional development initiative is the context in which the recontextualization of ways of mathematical reasoning is studied. This study shows discrepancies between teachers' self-reported noticing of students' process of justifying and generalizing and what the observers notice. This fact indicates the multiple factors which are conditioning the teachers' professional learning. Furthermore, it is an indicator of the complex nature of the processes

to enact a specific instruction with principles and ideas from the professional development initiatives. Melhuish et al. suggest two factors that can explain these discrepancies: the teachers' conceptions about justifying and generalizing and that noticing is a challenging skill that takes years to develop. The connection between noticing forms of mathematical reasoning, and enacting a practice supporting these forms of reasoning, underlines the development of teachers' awareness about what is notice in practice. (It is not possible to develop a specific practice if it is not noticed.) This study suggests that a tool to enhance the teachers' awareness of these connections is the prompts explicitly used by the facilitator in the professional development initiatives. In this sense, Melhuish et al. suggest that 'the ability to notice complex mathematical reasoning forms (a necessary prerequisite for fostering them in classrooms) may rely not only on providing teachers with mathematical definitions and rich examples but also on promoting explicit connections between definitions and the examples themselves'.

Tallman and Frank paper focuses on the connections between the mathematical ways of thinking as a component of teachers' professional knowledge base, and the quality and coherence of a secondary teacher's instruction. The connections studied are among teacher's knowledge and the mathematical meanings that teachers' instruction supported. Authors suggest that the nature of these connections was derived from the teacher's lack of awareness of the conceptual affordances of students' quantitative reasoning, on how mathematical meanings were constructed. This study suggests that teachers should be aware of the connections between how they know the mathematics processes and how this knowledge can shape their instruction. Tallman and Frank indicate that 'without an awareness of how reasoning quantitatively might contribute to students construction of coherent mathematical meanings, teachers will not develop a disposition to consistently emphasize quantitative reasoning in their instruction.' As Melhuish et al.'s paper, Tallman and Michael point out the mathematical ways of thinking as an essential aspect of mathematical knowledge for teaching that can inform the teachers' instructional practices.

Previous papers suggest that teacher educators need to use different tools to help preservice and practice teachers become aware of connections in mathematics teaching. From this perspective, Wickstrom and Langrall paper describes the efforts of a primary teacher in using a learning trajectory (LT) as a tool to enact the instruction of area measurement. Since the learning trajectory was introduced to the teacher during a professional development program, this study is also an example of how she struggles in using the knowledge provided in professional development when noticing and justifying the instructional practice. In this sense, the focus on the recontextualization of knowledge from teacher education (initial or professional development) into instruction complements what is reported in the papers of Kinser-Traut and Turner, and Melhuish and colleagues. The facts described here are features of the teacher's use of LT as a tool to notice some events in her practice, talk about these events—with a more or less elaborated discourse—and interpret them in terms of how her students were learning. Wickstrom and Langrall suggest that, although the teacher seems to have some troubles using the terminology from the LT, the LT becomes a frame through which this teacher notices her practice helping her to modify tasks to further more the students' thinking. However, the LT did not allow the teacher to differentiate her instruction considering individual students and to consider lesson goals in terms of broader curriculum goals. These facts show the complexity of the recontextualization of knowledge in practice, in terms of how the teacher can use the knowledge to notice and talk about her practice. The features of the use of LT as a tool described in this study address our attention on the ways in which teachers endow sense to knowledge in teacher education programs.

The papers in this issue of the J*ournal of Mathematics Teacher Education* show examples of connections in mathematics teaching and suggest tools that can be used by teacher educators to support the enhancement of the teachers' awareness of these connections.