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ARTICLE

Coordination, Convention and the Constitution of Physical Objects

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Abstract
In this paper, I address the significance of the key notions of coordination, constitution and convention. My aim in so doing is to provide a better understanding of their relation to conventionalism and to evaluate the prospects for a version of the relativized a priori based on a refinement of the notion of coordination. I stress the Kantian roots of all three concepts. Moreover, I argue that the link between the early logical positivist requirement for the uniqueness of coordination and the Kantian account of empirical objectivity provides an interpretive key that sheds light on the alleged incompatibility between constitutive principles and conventionalism.

Keywords Coordination · Constitutive principles · Conventionalism · A priori · Reichenbach: Kant

1 Introduction

The discussion about the foundations of geometry and their relation to the spatio-temporal character of physical theories reached a decisive high point at the end of the 19th century, with authors of the stature of Helmholtz, Hilbert, Poincaré, Einstein, Cassirer, Schlick, Reichenbach and Carnap, amongst others, participating in it. Some of the main reasons for this interest are easily found in the history of the disciplines involved. The study of non-Euclidean geometries and the formulation of relativity theory without a doubt triggered a particular concern for such foundational issues, especially once it was realized that certain innovations in the fields of mathematics and physics seem to contradict some generally accepted ideas about the status of geometry.

If there is a common philosophical framework within which this discussion takes place, it is to a great extent configured by Kant's account of (empirical) knowledge and the perspective he developed on the foundations of mathematical physics. All the aforementioned authors entered into dialogue with this Kantian approach in different and complex ways. Many works in recent decades have shown that the relation that the fathers of logical

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positivism (Schlick, Reichenbach and Carnap) maintained with the Kantian legacy was particularly complex.[1] The reactions of these authors to the question of what to do with a Kantian-inspired conception of physical geometry in light of the consequences for the notions of space and time of Einstein's two theories of relativity range from declaring the definitive death of the Kantian approach[2] to an attempt to revitalize it and vindicate it in the relativistic context.[3] The problem is that the different reactions do not always belong to different authors.

At the centre of the problem of how to combine, if at all possible, the Kantian legacy and relativity is the figure of Hans Reichenbach. In 1920, he published *The Theory of Relativity and A Priori Knowledge* (Reichenbach 1920) which has been recognized, especially thanks to the work of Michael Friedman over the last two decades, as one of the main contributions to the debate. A key notion that is necessary to understand Reichenbach's appropriation of Kantian elements in order to produce an understanding of the basic principles of relativity and, in particular, an account of the spatio-temporal character of this theory is his notion of 'coordination'. Reichenbach first introduces his notion of 'coordination' in his doctoral thesis, in the context of a discussion about the application of mathematics in physical knowledge, in which a principle of probability plays a central role.[4] With this notion in hand, Reichenbach proposes a way to understand the special status that, according to Kant, some mathematical and physical principles have: those that make empirical knowledge possible. This is a version of what Friedman dubs the "relativized a priori" (Friedman 2001).

One of the problems with Reichenbach's early proposal is that it revolves around a notion which, according to general consensus, is rather 'obscure' and characterized in a way that is insufficient for the purposes of capturing the core of the Kantian notion of the constitution of objects of experience.[5] Although I might agree with this general impression that the notion falls short of the mark, I think that it can be enhanced by contrasting the notions of coordination in Schlick and Reichenbach, thereby making its connections to some relevant Kantian notions more explicit; and then by discussing its complicated relation to the question of conventionalism. Although similar questions have been dealt with before, there is still room for progress. I consider that such connections, sometimes neglected in the discussion, are essential in order to understand the role that the notion of coordination originally played in Reichenbach's proposal. We must bear in mind that, at least initially, Reichenbach was trying to adapt the Kantian notion of synthetic a priori elements of knowledge to the context opened up by relativity theory; but that Kant's notion is itself extremely problematic: it involves the notions of synthesis, a priori and,

[1] See Coffa (1991), Darrigol (2020), Friedman (1999), Friedman (2001), Ryckman (2005), Eberhardt (2022).

[2] The development of logical empiricism, by authors like Schlick, Ayer or the late Reichenbach, can be read as advancing this message.

[3] Part of the neo-Kantian camp, particularly Cassirer, reacted by attempting to make the Kantian approach and relativity theory compatible; so, as we will see below, did the earlier Reichenbach.

[4] There seems to be a commonly shared misunderstanding in the literature on this point: namely, that Reichenbach's notion of coordination first appeared in 1920 inherited from Schlick. Padovani (Padovani 2011; 2015) dissolve this standard reading and offer a careful discussion of Reichenbach's early notion of coordination. I thank an anonymous reviewer for pointing out this same mistake in an earlier version of this paper and pressing me to correct it.

[5] Examples of this type of claim can be found in Coffa (1991), Friedman (2001), Friedman (1999) and Darrigol (2020).

in a somewhat more indirect way, intuition. Each of these concepts has been the object of interpretive quarrels. Even if this is not the place to enter into the details of such disputes, I will briefly recap what roles these concepts play in the Kantian scheme, in order to assess the notion of coordination.

Reichenbach can be praised for having rescued those elements of the Kantian perspective on empirical knowledge that, although originally elaborated using the template of Newtonian physics and Euclidean geometry, are compatible with physical knowledge as encoded in relativity theory. This is done through the reinterpretation of two central notions of the Kantian scheme: that of 'a priori' and the notion of 'constitution'. It is obvious that a reinterpretation of a conceptual scheme always presupposes a previous interpretation of it and this, when applied to Kant's philosophy, is far from trivial. One of the main problems in gauging the nature of the different proposals in this debate has to do with identifying the different interpretative starting points of the participants. The reactions of the logical positivists to the question of the compatibility of the Kantian proposal and relativity theory is exemplified in their interpretations of the key notions of a priori, intuition and synthetic a priori judgements. And, at least for the younger Schlick and Reichenbach, these converge on the concepts of coordination and convention.

So, in this paper, I address the mutual relations between the notions of coordination, convention and constitutivity in Schlick's and Reichenbach's early work, in particular in what these notions have to say about the establishment of the (chrono)-geometry of space(time) in physical theories. In order to do so, it will be necessary to take a special look at the connection these notions have with their Kantian predecessors. As a result of that inquiry, I hope to contribute to the task of providing a better understanding of the different stands that Schlick and Reichenbach adopt with respect to the question of convention, the extent of Reichenbach's so-called *conventional shift*, and the prospects for composing a version of constitutive principles that is compatible with, and illuminating for, contemporary physics. The basic idea behind my reading of such a well-known debate is that the notion of 'coordination' that the early logical positivists use, together with the criterion of uniqueness of coordination, bring us closer to the Kantian account of objectivity than is usually recognized. As a consequence of this, some very characteristic elements of Kant's transcendental approach, which are explicitly rejected by all the logical positivists at some point—namely, the role of intuition and the synthetic a priori—cannot be so easily dispensed with if we still want to have a workable version of something similar to Reichenbach's principles of coordination. Moreover, through an understanding of convention that incorporates some of these Kantian elements in a generalized fashion, there is hope of formulating a version of constitutive principles that illuminates the claim that there are a priori mathematical structures in physical experience; and a version that furthermore is compatible with relativity.[6]

[6] The main aim of this paper is not to provide an interpretation of these authors that competes with accepted ones. I believe that many of the main features of my characterization of the central notions that I discuss herein are grounded solidly in Reichenbach's views. Nevertheless, whatever the hermeneutics may be, my objective here is to use this inspiration to rehabilitate some of these notions, which are sometimes considered obsolete, for their incorporation into some recent debates on the nature of spacetime. If it turns out that this is not the way in which the authors I discuss actually understood the central notions, then I will take all the credit or the blame; but if their understanding was indeed as I portray it, then I willingly share it all with them. On the other hand, there are other projects that aim to defend the relevance of some kind of constitutive elements in physics and that share some insights with my perspective; Pap (1946), Friedman (2001), Stump (2015), Darrigol (2020) are some of these. I leave for another paper a full development of my version of constitutive principles as well as the discussion of the differences with the alternative proposals. I have initiated this work in Sus (2019; 2021; 2023).

The rest of the paper unfolds as follows. In Sects. 2, 3 and 4, I address the convergence and divergence of Schlick's and Reichenbach's attack on the notions of coordination and convention that are portrayed in Reichenbach's conventionalist shift. Then, Sect. 5 presents Friedman's version of the difficult relation between the logical positivists and Poincaré's conventionalism. In Sect. 6, I search for Kant's correlates of the notion of coordination to show the relevance of some neglected Kantian notions that, as I argue in Sect. 7, are essential to arrive at Reichenbach's idea of constitutive principles. Finally, in Sect. 8, I venture the thesis of there being constitutive elements in Reichenbach's conventionalist axiomatization programme. I end, in Sect. 9, with some conclusions.

