Journal of Intelligent & Robotic Systems (2023) 109:8
https://doi.org/10.1007/s10846-023-01941-z

REGULAR PAPER

Asymmetric Bipartite Consensus of Nonlinear Agents with Communication Noise

Sabyasachi Mondal[1] · **Antonios Tsourdos**[1]

Received: 16 July 2022 / Accepted: 24 July 2023 / Published online: 24 August 2023

Abstract

In this paper, the asymmetric bipartite consensus problem of a nonlinear multi-agent system is solved using Distributed Nonlinear Dynamic Inversion (DNDI) based controller. The application of DNDI is new in the context of asymmetric bipartite consensus, and it inherits all the advantages of NDI and works efficiently to solve the asymmetric bipartite problem. The mathematical details presented provide theoretical proof of its efficiency. A realistic simulation study is performed to establish the claims. The controller's performance has been tested in the presence of communication noise, and the results are promising.

Keywords Distributed nonlinear dynamic inversion · Asymmetric bipartite consensus · Communication noise

1 Introduction

The consensus among agents is a key mechanism for networked multi-agent operations. The consensus process brings the states of all agents to an agreement to achieve a common goal. The consensus control algorithm is designed with different branches of control theory and uses the information the neighbouring agents share. The widespread application of consensus includes distributed computation [1], cooperative mobile robotics [2], tracking [3], synchronization [4], sensory networks [5] etc. It can be mentioned that the application of consensus is not limited to the engineering domain only. It extends to application in biology, ecology and social sciences (for example, flocking [6, 7] and dynamics of opinion-forming [8]). There are many papers where various consensus problems have been solved, considering communication issues (switching topology, delays, noise), disturbance, and fault [9–19].

The consensus of the states of the agents is achieved when they interact on a nonnegative graph. However, cooperation and competition (antagonistic interactions) may exist among agents when they interact on a signed network with positive and negative edge weights. Altafini [20] showed cooperation and competition result in a division of a group into two with the same consensus value but opposite sign, i.e., two consensus values are symmetric. This type of consensus is named bipartite consensus. The bipartite consensus was applied to the social network and opinion dynamics [21]. Many papers exist where bipartite consensus was designed for agents with linear dynamics [22–33]. A few research were done for nonlinear agents [34–41]. However, the consensus does not need to be symmetric always because trust/distrust and cooperation/competition between the groups may be at different levels. Therefore, agents in one group may have a non-symmetric consensus value, i.e., the consensus values of two groups are different both in magnitude and sign. This type of bipartite consensus is named asymmetric bipartite consensus. This problem was addressed by Guo et al. [42], where a new class of general Laplacian matrices was proposed. The authors solved a finite time asymmetric bipartite problem [43] to guarantee that all agents could achieve the asymmetric bipartite consensus in a fixed time. Next, they proposed pinning asymmetric bipartite consensus protocol for multi-agent systems considering communication delays. Liang et al. [44] presented an iterative learning-based control protocol for nonlinear agents. It can be noticed that the bipartite consensus can be regarded as a special case of asymmetric bipartite consensus. Therefore, the asymmetric bipartite consensus is applied to solve problems which are solved using bipartite consensus.

✉ Sabyasachi Mondal
sabyasachi.mondal@cranfield.ac.uk

Antonios Tsourdos
a.tsourdos@cranfield.ac.uk

1 Aerospace, Cranfield University, College Rd, MK43 0AL Cranfield, Bedford, UK

The asymmetric consensus protocols proposed so far have been considered mostly for linear agents. The only work that considered nonlinear agent dynamics explained the iterative learning approach. We propose a nonlinear consensus controller based on Distributed Nonlinear Dynamic Inversion (DNDI) [17], which is designed using Nonlinear Dynamic Inversion (NDI) [45–47]. DNDI is used to solve consensus problems for nonlinear agents with external disturbance [41] and faults [19]. DNDI inherits the advantages of NDI, like closed-form control expression, easy mechanization, global exponential stability, the inclusion of nonlinear kinematics in the plant inversion, and minimization of the need for individual gain tuning or gain scheduling. The contribution of this work is summarized as follows.

- A variety of NDI-based distributed controllers (DNDI) is proposed in this paper to solve the asymmetric bipartite consensus problem of nonlinear agents. The application of an NDI-based controller for the asymmetric bipartite problem is new.
- Mathematical details for the controller's convergence are included, which provides a theoretical basis for its effectiveness.
- The performance of the proposed controller is evaluated using a simulation study in MATLAB.
- Moreover, we added communication noise to make the simulation scenario more realistic. Results are obtained with and without noise to prove the effectiveness of the controller.

The rest of the paper is organized as follows. Preliminaries are given in Section 2. The problem description is given in Section 3. The mathematical details of the DNDI for asymmetric bipartite consensus are shown in Section 4. The convergence study of the controller is presented in Section 5. The simulation results are discussed in Section 6. Finally, the conclusion is given in Section 7.

2 Preliminaries

In this section, we present a few topics to support the work in this paper.

2.1 Asymmetric Bipartite Consensus

Definition 1 A group of agents achieve asymmetric bipartite consensus if there exists a constant κ such that

- $\lim_{t\to\infty}(x_i(t)) = \rho, \forall i \in \Omega_1$.
- $\lim_{t\to\infty}\left(x_j(t)\right) = -\kappa\rho, \forall j \in \Omega_2$.

are satisfied, where $x_i(t)$ and $x_j(t)$ denote the states of the i^{th} and j^{th} agents respectively. $\kappa > 0$,
$\Omega_1 = \{v_1, v_2, \ldots v_m\}$, $\Omega_2 = \{v_{m+1}, v_{m+2}, \ldots v_N\}$, and
$\Omega = \Omega_1 \cup \Omega_2$.

2.2 Graph Theory

The communication topology is described using a weighted graph, given by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. The vertices $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ represent the agents. The edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent the connection among the agents. The elements of weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \Re^{N\times N}$ of $\mathcal{G}$ are $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$, otherwise $a_{ij} = 0$. We do not consider any self loop. Therefore, the adjacency matrix A has zero diagonal elements, i.e., $v_i \in V$, $a_{ii} = 0$. The degree matrix can be given by $D \in \Re^{N\times N} = diag\{d_1\ d_2\ \ldots d_N\}$, where $d_i = \sum_{j\in N_i} a_{ij}$. Therefore, the Laplacian matrix is written as $\mathcal{L} = D - \mathcal{A}$.

Generally, the synchronization of networked agents is explained using the Laplacian matrix $\mathcal{L}$ of a nonnegative graph. However, the Laplacian matrix for a signed graph is defined differently. In the case of a signed graph, we define the laplacian matrix as signed laplacian ($\mathcal{L}_s$) given by

$$\mathcal{L}_s = diag\left(\sum_{j=1}^{N} | a_{1j} |, \ldots, \sum_{j=1}^{N} | a_{1j} |\right) - \mathcal{A} \tag{1}$$

2.3 Communication Noise

The communication noise perturbs the information shared among the agents. Therefore the perturbed information is received by i^{th} agent from its neighbours. We present an additive noise model by $\tilde{X}_{ji} = X_{ji} + \sigma_{ji}\omega_{ji}$ which represents the noise added to information received by i^{th} agent from neighbouring j^{th} agents $j \in N_i$. $X_i, X_j \in \Re^n$ are states. ω_{ji}; $i, j \in 1, 2, \ldots, N$ denotes independent white noises, and σ_{ji} is the noise intensity.

2.4 Theorems and Lemmas

A few useful lemmas are presented here.

Definition 1 [20, 48] A signed graph is structurally balanced if it has a bipartition of the nodes $\mathcal{V}_1, \mathcal{V}_2$, i.e., $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ such that $a_{ij} \leq 0, \forall v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q$ where $p, q \in 1, 2,\ p \neq q$, and $\emptyset$ is empty set; otherwise $a_{ij} \geq 0$.

Lemma 1 [49] *A spanning tree is structurally balanced.*

Lemma 2 [50] *Let us consider that the signed graph $\mathcal{G}(\mathcal{A})$ has a spanning tree. The signature matrices set as*

$$\mathcal{D} = \{D = diag\,(\sigma_1,\ \sigma_2,\ \ldots, \sigma_N)\,; \sigma_i \in \{1,\ -1\}\} \tag{2}$$

Then the following statements are equivalent.

