

## Promoting teachers' mathematical and pedagogical awareness

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Published online: 23 March 2013  
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Mathematical and pedagogical awareness is crucial for a teacher to notice, articulate, interpret aspects of classroom practice, and make on the spot decisions. Awareness is related to teacher knowledge and is rooted in the context of the actual classroom practice. Mason (2002) talks about different levels of awareness both in mathematics and in mathematics teaching and relates them to the process of noticing that involves systematic reflection on acts or issues. A number of studies at both pre-service and in-service levels that have been published in this Journal of Mathematics Teacher Education [JMTE] focus on what the teachers “notice” in a classroom while teaching or reflecting on it and investigate ways that can support them in the process of developing this ability (e.g., Mellon 2011; Scherer and Steinbring 2007). Approaches that seem to be effective in teacher education and professional development for supporting the development of teacher awareness include teacher interaction and collaboration and the use of theoretical tools in analyzing and studying mathematics teaching.

This theme of teacher awareness is reflected in the articles of this issue of JMTE. The three articles in the research category and one article in the *Mathematics Teacher Education around the World* category provide further insights into the nature and development of mathematics teachers' awareness or noticing during practice. The article by Jimmy Sherrer and Mary Kay Stein and the article by Shari Stockero and Laura Van Zoest focus on teachers' noticing of critical classroom interactions that involves students' contributions and teachers' reactions. Sherrer and Stein propose and evaluate an intervention where a coding scheme was given to the teachers to analyze teaching while Stockero and Van Zoest examine teachers' actions and decisions in the context of actual teaching when pivotal teaching moments emerge. The article by Jennifer Tobias focuses on the development of prospective teachers mathematical awareness related to the fraction concept in the context of a teaching experiment that allowed mathematical communication and argumentation. Finally, the article by Su Liang, Sarah Glaz, Thomas DeFranco, Charles Vinsonhaler, Robin Grenier, and Fabiana Gardetti reports on mathematics teacher education and professional development in China and indicates ways that mathematical and pedagogical awareness of

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the teacher can be developed. The following summary of each article further highlights these connections to teachers' mathematical and pedagogical awareness or noticing.

The Jimmy Sherrer and Mary Kay Stein's article on "Effects of a coding intervention on what teachers learn to notice during whole-group discussion" reports a study that investigated in what ways the teachers can develop their ability to notice classroom interaction through the use of a research tool. The research tool is a coding scheme that the two authors and their collaborators had developed to address the characteristics of instructional practices in cases where the students work on high-level tasks. The scheme focuses on the interrelationships between the students' and the teacher's activity and captures the complexity of classroom interactions. The teachers in the treatment group were initiated in this coding system, and they also used it as a tool to analyze classroom discourse in the form of lesson transcripts. The effect of the intervention of the development of teachers' noticing capacity was studied initially through a pre- and post-test administered to both the experimental group and the control group of teachers. At a second stage, the analysis of argumentation during the intervention by the use of Toulmin's model aimed to go more deeply on the actual process of development. The results indicate that the teachers made certain shifts in their attention focusing on the interaction between teacher and students and in particular on the learning opportunities that these interactions create for the students. Moreover, the use of the coding scheme for analyzing teaching promoted teachers' reflections on their own teaching. A main contribution of this study to mathematics teacher education research and practice is that it indicates that it is possible to develop teachers' attention to significant issues of classroom mathematical discourse by providing theoretical and methodological tools. In this sense, the link between research and practice in mathematics teacher education and professional development becomes meaningful for the classroom teacher.

The Shari Stockero and Laura Van Zoest's article on "Characterizing pivotal teaching moments in beginning mathematics teachers' practice" focuses on the pivotal teaching moments that emerge in the classroom of beginning secondary school teachers, the teaching decisions that are taken and their likely impact on students' mathematical engagement. As in the Sherrer and Stein's article, the focus is on what the teachers notice in the classroom discourse. However, the character of this study is exploratory and the beginning teachers notice and react on events that take place in their own teaching during the actual teaching. The authors provide a framework that categorizes the pivotal teaching moments, the teacher's decision related to pivotal teaching moments and the likely impact on students' mathematical engagement. According to the classification, pivotal teaching moments emerged in five circumstances: (a) when students asked a question or made a comment beyond the mathematics that the teacher had planned to discuss; (b) when students were trying to make sense of the mathematics in the lesson; (c) when students made a mistake; (d) when a mathematical contradiction occurred; and (e) when students expressed mathematical confusion. Moreover, three productive teacher decisions were identified: extend the mathematics and make connections; pursue student mathematical thinking; and emphasize the meaning of the mathematics. In terms of the likely impact that certain teacher decisions have on students' mathematical activity, the findings suggest that the decisions a teacher makes in response to pivotal teaching moments have a significant effect on the way in which a pivotal teaching moment is likely to impact student learning. In particular, teachers' inability to notice pivotal teaching moments generally has a negative impact on student learning while active responses have a positive one. Moreover, focusing on mathematical trajectories and connections might produce more positive likely impacts on student learning. The main contribution of the study is that it provides a framework for deepening our understanding of the ways that a beginning teacher notices, interprets and reacts on students' contributions while teaching. Moreover, it

provides the mathematics teacher educator with tools that can improve teachers' ability to recognize these pivotal teaching moments and develop their teaching decisions and actions to meet the students' mathematical needs.

The Jennifer Tobias's article on "Prospective Elementary Teachers' Development of Fraction Language for Defining the Whole" focuses on the development of prospective elementary teachers' mathematical awareness in the context of a teaching experiment. In particular, the study investigates prospective teachers' understanding of the fraction concept with respect to defining the whole and ways that this understanding develops. The author uses Toulmin's model to analyze the arguments that the prospective teachers gave to support their claims while solving a number of fraction tasks in the context of pizza sharing. Shifts in the arguments were identified, and the concept of "taken-as-shared ideas" was used as an indication that an idea was understood by the class as a whole. Three mathematical ideas related to fraction emerged during the instruction and became taken-as-shared. The first was that fractions depend on a group or a whole. The second was related to the meaning of the expression "of what", while the third, to the meaning of the denominator. In general, the study demonstrates that prospective elementary teachers face difficulties with the meaning of fraction language similar to those that have been reported by studies with children. The main contribution of the study is that it examines the development of prospective teachers' understanding in the context of communication and suggests Toulmin's model as a way to address it. This offers another lens to talk about teacher mathematical knowledge paying more attention on the process of developing it.

Finally, in the *Teacher Education around the world* category, the article "An examination of the preparation and practice of grades 7-12 mathematics teachers from the Shandong Province in China" by Su Liang, Sarah Glaz, Thomas DeFranco, Charles Vinsonhaler, Robin Grenier, and Fabiana Gardetti reports on the characteristics of mathematics teacher education in China both at pre-service and in-service levels. At the pre-service level, the focus is on developing deep mathematical understanding, something that the teachers themselves seem to consider as foundational knowledge for the development of teaching throughout their professional life. In terms of professional development experiences, Chinese teachers have opportunities to develop their mathematical and pedagogical awareness by the following: one-to-one mentoring as beginning teachers; designing collaboratively and making public mathematics teaching to their colleagues (open class); researching together with their colleagues mathematics teaching (lesson preparation, seminars, research projects); and being committed to self-learning through personal readings and after-class reflections. In this context, we see that in a centralized educational system like the one in China, mathematics teacher education is a priority and teachers also value its importance in the continuous development of their teaching.

Overall, the articles presented offer to the field approaches and tools that can improve mathematics teacher education at the level of both research and practice.

## References

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