2 Coordination and Constitutivity

In *The Theory of Relativity and A Priori Knowledge* (Reichenbach 1920), Reichenbach presents and discusses his notion of cognition as coordination. The presentation can be seen as programmatic and incomplete. He clearly differentiates the epistemic situations in mathematics and physics: while in mathematics, the object "is uniquely determined by the axioms and definitions of mathematics" (Reichenbach 1920, 34), we might say that it is given by the mutual relations of concepts in the system of axioms, in a physical theory, the determination of the object needs some further element. According to Reichenbach this is provided by the coordination of physical things to equations: the coordination of physical objects to mathematical concepts.

The model of coordination is that of establishing a relation between the elements of two mathematical sets. But, at the same time, Reichenbach stresses that what he calls 'cognitive coordination' has the peculiarity—different from when one considers coordination of two mathematically defined sets—of one of the sides of the coordination not being previously defined. This means that the coordinated object in the context of physical theories, the physical thing, is not previously given but only determined through the coordination. This, by the way, does not mean that the direction of determination runs only from mathematical concept to physical object. He tells us:

> Thus we are faced with the strange fact that in the realm of cognition two sets are coordinated one of which not only attains its order through this coordination, but whose elements are defined by means of this coordination (Reichenbach 1920, 40).

The tension in this notion of coordination is patent. On the one hand, the reference to coordination in order to characterize the difference between mathematical and physical objectivity seems to suggest that such a difference can be accounted for in terms of how, in the second case, mathematical concepts connect with physical things, understood as specific physical objects. On the other hand, Reichenbach is at pains to explain that in cognitive coordination the object, one of the sides of the coordination, is not previously determined but defined through the coordination. The tension at this point, prior to the adoption of the language of conventionalism, is mitigated by recourse to the notion of the uniqueness of coordination as a criterion for the truth for empirical knowledge, as had previously been discussed by Schlick. It is the uniqueness of coordination that determines the physical thing. Let us see how this works.

Reichenbach understands the criterion of the uniqueness of coordination as essential for the definition of the elements of reality: it provides the only reasonable definition of *true* in the context of empirical knowledge. Perceptual content cannot by itself fulfil the criterion for reality but, as Reichenbach expresses it, experience "furnishes only a criterion for the uniqueness of the coordination - and not the coordination itself" (Reichenbach 1920, 63). The criterion is, properly, a criterion for truth. This translates into empirical content being valid insofar as it fits into a unique coordination; or if it is classified or ordered in such a way that the same physical variable is not assigned different values. Truth is regarded by Reichenbach as defined in terms of uniqueness of coordination, and this uniqueness is expressed in rules of consistency that restrict possible coordinations. It is important to stress that uniqueness of coordination is not guaranteed: as expressed by a system of coordinating principles, one such a system of principles might fail to provide a unique coordination that, in this sense, is not arbitrary.[7] One might say that perceptual content provides the raw material to which the criterion of uniqueness of coordination is applied, without which the coordination could not be performed, but that content is not to be understood as being one of the sides of the coordination: the coordination, we must not forget, is between physical objects (this is not perceptual content) and certain mathematical concepts.[8] And, according to Reichenbach, it might seem paradoxical that knowledge is determined by a physical object that is only determined through coordination. In his words:

> We notice the strange fact that it is the defined side that determines the individual things of the undefined side, and that, vice versa, it is the undefined side that prescribes the order of the defined side. The existence of reality is expressed in this mutuality of coordination (Reichenbach 1920, 42).

Before moving on, I think that it is important to stress and keep in mind three aspects of this characterization of cognitive coordination. The first has to do with the differences between the concepts in mathematical and physical theories, the former defined implicitly by the systems of axioms of mathematical theories, the latter defined through a coordination subjected to the criterion of uniqueness. There is no doubt, then, that Reichenbach (like other logical empiricists) considers that the way in which terms are defined in mathematical theories and in empirical ones is substantively different: in the former context, terms acquire meaning through implicit definitions; in the latter, by coordination with experience. That said, to establish this difference in clear terms is problematic in many aspects.[9] And, more importantly, there are elements in Reichenbach's characterization of the criterion of uniqueness of coordination that allow us to attenuate the differences between the two contexts in, I will argue later, a very fruitful way. To state it crudely, one might think that physical concepts are also, at least partially, ultimately defined by the physical principles according to which they occur, insofar as they form a consistent system that allows a unique coordination. This does not mean, clearly, that they are not empirically determined, as such principles also include the coordinating principles that operate on the empirical

[7] I am grateful to an anonymous reviewer for insisting on me stressing this aspect of the concept of coordination. As I mention below, this is also the key to understand Reichenbach's distance with Kant's synthetic a priori.

[8] These features, the mutuality of coordination and the fact that perceptual contents cannot be what Reichenbach takes that mathematical concepts are coordinated to in cognitive coordinations, are considered in Eberhardt (2022), Padovani (2015; 2021) as both essential characteristic of this Reichenbachian notion and partly responsible for its obscurity.

[9] I would say that the problematic character of this distinction is universally recognized by critics of logical positivism. A discussion of the repercussions of the tension between the notion of implicit definition and coordinative definitions for the notion of convention in Schlick and Reichenbach—particularly relevant for this paper—can be found in Belkind (2022).

material. So it is that Reichenbach claims: "The physical object cannot be determined by axioms and definitions. It is a thing of the real world, not an object of the logical world of mathematics" (Reichenbach 1920, 36). Moreover, as I will also argue later, it is possible to elaborate interesting interpretations of the notion of coordination according to which the determination of a physical object, formally or structurally, is not very different from that of a mathematical object. In both cases, the object becomes constituted in some way.

The second aspect, which complements the first, is that since the input of coordination accounts for all the empirical contents, the uniqueness of a given coordination is never definitely ensured and, consequently, the consistency of the principles that support the coordination is always open to question. Along these lines we can now draw a more precise initial difference between the definition of the mathematical object and the physical one, due to the presence of coordination as it can be understood from the characterization given by Reichenbach in 1920: the mathematical axioms fully define the object and the meaning of the mathematical concepts; in contrast, the physical principles can only partially define their object, at most. Expressed this way, there is a clear-cut difference between the two domains; the difference can be expressed in terms of features of structurally analogous procedures; further explanations follow after the subsequent third comment.

The third comment is that Reichenbach explicitly claims that his axioms of coordination, which express the criteria that ensure a unique coordination, are equivalent to what Kant calls synthetic a priori judgements and, in this sense, constitutive principles of experience (Reichenbach 1920, 47–49). This is not just an empty declaration and it is fully consistent, according to the way coordination is characterized by Reichenbach, with the role that the synthetic a priori principles play for Kant in physical knowledge[10]: only through these principles of coordination and with the input of empirical content is the physical object defined. Another thing is that, as it is well known, Reichenbach corrects the notion of a priori used by Kant and argues that the Kantian principles must be replaced by different ones. According to Reichenbach, a priori principles are so in the sense of being constitutive but they are revisable, thus contesting one of the Kantian marks of the a priori: its necessity. In the last part of this paper I will have more to say about these aspects of the programmatic characterization of coordination that Reichenbach offers in 1920 and their relation to the Kantian programme. Nevertheless, as the last two comments are directly related to Reichenbach's critique of what he calls the Kantian idea of the arbitrariness of coordination, and this has a clear impact on how to understand the tension between implicit definitions and coordinative ones, I must say something about it at this point, even if just schematically.