1. *$\mathcal{G}(\mathcal{A})$ is structurally balanced.*
2. *$a_{ij}a_{ji} \geq 0$ and the associated undirected graph $\mathcal{G}(\mathcal{A}_u)$ is structurally balanced, where $\mathcal{G}(\mathcal{A}_u) = \frac{\mathcal{A}+\mathcal{A}^T}{2}$.*
3. *$\exists D \in \mathcal{D}$, such that $\bar{\mathcal{A}} = [\bar{a}_{ij}] = D\mathcal{A}D$ is a nonnegative matrix.*

Lemma 3 [51] *Suppose the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree. If the graph is structurally balanced, then 0 is a simple eigenvalue of its Laplacian matrix, and all its other eigenvalues have positive real parts, but not vice versa.*

Corollary 1 [28] *Suppose the signed graph $\mathcal{G}(\mathcal{A})$ is undirected and connected. The graph is structurally balanced, if and only if 0 is a simple eigenvalue of $\mathcal{L}$ and all other eigenvalues have positive real parts.*

3 Problem Description

The objective of this work is to design a consensus protocol to solve asymmetric bipartite consensus problems among the agents having nonlinear dynamics. The consensus controller designed here is the modification of Distributed NDI (DNDI), which is derived in the following section. The dynamics of i^{th} agent is given in Eqs. 3-4.

$$\dot{X}_i = f(X_i) + g(X_i)U_i \tag{3}$$
$$Y_i = X_i \tag{4}$$

The state and control of i^{th} agent is given by $X_i \in \Re^n$ and $U_i \in \Re^n$ respectively. The output of i^{th} agent is given by

$$Y_i = X_i \in \Re^n \tag{5}$$

Assumption 1 *The system is perfectly known and the matrix $g(X)$ is invertible for all time.*

Assumption 2 *The agents share information over the communication topology described by a signed digraph $\mathcal{G}(\mathcal{A})$ having a spanning tree, and the topology is structurally balanced.*

Also, the controller's performance is tested in the presence of noise.

4 Asymmetric Bipartite consensus using Distributed Nonlinear Dynamic Inversion (DNDI) Controller

The Distributed Nonlinear Dynamic Inversion (DNDI) controller [17] is designed to solve the ordinary consensus problem of nonlinear agents. Also, DNDI can handle communication issues and external disturbance [18] in consensus problems. DNDI is modified to solve the bipartite consensus problem for nonlinear agents with communication noise [41]. We consider an undirected signed graph in this study to analyse the asymmetric bipartite problem and the convergence of the controller.

The agents are split in two groups $\Omega_1 = \{v_1, v_2, \ldots v_m\}$, $\Omega_2 = \{v_{m+1}, v_{m+2}, \ldots v_N\}$, and $\Omega = \Omega_1 \cup \Omega_2$. The error in states $X_i \in \Re^n; n > 1$ of i^{th} agent is given as follows.

$$\boldsymbol{\delta}_{i_{\Omega_1}} = |\ \bar{d}_i\ |_{\Omega_1} X_{i_{\Omega_1}} - \bar{a}_{i_{\Omega_1}}\mathbf{X}_{\Omega_1},\ \text{if } i \in \Omega_1. \tag{6}$$
$$\boldsymbol{\delta}_{i_{\Omega_2}} = |\ \bar{d}_i\ |_{\Omega_2} X_{i_{\Omega_2}} - \bar{a}_{i_{\Omega_2}}\mathbf{X}_{\Omega_2},\ \text{if } i \in \Omega_2. \tag{7}$$

where $|\ \bar{d}_i\ |_{\Omega_1} = \left(|\ d_i\ |_{\Omega_1} \otimes \mathbf{I}_n + \kappa\ |\ d_i\ |_{\Omega_2} \otimes \mathbf{I}_n\right) \in \Re^{n\times n}$, $|\ d_i\ |_{\Omega_2} = \left(\frac{1}{\kappa}\ |\ d_i\ |_{\Omega_1} \otimes \mathbf{I}_n + |\ d_i\ |_{\Omega_2} \otimes \mathbf{I}_n\right) \in \Re^{n\times n}$, scaling factor $\kappa > 0$, $\bar{a}_{i_{\Omega_1}} = (a_{i_{\Omega_1}} \otimes \mathbf{I}_n) \in \Re^{n\times nm}$, $\bar{a}_{i_{\Omega_2}} = (a_{i_{\Omega_2}} \otimes \mathbf{I}_n) \in \Re^{n\times n(N-m)}$, $\bar{a}_{i_\Omega} = [a_{i_{\Omega_2}} \otimes \mathbf{I}_n\ a_{i_{\Omega_2}} \otimes \mathbf{I}_n]$, $\mathbf{X}_{\Omega_1} = \begin{bmatrix} X_1\ X_2\ \ldots\ X_m \end{bmatrix} \in \Re^{mn}$, $\mathbf{X}_{\Omega_2} = \begin{bmatrix} X_{m+1}\ X_{m+2}\ \ldots\ X_N \end{bmatrix} \in \Re^{n(N-m)}$. $\mathbf{I}_n$ is $n \times n$ identity matrix. $|\ d_i\ |_{\Omega_1} = \sum_{j=1}^{m} |\ a_{ij}\ | \in \Re$, $|\ d_i\ |_{\Omega_2} = \sum_{j=m+1}^{N} |\ a_{ij}\ | \in \Re$, $a_{i_{\Omega_1}} = [a_{i1}\ a_{i2} \ldots\ a_{im}] \in \Re^m$, $a_{i_{\Omega_2}} = [a_{im+1}\ a_{im+2} \ldots\ a_{iN}] \in \Re^{N-m}$, $a_{i_\Omega} = [a_{i1}\ a_{i2} \ldots\ a_{im}\ a_{im+1} \ldots\ a_{iN}] \in \Re^N$.

The agents have different control expression depending on which group they belong Ω_1 or Ω_2. Let us construct a Lyapunov function $V_{i_{\Omega_1}}$ as given below.

$$V_{i_{\Omega_1}} = \frac{1}{2}\boldsymbol{\delta}_{i_{\Omega_1}}^T \boldsymbol{\delta}_{i_{\Omega_1}} \tag{8}$$

Differentiation of Eq. 8 yields

$$\dot{V}_{i_{\Omega_1}} = \boldsymbol{\delta}_{i_{\Omega_1}}^T \dot{\boldsymbol{\delta}}_{i_{\Omega_1}} \tag{9}$$

According to the Lyapunov stability theory, the time derivative of the Lyapunov function should be

$$\dot{V}_{i_{\Omega_1}} = -\boldsymbol{\delta}_{i_{\Omega_1}}^T \kappa_{i_{\Omega_1}} \boldsymbol{\delta}_{i_{\Omega_1}} \tag{10}$$

where, $\kappa_{i_{\Omega_1}} \in \Re^{n\times n} > 0$ is a diagonal gain matrix. The expression of $\dot{V}_{i_{\Omega_1}}$ in Eqs. 9 and 10 are equated to obtain

$$\boldsymbol{\delta}_{i_{\Omega_1}}^T \dot{\boldsymbol{\delta}}_{i_{\Omega_1}} = -\boldsymbol{\delta}_{i_{\Omega_1}}^T \kappa_{i_{\Omega_1}} \boldsymbol{\delta}_{i_{\Omega_1}} \tag{11}$$

Equation 11 is simplified as follows.

$$\dot{\boldsymbol{\delta}}_{i_{\Omega_1}} + \kappa_{i_{\Omega_1}} \boldsymbol{\delta}_{i_{\Omega_1}} = 0 \tag{12}$$

Differentiation of Eq. 6 yields

$$\begin{aligned}\dot{\boldsymbol{\delta}}_{i_{\Omega_1}} &= \mid \bar{d}_i \mid_{i_{\Omega_1}} \dot{X}_{i_{\Omega_1}} - \bar{a}_{i_{\Omega_1}} \dot{\mathbf{X}}_{\Omega_1} \\ &= \mid \bar{d}_i \mid_{i_{\Omega_1}} \Big[f(X_{i_{\Omega_1}}) + g(X_{i_{\Omega_1}}) U_{i_{\Omega_1}} \Big] - \bar{a}_{i_{\Omega_1}} \dot{\mathbf{X}}_{\Omega_1}\end{aligned} \tag{13}$$

