Robust Position/Force Control of Constrained Flexible Joint Robots with Constraint Uncertainties

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Abstract

A novel robust control method for simultaneous position/force control of constrained flexible joint robots is proposed. The facts that the uncertainties make the usual control task unsolvable and that the equations of the controlled system are differential-algebraic make the problem dealt with considerably demanding. In order to overcome the unsolvability problem due to the constraint uncertainties the position control task is redefined in a practical way such that only a suitable subgroup of the link positions are driven to their desired trajectories. To determine the elements of the subgroup a simple algorithm of practical relevance is proposed. Under certain smoothness conditions to the contact surfaces, it is demonstrated that the position control problem can dynamically be isolated from the force control. Thus, it becomes possible to handle the position and force control tasks separately. The most significant advantage of the separation of the position and force control tasks is that it makes possible to adapt the position control methods known from free robots. Each joint is used in either position control or force control. The proposed position controller has a cascaded structure: First, trajectories for joint positions that drive the link positions to their desired values are calculated. Then, the joint torques that drive the joint positions to their calculated values are determined. A further significant benefit of the separation of the position and force control tasks arises in the force control such that the transformed equations are linear and any linear robust control approach can be used for the force control. The whole controller requires the measurement of the link and joint positions, the link and joint velocities and the contact forces, and allows modeling uncertainties in the equations of both the robot dynamics and the contact surfaces. The proposed control method is also confirmed by simulations.

Keywords Constrained flexible joint robots · Position and force control · Robust control · Constraint uncertainties

1 Introduction

In diverse robotic applications such as cleaning, assembling, writing, welding or surgery, end-effectors of robot manipulators have contact with their environments. Simultaneous control of the position of the end-effector and the force between the end-effector and the environment is usually mandatory to perform the task. Applicable position/force control algorithms are strongly needed and will further expand the application areas of robots. It can be estimated that robotic tasks will include more contacts with environment in more sophisticated future applications such as service robotics.

If the contact surface can be modeled as a mass-spring-damper system, the force control task can be transformed into a position control task, since in this case the contact forces are roughly proportional to the deformations of the contact surfaces and can be expressed as a function of the end-effector kinematic variables. In the literature, studies in this context are called impedance control [15–21]. Impedance control has been extensively studied since the 1980s [14].

Another widespread approach used for simultaneous position/force control is hybrid control, in which the position and force control tasks are separated from each other in the task space [11–13, 22–27]. Hybrid control has been controversially and intensively discussed in the literature, since, on the one hand, it provides an attractive procedure for separation of position and force control tasks, on the other hand, it can cause unstable control loops. The criticism of hybrid control is focused on its kinematic nonconsistency [23].

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The most challenging system model for control purposes emerges if the contact surfaces are rigid and the contact forces are expressed by Lagrange multipliers, since the resulting system model in this case has a differential-algebraic form. Robots modeled by differential-algebraic equations are called constrained robots [28].

Although the issue of position/force control of rigid joint robots is well understood [1, 9–12], position/force control methods for flexible joint robots are rarely encountered in the literature. A straightforward application of the control methods from rigid joint robots to flexible joint case does not lead to a satisfactory performance. The significance of joint flexibility for robot control has widely been discussed [2–6]. The fact that the joint flexibility creates a natural compliance between the end-effector and the contact surfaces motivates the use of flexible joint robot manipulators for simultaneous position/force control.

In [7, 8, 16, 18] and [23], a few rarely encountered works on position/force control of flexible joint robots are given.

The work [7] addresses adaptive position/force control of uncertain constrained flexible joint robots. The controller is based on singular perturbation approach and achieves the position tracking and the boundedness of the force errors. Unfortunately, the method cannot handle robots with strong joint elasticities and requires the measurement of the joint accelerations.

The position/force control law in [8] is based on solving the singular acceleration-level inverse dynamic equations. The controller does not require the measurement of the contact forces. However, unmeasured contact forces can reduce the performance of the force control in the case of modeling uncertainties.

In [16], a passivity based force and impedance controller for rigid and flexible joint robots is proposed, where the concept of energy tanks is applied such that the force tracking controller, the impedance controller and the motor dynamics together form a passive system. The approach is also capable to stabilize unwanted contact losses. It does not deal with the robustness issues which arise due to modeling uncertainties. The work suffers under the assumption that the time derivatives of the contact forces are obtainable.