As stressed above, Reichenbach identifies the criterion of uniqueness of coordination as the essential element for the definition of real objects (fixing the meaning of physical concepts). Such a criterion imposes restrictions on the possible coordinations expressed by what Reichenbach calls **axioms** of coordination which, due to their being part of the determination of the object via cognitive coordination, can also be thought of as constitutive principles. Now the question is whether the uniqueness of cognitive coordination is ensured and how. Reichenbach provides what, according to him, are systems of principles of coordination that are inconsistent. This shows that uniqueness of coordination is not guaranteed. In contrast, Kant argues for the existence of synthetic a priori principles,

[10] See my discussion in Sect. 6.

understanding that a priori is tantamount to necessary.[11] This means that Kant seems to be assuming that those principles, if they are part of a complete system, must be consistent.[12] In Reichenbach's language, Kant's synthetic a priori principles are principles of coordination and the uniqueness of coordination, according to Kant, must be determined independently of (prior to) experience. And if uniqueness is ensured independently of experience, according to Reichenbach's reading of Kant, it can only be *reason* what determines the principles, together with their uniqueness of coordination. And through a shaky inference supported by some Kantian fragments,[13] Reichenbach claims that such principles, for Kant, must be self-evident. To sum up this chain of reasoning, Kantian cognitive coordination would assume that any system of (synthetic a priori) principles does not contain contradictions or, in other words, any one such system provides a unique coordination. This is what Reichenbach calls the *thesis of arbitrariness of coordination*. This is not the place to discuss whether this is or is not a good rendition of Kant's argument; what must be kept in mind is that Reichenbach's reasoning depends on regarding the a priori principles as necessary together with the idea that the determination of the system of principles obeys some internal criteria linked to self-evidence. I will argue later that, even if this is for many fragments of Kant a correct material reading, something that I have no intention of questioning here, the core of the Kantian scheme is not committed to the strong thesis of arbitrariness of coordination.

At this point, I must refer to an element that is key for the difference between empirical and mathematical concepts, and which is central to Reichenbach's 1920 notion of coordination and its difference from the equivalent Kantian notion: the role that what Reichenbach calls the *principle of normal induction* plays in his scheme. Reichenbach wants to give a general argument showing why the claim that the coordinating principles in the Kantian system are incompatible with the principles of relativity theory does not involve the abandonment of a sense of a priori as constitutivity. One might think, as apparently Kant did, that having principles that are constitutive of an object of experience is incompatible with having empirical content that contradicts such principles. So, a claim of incompatibility between an empirically valid theory like relativity and some constitutive principles would be absurd. But Reichenbach notes that amongst the principles of coordination there must always be one that fixes the procedure for extrapolating empirical data: the aforementioned principle of normal induction. In a case of incompatibility of data with the system of coordinating principles, Reichenbach says, one could always renounce this principle of normal induction. But, as he is at pains to show, this would imply renouncing the uniqueness of coordination. This is why Reichenbach ends up claiming: "The principle of normal induction, above all other coordinating principles, is distinguished by the fact that it defines the uniqueness of the coordination" (Reichenbach 1920, 66–67). The argument is convoluted and not easy to examine briefly but, for the purposes of the present discussion, we can take one of its consequences for granted: Reichenbach considers that his understanding of

[11] Kant already asserts in the Introduction to KrV that necessity is one of the signs of the a priori.

[12] The technical notion of 'consistency' refers to the impossibility of deriving contradictions in a formal language with a calculus. Reichenbach (Reichenbach 1920) use is, let us say, more informal and semantic: principles with non-unique coordination are inconsistent in the sense of assigning different values to the same term.

[13] See Reichenbach (1920, 57). In the footnote, Reichenbach quotes some passages from Kant to tain the inference; but I do not see how the claim of self-evidency follows from them. Nonetheless, the rationale of the argument that he claims supports this conclusion might be as follows. For Kant, such principles are a priori (necessary) but not fictions; their determination must be referred to a thing-in-itself (p-50) and thus manifested as self-evident.

coordinating principles, for which the notion of uniqueness of coordination is essential, is dependent on a principle of normal induction (based on a principle of probability that is indispensable for the treatment of errors inherent in empirical data) and this is a feature that distinguishes it from Kant's system and makes it compatible with relativity. Now, bracketing any complexity of this intervention of probability and induction in the extrapolation of empirical data, one very simple effect of Reichenbach's reference to the principle of normal induction is that it brings to the fore the main feature that distinguishes between the pure mathematical and the physical contexts: due to the fact that actual empirical data are necessarily finite while potentially infinite, the coordination of physical concepts to experience must always involve some kind of extrapolation that leaves space for movements to try to accommodate a set of data to a given system of principles. This is what Reichenbach rejects and attributes to Kant by arguing that it would involve renouncing the uniqueness of coordination. But beyond this, what seems clear is that this is not a problem in the purely mathematical context: there, because there is no coordination with experience and, therefore, no operation of a principle of induction, the only requirement for the definition of mathematical concepts and, one might say, the constitution of mathematical objects, is the consistency of the axioms. I will discuss this difference further, together with the common root, at the end of the paper.[14]

This characterization of the notion of coordination which, as we have seen, involves the definition of physical objects through the requirement for the uniqueness of coordination, justifies Reichenbach's use of the term 'constitutive principles'. The criterion for successful coordination is its uniqueness which, Reichenbach says, must be secured by the existence of some principles of coordination. Insofar as such principles "ultimately define real objects and events" (Reichenbach 1920, 49), they deserve to be called constitutive principles.

3 Schlick on Coordination

To complete this first approach to the notion of coordination, we must look at Schlick's discussion in his *General Theory of Knowledge* (Schlick 1918).

The first essential component of Schlick's position is his characterization of the notion of an implicit definition and its differences from ordinary definitions. The model that inspires this notion of definition is Hilbert's axiomatization of geometry, in which the concepts are defined, not by reference to some external reality, but just through their mutual

[14] There is obviously much more to say about the role of the principles of normal induction and probability in Reichenbach's early account of coordination. Eberhardt (2011) and Padovani (2011), for instance, stress the importance of Reichenbach's work on probability in his doctoral thesis, prior to his engagement with the notion of coordination, for the development of that notion. I think that my somewhat simplified account of this connection does not contradict one of Padovani's theses, that is, that some principles like probability/induction or genidentity, play a more fundamental constitutive role in Reichenbach's scheme than others, at least very clearly in his early work; they are, somehow, constitutive of the constitutive principles (Padovani 2011, 54). There are many suggestive threads in that perspective that I cannot follow up on now, but that seem very promising for an attack on the status of the problem of induction from a Kantian perspective. Another point of coincidence—which I can also only mention here—has to do with the detection of Cassirer elements in Reichenbach's discussion: his stress on the role of induction and the inexhaustibility of experience in empirical coordination immediately makes one think of the regulative character of the notion of an object of experience. All these, I would say, are seeds that can also be seen as part of one of the plausible interpretations of Kant's proposal.

relations, established in the axioms. Schlick stresses that this characteristic is what provides the central differences with respect to ordinary definitions: concepts are defined by concepts and the certainty of the axioms does not refer to anything beyond the concepts. This is opposed to ordinary definitions, which must make contact with some non-conceptual empirical content in order to be true. Systems of axioms in which concepts are implicitly defined "float freely", as Schlick expresses it (Schlick 1918, 37). And this is mainly what would lead, according to Schlick, to the rejection of two of the pillars that sustain the Kantian account of mathematical certainty: the connection with intuition and the synthetic a priori character of mathematical judgements.

I think that it is important to make clear these consequences which, according to Schlick, has the notion of implicit definitions, as they give rise to the opposition to Kant and the neo-Kantians that is assumed at different stages by authors working within and around logical positivism. Schlick propounds an understanding of implicit definitions that is committed to the view that intuition does not play any role in the validity of mathematical judgements or the meaning of mathematical concepts: "the intuitive meaning of the basic concept is of no consequence whatsoever" (Schlick 1918, 34). Implicit definitions are, then, instruments that permit full determination of concepts without recourse to any non-conceptual element. Concepts that are determined in this way do not designate anything real and, accordingly, "the construction of a strict deductive science has only the significance of a game with symbols" (Schlick 1918, 37). As a final remark on this characterization of implicit definitions, Schlick notes that there are certain conditions that the defining axioms must fulfil: they must be consistent or, equivalently, must not contain any contradictions.

Together with this, we obtain a theory of judgements: a judgement establishes relations between concepts, and concepts are determined through judgements. Some judgements are taken to be definitions of some of the concepts involved (either implicitly or explicitly), while others are either descriptions of facts or hypotheses about unknown future facts. These two categories (definitions and hypotheses) exhaust the types of judgements. What is relevant for our discussion is that this distinction can be reduced to the analytic/synthetic distinction and therefore lends support to the rejection of the category of synthetic a priori judgements. Let us take a closer look at this. Schlick interprets one of Kant's characterizations of analytic judgements, that the concept of the predicate is contained in the concept of the subject, as saying that the predicate is included in the definition of the concept of the subject.[15] There can be no question that empirical judgements (hypotheses) are not analytical in this sense. Nonetheless, Schlick's substantive claim is that his class of definitions (implicit and explicit) coincides with the class of analytic judgements. Is this uncontroversial? It seems unproblematic in the case of explicit definitions: if the concept of the subject is defined in terms of other concepts and the predicate is one of them, we have here an example of a predicate concept that is contained in the definition of the subject concept and, therefore, an analytic judgement. But things are not that simple for implicit definitions.

