Differential Geometry and Orientation Analysis in Image Processing

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The synergy between the mathematical fields of partial differential equations, geometric control, Lie group analysis, harmonic analysis, variational methods and the applied fields of image analysis, numerical analysis, neurogeometry, and neuroimaging is increasing rapidly and has attracted many researchers.

An exciting playground for interaction between these fields is multi-orientation image analysis. Here, differential geometrical concepts play a fundamental role in PDE- and ODE-based techniques for pattern recognition and image analysis. Image processing applications are then provided with effective differential geometrical tools to deal generically with complex structures in images.

This JMIV special issue aims to further stimulate this synergy and to provide a showcase for recent state-of-the-art methods combining differential geometry and image processing. The 11 articles are high-quality works in accordance with the high standards of JMIV. The submissions can be categorized into:

1. Geometric models for symmetric and asymmetric geodesic front propagation above the base manifold of positions *and* orientations. Such geodesically equidistant front propagations yield (data-driven) sub-Finslerian, sub-Riemannian spheres, or value functions for elastica models. They allow for globally optimal geodesic tracking and segmentation in the combined space of positions and orientations. The main advantage over existing geodesic tracking methods acting in the image domain (position space) is that such methods deal with complex structures (crossings and bifurcations), as geodesic fronts do not leak at such structures.
2. Geometric perceptual organization models of brightness and orientation. This area aims for perceptual organi-

zation methods for orientation and brightness perception that live on higher dimensional manifolds including orientation, color, and scale. The advantage over existing geodesic tracking methods acting directly in the image domain is that the geometry in higher dimensional manifolds allows for better results in correction and denoising of color images and in perceptual grouping assignments (e.g., of blood vessel patches). It also puts established practical imaging techniques and cortical line perception models into mathematical context.

3. Geometric models for efficient shape and multi-orientation analysis. This research area aims for analysis of shapes and includes shape registration models and shape deformation models. Here, sparse encoding of the data leads to efficient differential geometric multi-orientation methods for local shape analysis, path following and visual tracking.

Next, we address each of these items, where we categorize the submissions in our special issue and where we highlight per category both the novel image analysis techniques and the novel mathematical results.

The first category covers various new image processing techniques and mathematical results, for example on:

- effective image segmentation via antisymmetric front propagation in position and orientation space in the work “*Fast Asymmetric Fronts Propagation for Image Segmentation*” by Da Chen and Laurent Cohen. This work contains new (analysis of) data-driven Randers metrics for effective image segmentation, with clear advantages for image segmentation over (anisotropic) Riemannian approaches.
- efficient and new numerical implementations of front propagation in position and orientation space in the work “*Fast Marching methods for Curvature Penalized Shortest Paths*” by Jean-Marie Mirebeau. This work contains new theoretic results on monotonicity, causality, and convergence of new discretization schemes for anisotropic fast-marching methods to compute distance maps and

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geodesics in Finslerian models on the combined manifold of positions and orientations.

- tracking of complex vasculature and neural fibers via new models and solutions of the Reed-Shepp car in “*Optimal Paths for Variants of the 2D and 3D Reeds-Shepp Car with Applications in Image Analysis*” by Remco Duits, Stephan Meesters, Jean-Marie Mirebeau, and Jorg Portegies. This work improves geodesic vessel/fiber tracking in lifts of image data on the manifold M of positions and orientations. It contains new theorems on:
 - strong convergence of Finslerian minimizing geodesics to sub-Finslerian minimizing geodesics on M .
 - the presence/absence of topological interest points on Finslerian geodesics, and a tangible solution to the problem of cusps in sub-Riemannian image analysis models.
 - the back-tracking of optimal asymmetric Finsler geodesics via manifold splitting into symmetric manifolds (explaining automatic placement of in-place rotations at corners and bifurcations).