Substitution of the expressions $\dot{\boldsymbol{\delta}}_{i_{\Omega_1}}$ in Eq. 12 yields

$$\begin{aligned}\mid \bar{d}_i \mid_{i_{\Omega_1}} \Big(f(X_{i_{\Omega_1}}) + g(X_{i_{\Omega_1}}) U_{i_{\Omega_1}} \Big) - \bar{a}_{i_{\Omega_1}} \dot{\mathbf{X}}_{\Omega_1} \\ + \kappa_{i_{\Omega_1}} \Big(\mid \bar{d}_i \mid_{\Omega_1} X_{i_{\Omega_1}} - \bar{a}_{i_{\Omega_1}} \mathbf{X}_{\Omega_1} \Big) = 0\end{aligned} \tag{14}$$

The resulting expression of $U_{i_{\Omega_1}}$ for i^{th} agent is obtained by simplifying Eq. 14 as follows

$$\begin{aligned}U_{i_{\Omega_1}} &= \Big(g(X_{i_{\Omega_1}}) \Big)^{-1} \Big[- f\Big(X_{i_{\Omega_1}}\Big) \\ &+ \mid \bar{d}_i \mid_{\Omega_1}^{-1} \Big(\bar{a}_{i_{\Omega_1}} \dot{\mathbf{X}}_{i_{\Omega_1}} - \kappa_{i_{\Omega_1}} \Big(\mid \bar{d}_i \mid_{\Omega_1} X_{i_{\Omega_1}} - \bar{a}_{i_{\Omega_1}} \mathbf{X}_{\Omega_1} \Big)\Big)\Big]\end{aligned} \tag{15}$$

Similar expression can be obtained for $U_{i_{\Omega_2}}$. It is given as follows.

$$\begin{aligned}U_{i_{\Omega_2}} &= \Big(g(X_{i_{\Omega_2}}) \Big)^{-1} \Big[- f(X_{i_{\Omega_2}}) \\ &+ \mid \bar{d}_i \mid_{\Omega_2}^{-1} \Big(\bar{a}_{i_{\Omega_2}} \dot{\mathbf{X}}_{i_{\Omega_2}} - \kappa_{i_{\Omega_2}} \Big(\mid \bar{d}_i \mid_{\Omega_2} X_{i_{\Omega_2}} - \bar{a}_{i_{\Omega_2}} \mathbf{X}_{\Omega_2} \Big)\Big)\Big]\end{aligned} \tag{16}$$

where, $\kappa_{i_{\Omega_2}} \in \Re^{n \times n} > 0$ is a diagonal gain matrix.

5 Convergence of DNDI for Asymmetric Bipartite Consensus

Theorem 1 *A group of nonlinear agents (dynamics given in Eqs. (3)-(4) along with the assumptions 1 and 2, achieve asymmetric bipartite consensus using the consensus protocol obtained in Eqs. (15)-(16). The Uniformly Ultimate Boundedness (UUB) of the consensus error ($\boldsymbol{\delta}_{i_{\Omega_1}}$ and $\boldsymbol{\delta}_{i_{\Omega_2}}$) ensures the convergence and asymmetric bipartite consensus.*

Proof:

In this section we present the convergence proof of the proposed controller. We define a Lyapunov function

$$\Psi = \frac{1}{2} \mathbf{X}_{\Omega}^T \Big(L_s \otimes \mathbf{I}_n \Big) \mathbf{X}_{\Omega} \tag{17}$$

where, $\mathbf{X}_{\Omega} = [\mathbf{X}_{\Omega_1} \, \mathbf{X}_{\Omega_2}]^T \in \Re^{nN}$. In this study, a undirected and connected graph considered. Therefore, we write $L_s \otimes \mathbf{I}_n$ as

$$L_s \otimes \mathbf{I}_n = \Theta \Pi \Theta^T \tag{18}$$

where, $\Theta \in \Re^{nN \times nN}$ is the left eigenvalue matrix of $L_s \otimes \mathbf{I}_n$, $\Pi = (diag\{0, \lambda_2(L_s), \lambda_3(L_s), \ldots, \lambda_N(L_s)\} \otimes \mathbf{I}_n) \in \Re^{nN \times nN}$ is eigenvalue matrix, $\Theta^T \Theta = \Theta \Theta^T = \mathbf{I}_{nN \times nN}$.

$$\begin{aligned}\Psi &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Big(L_s \otimes \mathbf{I}_n \Big) \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Theta \Pi \Theta^T \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Theta \sqrt{\Pi} \sqrt{\Pi} \, \Theta^T \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Theta \sqrt{\Pi \bar{\Pi}} \sqrt{\bar{\Pi}^{-1}} \sqrt{\bar{\Pi}^{-1}} \sqrt{\bar{\Pi} \Pi} \, \Theta^T \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Theta \Pi \bar{\Pi}^{-1} \Pi \Theta^T \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Theta \Pi \Big(\Theta^T \Theta \Big) \bar{\Pi}^{-1} \Big(\Theta^T \Theta \Big) \Pi \Theta^T \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T \Big(\Theta \Pi \Theta^T \Big) \Big(\Theta \bar{\Pi}^{-1} \Theta^T \Big) \Big(\Theta \Pi \Theta^T \Big) \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \mathbf{X}_{\Omega}^T (L_s \otimes \mathbf{I}_n) \Upsilon (L_s \otimes \mathbf{I}_n) \mathbf{X}_{\Omega} \\ &= \frac{1}{2} \Xi^T \Upsilon \Xi\end{aligned} \tag{19}$$

where $\bar{\Pi} = (diag\{\lambda_2(L_s), \lambda_2(L_s), \lambda_3(L_s), \ldots, \lambda_N(L_s)\} \otimes \mathbf{I}_n) \in \Re^{nN \times nN}$, $\Xi = \Big[\boldsymbol{\delta}_{1_{\Omega_1}}^T \; \boldsymbol{\delta}_{2_{\Omega_1}}^T \ldots \boldsymbol{\delta}_{m_{\Omega_1}}^T \; \boldsymbol{\delta}_{m+1_{\Omega_2}}^T \ldots \boldsymbol{\delta}_{N_{\Omega_2}}^T \Big]^T \in \Re^{nN}$, and $\Upsilon = \Theta \bar{\Pi}^{-1} \Theta^T \in \Re^{nN \times nN}$.

Remark 1 Using Eqs. 17 and 19, we can write

$$\frac{\lambda_{\min}(\Upsilon)}{2} \parallel \Xi \parallel^2 \leq \Psi \leq \frac{\lambda_{\max}(\Upsilon)}{2} \parallel \Xi \parallel^2 \tag{20}$$

$$\Psi = \frac{1}{2} \mathbf{X}_{\Omega}^T (L_s \otimes \mathbf{I}_n) \mathbf{X}_{\Omega} = \frac{1}{2} \mathbf{X}_{\Omega}^T \Xi \tag{21}$$

Remark 2 According to Lemma 4, $\lambda_2 > 0$. Hence, $\bar{\Pi}$ is invertible.

Remark 3 It can be observed that $\Upsilon = \Theta \bar{\Pi}^{-1} \Theta^T$ is positive definite matrix. Therefore, Ψ is positive definite subject to consensus error and qualify for a Lyapunov function.

Differentiation of Eq. 17 yields

$$\begin{aligned}\dot{\Psi} &= \mathbf{X}_{\Omega}^T (L_s \otimes \mathbf{I}_n) \dot{\mathbf{X}}_{\Omega} = \Xi^T \dot{\mathbf{X}}_{\Omega} \\ &= \sum_{i=1}^{m} \boldsymbol{\delta}_{i_{\Omega_1}}^T \Big[f(X_{i_{\Omega_1}}) + g(X_{i_{\Omega_1}}) U_{i_{\Omega_1}} \Big]\end{aligned}$$

$$+\sum_{i=m+1}^{N}\boldsymbol{\delta}_{i\Omega_2}^T\left[f(X_{i\Omega_2})+g(X_{i\Omega_2})U_{i\Omega_2}\right] \tag{22}$$

Substitution of the control expression of $U_{i\Omega_1}$ and $U_{i\Omega_2}$ in Eq. 22 gives

$$\dot{\Psi}=\sum_{i=1}^{m}\boldsymbol{\delta}_{i\Omega_1}^T\left[\mid \bar{d}_i^{-1}\mid_{\Omega_1}(\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}-\kappa_{i\Omega_1}\boldsymbol{\delta}_{i\Omega_1})\right]+\sum_{i=m+1}^{N}\boldsymbol{\delta}_{i\Omega_2}^T\left[\mid \bar{d}_i^{-1}\mid_{\Omega_2}(\bar{a}_{i\Omega_2}\dot{\mathbf{X}}_{i\Omega_2}-\kappa_{i\Omega_2}\boldsymbol{\delta}_{i\Omega_2})\right] \tag{23}$$

We have two similar terms in Eq. 23 for two groups. We show the convergence of controller for group Ω_1 . For group Ω_2, the convergence can be obtained similarly. The term for group Ω_1 is written as

$$\sum_{i=1}^{m}\boldsymbol{\delta}_{i\Omega_1}^T\left[\mid \bar{d}_i^{-1}\mid_{\Omega_1}\left(\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}-\kappa_{i\Omega_1}\boldsymbol{\delta}_{i\Omega_1}\right)\right]$$

$$=\sum_{i=1}^{m}\boldsymbol{\delta}_{i\Omega_1}^T\mid \bar{d}_i^{-1}\mid_{\Omega_1}\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}-\sum_{i=1}^{m}\boldsymbol{\delta}_{i\Omega_1}^T\mid \bar{d}_i^{-1}\mid_{\Omega_1}\kappa_{i\Omega_1}\boldsymbol{\delta}_{i\Omega_1} \tag{24}$$

We have mentioned a few Lemmas (4-6) which help us to proceed further with the derivation.