[15] This seems a natural way of reading Kant's definition of analytic judgements. Strictly speaking when Kant introduces his famous distinction analytic-synthetic, the definitions that he seems to be taking as characteristic of analytic judgements would be explicit definitions in which the predicate is one of the concepts that define the predicate. If this is the case, Schlick's definition would be generalizing Kant's characterization to include also implicit definitions. Nevertheless, this way of presenting things might be too simplistic to capture the differences between the notions of analyticity in Kant and the logical positivists.

One might say that implicit definitions are analytical because the set of axioms, and nothing else, determines the meaning of the concepts and, in this sense, once the system of axioms is given, anything that can be predicated of the concepts so defined is already contained in the axioms. It is hard to believe that a characterization of analyticity of this type might be seen as a refutation of the Kantian notion of the synthetic a priori without it begging the question. What seems clear, nonetheless, is that such a rendition of analyticity differs wildly from whatever is effective for explicit definitions. Without going deeper into the question of the viability of this analytic/synthetic distinction at this point, we must next see how Schlick introduces the notion of 'convention'.

In principle, in a broad sense, all definitions are conventions (in the general sense of their being stipulations). Nevertheless, Schlick introduces a more specific use of the term 'convention', one that is needed when we try to answer the question of how concepts that are implicitly defined by systems of axioms can be applied, i.e. be uniquely coordinated, to facts. This is far more complicated than for concepts that are individually connected to reality; in these latter cases, the coordination is realized through what Schlick calls "concrete definitions", which involve free stipulations of meaning by means of direct connections between concepts and objects of reality, producing judgements that belong to descriptive or historical knowledge (Schlick 1918, 70). But something quite different happens in the physical sciences where concepts that are implicitly defined are applied to experience. Schlick points out that it is remarkable that objects, which might originally be selected by direct definitions, fit a system of concepts that are implicitly defined. He comments:

> Now the remarkable thing is that for a suitable choice of objects (singled out by means of concrete definitions), we can find implicit definitions such that the concepts defined by them may be used to designate uniquely those same real objects. […] Obviously, to suppose that the world is intelligible is to assume the existence of a system of implicit definitions that correspond exactly to the system of empirical judgements (Schlick 1918, 70).

Here we have what we could call Schlick's concrete formulation of the classical problem of the applicability of mathematical concepts to reality.

Let us have a closer look at the notion of convention sketched out by Schlick in these passages. Narrowly understood, convention has a fairly restricted sense that connects with the procedure of implicit conceptual definitions and the coordination of the corresponding concepts to the empirical domain. There are then at least two different senses that concur in this characterization: the free stipulation involved in concrete definitions and the possibility that alternative systems of concepts, that are implicitly defined, be compatible with experience.[16] In this sense, convention plays the role of the missing link that solves the problem of applicability. The question is how the mathematical concepts, implicitly defined by the axioms, can fit in with the web of empirical relations that we have in the world of objects. The answer is that it is thanks to convention, to the possibility of freely choosing how to connect concepts and objects, that this can be done most of the time. The following passage is revealing of Schlick's position on convention and coordination.

[16] The combination of these two senses is what Schlick calls convention in the narrower sense that he attributes to Poincaré and that can be understood as coordination by convention. Belkind (2022) contends that in Schlick these two axes are independent. As I will discuss below, partly the question about the constitutive character of coordination depends on how one understand the relation between these two senses of convention.

> To define a concept implicitly is to determine it by means of its relations to other concepts. But to apply such a concept to reality is to choose, out of the infinite wealth of relations in the world, a certain complex or grouping and to embrace this complex as a unit by designating it with a name. By a suitable choice, it is always possible under certain circumstances to obtain an unambiguous designation of the real by means of the concept. Conceptual definitions and coordinations that come into being in this fashion we call *conventions* […] (Schlick 1918, 71, emphasis in the original).

We have then, according to Schlick, two clearly differentiated domains, the mathematical one with the implicit conceptual definitions and the physical/empirical one in which concepts are directly defined by coordinating them, pragmatically we might say, to empirical objects. The problem of applicability has to do with the matching of concepts heterogeneously defined. In this context, Schlick's rendition of coordinating by convention can be understood as a solution to the problem: starting from a presumption of ineligibility—a correlation between the structures of the system of axioms and certain fragments of reality—convention will involve a selection of the relations given by nature in the act of coordination. In principle, as we will discuss next, Schlick's notion of coordination is substantially different from the one attributed to Reichenbach in the previous section: there is no sign of constitution of the object of experience in Schlick's notion, which is somehow supplanted by the reference to convention (absent in Reichenbach's characterization). Nonetheless, even if the term possesses different meanings, both authors recognize that coordination must be unequivocal. And this will imply, as I will make explicit later in the paper, that conventions must be conditioned or constrained. This theme about the status of the paradoxical constraints on free conventions unavoidably pervades the discussions around the different notions of conventions involved in conventionalism.[17]

4 Reichenbach's Conventionalist Shift

The preceding discussion shows that there are a number of tensions in the notions of coordination and convention that Reichenbach and Schlick discuss in their early work. So, it is of utmost importance to consider whether Schlick and Reichenbach, in their discussion of these notions, both understand the same things by *coordination* and *convention*, and whether their use is consistent with the notion of convention as Schlick introduces it in his *General Theory of Knowledge*. As is well known, in their epistolary exchange of 1920,[18] Schlick concedes to Reichenbach that the existence of constitutive principles is obvious, but urges him to regard them as conventions.[19] Schlick goes as far as to say to Reichenbach: "The decisive places where you describe the character of your a priori correspondence principles seem to me to be nothing short of accomplished definitions of the concept of convention" (Schlick 1918, 2–3). This seems to be in line with the narrower sense of convention that he offers in his *General Theory of Knowledge*, as something that takes place in the coordination of concepts that are implicitly defined. Nonetheless, Reichenbach's reply

17 See Belkind (2022) and Ben-Menahem (2006) for clear manifestations of this.

18 Discussed, for instance, in Friedman (1999), Coffa (1991), Ryckman (2005).

19 We must bear in mind that Schlick's previous discussion had divided judgements into hypotheses and conventions.

to Schlick is certainly revealing. After assuring Schlick that they can reach agreement on this question easily, he writes (Coffa 1991, 203):

> Even though several systems of principles are possible, it is always only a group of system principles that is possible; and in this limitation lies an epistemic content. Each possible system signifies through its possibility a property of reality. What I miss in Poincaré is an emphasis on the fact that the arbitrariness of principles is circumscribed as soon as one combines principles. That is why I cannot accept the name "convention." Moreover, we are not certain that two principles, that today we put together as constitutive principles and that are both, according to Poincaré, conventions, will not tomorrow be separated because of new experiences, so that the alternative between these two conventions will end up being synthetic knowledge.

There is agreement about the general conception of theories as systems of axioms which, to a certain extent, implicitly define the concepts involved. Reichenbach's resistance to accept Schlick's point of view seems to come from stressing one of the senses of convention referred above: the principles would be conventions in the sense of it being possible that alternative systems of them be chosen arbitrarily (he assumes this to be the moral of Poincaré's conventionalism). Reichenbach accepts the possibility of coordinating different of these systems (involving different mathematical concepts) with the objects and relations of reality but for him, at this point, the fact that only one of these systems, at each time, is compatible with experience, and it is therefore possible, makes it incompatible with the notion of convention.

This way of understanding conventionalism, as I stressed above, seems to presuppose that any formal system of axioms that is itself consistent is applicable to the set of relations between physical objects. Coordination would be conventional but only by assuming from the beginning that coordination is always possible, which it is not clear at all is always the case. (In other words, once the coordination is secured, meaning that it is possible to establish an unequivocal correlation between the formal concepts and the real objects, there is still room to choose the final coordination as a matter of convention.) The claim that this is possible seems to be grounded in the insights put forward by Poincaré's famous example of the spherical world and the temperature field. The more important aspect of convention, as Reichenbach is reading it, assumes that coordination for a given system of axioms can be implemented for all the empirical contents. By this I mean that coordination by convention, as Reichenbach seems to think that Schlick understands it, allows us to coordinate different systems of principles with objects in reality, constrained by the condition of uniqueness. This would not seem, in principle, to endorse what Reichenbach calls, referring to Kant, the arbitrariness of coordination but, at the same time, this conventionalism amounts to the claim that any consistent system of principles can be coordinated with experience once and for all, because its conventional character will always make room for it. This last claim, paradoxically, seems to fully capture what Reichenbach despises about Kantian coordination. So, according to this way of understanding Schlick's notion, even if convention is constrained by the possibility of uniqueness of coordination, this fact is hidden by assuming that uniqueness is always achievable.