In [18], a passivity based impedance controller for flexible joint robots is proposed, where the controller possesses a cascaded structure with an inner torque feedback loop and an outer impedance control loop. The controller contains a gravity compensation term and a PD control term and does not treat the modeling uncertainties.

The work [23] is based on the concept of singular perturbation theory, which works with the restrictive assumption that the dynamics of the controlled system can be divided into a fast and a slow subsystem. The controller for the

fast subsystem contains an additional robustness term compensating the effects of the parameter mismatching. The unknown system parameters are estimated in real-time.

The present work proposes a novel robust control method for position/force control of constrained flexible joint robots. Its position control part has a cascaded structure: First, trajectories for joint positions that drive the link positions to their desired values are calculated. Then, the joint torques that drive the joint positions to their calculated values are determined. The phenomenon that the controlled system with respect to the force is linear, leads to the conclusion that any linear robust control approach can be used for the force control. The force control law in the simulations of Section 5 consists of a linear high gain PID term. The contributions of the work are given as follows:

- To the best of the author’s knowledge, this is the first work that allows modeling uncertainties in the contact surfaces. Dealing with the uncertainties in the contact surface is quite challenging, since the constraint uncertainties can make the position control task unfeasible [1]. For this purpose, the position control task is redefined in a pragmatic way in Section 2.
- The present work allows not only parameter inaccuracies in the system model but also unmodelled dynamics, which is of considerable practical relevance.
- The proposed control law does not require the measurement of the joint and link accelerations and the time-derivatives of the contact forces. It requires only the measurement of the link and joint angles, the link and joint velocities and the contact forces.
- It is demonstrated that under certain smoothness conditions to the contact surfaces the position control can dynamically be isolated from the force control. This result is significant since it makes possible to handle the position and force control tasks separately and to adapt the position control methods known from free robots. Each joint is used in either position control or force control. A simple algorithm is developed to determine whether a joint can be used in position control or force control. Note that by contrast with the present work, the separation of the position and force control tasks in classical hybrid control is realized kinematically.

The present work is organized as follows: In Section 2, the dynamics of constrained flexible joint robots and the robust position/force control problem are introduced. In Section 3, the dynamic equations of the controlled system is first transformed into a new form that enables to treat the position control independently from the force control. Then, a robust position control law is developed. In Section 4, the linear force control law is derived. The success of the

force control depends on the success of the position control, although the position control is independent of the force control. In Section 5, the proposed control method is also confirmed by simulations.

2 Problem Statement

Constrained flexible joint robots of the dynamics [3]

$$M(q)\ddot{q} + h(q, \dot{q}) + K(q - \theta) + J(q)\lambda = 0 \quad (1)$$

$$T\ddot{\theta} + F\dot{\theta} - K(q - \theta) = \tau \quad (2)$$

$$\phi(q) = 0 \quad (3)$$

are considered, where the vectors $q \in \mathbb{R}^n$, $\theta \in \mathbb{R}^n$ and $\tau \in \mathbb{R}^n$ denote the link angles, the motor angles and the motor torques at the joints, respectively. The symmetric and positive definite matrix $M(q) \in \mathbb{R}^{n \times n}$ denotes the mass matrix of the rigid links. The vector $h(q, \dot{q}) \in \mathbb{R}^n$ consists of the centrifugal, Coriolis and gravitational torques. The diagonal and positive definite matrices $K \in \mathbb{R}^{n \times n}$, $T \in \mathbb{R}^{n \times n}$ and $F \in \mathbb{R}^{n \times n}$ denote the stiffness, inertia and viscous friction matrices of the joints, respectively. Equation 3 describes the contact surfaces of the end-effector in the link coordinates q , where $\phi(q) \in \mathbb{R}^m$ is a vector function with independent components except at some isolated values of q . Finally, the matrix $J(q) \in \mathbb{R}^{n \times m}$ and the vector $\lambda \in \mathbb{R}^m$ denote the transpose of the Jacobian matrix of the constraints $J(q) = (\partial\phi(q)/\partial q)^T$ and the Lagrange multipliers, respectively. The Lagrange multipliers λ can be interpreted as the constraint forces, since they are determined by the constraint forces and vice versa. It is assumed that the number of the constraints m is less than the number of the joints n .