Lemma 4 *The Laplacian matrix L in an undirected graph is semi-positive definite, it has a simple zero eigenvalue, and all the other eigenvalues are positive if and only if the graph is connected. Therefore, L is symmetric and it has N non-negative, real-valued eigenvalues $0=\lambda_1<\lambda_2\leq\ldots\leq\lambda_N$* [43].

Lemma 5 [52] *Let $\psi_1(t),\psi_2(t)\in R^m$ be continuous positive vector functions, by Cauchy inequality and Young's inequality, there exists the following inequality:*

$$\psi_1(t)\psi_2(t)\leq\parallel\psi_1(t)\parallel\parallel\psi_2(t)\parallel\leq\frac{\parallel\psi_1(t)\parallel^{\lambda}}{\lambda}+\frac{\parallel\psi_2(t)\parallel^{\zeta}}{\zeta} \tag{25}$$

where

$$\frac{1}{\lambda}+\frac{1}{\zeta}=1.$$

Lemma 6 *[53] Let $R(t)\in\Re$ be a continuous positive function with bounded initial $R(0)$. If the inequality holds $\dot{R}(t)\leq-\beta R(t)+\eta$ where, $\beta>0,\eta>0$, then the following inequality holds.*

$$R(t)\leq R(0)e^{-\beta t}+\frac{\eta}{\beta}\left(1-e^{-\beta t}\right). \tag{26}$$

Using Lemma 5, the first term of Eq. 24 can be written as

$$\boldsymbol{\delta}_{i\Omega_1}^T\mid \bar{d}_i^{-1}\mid_{\Omega_1}\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}\leq\parallel\boldsymbol{\delta}_{i\Omega_1}\parallel\parallel\mid \bar{d}_{i\Omega_1}^{-1}\mid\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}\parallel$$

$$\leq\frac{\parallel\boldsymbol{\delta}_{i\Omega_1}\parallel^2}{2}+\frac{\parallel\mid \bar{d}_i^{-1}\mid_{\Omega_1}\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}\parallel^2}{2} \tag{27}$$

Substituting $\sum_{i=1}^{m}-\boldsymbol{\delta}_{i\Omega_1}^T\mid \bar{d}_i^{-1}\mid_{\Omega_1}\kappa_{i\Omega_1}\boldsymbol{\delta}_{i\Omega_1}$ in Eq. 23 with inequality relation, we get

$$\dot{\Psi}\leq\sum_{i=1}^{m}\left[-\boldsymbol{\delta}_{i\Omega_1}^T\mid \bar{d}^{-1}\mid_{i\Omega_1}\kappa_{i\Omega_1}\boldsymbol{\delta}_{i\Omega_1}+\frac{\parallel\boldsymbol{\delta}_{i\Omega_1}\parallel^2}{2}+\frac{\parallel\mid \bar{d}^{-1}\mid_{i\Omega_1}\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}\parallel^2}{2}\right] \tag{28}$$

By designing the gain $\kappa_{i\Omega_1}$ as

$$\kappa_{i\Omega_1}=\mid\bar{d}\mid_{i\Omega_1}\left(\frac{1}{2}+\frac{\rho_{i\Omega_1}}{2}\lambda_{max}(\Upsilon)\right). \tag{29}$$

$\rho_{i\Omega_1}>0$. Equation 28 can be written as

$$\dot{\Psi}\leq\sum_{i=1}^{m}\left[-\frac{\rho_{i\Omega_1}}{2}\lambda_{max}(\Upsilon)\parallel\boldsymbol{\delta}_{i\Omega_1}\parallel^2+\frac{\parallel\mid \bar{d}^{-1}\mid_{i\Omega_1}\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}\parallel^2}{2}\right]\leq-\rho_{i\Omega_1}\Psi+\Gamma_{i\Omega_1}. \tag{30}$$

where, $\Gamma_{i\Omega_1}=\sum_{i=1}^{m}\frac{\parallel\mid\bar{d}^{-1}\mid_{i\Omega_1}\bar{a}_{i\Omega_1}\dot{\mathbf{X}}_{i\Omega_1}\parallel^2}{2}$. Applying Lemma 6 we get

$$\Psi\leq\frac{\Gamma_{i\Omega_1}}{\rho_{i\Omega_1}}+\left(\Psi(0)-\frac{\Gamma_{i\Omega_1}}{\rho_{i\Omega_1}}\right)e^{-\rho_{i\Omega_1}t} \tag{31}$$

Hence, we conclude that Ψ is bounded as $t\to\infty$. In addition, we show the Uniformly Ultimate Boundedness (UUB) here. Using Eqs. 20, 31, and Lemma 6 presented by Ge et al. [53] we can write

$$\frac{\lambda_{\min}(\Upsilon)}{2}\parallel\Xi\parallel^2\leq\Psi\leq\frac{\Gamma_{i\Omega_1}}{\rho_{i\Omega_1}}+\left(\Psi(0)-\frac{\Gamma_{i\Omega_1}}{\rho_{i\Omega_1}}\right)e^{-\rho_{i\Omega_1}t} \tag{32}$$

Equation 32 is simplified as

$$\frac{\lambda_{\min}(\Upsilon)}{2}\parallel\Xi\parallel^2\leq\frac{\Gamma_{i\Omega_1}}{\rho_{i\Omega_1}}+\left(\Psi(0)-\frac{\Gamma_{i\Omega_1}}{\rho_{i\Omega_1}}\right)e^{-\rho_{i\Omega_1}t}$$

Therefore,

$$\parallel \Xi \parallel \leq \sqrt{\frac{2\frac{\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}}} + 2\left(\Psi(0) - \frac{\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}}}\right) e^{-\rho_{i_{\Omega_1}} t}}{\lambda_{\min}(\Upsilon)}} \tag{33}$$

If $\Psi(0) = \frac{\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}}}$ then we can write

$$\parallel \Xi \parallel \leq \sigma^*. \tag{34}$$

$\forall t \geq 0$ and $\sigma^* = \sqrt{\frac{2\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}} \lambda_{\min}(\Upsilon)}}$. If $\Psi(0) \neq \frac{\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}}}$ then for any given $\sigma > \sigma^*$ there exist a time $T > 0$ such that $\forall t > T$, $\parallel \Xi \parallel \leq \sigma$.

$$\sigma = \sqrt{\frac{2\frac{\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}}} + 2\left(\Psi(0) - \frac{\Gamma_{i_{\Omega_1}}}{\rho_{i_{\Omega_1}}}\right) e^{-\rho_{i_{\Omega_1}} T}}{\lambda_{\min}(\Upsilon)}}. \tag{35}$$

Therefore, we can conclude

$$\lim_{t\to\infty} \parallel \Xi \parallel = \sigma^*. \tag{36}$$

i.e., the consensus error for the proposed controller converges to σ^*. Hence, the consensus error converges, and the agents achieve the asymmetric bipartite consensus.

6 Simulation Study

The simulation results are presented here. We have used a signed graph to represented the communication topology given as follows.

6.1 Communication topology

The communication topology is represented by a signed graph. The adjacency matrix corresponding to the graph is given in Eq. 37.

$$\mathcal{A} = \begin{bmatrix} 0 & 3 & 0 & -.5 & 0 & -1 \\ 3 & 0 & .4 & 0 & 0 & -1 \\ 0 & .4 & 0 & -1.5 & 0 & 0 \\ -.5 & 0 & -1.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 2.5 & 0 & 1 \\ -1 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{37}$$

The graph obtained using the adjacency matrix is shown in Fig. 1. The weights are assigned to each edge. It can be observed that the adjacency matrix gives an undirected signed graph. In Fig. 2, we have shown the distinct eigenvalues of the Laplacian matrix (L_s) of this signed graph. It is clear that one eigenvalue is zero, and the rest have a positive real part. Therefore, we can conclude that the graph has a spanning tree and is structurally balanced (according to Corollary 1).