Reichenbach's reading of Schlick's coordination by convention and of Kant's synthetic a priori principles can be seen as two sides of the same coin: as manifestations of the thesis of the arbitrariness of coordination. In both cases, the underlying assumption is that any consistent system of principles is compatible with experience, the difference being that Kant would allegedly also incorporate a criterion of self-evidence to select the right a priori principles, while in Schlick's case, after the rejection of any intuitive element, we are left with conventional coordination. This is a manifestation of what a particular reading of Kant (one that was sometimes assumed by logical positivists) leads to when stripped of some essential elements.

On the other hand, what Reichenbach seems to be doing in his reply to Schlick is pointing to the fact that conventions are always constrained by the requirement of uniqueness of coordination of the conceptual system to objects through the empirical contents which, by definition, are never exhausted. This means that the notion of coordination that Reichenbach is presupposing here must be different from that assumed by Schlick; as Reichenbach has made clear in Reichenbach (1920), the cognitive coordination is always between two sets, one of which is not determined prior to the coordination. All this, so thinks Reichenbach, makes the use of the term 'convention' highly questionable; he would prefer an explicit recognition of the principles as synthetic a priori principles and, in this sense, constitutive.[20]

To sum up, Schlick invokes a Poincaréan notion of convention to characterize his idea of coordination, that is eventually understood by Reichenbach as a correlation between two sets that are defined previously to the coordination.[21] This means that the uniqueness of coordination must be assumed from the beginning for all possible empirical contents. From this point of view, Poincaré's conventionalism is meant to settle the final coordination. This interpretation reminds us what Einstein would later say about Poincaré's conventionalism, in *Geometry and Experience*: it is right, sub specie aeterni. This would mean that such a position regarding conventionalism can only be held from the perspective of a final theory that is empirically adequate. Nonetheless, Einstein thinks, in the intermediate position we always find ourselves in, geometry must be understood as an empirical theory describing the behaviour of material rods, practically assumed to be rigid bodies. As I will stress next, following along the lines of Friedman, this Helmholtzian conception is hidden inside a deeper notion of convention and a better interpretation of Poincaré's position. Reichenbach, while apparently assuming Schlick's reading of Poincaréan conventionalism, notices that it is incompatible with his notion of coordination which, as it does not assume a previously defined set of physical objects, must result in their constitution.

A different question is whether this way of interpreting conventionalism makes justice to Schlick's position. As I mentioned above, there are different senses of convention converging in his General Theroy of Knowledge: one of them arising from concrete or coordinative definitions, involving different possible links between the concepts of the theory and *the real*; another one indicating that different systems of axioms, understood as providing implicit definitions, are compatible with experience. It seems clear that these two senses of convention are interrelated in the notion of uniqueness of coordination. This, according to Belkind, is in line with Schlick's position in his early discussion of Einstein's theory of special relativity: "The conventional nature of sign-systems is both conceptual and coordinative at once" (Belkind 2022, 8). Belkind's thesis is that Schlick later distinguishes between these two senses of convention, resulting, I contend, in the interpretation that Reichenbach ends up taking as the version of conventionalism that he takes to be incompatible with his initial idea of coordination. As I will develop below, this reflects a tension between the

[20] Friedman (1999).

[21] It must be pointed out here that it is highly questionable whether this is a fair interpretation of Poincaré's complex take on convention. Authors with different views on the discussion about conventionalism, like Friedman (2001; 2010; 1999), Ben-Menahem (2006), Stump (2015) and Belkind (2022) would agree with this, I believe.

notions of convention and coordination, regarding whether they are more or less independent, that is reflected in the question about the existence and relevance of constitutive principles. The tendency to separate the two senses of convention ends up, one might argue, in the idea that the meaning of the concepts are first determined by the implicit definitions provided by the systems of axioms (analytically) and only later coordinated with experience, leaving out any sign of constitution of the object.

5 Friedman's Take on the Conventionalist Shift

Friedman argues in *Reconsidering Logical Positivism* that the disagreement between Reichenbach and Schlick on coordinating principles and convention is much more than a mere terminological quibble. No doubt my preceding sketch of the different interpretations of the fundamental notions of coordination and convention is in agreement with this general point. In fact, this discussion might be seen as an attempt to extend the elucidation of the substance of the dispute started by Friedman. Notwithstanding, in this section, I intend to clarify some differences between Friedman's diagnosis of the problem and mine.

Friedman presents Schlick's position as being derived from two general epistemological doctrines: Hilbert's treatment of implicit definitions and a version of conventionalism inspired by Poincaré that, according to Friedman, can be reduced down to Duhemian holism. Under the first doctrine, the definition of geometrical concepts is determined by the axioms that, according to the second, would express conventions: free choices that can configure different geometries. Physical geometry, consequently, is understood as the result of coordinating the formal system of axioms to physical objects; this is due to the fact that, inspired by Poincaré's discussion, only the conjunction of geometry and physics is subject to the tribunal of experience, while physical geometry is determined by convention. The combination of these two tenets thus yields a holistic view of physical theories that cannot distinguish between constitutive and connection axioms. As Friedman clearly says, this strategy completely fails to provide a workable version of the relativized a priori. Even worse than this, it does not seem to offer an illuminating way of accounting for the role that mathematical principles in general—and geometry in particular—play in physical theories.

Reichenbach's starting position in 1920 is quite different. He rejects Poincaré's geometric conventionalism while agreeing with Schlick on the doctrine of implicit definitions for pure geometry and the idea of physical geometry being non-necessary. This is done, according to Friedman,[22] by endorsing an account of physical geometry that is reminiscent of Klein's Erlanger Programme: the geometry of each physical theory is identified with the elements that are invariant under the transformations of the invariance group associated with the theory. This would also provide the constitutive a priori elements for each theory. All this, nonetheless, changes after his famous correspondence with Schlick and Reichenbach's subsequent adoption of Schlick's conventionalist perspective. In Friedman's words, only Reichenbach's 1920 version "yields a true relativized a priori, and so, when Reichenbach accepts Schlick's view in 1922, he in fact gives up the relativized a priori" (Friedman 1999, 68).

It must also be noted that, in spite of his apparently positive view of Reichenbach's original account of the relativized a priori, Friedman also argues that it falls short in different

[22] See Friedman (1999), 65–66.

respects which, according to him, are better dealt with in Carnap's later account of the a priori in terms of linguistic frameworks. Of special interest for our discussion is Friedman's claim to the effect that Reichenbach's account does not provide a general explanation of what constitutivity amounts to. In this respect, what Reichenbach's account would be lacking is an explanation of the link between the intuitive notion of constitutive principles and the machinery of invariance groups (Friedman 1999, 70).

It is difficult not to feel puzzled by this presentation of the state of affairs. In 1920, Reichenbach had given a version of the a priori elements of physical knowledge which, revising Kant, could provide a version of the synthetic a priori that accommodates relativity theory. He then abandons it two years later supposedly because of the irresistible allure of conventionalism, which is completely ineffective as an account of constitutivity and looks very problematic for general relativity (GR). Furthermore, it is unavoidable that the shift to conventionalism involves essentially changing his understanding of some of the main elements that enter into a physical theory. It seems clear that if the original disagreement was genuine and Reichenbach's shift substantive, then different notions of coordination, implicit definition and convention must have been at work.