The system model matrices and vectors are denoted by $\hat{M}(q)$, $\hat{h}(q, \dot{q})$, $\hat{K}$, $\hat{T}$, $\hat{F}$, $\hat{\phi}(q)$ and $\hat{J}(q)$. They may differ from the relating system matrices and vectors $M(q)$, $h(q, \dot{q})$, K , T , F , $\phi(q)$ and $J(q)$, which are supposed to be unknown, and therefore, not allowed to be used in the control law. The differences between the system and model components may arise due to both parameter mismatching and unmodeled (or neglected) parts of the system dynamics.

In the following, the arguments of the vector and matrix functions will often be neglected for the sake of brevity if they are obvious from the context.

Notation Let Z be a $k \times l$ matrix and v and μ be subsets of $\{1, \dots, k\}$ and $\{1, \dots, l\}$ with i and j number of elements, respectively. Z_v denotes the $i \times l$ sub-matrix of Z which includes only the v -rows of Z . $Z_{v\mu}$ denotes the $i \times j$ sub-matrix of Z_v which includes only the μ -columns of Z_v .

Assumption 1 Vector functions $\phi(q)$ and $\hat{\phi}(q)$ are of class $\mathcal{C}^2$.

Assumption 2 There exists a subset $\sigma \subset \{1, \dots, n\}$ with m elements such that the equalities

$$\text{rank } J_\sigma = \text{rank } \hat{J}_\sigma = m$$

hold along the system trajectories.

Assumption 1 is required by the proposed control law and satisfied by sufficiently smooth contact surfaces. Assumption 2 is required to separate the position control problem from the force control and is only slightly restrictive since the constraint surfaces are supposed to be independent of each other.

Let the subset α be $\alpha = \{1, \dots, n\} - \sigma$. In Section 3, α -rows of the system dynamics Eqs. 1 and 2 will be used for the position control, and in Section 4, σ -rows for the force control.

The control problem is given as follows: Specify the system inputs τ such that the system and controller variables remain bounded in time and q_α and λ converge to their desired trajectories $q_\alpha^d(t)$ and $\lambda^d(t)$, respectively. Note that the remaining link coordinates q_σ , which are not controlled, are not independent variables. For a given trajectory of q_α , the trajectory of q_σ is determined by the contact surfaces (3) and the initial conditions of (1) and (2) [1].

3 Position control

In the following, the α -rows of the system Eqs. 1 and 2 are first transformed into a structure which is more convenient for design of position controllers, then the robust position control law is given. The α -rows of Eqs. 1 and 2 are

$$M_\alpha \ddot{q} + h_\alpha + K_{\alpha\alpha}(q_\alpha - \theta_\alpha) + J_\alpha \lambda = 0 \quad (4)$$

$$T_{\alpha\alpha} \ddot{\theta}_\alpha + F_{\alpha\alpha} \dot{\theta}_\alpha - K_{\alpha\alpha}(q_\alpha - \theta_\alpha) = \tau_\alpha. \quad (5)$$

For separating the position control from the force control, the vectors λ and $\ddot{q}_\sigma$ will be eliminated from Eq. 4. The solution of the σ -rows of Eqs. 1 and 2

$$M_\sigma \ddot{q} + h_\sigma + K_{\sigma\sigma}(q_\sigma - \theta_\sigma) + J_\sigma \lambda = 0 \quad (6)$$

$$T_{\sigma\sigma} \ddot{\theta}_\sigma + F_{\sigma\sigma} \dot{\theta}_\sigma - K_{\sigma\sigma}(q_\sigma - \theta_\sigma) = \tau_\sigma. \quad (7)$$

for λ is

$$\lambda = -J_\sigma^{-1}[M_\sigma \ddot{q} + h_\sigma + K_{\sigma\sigma}(q_\sigma - \theta_\sigma)]. \quad (8)$$

Substituting Eqs. 8 into 4 yields

$$M_\alpha \ddot{q} + h_\alpha + K_{\alpha\alpha}(q_\alpha - \theta_\alpha) - J_\alpha J_\sigma^{-1}[M_\sigma \ddot{q} + h_\sigma + K_{\sigma\sigma}(q_\sigma - \theta_\sigma)] = 0. \quad (9)$$