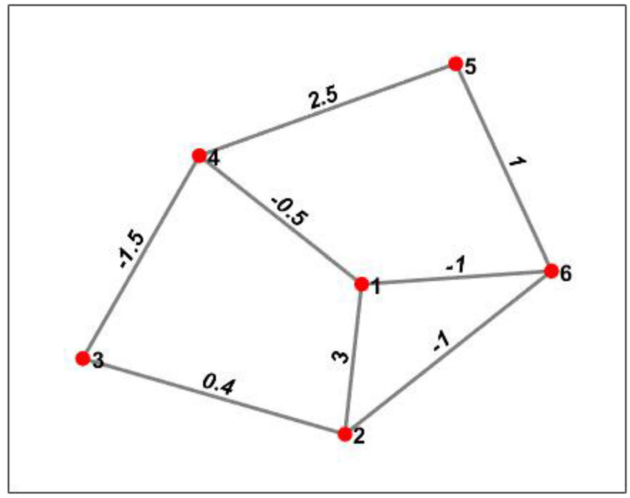

Fig. 1 Signed graph corresponding to $\mathcal{A}$

We have considered two cases to demonstrate the performance of the controller. They are given below.

- Case 1: Without noise
- Case 2: With noise

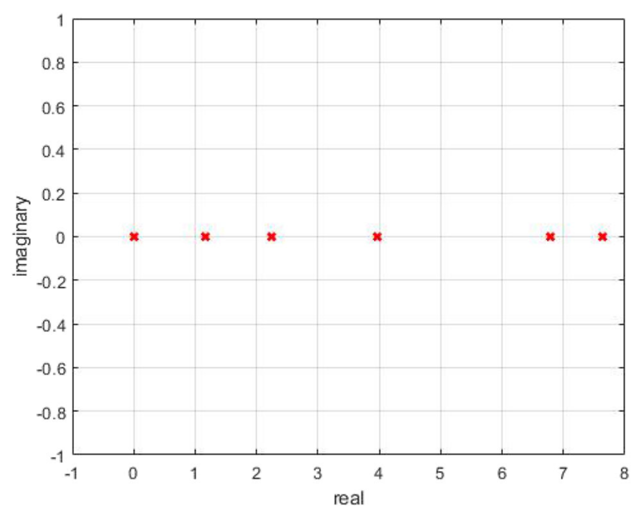

Fig. 2 Eigen values of signed graph. One eigenvalue is zero and the rest have a positive real part

6.2 Agent Dynamics considered for Simulation Study

The nonlinear dynamics considered for the i^{th} agents is given by [41]

$$\dot{X}_{i_1} = X_{i_2} \sin(2X_{i_1}) + U_{i_1} \tag{38}$$
$$\dot{X}_{i_2} = X_{i_1} \cos(3X_{i_2}) + U_{i_2} \tag{39}$$

where $X_i = \left[X_{i_1} \; X_{i_2}\right]^T$. Substituting the dynamics of Eqs. 38 and 39 in the form given in Eqs. 3 and 4 we write

$$f(X_i) = \begin{bmatrix} X_{i_2} \sin(2X_{i_1}) \\ X_{i_1} \cos(3X_{i_2}) \end{bmatrix} \tag{40}$$

and

$$g(X_i) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{41}$$

and

$$U_i = \begin{bmatrix} U_{i_1} \\ U_{i_2} \end{bmatrix} \tag{42}$$

where $X_i \in \Re^2$. The states X_{1_i} of all the agents are denoted by $\mathbf{X}_1 = [X_{1_1} \; X_{2_1} \; \ldots \; X_{6_1}]$. Similarly, we denote $\mathbf{X}_2 = [X_{1_2} \; X_{2_2} \; \ldots \; X_{6_2}]$, $\mathbf{U}_1 = [U_{1_1} \; U_{2_1} \; \ldots \; U_{6_1}]$, and $\mathbf{U}_2 = [U_{1_2} \; U_{2_2} \; \ldots \; U_{6_2}]$. The errors in $\mathbf{X}_1$ and $\mathbf{X}_2$ is given by $\mathbf{e}_i \; in \; \mathbf{X}_1$ and $\mathbf{e}_i \; in \; \mathbf{X}_2$ respectively.

The initial conditions for the agents ($\mathbf{X}_1$ and $\mathbf{X}_2$) are given in the Table 1.

6.3 Case 1: Without noise

The results for the asymmetric bipartite consensus without noise are presented here. The control signals of the agents, i.e., $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$, obtained using DNDI, are shown in Figs. 3 and 4, respectively. They drive the states of the agents, i.e., $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$, to achieve the asymmetric bipartite consensus.

The states $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are shown in Figs. 5 and 6 respectively. The proposed consensus protocol divides the agents into two groups depending on the antagonistic interaction on the graph. According to the graph, one group consists of agents 1, 2, and 3. They reached a consensus value of -0.4 and 0.458 for states $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ respectively. On the other hand, agents 4, 5 and 6 form another group. These agents achieve the consensus values of 3.21 and -3.64 for the states $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$, respectively. It can be noticed that the consensus value of each state of one group is eight times the multiple ($\kappa = 8$) of the other group, which is in accordance with the scaling factor we selected.

The convergence capability of the proposed controller is understood by consensus errors in states $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ of the agents (shown in Figs. 7 and 8 respectively. These figures explain the efficiency of the proposed DNDI controller.

Table 1 Initial conditions of the agents

Agents	1	2	3	4	5	6
$\mathbf{X}_{10}$	6	−3.4	4.42	3.47	4.15	7.16
$\mathbf{X}_{20}$	7.73	2.9	7.4	2.93	−4.26	5.62

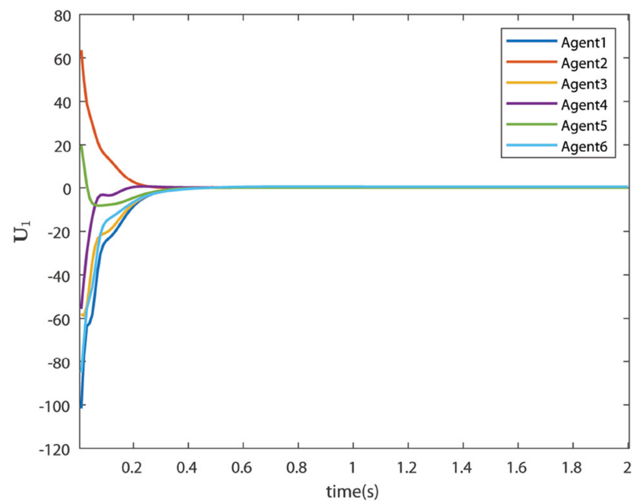

Fig. 3 Control $\boldsymbol{U}_1$ (Case 1)

6.4 Case 2: With noise

In this case, we consider the communication noise in the simulation study. The noise is added to all agents according to

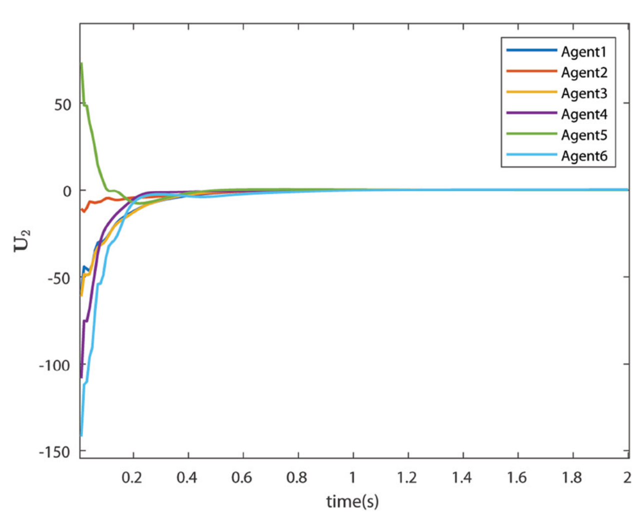

Fig. 4 Control $\boldsymbol{U}_2$ (Case 1)