I would also like to consider Friedman's analysis of the difficult relation between logical positivists (Schlick and Reichenbach in particular) and Poincaré's conventionalism. In his *Reconsidering Logical Positivism* (Ch. 4), Friedman engages with the theoretical foundations of Poincaré's conventionalism and the partial reading that logical positivists make of it. His main thesis is that the argument that Schlick uses for conventionalism, inspired by Poincaré's famous example of the sphere and the temperature field, is a bad argument according to Poincaré's own conception of geometry and his justification of conventionalism. Schlick's argument, that might be seen as exploiting a version of the holistic Duhem–Quine thesis, has no particular importance for geometry and is based on a conception of geometry that is incompatible with Poincaré's synthetic perspective. As we have seen from the preceding discussion of Schlick's conventionalist thesis, one natural interpretation of it, that probably does not capture the full complexity of Schlick's position, is that from the presupposition of there being a unique coordination of mathematical concepts with physical objects, one is left with some freedom to choose the mathematical structure that counts as geometry and that is compatible with the same empirical results. This argument is lacking something that, as Friedman stresses, is provided by Poincaré's version of the conventionality thesis, namely, the origin of the conventional possibilities. According to Poincaré, this is directly connected to the group theoretic considerations contained in the Hemholtz–Lie theorem.[23]

[23] In a similar vein, Ben-Menahem (2006) distinguishes two senses of conventionalism: one directly linked to the thesis of underdetermination of theory by experience (some version of the Duhem–Quine thesis); and the other, primarily applied to conventionalism in the determination of the geometry of physical space, that is directly linked to the classical Helmholtz–Lie problem of space and that in Poincaré takes the form of a specific thesis about the intertranslatability of different geometries. Ben-Menahem defends the idea that Poincaré's conventionalism, applied to the question of spatial geometry, must be understood in the latter sense. And she would also agree with Friedman that a reading of Poincaré's conventionalism that presents it as a version of the Duhem–Quine thesis is, if not completely incorrect, at least partially. Stump (2015, Ch. 4), along similar lines, claims that interpreting Poincaré's conventionalism as equivalent to the Duhem–Quine thesis makes Poincaré's position seem inconsistent. According to him, Poincaré's geometric conventionalism is linked to his relational view of space. In contrast, Belkind (2022) distinguishes between three different senses of conventionalism in early logical positivism: two broadly correspond with Ben-Menahem's classes, plus conventionalism in the sense of coordinating theoretical terms with physical or empirical facts (equivalent to coordinative definitions). He denies the possibility of a conventionalist interpretation of the principle of relativity based on any of these three senses. As I understand it, these three senses of convention are interrelated: the holistic notion of convention and the more specific geometric one can be

Poincaré's argument starts from the idea that the object of geometry is linked to the study of a particular group of transformations and that this particular group is connected to certain possible physical operations: free mobility. From the choice of a physical body as a standard of length and the idealized assumption of free mobility, one can derive the geometries of constant curvature, which include Euclidean, Lobatcheschian and spherical geometries. From here, according to Poincaré, it is a matter of convention which one is the geometry of space. As Friedman remarks, this is a good example of a transcendental Kantian argument, where the faculty of sensibility has now been replaced by the description of 'our capacities' given by the principle of free mobility and, consequently, the geometry of space is not constrained to be Euclidean. I will have more to say about the character of this generalized transcendental argument later; for now, what is relevant is that Friedman says that this is a plausible argument for conventionalism, but it fails in the context of GR precisely because the Hemholtz–Lie theorem is not applicable there. Due to the fact that, on Friedman's account, the logical positivists failed to see the grounds for Poincaré's argument, they did not extract the right conclusions in relation to GR either, and continued to think that their conventionalist stance was available in that context. Only Reichenbach, but before becoming a conventionalist, seems to have been aware of the incompatibility of Poincaré's conventionalism and GR.

There is an alternative account of the relation of Reichenbach with Poincaré's conventionalism which would help us to understand his exchange with Schlick better. The interpretational key is suggested in my preceding presentation of the controversy: Reichenbach's understanding of the conventional thesis would change together with his conventionalist shift, starting in a similar way to Friedman's rendition of Schlick's understanding it would then end up closer to the interpretation of Poincaré based on the Hemholtz–Lie theorem. Let me expand this idea a little bit. There are two main notions playing a part in the so-called conventionalist shift: coordination and convention. A traditional reading of Reichenbach's transition is that after the shift he embraces coordination as convention abandoning completely the idea of cognitive coordination as constitution. The more nuanced reading that I am proposing implies, first, to acknowledge that Reichenbach's initial rejection of Schlick's conventionalist thesis stems from a narrow reading of the notion of 'convention', heavily weighted by the idea of arbitrariness, that is seen as incompatible with his idea of coordinative principles. Eventually, he ends up accepting the conventional character of coordination, but this is not only at the cost of having to revise the concept of 'coordination'—there is no doubt that he starts talking of coordinative definitions, which is not a merely innocent terminological innovation—but also due to incorporating a more sophisticated notion of 'convention'. This reading offers a different perspective on how to interpret the evolution of coordination in Reichenbach's trajectory: the acceptance of conventional elements, which enter in the form of coordinative definitions, is not necessarily incompatible with and idea of coordination that still preserves its constitutive character. As I discuss below when I comment on *The axiomatization of the theory of relativity*, this offers a more nuanced perspective on the interplay between coordination and convention and, so I think,

Footnote 23 (continued)

seen as resulting from a previous conventional coordination between some mathematical concepts and some dynamical systems that are taken to act as measuring devices. But this is another proposal to be fully developed elsewhere, see (Sus 2023).

opens the door for an actualization of the project of elaborating a version of constitutive principles compatible with present physics.[24]

This would explain why Reichenbach initially rejected the use of conventional terminology—it does not seem plausible to him that his constitutive principles should be read in terms of the conventional holistic thesis, which is useless from the constitutive point of view (recall that at this time he defends a version of the synthetic a priori similar to Poincaré's)—and, more importantly, it makes the shift less puzzling: understood à *la* Poincaré, conventionalism is fully compatible with his notion of a priori constitutive principles. Furthermore, what he accomplishes in his axiomatic reconstruction of relativity theory, once he has accepted conventionalism and rejected the synthetic a priori, might be best understood as a realization of the transcendental Kantian programme shifted through Poincaré's conventionalism. A bonus that comes with this way of reading things is that it opens up a path towards providing what Friedman's finds lacking in Reichenbach's presentation of the relativized a priori: a general explanation of what constitutivity amounts to.[25]

Let us make how this interpretation works explicit by bringing to the fore some allegedly Kantian elements in Reichenbach's approach.

6 The Kantian Programme

Reichenbach's 1920 monograph contains an attempt at revitalizing the Kantian project in light of relativity theory.[26] As we know, Reichenbach was not the only one at the time thinking about how the core of Kant's way of understanding physical knowledge, designed to account for the validity of Newtonian mechanics, might be adapted to the context opened up by relativity theory. An ineludible question, however, is whether there is some kind of a shared core to the Kantian programme. As might be expected, due to the different philosophical sensibilities that were involved in dealing with this problem at the beginning of the 20th century, general agreement on this was highly unlikely. In fact, it can be argued that the different perspectives with respect to the impact that relativity might have on Kant's philosophy could be traced back to different ways of extracting the main commitments of the Kantian critique. Nevertheless, in order to shed light on our discussion of the notions of coordination and convention, I will venture here some aspects of what can be taken as a minimal nucleus of a transcendental approach to theories, which I also believe

[24] I thank an anonymous referee for pressing me to clarify further my position on the conventionalist shift. As s/he rightly remarks, there are two issues involved in the question about the continuity of Reichenbach's notions of 'coordination' before and after the shift: the inclusion of concrete definitions and the question about arbitrariness of the conventions. I hope to have made clear that there is a plausible reading of the shift in which, these two issues nicely combine in a way so the resulting notion of coordination is not incompatible with the idea behind constitutive principles.

[25] Padovani (2011, 58–60) exploits her distinction between different levels of constitutivity to defend a view on Reichenbach's conventionalist shift that I think is congenial to my own. According to it, Reichenbach's truly constitutive principles, even after the shift, would be immune to being treated as conventions. There is an open question, though, as to how deep this protection against trivial conventionalism goes. Beyond the question regarding the interpretation of Reichenbach's position, my take on this would be that it opens the door for a richer interpretation of conventionalism that is not incompatible with the idea of constitutive coordination.

[26] This is certainly at least one plausible interpretation of this work. Friedman (2001) and de Boer (2011) lean in this direction.

not to be incompatible with the historical Kant, and that can be defended in the context of relativity and other contemporary physical theories.[27]

6.1 Kant's Synthetic a Priori

Kant's main epistemic problem, one can argue, is how to account for the notion of empirical validity. He argues that this *empirical certainty*[28] cannot proceed from experience alone but needs some elements—those which are constitutive of, and that play the role of rules of, empirical validity—that are not of empirical origin, being, therefore, a priori. Moreover, insofar as they are criteria for validity, with the effect of discriminating between contents, they must be both formal[29] and not expressible in analytic discourse (which for Kant is empty and unable to make a difference). So, we have a need for some type of discourse, a type of judgement, that must be both a priori and non-analytical. This is the seed of Kant's notion of the synthetic a priori. Nonetheless, this is very abstract argumentation. To evaluate the viability of Kant's programme in light of relativity, as the logical positivists did, one needs to consider specific a priori components. Where does Kant find such discursive elements? His template is geometrical knowledge, which is, according to him, apodictic, therefore a priori, and clearly non-analytical to his eyes.