The vector $\ddot{q}_\sigma$ depending on the vectors q , $\dot{q}$ and $\ddot{q}_\alpha$ is obtained by differentiating the constraints Eq. 3 according to time twice [1]:

$$\begin{aligned} \frac{d^2}{dt^2}[\phi(q)] &= \frac{d}{dt}[J^T \dot{q}] \\ &= \dot{J}^T \dot{q} + J^T \ddot{q} \\ &= \dot{J}^T \dot{q} + J_\sigma^T \ddot{q}_\sigma + J_\alpha^T \ddot{q}_\alpha = 0 \end{aligned} \quad (10)$$

The solution of Eq. 10 for $\ddot{q}_\sigma$ is

$$\ddot{q}_\sigma = -J_\sigma^{-T}(\dot{J}^T \dot{q} + J_\alpha^T \ddot{q}_\alpha). \quad (11)$$

Substituting (11) into (9) converts the α -rows of Eq. 1 into

$$D(q) \ddot{q}_\alpha + g(q, \dot{q}, \theta_\sigma) = \theta_\alpha \quad (12)$$

where $D(q)$ and $g(q, \dot{q}, \theta_\sigma)$ are defined by

$$\begin{aligned} D(q) &= K_{\alpha\alpha}^{-1}[M_{\alpha\alpha} - M_{\alpha\sigma} J_\sigma^{-T} J_\alpha^T \\ &\quad - J_\alpha J_\sigma^{-1}(M_{\sigma\alpha} - M_{\sigma\sigma} J_\sigma^{-T} J_\alpha^T)] \\ g(q, \dot{q}, \theta_\sigma) &= K_{\alpha\alpha}^{-1}\{-M_{\alpha\sigma} J_\sigma^{-T} \dot{J}^T \dot{q} + h_\alpha \\ &\quad - J_\alpha J_\sigma^{-1}[-M_{\sigma\sigma} J_\sigma^{-T} \dot{J}^T \dot{q} + h_\sigma \\ &\quad + K_{\sigma\sigma}(q_\sigma - \theta_\sigma)]\} + q_\alpha. \end{aligned}$$

Equations 5 and 12 remind the dynamics of flexible joint robots without constraints, for which many control methods have been proposed so far [2]. In the following, the robust position control method for free flexible joint robots in [2] is adapted to the constrained case.

Let $\hat{D}(q)$ and $\hat{g}(q, \dot{q}, \theta_\sigma)$ be the models of $D(q)$ and $g(q, \dot{q}, \theta_\sigma)$, respectively.

Assumption 3 rank $D(q) = \text{rank } \hat{D}(q) = n - m$ along the trajectories.

Assumption 3 is fundamental since Eq. 12 may have infinitely large solutions for the components of $\ddot{q}_\alpha$ if Assumption 3 does not hold at some time instants. On the other hand, Assumption 3 is only slightly restrictive, since the matrices $D(q)$ and $\hat{D}(q)$ can be influenced by the reasonable placement of the robot in relation to the contact surfaces.

The error variables regarding the position control are defined by

$$e_1 = q_\alpha - q_\alpha^d \quad (13)$$

$$e_2 = \dot{q}_\alpha - \dot{q}_\alpha^d \quad (14)$$

$$r_1 = [e_1^T \ e_2^T]^T \quad (15)$$

$$e_3 = \theta_\alpha - \theta_\alpha^d \quad (16)$$

$$e_4 = \dot{\theta}_\alpha - \dot{\theta}_\alpha^d \quad (17)$$

where θ_α^d and $\dot{\theta}_\alpha^d$ depend on t , q , $\dot{q}$, θ and $\dot{\theta}$ and denote the desired values of θ_α and $\dot{\theta}_\alpha$, respectively. θ_α^d is given by

$$\theta_\alpha^d = \hat{D}(\ddot{q}_\alpha^d - K_1 e_1 - K_2 e_2) + \hat{g} - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} \quad (18)$$

where K_1 and K_2 are $(n-m) \times (n-m)$ diagonal and positive definite controller matrices, ε_1 is a positive real number, B_1 is a $2(n-m) \times (n-m)$ matrix given by

$$B_1 = \begin{bmatrix} 0 \\ \hat{D}^{-1} \end{bmatrix}, \quad (19)$$

$\bar{\Delta}_1$ is an upper bound function for the Euclidean norm of the unknown $n - m$ dimensional vector Δ_1