The second category covers various new image processing techniques and mathematical results, for example, on:

- differential geometrical models for brightness perception that integrate covariant derivatives to model color appearance phenomena for improved correction tasks of color images, as explained in “*A Geometric Model of Brightness Perception and its Application to Color Images Correction*” by Thomas Batard and Marcelo Bartelmio. Theoretical results are obtained by inclusion of Weber’s/Weber-Fechner’s laws in covariant derivatives in associated vector bundles that are solutions of a variational problem parametrized by a scalar and a Lie group. Here, conditional vanishing of the curvature of the underlying connection one forms has been shown.
- improved perceptual grouping of local orientations. Local fragments of blood vessels are grouped both efficiently and correctly via nilpotent approximations of sub-Riemannian distances in the roto-translation groups $SE(2)$ and $SE(3)$ in “*Nilpotent Approximations of Sub-Riemannian Distances for Fast Perceptual Grouping of Blood Vessels in 2D and 3D*” by Erik Bekkers, Da Chen, and Jorg Portegies. Here, simple local analytic approximations of sub-Riemannian distances on position and orientation space with improved anisotropies and constants in existing estimations are used for better and faster perceptual grouping algorithms.
- improved modeling of the generation of orientation preference maps in the primary visual cortex, considering both orientation and scale features in “*A Geometrical Model of Multiscale Orientation Preference Maps via*

Gabor Functions” by Emre Baspinar, Giovanna Citti, and Alessandro Sarti. Here, the functional architecture of the orientation preference maps in the primary visual cortex is considered as a principal fiber bundle over the 2D retinal plane. The visual stimulus is then lifted in a four-dimensional space with coordinate variables, position, orientation, and scale, through a linear filtering with Gabor functions. Simulation results are compared to relevant neurophysiological findings in the literature.

The third category covers various new techniques and mathematical results:

- shape registration in the work “*Fast Blended Transformations for Partial Shape Registration*” by Alon Shtern, Matan Sela, and Ron Kimmel. Here, over-complete dictionary representations are integrated in shape deformation problems. The approach relies on blend skinning models on the space of positions and orientations and extends the subspace of deformations and controls the smoothness of the reconstructed mesh. It allows for a computation of a new pose from a small number of known feature points. Analysis, splitting, and sparsification of variational problems for as-rigid-as-possible models for shape registration are achieved.
- efficient algorithms for channel representations, which can be used in computer vision applications such as path following and visual tracking, are presented in the work “*Approximative Coding Methods for Channel Representations*” by Kristoffer Öfjäll and Michael Felsberg. In particular, they present new approaches for continuous density reconstruction, for nonlinear gridding, and for estimating Copulas.
- a method for representation of 3D meshes using the spectral geometry of the Hamiltonian operator is presented in the work “*Sparse Approximation of 3D Meshes Using the Spectral Geometry of the Hamiltonian Operator*” by Yoni Choukroun, Gautam Pai, and Ron Kimmel. Compression performance is improved by sparsely encoding the shape geometry using a data-dependent basis, compared to previous results using the standard Laplacian basis.
- the stochastic version of the Beg algorithm is derived and applied to shape and image matching in the paper “*String Methods for Stochastic Image and Shape Matching*” by Alexis Arnaudon, Darry Holm, and Stefan Sommer. The similarity between the presented algorithm and the finite temperature string method used in rare event sampling is also shown.
- a linear algebraic and tensorial framework for the shape-from-shading problem is presented in the work “*Tensors, Differential Geometry and Statistical Shading Analysis*” by Daniel Niels Holtmann-Rice, Benjamin S. Kuns-

berg, and Steven W. Zucker. Using this framework, they investigated when image derivatives exhibit invariance to changing illumination and build a boot-strapping algorithm to find the surface solutions.

Altogether, this special issue provides a comprehensive overview of state-of-the-art differential geometrical methods in image processing, with an emphasis on image processing that takes place on the base manifold of positions and orientations. To avoid any conflicts of interest, manuscripts that

have been co-authored by one guest editor have been handled by another guest editor.

This special issue was preceded by an international workshop on “Geometry, PDE’s and Lie Groups in Image Analysis”, URL:<http://bmia.bmt.tue.nl/people/RDuits/workshop/index.html>, held at Eindhoven University of Technology (August 24–26, 2016, Eindhoven, the Netherlands).

The guest editors, Giovanna Citti, Remco Duits, Andrea Fuster, Thomas Schultz.