This last remark deserves more careful consideration, as it became one of the most disputed questions in the foundations of geometry. Kant's ultimate reasons for not taking geometrical statements to be analytical are as follows. First, formally they do not fit into what he considers to be the characterization of analytical judgements, namely, that the concept of the predicate must be included in that of the subject. What criterion of 'inclusion' is at play here is a contentious matter. Nonetheless, there is an obvious characterization of it for which the claim that significant mathematical statements are not analytical is true, which is consistent with Kant's discussion of the analytic/synthetic difference. According to such a criterion, the predicate is included in the subject if the concept-predicate belongs to (is one of the concepts of) the explicit definition of the concept-subject. Kant thinks that judgements that express geometrical (and arithmetical) knowledge are not of this type and he takes statements from geometry and arithmetic as examples of this. The second reason for the non-analyticity of mathematical judgements, obviously related to the first, is that the validity of such judgements requires recourse to something that is not included in, does not belong to, the concepts themselves. Again the meaning of this must be evaluated by contrasting it to what happens in analytic judgements: there, the *mere concepts*, coherently with this idea of inclusion of the predicate in the subject, are, trivially, sufficient to determine the validity of the judgements. However, in synthetic judgements, this cannot be the case: which means that the connection of subject and predicate effected by the judgement goes beyond the mere concepts. At this point, Kant invokes the faculty of intuition, another extremely controversial notion. And, without intending to imply that it is not problematic, I think that it is crucial to be as clear as possible about how such a notion enters into

[27] I believe that what I present below is compatible with the Kantian corpus and I know that it is inspired by it, but I do not intend to present here a contribution to Kant scholarship. The sensitive reader who feels uncomfortable with connecting these views with Kant can think of them as an original proposal linked to the transcendental view.

[28] Kant uses this label in the Introduction to KrV, for instance.

[29] Kant sometimes uses the expression *form of experience/knowledge* to refer to the elements that constitute experience.

this argumentation, leaving aside any psychological connotations as far as possible. Kant regards the recourse to intuition as an essential element to account for a kind of validity that is able to go beyond empty analytical validity. Together with this we necessarily have a definition of concepts that is different from that which operates in analytical judgements, namely, that which is determined by what would later be dubbed ‘explicit definitions’.[30] Therefore, this points towards a different characterization of conceptual determination, one which takes us straightforwardly to the binomial structure of concept and intuition.

Let us take stock. Kant’s distinction between analytic and synthetic judgements leads, in the context of mathematics, to a notion of intuition that is linked to a characterization of the determination of concepts[31] that is different from that given in explicit definitions. In Kant’s work,[32] he specifies that this different determination has the form of a characterization that regards concepts as rules of construction for figures in intuition. What affords this binomial of intuition and concept such a central position in Kant’s thought is the fact that this structure, which I have introduced here in relation to mathematical knowledge, is also replicated in the empirical context; this, at least, is what Kant’s analysis intends to show. And this will be fundamental for the Kantian view on the question of the mathematical character of our experience in general and, in particular, of the spatio-temporal structures of our physical theories.[33] This is in fact how objectivity, as much in mathematics as in experience, is understood in Kantian terms: as the conceptual synthesis of intuition.

6.2 Kantian Notion of Objectivity

One key to understanding the essence of the Kantian programme, and which is introduced as such from the beginning of Kant’s *Critique of Pure Reason*, is the aforementioned double character of experience or of knowledge in general; each valid empirical content, every *representation* as Kant sometimes calls general empirical content, is *composed* of intuition and concepts. The nature of this double dimension of empirical knowledge can be thought as emerging from the Kantian notion of objectivity, in the sense of reference to objects. At the risk of trivializing what Kant develops, certainly with difficulty, through dozens of pages mainly concentrated in the *Transcendental Analytics* of his first *Critique*, one can say that the role that the two poles of intuition and concept play in the Kantian scheme consists mainly of providing a way of explicating the reference of sensible contents to an object. From the Kantian perspective, this is explicated in terms of construction (of objects) in intuition: the conceptual side, we might say, encodes the rule operating on each construction while the intuitive side would provide, using a very imperfect metaphor, the material. So, the key to what is involved in the claim that one of our empirical contents is objective

[30] It can be argued that a key difference between analytic and synthetic judgements is given by the notion of concept that is operative in each type of judgement. In analytic judgements subject-concepts are defined by an addition of other concepts, one of which is the predicate-concept. In synthetic judgments, otherwise, the subject-concept is constructed in intuition. The notion of concept operative in the latter is, then, that of a rule of construction that is universal.

[31] This expression needs some clarification: by the determination of concepts I mean the determination of which concept is operative in a given epistemic situation which, according to Kant, must be expressible in a judgement in which the concept is its predicate. The determination is then of what concept applies to the object. This is not possible without the participation of intuition.

[32] See, for instance, the discussion in the Schematism in *Critique of Pure Reason.*

[33] It is almost common knowledge that Kant, in Kant (1998), refers to space and time as the forms of intuition or pure intuitions.

is explicated in terms of the uniqueness of conceptual construction of the object of experience (the fact that there is one and only one rule operating in a given situation) on the raw material given in intuition; that only one of the, in principle, possible constructions is in fact realizable is what is meant by the notion of representation or valid empirical content. This is not far from what the logical positivists call 'uniqueness of determination'.

Kant uses the term synthesis to refer to the realization of such construction and—more confusingly but in line with the terminology of his time—he uses terms like sensibility and understanding, or even receptivity and spontaneity, to talk about the human faculties responsible for these two sides of knowledge. This exuberant terminology opens up a complex panorama of interpretations. Yet here, I intend to stress only one minimal aspect of the Kantian approach, namely, that the notion of objectivity, understood as determination of the object of knowledge (and therefore of truth) in his epistemological proposal is characterized in terms of what I am calling uniqueness of conceptual construction of intuition. And this is the predecessor, I claim, of the notion of uniqueness of coordination used by Schlick and Reichenbach.

There are two important aspects of this characterization of objectivity that I would like to stress. The first is the role of what Kant calls 'intuition'. In general terms, Kant introduces the notion of intuition to refer to the immediate representation of the object of experience, understood as the presence of contents, one next to another, before or after one another. From here, the spatio-temporal character of intuition (space and time are naturally regarded as the form of (or pure) intuition) and its characterization as an infinite manifold or plurality follows. According to Kant, this intuitive dimension is always involved in any objective content; without it, all we can do is simply play with mere concepts, which according to Kant is (empty) analytic discourse. Neither empirical nor mathematical knowledge is deemed knowledge without the intuitive component, and insofar as it is knowledge, properly speaking, it can only be synthetic. The second aspect, correlated to the first, has to do with the notion of concept which is involved in knowledge according to this perspective. As the concept, so understood, must always be on the other side from intuition in the determination of the object of knowledge, its role can be characterized as a rule that unifies the manifold of intuition producing a certain synthesis. Concept, understood in this way, is not just the predicate of a possible judgement, something that belongs to its formal characterization and that by itself only supports analytical (therefore, empty) judgements, but the rule of construction in the synthesis.[34] Kant's idea is that only through the determination of the plurality of intuition by means of concepts can empirical contents refer to objects.