$$\Delta_1 = (D - \hat{D})\ddot{q}_\alpha + g - \hat{g}, \quad \|\Delta_1\| \leq \bar{\Delta}_1 \quad (20)$$

and P_1 is the $2(n-m) \times 2(n-m)$ symmetric and positive definite solution of the following Lyapunov equation for a given $2(n-m) \times 2(n-m)$ dimensional symmetric and positive definite matrix Q_1

$$A_1 = \begin{bmatrix} 0 & I \\ -K_1 & -K_2 \end{bmatrix}, \quad A_1^T P_1 + P_1 A_1 = -Q_1. \quad (21)$$

The vector β is defined by

$$\beta = -B_1^T P_1 r_1 - K_3 e_3 - e_3 \frac{\bar{\Delta}_2^2}{\varepsilon_2} \quad (22)$$

where K_3 is a $(n-m) \times (n-m)$ diagonal and positive definite matrix, ε_2 is a positive real number and $\bar{\Delta}_2$ is an upper bound function for the Euclidean norm of the unknown variable $\dot{\theta}_\alpha^d$

$$\|\dot{\theta}_\alpha^d\| \leq \bar{\Delta}_2. \quad (23)$$

The position control law is given by

$$\tau_\alpha = \hat{T}_{\alpha\alpha}(-e_3 - K_4 e_4 - e_4 \frac{\bar{\Delta}_3^2}{\varepsilon_3}) + \hat{F}_{\alpha\alpha} \dot{\theta}_\alpha - \hat{K}_{\alpha\alpha}(q_\alpha - \theta_\alpha) \quad (24)$$

where K_4 is a $(n-m) \times (n-m)$ diagonal and positive definite matrix, ε_3 is a positive real number and $\bar{\Delta}_3$ is an upper bound function for the Euclidean norm of the unknown $n - m$ dimensional vector Δ_3

$$\begin{aligned} \Delta_3 &= T_{\alpha\alpha}^{-1}[\tau_\alpha - F_{\alpha\alpha} \dot{\theta}_\alpha + K_{\alpha\alpha}(q_\alpha - \theta_\alpha)] \\ &\quad - \hat{T}_{\alpha\alpha}^{-1}[\tau_\alpha - \hat{F}_{\alpha\alpha} \dot{\theta}_\alpha + \hat{K}_{\alpha\alpha}(q_\alpha - \theta_\alpha)] - \dot{\beta} \\ \|\Delta_3\| &\leq \bar{\Delta}_3. \end{aligned} \quad (25)$$

To demonstrate the efficiency of the position control law Eq. 24, the error dynamics regarding r_1 , e_3 and e_4 is

required. Substituting Eqs. 16, 18 and the time derivative of Eqs. 14 into 12 yields

$$D\ddot{q}_\alpha + g = \theta_\alpha^d + e_3 \quad (26)$$

$$= \hat{D}(\ddot{q}_\alpha^d - K_1 e_1 - K_2 e_2) + \hat{g} - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + e_3 \quad (27)$$

$$= \hat{D}(\ddot{q}_\alpha - \dot{e}_2 - K_1 e_1 - K_2 e_2) + \hat{g} - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + e_3 \quad (28)$$

$$\hat{D}(\dot{e}_2 + K_1 e_1 + K_2 e_2) + \underbrace{(D - \hat{D})\ddot{q}_\alpha + g - \hat{g}}_{\Delta_1} - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + e_3. \quad (29)$$

The equality

$$\dot{r}_1 = A_1 r_1 + B_1(-\Delta_1 - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + e_3) \quad (30)$$

follows from Eqs. 13–15 and 29. Note that the first $n - m$ rows of Eq. 30 includes only the equation $\dot{e}_1 = e_2$ and the last $n - m$ rows of Eq. 30 is a reshaping of Eq. 29. Equation 30 forms the first part of the error dynamics.

