Understanding mathematics teaching and learning “in their full complexity”

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When I did my master’s in mathematics education in the 1980s in Copenhagen, Denmark, my professor, Bent Christiansen, used to say that the aim of mathematics education research is “to understand mathematics teaching and learning in their full complexity and contribute to their further development” (personal communication). Except for the somewhat absolutist connotations of the way he phrased it, I think most in our field today would agree with Christiansen’s intention: we are to develop comprehensive understandings of the problems of mathematics teaching and learning, and the ambition is, at least in the long run, to support the development of institutionalised teaching–learning practices.

There is, of course, a history and background to Christiansen’s wording about research in mathematics education. In the 1980s, he was vice president of ICMI, and he had been actively involved in research and development in school mathematics for decades. For instance, he was a prominent contributor to and exponent of a Scandinavian version of New Maths in the 1960s. Like many others, who had been involved in New Maths, he now looked back at the attempts to structure syllabi and other curricular materials and to study student learning in clinical settings, and—like many others—he was disappointed with what he in retrospect saw as the dismal results of the efforts. The question he asked himself, I think, was how it could go so wrong, when the people involved were highly qualified mathematicians and competent and experienced psychologists, who joined forces to create the best possible suggestions for school mathematics? The answer, he said, was that there is “a complexity” involved in mathematics education that goes beyond the subject itself and beyond individual students’ cognition.

Obviously, Christiansen was not the only one to point to the limitations of previous approaches to mathematics education research in the 1980s, and there were two closely related reactions to that. First, there was an extension of the field of interest first to teachers and teaching, then to classroom interaction, and later to broader cultural and political issues. Second, and because of the broader research interests, other theoretical frameworks were needed than the ones that had been used before. As an early indication of that, Christiansen was one of the first in mathematics education to refer to the work of Vygotsky and Leont’ev (Christiansen and Walther 1986). Other frameworks that take social issues into account in different ways, e.g. social practice theory, have later come to play a prominent role.

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The papers in this issue of JMTE

The papers in the present issue of JMTE in different ways and to different extents reflect this move beyond an interest in mathematics per se and in individual students' learning. The paper by Diamond is on teachers' beliefs, a field that gained momentum in the 1980s as part of the shift of focus towards teachers and teaching and that continues to attract much attention. Diamond is interested in the notion of transfer, which she defines as how a learner "generalizes, extends, applies, or in some way makes use of his or her learning while engaging in a novel situation". Based on the assumption that teachers' beliefs significantly influence instruction, she addresses the question of what teachers believe about transfer. She presents a qualitative study of eight practising teachers from the USA, who have recently taught linear functions, and she focuses on how the teachers think of students' transfer as it relates to slope. The result of the analysis is six categories of beliefs about students' transfer, which are organised as three pairs dealing, respectively, with the mathematical contents (procedures or concepts), with students' dispositions towards problem-solving (individual sense making or collaborative learning) and with affective issues (students' beliefs about themselves or their beliefs about mathematics). Some of the belief categories resemble research perspectives on transfer. For instance, Diamond likens the first category, in which task contexts are expected to lead to particular reactions on the part of the students, to early work on transfer, especially to Thorndike's view that transfer is triggered by observable elements of the new situation that resemble those of the original setting. Diamond concludes that teachers' beliefs about transfer of mathematical knowledge is closely connected to how they themselves understand the mathematics in question.

The paper by Norton is on teachers' knowledge, another field that also became prominent in mathematics education as a response to the developments in the 1980s. Norton's study is on prospective primary teachers at a prestigious university in Australia. At the beginning of the study, the research participants have completed two courses in mathematics, and Norton asks what mathematical content knowledge they bring to the third and final course in the subject and how that knowledge changes because of their participation in the third course. He also wants to know what the relations are between their content knowledge and pedagogical content knowledge in mathematics (MPCK) at the end of the course. One part of the conclusion is disappointing. Way into their teacher education programmes, the research participants face significant problems with tasks that their future students are to solve. Another and more optimistic conclusion is that an intervention—the third course—that focuses on school mathematics and on related instructional issues may alleviate the problem: Pre-service teacher education is not necessarily in vain. With regard to the last question, the research participants were generally better at identifying students' errors than at solving the tasks. Also, and less surprisingly, prospective teachers who had difficulties with the contents themselves did not do well when seeking to address elementary students' learning problems. Norton's findings support claims that although primary teachers may not need strong academic backgrounds in mathematics, a threshold level of content knowledge is required. Norton also suggests that this level of content knowledge may be developed in connection with MPCK, if the structure of pre-service teacher education allows for sufficient specialisation mathematics.