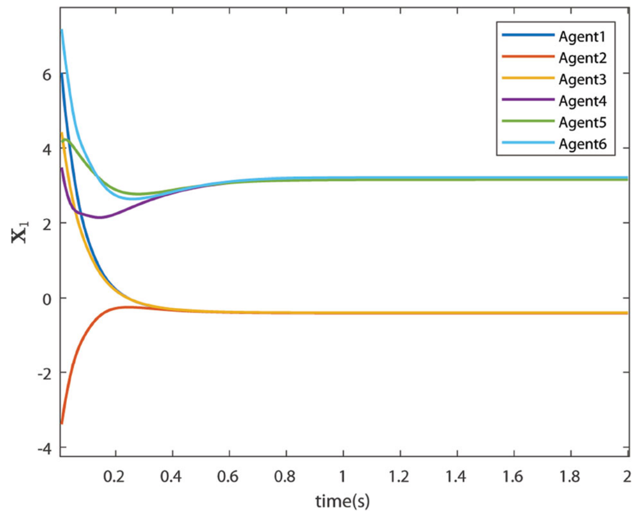

Fig. 5 Consensus of state X_1 (Case 1)

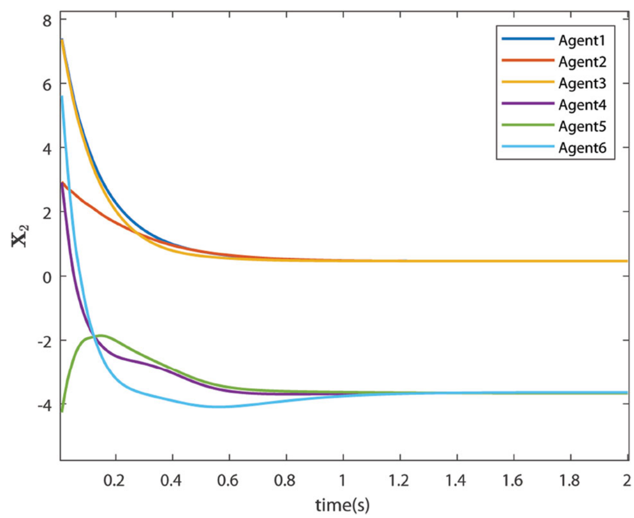

Fig. 6 Consensus of state X_2 (Case 1)

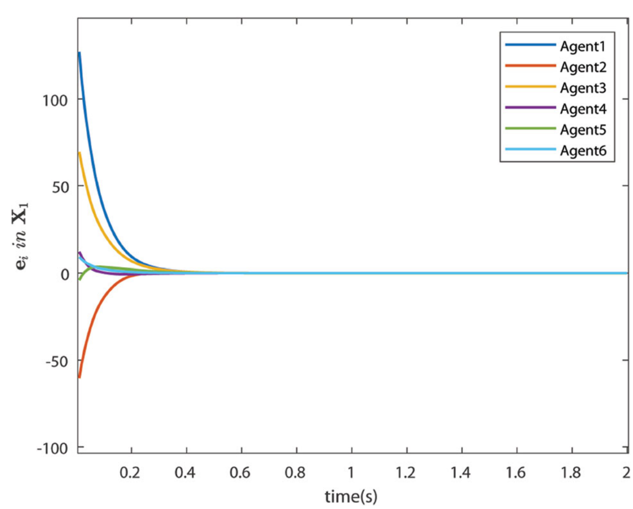

Fig. 7 Consensus error in state X_1 (Case 1)

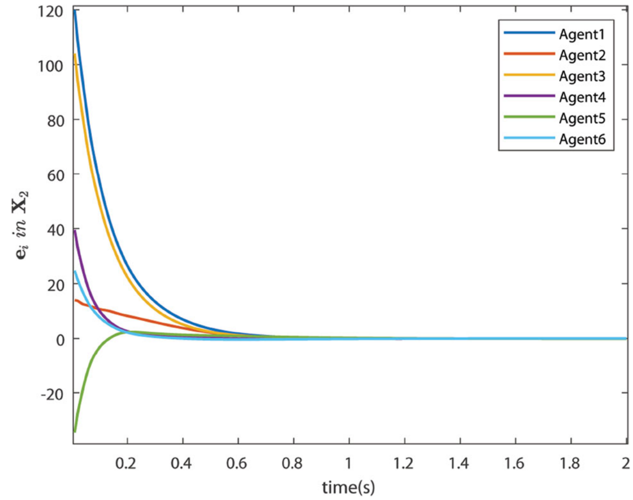

Fig. 8 Consensus error in state X_2 (Case 1)

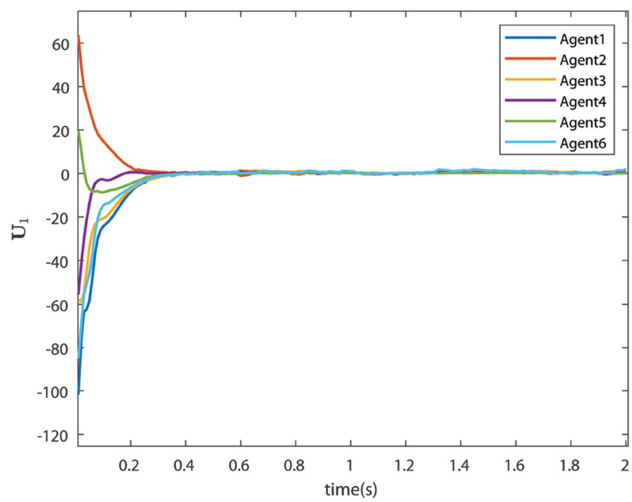

Fig. 9 Control U_1 (Case 2)

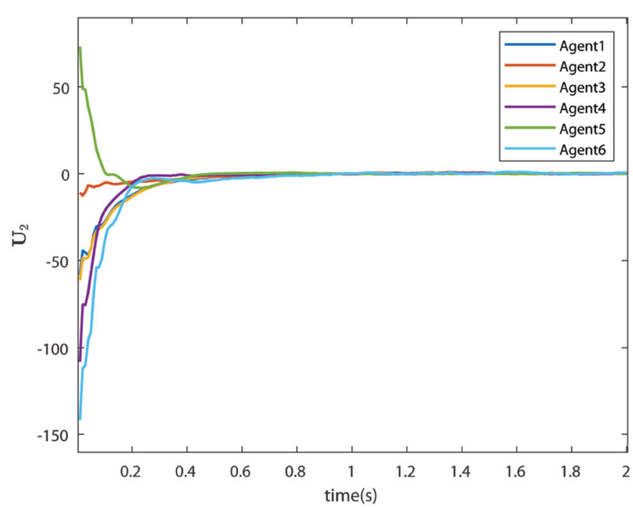

Fig. 10 Control U_2 (Case 2)

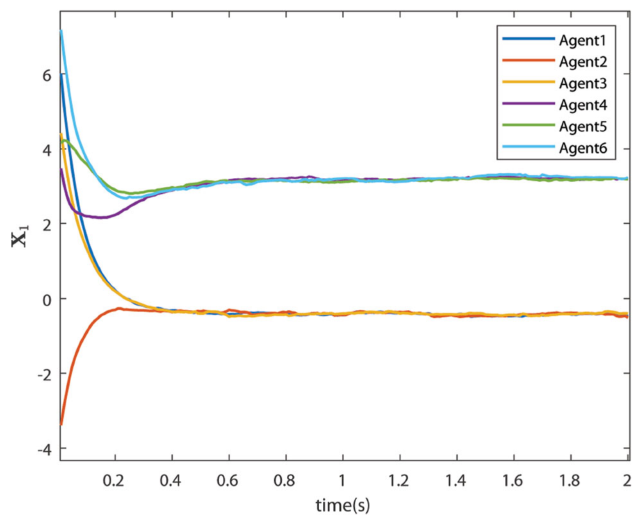

Fig. 11 Consensus of state X_1 (Case 2)

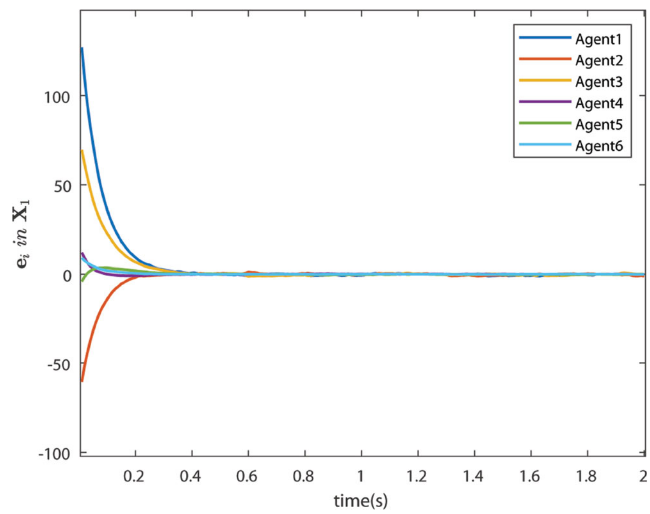

Fig. 13 Consensus error in state X_1 (Case 2)

the model provided in the preliminaries section. Moreover, the noise started at the same time. It can be regarded as the worst scenario. This study will help to understand the controller's efficiency in the presence of noise. The magnitude of the white Gaussian noise is considered random and generated by $\sigma = rand * 0.025$. The control signals U_1 and U_2 are shown in Figs. 9 and 10, respectively. It can be observed that the control signals are affected by random noise. Therefore, the states are also affected by the noise.