The time derivative of Eqs. 16, 17 and 22 yield the second part of the error dynamics

$$\dot{e}_3 = \dot{\theta}_\alpha - \dot{\theta}_\alpha^d \quad (31)$$

$$= e_4 + \beta - \dot{\theta}_\alpha^d \quad (32)$$

$$= e_4 - B_1^T P_1 r_1 - K_3 e_3 - e_3 \frac{\bar{\Delta}_2^2}{\varepsilon_2} - \dot{\theta}_\alpha^d. \quad (33)$$

The last part of the error dynamics follows from the time derivative of Eq. 17 and Eqs. 5, 24 and 25

$$\dot{e}_4 = \ddot{\theta}_\alpha - \dot{\beta} \quad (34)$$

$$= \hat{\tau}_{\alpha\alpha}^{-1}[\tau_\alpha - \hat{F}_{\alpha\alpha}\dot{\theta}_\alpha + \hat{K}_{\alpha\alpha}(q_\alpha - \theta_\alpha)] + \Delta_3 \quad (35)$$

$$= -e_3 - K_4 e_4 - e_4 \frac{\bar{\Delta}_3^2}{\varepsilon_3} + \Delta_3. \quad (36)$$

Equations 30, 33 and 36 constitute the error dynamics for the position control.

Theorem 1 *The solutions of the error dynamics consisting of Eqs. 30, 33 and 36 are globally uniformly ultimately bounded.*

Proof Consider the Lyapunov function candidate

$$V = \frac{1}{2}(r_1^T P_1 r_1 + e_3^T e_3 + e_4^T e_4). \quad (37)$$

The time derivative of Eq. 37 is

$$\dot{V} = \frac{1}{2}(\dot{r}_1^T P_1 r_1 + r_1^T P_1 \dot{r}_1 + \dot{e}_3^T e_3 + e_3^T \dot{e}_3 + \dot{e}_4^T e_4 + e_4^T \dot{e}_4). \quad (38)$$

Substituting Eqs. 21 and 30 into Eq. 38 yields

$$\begin{aligned} \dot{V} &= \frac{1}{2}r_1^T (A_1^T P_1 + P_1 A_1)r_1 + r_1^T P_1 B_1(-\Delta_1 \\ &\quad - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + e_3) + e_3^T \dot{e}_3 + e_4^T \dot{e}_4 \\ &= -\frac{1}{2}r_1^T Q_1 r_1 + r_1^T P_1 B_1(-\Delta_1 - B_1^T P_1 r_1 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + e_3) \\ &\quad + e_3^T \dot{e}_3 + e_4^T \dot{e}_4 \end{aligned} \quad (39)$$

$$\leq -\frac{1}{2}r_1^T Q_1 r_1 + \|r_1^T P_1 B_1\| \bar{\Delta}_1 - \|r_1^T P_1 B_1\|^2 \frac{\bar{\Delta}_1^2}{\varepsilon_1} + r_1^T P_1 B_1 e_3 + e_3^T \dot{e}_3 + e_4^T \dot{e}_4 \quad (40)$$

$$\leq -\frac{1}{2}r_1^T Q_1 r_1 + \frac{\varepsilon_1}{4} + r_1^T P_1 B_1 e_3 + e_3^T \dot{e}_3 + e_4^T \dot{e}_4. \quad (41)$$

Inequality Eq. 41 follows from the fact

$$a - \frac{a^2}{\varepsilon} \leq \frac{\varepsilon}{4}, \quad a \in \mathbb{R}, \quad \varepsilon \in \mathbb{R}^+. \quad (42)$$

Substituting Eqs. 33 and 36 into Eq. 41 yields

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2}r_1^T Q_1 r_1 + \frac{\varepsilon_1}{4} + r_1^T P_1 B_1 e_3 \\ &\quad + e_3^T (e_4 - B_1^T P_1 r_1 - K_3 e_3 - e_3 \frac{\bar{\Delta}_2^2}{\varepsilon_2} - \dot{\theta}_\alpha^d) \\ &\quad + e_4^T (-e_3 - K_4 e_4 - e_4 \frac{\bar{\Delta}_3^2}{\varepsilon_3} + \Delta_3) \\ &\leq -\frac{1}{2}r_1^T Q_1 r_1 + \frac{\varepsilon_1}{4} - e_3^T K_3 e_3 - e_4 K_4 e_4 - \|e_3\|^2 \frac{\bar{\Delta}_2^2}{\varepsilon_2} \\ &\quad + \|e_3\| \bar{\Delta}_2 - \|e_4\|^2 \frac{\bar{\Delta}_3^2}{\varepsilon_3} + \|e_4\| \bar{\Delta}_3 \\ &\leq -\frac{1}{2}r_1^T Q_1 r_1 - e_3^T K_3 e_3 - e_4 K_4 e_4 \\ &\quad + \frac{1}{4}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3). \end{aligned} \quad (43)$$