The paper by Wallin and Amador presents a study of teacher noticing, a theme that focuses on the challenges of dealing competently with classroom interactions in situ. In particular, Wallin and Amador are concerned with how rural secondary teachers in the USA may be supported in noticing student thinking through their participation in video clubs. The participants in Wallin and Amador's study are the three secondary teachers of mathematics

in a rural school district. The teachers have all been teaching for many years, but none of them were initially certified to teach secondary mathematics. Wallin and Amador use a range of qualitative methods and interpret the data from a situative perspective to document that all three teachers develop their capacity to notice student thinking in the course of the intervention. As a result, the teachers also begin to use curricular materials more flexibly and in this sense develop their capacity for pedagogical design. Working in a rural context, Wallin and Amador explicitly address issues that are different from most urban and suburban areas in the USA, and acknowledge that the complexities of teaching may be different there. One moral of the story, then, is that video clubs have potential for professional development, also when working with secondary teachers in rural school districts in the USA.

It is increasingly acknowledged that the student population in many countries in the West is becoming increasingly diverse, and that mathematics plays a significant societal and political role in and beyond educational contexts. If these equity issues are to be taken seriously, it further adds to the complexities of teaching mathematics. In part, equity was addressed already in the 1980s under the heading of mathematics for all, but it has attracted more attention and been reinterpreted since then. In the paper by Mintos et al., it is addressed in relation to mathematics teacher education. Mintos et al. investigate the opportunities pre-service secondary teachers have to learn about equity at five teacher education programmes in the USA. They use a framework developed by Gutiérrez that focuses on two axes of equity: one consisting of access and achievement (the dominant axis) and the other of identity and power (the critical axis). Mintos et al. use qualitative methods to address the question of what opportunities to learn about the altogether four aspects of equity prospective teachers have in courses on education, mathematics education, and algebra in the five programmes. Unsurprisingly, the five programmes differ with regard to how and how much they provide prospective teachers with equity-related learning opportunities. In general, however, access was the most prominent aspect, and “the dominant axis” was also dominant in this study: prospective teachers in these five programmes are offered more opportunities to learn about access and achievement as aspects of equity than about identity and power. This leads Mintos et al. to suggest that critical perspectives should play a stronger part and be incorporated in all courses in mathematics teacher education.

Revisiting “the full complexity” of mathematics teaching and learning

Between them, the four articles in the present issue of JMTE suggest that as a field we are addressing many of the complexities of mathematics teaching and learning that Bent Christensen talked about 35 years ago. Research on teachers’ knowledge and beliefs has developed still more nuanced perspectives on the role of teacher characteristics for classroom practice, and the articles by Diamond and Norton contribute further to that. Teacher noticing is a construct that focuses on the dynamic and interactive nature of classroom practice and the necessity for the teacher to make on-the-spot decisions about how to support student learning. The article by Wallin and Amador analyses the development of teacher noticing in a development initiative in a rural district of the USA and seeks to take into consideration how contextual issues matter. In addition, the equity perspective adopted by Mintos et al. adds further to the complexity by addressing issues of student diversity and of the societal and political role of mathematics in society.

The complexities involved in all of this are in many ways beyond what many mathematics educators imagined in the 1980s. At the time, a dominant concern was how to

overcome what was considered the main problem of implementation for educational development: the teacher as an obstacle to change. The current emphasis on, for instance, classroom dynamics and societal and political issues extends “the full complexity” beyond that. Incidentally, I am convinced that Christiansen and many of his contemporaries would have supported this extension of our research agenda wholeheartedly.

In spite of that, I think there are other ways in which our field as whole does not take “complexity” sufficiently into account. This point does not relate in particular to the articles in the present issue of JMTE. It is a consideration about what is published and not published in JMTE in general and even in other major journals.