The state trajectories X_1 and X_2 are shown in Figs. 11 and 12 respectively. The effect of noise can be observed on the consensus of states shown in Figs. 11 and 12. Agents 1, 2, and 3 reached a consensus value of -0.395 and 0.447 for states X_1 and X_2 respectively. On the other hand, the agents 4, 5 and 6 form another group with the consensus value of 3.18 and -3.66 for the states X_1 and X_2, respectively. The consensus values for X_1 and X_2 of one group are almost eight times ($\kappa = 8$) multiple of the other. The effectiveness of the controller can be verified by looking at the consensus errors shown in Figs. 13 and 14. The errors converged to zero within seconds, which gives proof of its capability.

7 Conclusion

The asymmetric bipartite consensus of nonlinear agents is studied in the presence of communication noise. The important modification of Distributed NDI (DNDI) is proved to be effective in achieving the asymmetric bipartite consensus. The mathematical details in this paper provide theoretical proof of robustness, and the results are generated by realistic simulation, supporting our claim. Moreover, the performance

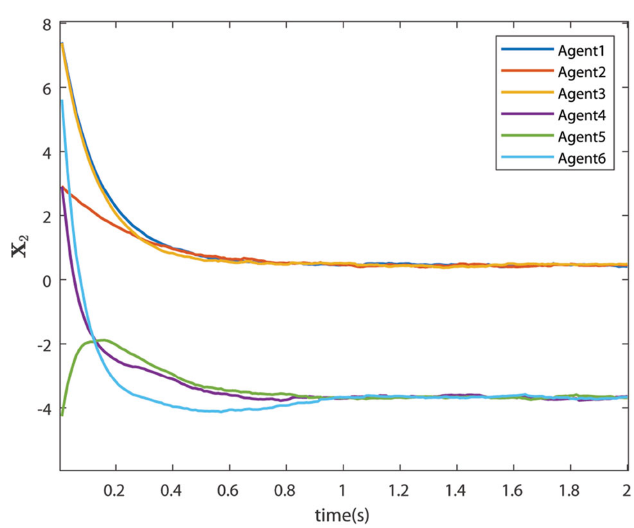

Fig. 12 Consensus of state X_2 (Case 2)

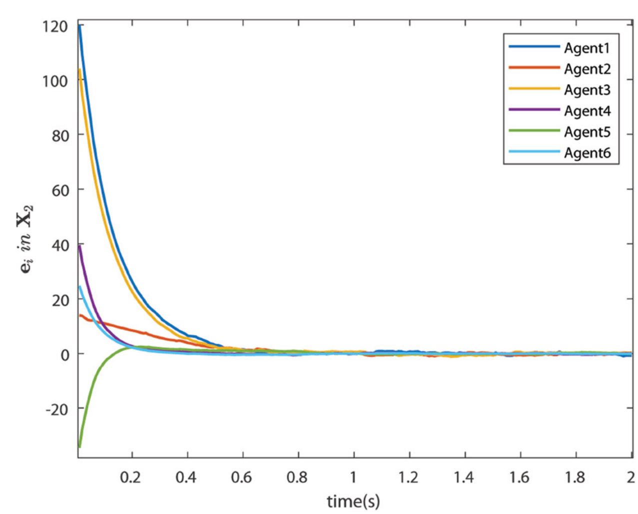

Fig. 14 Consensus error in state X_2 (Case 2)

of DNDI has been tested in the presence of random communication noise, and the results are very satisfactory. Therefore, DNDI has been proved to be a powerful controller for solving the asymmetric bipartite consensus of nonlinear agents with noise.

Acknowledgements This research was partially funded by an Engineering and Physical Sciences Research Council (EPSRC) project CASCADE (EP/R009953/1).

Author Contributions Conceptualization, writing–review and editing by Sabyasachi Mondal and Antonios Tsourdos. Methodology and validation by Sabyasachi Mondal. Writing–original draft preparation by Sabyasachi Mondal. Supervision by Antonios Tsourdos. Project administration by Antonios Tsourdos. Funding acquisition by Antonios Tsourdos. All authors have read and agreed to the published version of the manuscript.

Funding This research was partially funded by an Engineering and Physical Sciences Research Council (EPSRC) project CASCADE (EP/R009953/1).

Data Availibility Statement Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Statements & Declarations

Conflicts of interest The authors have no relevant financial or non-financial interests to disclose.

Ethics approval Not Applicable.

Consent to participate Not Applicable.

Consent to publish Consent to publish has been received from all authors.

References

1. Yan, D., Cheng, J., Lu, Y., Ng, W.: Blogel: A block-centric framework for distributed computation on real-world graphs. Proc. VLDB Endowment **7**(14), 1981–1992 (2014)
2. Al Khawaldah, M., Al-Khedher, M., Al-Adwan, I., Al Rawashdeh, A.: An autonomous exploration strategy for cooperative mobile robots. J. Softw. Eng. Appl. **2014** (2014)
3. Liang, H., Chen, L., Pan, Y., Lam, H.K.: Fuzzy-based robust precision consensus tracking for uncertain networked systems with cooperative-antagonistic interactions. IEEE Trans. Fuzzy Syst. (2022)
4. Wang, Y., Lu, J., Liang, J., Cao, J., Perc, M.: Pinning synchronization of nonlinear coupled lur'e networks under hybrid impulses. IEEE Trans. Circ. Syst. II: Express Briefs **66**(3), 432–436 (2018)
5. Couzin, I.D.: Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems (2014), pp. 1–2
6. Zhang, X., Duan, H.: Altitude consensus based 3d flocking control for fixed-wing unmanned aerial vehicle swarm trajectory tracking. Proc. Inst. Mech. Eng. G: J. Aerosp. Eng. **230**(14), 2628–2638 (2016)
7. Saulnier, K., Saldana, D., Prorok, A., Pappas, G.J., Kumar, V.: Resilient flocking for mobile robot teams. IEEE Robot. Autom. Lett. **2**(2), 1039–1046 (2017)
8. Altafini, C.: Dynamics of opinion forming in structurally balanced social networks. PloS One **7**(6), e38,135 (2012)
9. Wen, G., Duan, Z., Chen, G., Yu, W.: Consensus tracking of multi-agent systems with lipschitz-type node dynamics and switching topologies. IEEE Trans. Circ. Syst. I: Regular Papers **61**(2), 499–511 (2013)
10. Kim, J.M., Park, J.B., Choi, Y.H.: Leaderless and leader-following consensus for heterogeneous multi-agent systems with random link failures. IET Control Theory & Appl. **8**(1), 51–60 (2014)
11. Morita, R., Wada, T., Masubuchi, I., Asai, T., Fujisaki, Y.: Multi-agent consensus with noisy communication: Stopping rules based on network graphs. IEEE Trans. Control Netw. Syst. **3**(4), 358–365 (2015)
12. Ming, P., Liu, J., Tan, S., Wang, G., Shang, L., Jia, C.: Consensus stabilization of stochastic multi-agent system with markovian switching topologies and stochastic communication noise. J. Frankl. Inst. **352**(9), 3684–3700 (2015)
13. Liu, W., Zhou, S., Qi, Y., Wu, X.: Leaderless consensus of multi-agent systems with lipschitz nonlinear dynamics and switching topologies. Neurocomputing **173**, 1322–1329 (2016)
14. Liu, J., Ming, P., Li, S.: Consensus gain conditions of stochastic multi-agent system with communication noise. Int. J. Control, Autom. Syst. **14**(5), 1223–1230 (2016)
15. Xia, H., Dong, Q.: Dynamic leader-following consensus for asynchronous sampled-data multi-agent systems under switching topology. Inf. Sci. **514**, 499–511 (2020)
16. Li, M., Deng, F., Ren, H.: Scaled consensus of multi-agent systems with switching topologies and communication noises. Nonlinear Anal. : Hybrid Syst. **36**, 100,839 (2020)
17. Mondal, S., Tsourdos, A.: The consensus of non-linear agents under switching topology using dynamic inversion in the presence of communication noise and delay. Proc. Inst. Mech. Eng. G J. Aerosp. Eng. p. 09544100211012795 (2021)
18. Mondal, S., Tsourdos, A.: Neuro-adaptive augmented distributed nonlinear dynamic inversion for consensus of nonlinear agents with unknown external disturbance. Sci. Reports **12**(1), 1–15 (2022)
19. Mondal, S., Tsourdos, A.: Fault-tolerant consensus of nonlinear agents considering switching topology in the presence of communication noise. Proc. Inst. Mech. Eng. G J. Aerosp. Eng. p. 09544100211069179 (2022)
20. Altafini, C.: Consensus problems on networks with antagonistic interactions. IEEE Trans. Autom. Control **58**(4), 935–946 (2012)
21. Altafini, C., Lini, G.: Predictable dynamics of opinion forming for networks with antagonistic interactions. IEEE Trans. Autom. Control **60**(2), 342–357 (2014)
22. Qin, J., Fu, W., Zheng, W.X., Gao, H.: On the bipartite consensus for generic linear multiagent systems with input saturation. IEEE Trans. Cybern. **47**(8), 1948–1958 (2016)