The Lyapunov function V decreases as long as the error variables satisfy the inequality

$$\frac{1}{2}r_1^T Q_1 r_1 + e_3^T K_3 e_3 + e_4 K_4 e_4 \geq \frac{1}{4}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3).$$

Let $\mathcal{S}_1$ denote the smallest level surface of the Lyapunov function V_1 including the ellipsoid

$$\frac{1}{2}r_1^T Q_1 r_1 + e_3^T K_3 e_3 + e_4 K_4 e_4 = \frac{1}{4}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3).$$

The solutions of the error dynamics enter $\mathcal{S}_1$ in a finite time and then remain in it. $\square$

The block diagram in Fig. 1 illustrates the structure of the position controller.

Fig. 1 Closed loop position control scheme

4 Force Control

Multiplying Eq. 6 by $K_{\sigma\sigma}$ converts Eq. 6 into

$$K_{\sigma\sigma}^{-1}(M_\sigma \ddot{q} + h_\sigma + J_\sigma \lambda) + q_\sigma = \theta_\sigma. \quad (44)$$

Equations 7 and 44 (the σ -rows of the system dynamics) are used for the force control. For this purpose, a desired value for θ_σ is first determined such that λ converges to λ^d if θ_σ converges to its desired value θ_σ^d . Then, the control inputs τ_σ are given such that θ_σ converges to θ_σ^d .

Let e_5 and e_6 denote the force error

$$e_5 = \lambda - \lambda^d \quad (45)$$

and the position error regarding the σ components of the motor angles

$$e_6 = \theta_\sigma - \theta_\sigma^d \quad (46)$$

respectively. The desired value of θ_σ is given by

$$\theta_\sigma^d = K_5 e_5 + q_\sigma \quad (47)$$

where K_5 an $m \times m$ dimensional diagonal and positive definite matrix. Substituting Eqs. 47 and 46 into Eq. 44 yields

$$K_{\sigma\sigma}^{-1}(M_\sigma \ddot{q} + h_\sigma + J_\sigma \lambda) = K_5 e_5 + e_6. \quad (48)$$

The solution of Eq. 48 for e_5 is

$$e_5 = (K_5 - K_{\sigma\sigma}^{-1} J_\sigma)^{-1} [K_{\sigma\sigma}^{-1}(M_\sigma \ddot{q} + h_\sigma + J_\sigma \lambda^d) - e_6]. \quad (49)$$

It is obvious from Eq. 49 that the absolute values of the components of e_5 can be made arbitrarily small at any time instant by means of the diagonal elements of K_5 if the components of e_6 remain bounded in time. It means that the success of the force control depends on the boundedness of e_6 . Thus, the force control task is reduced to a position control problem at which the position error e_6 has to be kept bounded. A linear controller keeping e_6 bounded can easily be determined by means of linear control theory since the controlled system (7) with the input τ_σ and the output θ_σ is linear. In this section, a linear controller for the linear system (7) is not given explicitly, since design of linear controllers is a well-known topic and does not contribute to the present

Fig. 2 Closed loop position and force control scheme

Fig. 4 Time trajectories of q_1 , q_1^d , q_2 , θ_1 , θ_1^d , θ_2 , θ_2^d , λ , λ^d , τ_1 , τ_2 and the calculated distance of the wall from the robot base a during the simulation, respectively

arbitrarily. They must satisfy the constraint Eq. 3 and its time derivative $J^T \dot{q} = 0$.

6 Conclusions

A robust control method for the simultaneous position/force control of constrained flexible joint robots has been proposed. For this purpose, the system model has been first converted into a new form that allows to separate the position and force control tasks from each other and to group the joints: The joints in the set α are used at the position control and the joints in the set σ at the force control. The success of the force control is dependent on the success of the position control. Nevertheless, the success of the position control is independent on the success of the force control.

The applicability of the controller in practice seems to depend on overcoming the difficulty that the proposed control law requires upper bound functions for the modeling uncertainties. The necessity of such upper bound functions greatly increases the modeling effort.

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