To make my point, I need to return to Bent Christiansen and his professional life. Apart from his purely academic career, he was involved in development initiatives in different parts of the world. He worked in Liberia for a few years and was associated with UNESCO in different periods of time. These experiences seemed to convince him that different living conditions and cultural traditions are not just minor intervening variables between planned educational initiatives and student learning. Rather, he saw them as fundamentally forming students’ and teachers’ educational experiences and opportunities to learn.

In practice, I am not sure that we—as a field—sufficiently acknowledge and reflect that, in spite of the broadened research interests and the different theoretical frameworks that we use. To make my point, I want to mention some of what is generally not published in international journals in mathematics education.

What is generally *not* published in major international journals?

I think it is beyond doubt that articles published in the most widely read international journals in mathematics education are written primarily by colleagues working in relatively few countries. To illustrate the point, I looked at articles published in JMTE over the last 5 years, that is, from issue 5 in 2014 to issue 4 in 2019. Over these 5 years, the journal has published 121 articles, not including editorials. The vast majority of these articles, 108 in fact, are written by colleagues from USA, Europe, Australia, Israel, and Canada. Nine other articles are written by authors from one or more of these dominant countries in collaboration with colleagues from another country, in seven cases with colleagues from East Asia. There are, then, only four papers written exclusively by colleagues from Africa, Asia (except Israel), Australasia (except Australia), and Latin America.

There are probably many reasons for this, the most obvious ones being that there are more people involved in mathematics education research in the dominant countries than in other parts of the world and that articles for JMTE—and many other international journals—have to be in English. In spite of that, I wonder if there is an agenda set by the dominant countries that makes it difficult for colleagues from elsewhere to publish for instance in JMTE. I also wonder what it does to our understanding of “the full complexity” of mathematics teaching and learning that most of what is published internationally is from relatively few countries. Much of what is on our research agenda may look very different under different circumstances. For instance:

- What does it mean to attend to student thinking, if there are 50, 70 or more students in the classroom? In what sense is it reasonable to require teachers to do so? And how does the requirement relate to cultural issues such as the general relationships between adults and children?

- What is required in terms of teachers’ capacity for educational design, when there are no or few teaching–learning materials available and the ones that are available are not in the students’ or the teacher’s mother tongue?
- What may be expected in terms of knowledge and beliefs relevant to the teaching mathematics, if the teacher—at best—has completed a 1-year teacher education programme in a different language than the language of instruction and in a setting very different from the village school where she or he takes up teaching?
- What is different in relation to equity, if half the students are likely to drop out before the end of elementary school in order to help with family farming, take up unskilled ad hoc jobs (if there are jobs to get), or live on the streets?
- What does power mean and how do power relations function in schools in areas with violent ethnic conflict, for instance if the teacher represents one ethnic group and the students another?

Obviously, I do not raise the point, because I expect most colleagues from what I referred to as the dominant countries to conduct their research elsewhere. However, I think it is important that we—editors, reviewers, and authors—acknowledge that the issues discussed in our journals may look very different elsewhere and that there may be entirely different and more important issues on the agenda under other circumstances. This means that most of what is published may not be helpful for understanding or contributing to the further development of mathematics teaching and learning in other parts of the world. It also means that important research from elsewhere may not be published, even if it challenges dominant understandings of mathematics education in significant ways.

What about what is published?

I chose to tell the above story with reference to what is generally *not* published in international journals. I do think this is important in and by itself. However, I also think that a somewhat similar argument may be made even in relation to what *is* published: Contexts are not merely external frames for the otherwise autonomous functioning of teaching–learning processes. However, we sometimes tend to write about teachers’ knowledge and beliefs, about noticing, and even about equity as if contextual issues do not matter much. But they do matter, also within the countries that dominate our international journals at present. For instance, as Wallin and Amador point out in their article in the present issue, conditions in rural secondary schools in the USA are different in significant ways from those elsewhere in the country, let alone in other countries. When we conduct our research and write up the results, we need to take that into account. The “full complexity” of mathematics teaching and learning is to a great extent contextual complexity.

Reference

- Christiansen, B., & Walther, G. (1986). Task and activity. In B. Christiansen, A. G. Howson, & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 243–307). Dordrecht: Reidel.

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