23. Liu, M., Wang, X., Li, Z.: Robust bipartite consensus and tracking control of high-order multiagent systems with matching uncertainties and antagonistic interactions. IEEE Trans. Syst. Man. Cybern. Syst. **50**(7), 2541–2550 (2018)
24. Hu, J., Xiao, Z., Zhou, Y., Yu, J.: Proceedings of the 32nd Chinese Control Conference (IEEE, 2013), pp. 6879–6884
25. Li, H.: H-infinity bipartite consensus of multi-agent systems with external disturbance and probabilistic actuator faults in signed networks. AIMS Math. **7**(2), 2019–2043 (2022)
26. Hu, J., Zheng, W.X.: 52nd IEEE Conference on Decision and Control (IEEE, 2013), pp. 3451–3456
27. Valcher, M.E., Misra, P.: On the consensus and bipartite consensus in high-order multi-agent dynamical systems with antagonistic interactions. Syst. & Control Lett. **66**, 94–103 (2014)
28. H. Zhang, J. Chen, in: *2014 American Control Conference* (IEEE, 2014), pp. 808–812
29. Meng, D., Du, M., Jia, Y.: Interval bipartite consensus of networked agents associated with signed digraphs. IEEE Trans. Autom. Control **61**(12), 3755–3770 (2016)
30. Cheng, M., Zhang, H., Jiang, Y.: 2016 35th Chinese Control Conference (CCC) (IEEE, 2016), pp. 8287–8291
31. Zhang, H., Chen, J.: Bipartite consensus of multi-agent systems over signed graphs: state feedback and output feedback control approaches. Int. J. Robust and Nonlinear Control **27**(1), 3–14 (2017)
32. Bhowmick, S., Panja, S.: Leader-follower bipartite consensus of uncertain linear multiagent systems with external bounded disturbances over signed directed graph. IEEE Control Syst. Lett. **3**(3), 595–600 (2019)
33. Li, E., Ma, Q., Zhou, G.: Bipartite output consensus for heterogeneous linear multi-agent systems with fully distributed protocol. J. Frankl. Inst. **356**(5), 2870–2884 (2019)
34. Yu, T., Ma, L.: Chinese Intelligent Systems Conference (Springer, 2017), pp. 397–406
35. Li, H.: Event-triggered bipartite consensus of multi-agent systems in signed networks. AIMS Math. **7**(4), 5499–5526 (2022)
36. Liang, H., Guo, X., Pan, Y., Huang, T.: Event-triggered fuzzy bipartite tracking control for network systems based on distributed reduced-order observers (revised manuscript of tfs-2019-1049). IEEE Trans. Fuzzy Syst. (2020)
37. Wu, Y., Pan, Y., Chen, M., Li, H.: Quantized adaptive finite-time bipartite nn tracking control for stochastic multiagent systems. IEEE Trans. Cybern. **51**(6), 2870–2881 (2020)
38. Wang, D., Ma, H., Liu, D.: Distributed control algorithm for bipartite consensus of the nonlinear time-delayed multi-agent systems with neural networks. Neurocomputing **174**, 928–936 (2016)
39. Zhai, S., Li, Q.: Practical bipartite synchronization via pinning control on a network of nonlinear agents with antagonistic interactions. Nonlinear Dyn. **87**(1), 207–218 (2017)
40. Du, M., Ma, B., Meng, D.: Distributed control for signed networks of nonlinear agents. Int. J. Control, Autom. Syst. **18**(2), 271–281 (2020)
41. Mondal, S., Tsourdos, A.: Bipartite consensus of nonlinear agents in the presence of communication noise. Sensors **22**(6), 2357 (2022)
42. Guo, X., Liang, J., Lu, J.: Asymmetric bipartite consensus over directed networks with antagonistic interactions. IET Control Theory & Appl. **12**(17), 2295–2301 (2018)
43. Guo, X., Liang, J., Lu, J.: Finite-time asymmetric bipartite consensus for signed networks of dynamic agents. Int. J. Control, Autom. Syst. **17**(4), 1041–1049 (2019)
44. Liang, J., Bu, X., Cui, L., Hou, Z.: Finite time asymmetric bipartite consensus for multi-agent systems based on iterative learning control. Int. J. Robust and Nonlinear Control **31**(12), 5708–5724 (2021)
45. Enns, D., Bugajski, D., Hendrick, R., Stein, G.: Dynamic inversion: an evolving methodology for flight control design. Int. J. control **59**(1), 71–91 (1994)
46. Mondai, S., Padhi, R.: Formation flying using genex and differential geometric guidance law. IFAC-PapersOnLine **48**(9), 19–24 (2015)
47. Caverly, R.J., Girard, A.R., Kolmanovsky, I.V., Forbes, J.R.: Nonlinear dynamic inversion of a flexible aircraft. IFAC-PapersOnLine **49**(17), 338–342 (2016)
48. Harary, F.: On the notion of balance of a signed graph. Mich. Math. J. **2**(2), 143–146 (1953)
49. Wen, G., Chen, C.P., Liu, Y.J., Liu, Z.: Neural network-based adaptive leader-following consensus control for a class of nonlinear multiagent state-delay systems. IEEE Trans. Cybern. **47**(8), 2151–2160 (2016)
50. Ren, C.E., Chen, C.P.: Sliding mode leader-following consensus controllers for second-order non-linear multi-agent systems. IET Control Theory & Appl. **9**(10), 1544–1552 (2015)
51. Hu, J., Zheng, W.X.: Emergent collective behaviors on coopetition networks. Phys. Lett. A **378**(26–27), 1787–1796 (2014)
52. Ma, H., Wang, Z., Wang, D., Liu, D., Yan, P., Wei, Q.: Neural-network-based distributed adaptive robust control for a class of nonlinear multiagent systems with time delays and external noises. IEEE Trans. Syst. Man. Cybern. Syst. **46**(6), 750–758 (2015)
53. Ge, S.S., Wang, C.: Adaptive neural control of uncertain mimo nonlinear systems. IEEE Trans. Neural Netw. **15**(3), 674–692 (2004)

Sabyasachi Mondal received the Ph.D. degree in Aerospace Engineering from Indian Institute of Science, Bangalore, India. He is currently working as a Research fellow at Cranfield University. His research interests include Guidance of Aerospace Vehicles, Control of Multi-Agent systems, Autonomous Platform design, and Path Planning.

Antonios Tsourdos received the M.Eng. degree in electronic, control and systems engineering from the University of Sheffield, Sheffield, U.K., in 1995, and the M.Sc. degree in systems engineering and the Ph.D. degree in nonlinear robust missile autopilot design and analysis from Cardiff University, Cardiff, U.K., in 1996 and 1999, respectively. In 2013, he joined the Center for Cyber-Physical systems, Cranfield University, as the Head. He is currently a Professor of control engineering with Cranfield University. His research interest includes autonomous systems, guidance, control and navigation, decision making. He has published over 100 journal papers.