Let us take some more distance from Kant's text. What some post-Kantian discussions seem to make explicit is an unresolved tension in how to account for the difference between objectivity in the mathematical and empirical contexts. According to Kant, such a difference must be rooted in the kind of justification that is invoked in a given judgement: whether it needs just the form of intuition, for the mathematical, or it also involves the contents, the sensation, for the empirical ones. Furthermore, this is reflected in the fact that empirical contents are always subject to eventual correction. The object referred in a mathematical judgment, again according to Kant, would be determined just by the form of intuition (pure intuition), meaning that any mathematical concept would just effect the synthesis/constructions that the pure intuition allows. Nonetheless, and this is an elaboration

[34] A concentrated, although famously obscure, presentation of this dynamics of the, unhelpfully called, *application* of concept to intuition can be found in the section called the Schematism in Kant (1998).

on the afore-mentioned tension that goes beyond Kant, this gives us two possible ways of understanding the determination of mathematical objects in intuition, due to the fact that in this case it is only the form of intuition that is at play (with no specific content): even though intuition always refers to the given, the passive, receptive side of the synthesis and, in this sense, is often naturally connected with the notion of perceptions (Kant links it to the faculty of sensibility), it is clear from the previous characterization of the intuition/concept structure that determination always involves concepts (the side of unity).[35] So, the afore-mentioned tension can be expressed in terms of these two readings of, or further elaborations from, the Kantian programme: one that gives more weight to the idea that pure intuition, as given by being the form of any possible intuition, must come equipped with a fixed pre-conceptual structure, and another that stresses that, according to the sketched notion of objectivity or synthesis, every determination must always be conceptual. This tension is substantive and consubstantial to the Kantian characterization of empirical validity. And, I think it is fair to say, it is not done full justice either by Kant or by most of his commentators. Because if one takes the idea that determination is always conceptual seriously,[36] then one should conclude that intuition, by itself, without the concept, is never, in fact, fully determined and, therefore, that mathematics cannot be fully given previous to the intervention of the concept. This means that any claim to the effect that empirical intuition has such and such form should be taken as provisional and dependent on the viability of certain conceptual constructions. So, in the empirical context, one should think that whatever is a priori for any content and, in this sense, valid for any empirical content, must be such that it is conjointly given for intuition and concept, and compatible with the revisability of any empirical content.[37] But, then, how should we interpret what goes on for mathematical knowledge? Mathematics for Kant is intuitive but, in a reading in which the form of intuition is always given as the other side of the form of concepts in the determination of the object of knowledge, mathematical knowledge is just the acquaintance with these formal rules that constitute pure intuition. Not in vain part of Kant's reasoning in KrV is dedicated to showing that the different modes of time (pure intuition) correspond to the different classes of the categories (pure concepts). This can be easily stretched to the claim that whatever fixes the rules of construction for the mathematical objects, do also for the objects of (physical) experience.

A slightly different way of trying to present this dichotomy, as it is assimilated by Kant's heirs, is through the question about the origin of the a priori elements in the determination: as being, let us say, more intuitive or more conceptual. This way of presenting things can be, again, misleading in the sense that it does not take into account the fact that the empirical determination always requires conceptual rules to be effected on an intuitive background: the rule comes from the conceptual pole (the unity) but is always effected on the intuitive pole (the plurality). What determines which rule to effect, which concept to apply? Here is where a difference between the two contexts, when one takes intuition and concept as the product of two independent faculties, as logical positivists interpreted

[35] Here again, the wording is not too rigorous. For Kant intuition and concept are the two sides of the same coin—the synthesis—which is where the determination of content occurs. There is no determination of either intuition or concept separately. What I just want to stress here is that, according to this scheme, there is no determination without concept.

[36] This must be understood as saying that determination of the object of knowledge is always done through the concept that, in Kant's scheme, represents the unity side of the determination effected in the synthesis.

[37] I insist in stressing that this is not Kant any more. This is a further elaboration on the framework provided by Kant.

Kant, can be drawn: in the empirical context, the rule dictated is given by the compatibility of every eventual different empirical content. Meanwhile, in the mathematical context, it should be determined, as I said above, once and for all and, in a sense, in an arbitrary manner: as there is only, according to Kant, pure intuition in this context to construct on, if one rejects intuition as a possible source of mathematical knowledge, as logical positivists do, it must be determined just by concepts. The characteristic of the mathematical context, in the abstract, is that it seems to be dominated by the conceptual side and that this is linked to its conventional character. In other words, in the mathematical context the determination seems both conceptual and conventional. In experience, on the other hand, the weight of the determination is located in intuition (which in a naive empiricist fashion can be replaced by perception or something of the sort) and the conventional element is displaced to the relation between concept and experience. This last condensed presentation sketches a picture of the Kantian framework that the logical positivists inherit from the discussions in the 19th century, and that announces some of the consequences that renouncing to intuition as part of the understanding of mathematical knowledge has.

How is this tension reflected in the different proposals? What, precisely, the Kantian resolution of this tension, intrinsic to his perspective, consists of is controversial, a matter of continuing interpretive discussion and, perhaps, not essential for the purposes of this paper. What directly concerns me here is how the logical positivists understood the Kantian response, how they adapt its valuable elements and how they react to what they consider to be defective in the proposal.

7 Constitution by Means of Coordination

Without any doubt, there are controversial claims in the previous telegraphic presentation of Kant's notion of objectivity and its reception, but this is not the place to navigate further into interpretive waters. Nevertheless, I would like to address the impact of this reading on some of the issues that are central to our current discussion. Firstly, one of the things on which all the logical positivists seem to have agreed is their rejection of anything that has to do with intuition for the account of mathematical knowledge and, together with this—although perhaps more shyly and not at the same pace—their deprecation of the synthetic a priori.[38] In many passages in Kant, without a doubt, intuition has a psychological flavour. In this sense, the authors writing after the inception of relativity were right to reject intuition as a source of mathematical knowledge. Nonetheless, as discussed above, intuition can also be interpreted as playing the epistemic role of being an indispensable part of understanding what the reference of concepts to objects consists of, in mathematics as well as in physics. And it is not clear that the standard logical positivist account of the definition of mathematical concepts, their implicit determination by means of axioms, leaves out intuition in its more general non-psychological sense. Hilbert's axioms of geometry and the consequent definition of concepts produced by them is not necessarily opposed to the idea of operations on a manifold that has the formal character of intuition. And although this is no argument for the relevance of intuition to geometrical knowledge, it is significant that neither Hilbert nor Poincaré thought that the formulation of theories as systems of axioms

[38] A paramount example where this aspect of logical positivism is particularly stressed is Coffa (1991).

meant that some form of intuition in the characterization of mathematical knowledge was irrelevant.

According to this way of reading Kant's programme, the reference to intuition precisely furnishes one mode of conceiving the definition of concepts as opposed to what goes on in the case of what Kant names the *mere concept* and which would play a role analogous to Schlick's explicit definitions.[39] So, from this point of view, I claim that intuition can at least be regarded as an essential component in the characterization of what the definition of concepts mean that, in principle, can be related to Schlick's and Reichenbach's instrument of the implicit definitions of concepts. Beyond the more or less fortunate connection with a human psychological faculty, that is of course justified by the history of the notion, there are reasons that support referring to intuition in the definitions of concepts through axioms. One might say that these definitions allude to certain operations constrained by the axioms, something which has little to do with what goes on in the explicit scholastic definition of concepts by means of other concepts that express genre and specific difference. The material, one might continue, on which such operations work is one of the senses that the traditional notion of intuition has. Another way of pointing to this is by noting that systems of axioms implicitly define concepts insofar as they are realized in certain models; the idea that a system of axioms is just a game with symbols is a fiction that is of little use when it comes to understanding mathematical knowledge. Systems of axioms, as they are given in geometry, determine manifolds which, in a generalized way, correspond to the formal notion of intuition. The connection between this notion and something rooted in human psychology is suggested, for instance, in Helmholtz' and Poincaré's linking of geometry to the operations of free mobility.[40]

So, my claim at this point is that, despite the explicit rejection of the Kantian recourse to intuition in mathematical knowledge by the logical positivists, and also of the synthetic a priori character of the constitutive principles of physics, there are essential aspects of their proposals that can be regarded as reflecting these neglected elements. This is particularly clear with respect to Schlick's and Reichenbach's talk of the uniqueness of coordination as the means of defining objectivity in empirical theories (or reference to real objects), and which they claim would go beyond the mere consistency that marks the validity of mathematical theories. It is important to bear in mind that in the original Kantian programme (and as it is also explicitly alluded to in some passages of Reichenbach's 1920) the claim of uniqueness or simply determination is understood as an essential requisite for the constitution of the physical object. In the Kantian context this is certainly so, as I stressed above, insofar as the necessary ingredient of knowledge provided by the conceptual side is understood as a rule of construction. In other words, for this generalized Kantian framework, why concepts (together with intuition) are necessary components for valid experience is because only through concepts can the manifold of intuition be regarded as objective (referring to something that is beyond the immediate content itself). This means, if we translate it to the language of coordination, that only through the constitution of the

[39] I am not claiming that the way in which Schlick conceives of the explicit definitions is the same as Kant's notion of analytical judgement. Arguably, Schlick's perspective is different in many aspects, principally due to the pragmatic dimension that is always involved in specific definitions. What I do want to claim though, is that there are common structural features in both views and that such features are essential to understanding their difference from the conceptual determination involved in implicit definitions.

[40] The idea of somehow rehabilitating some notion of intuition, of Kantian inspiration, to replace the, partly, logical positivists idea that mathematical knowledge, as given by implicit definitions, is merely conceptual is not new. An example of it is given by Hintikka (1974).

object (given by this construction of the meaning of concepts through principles that can be regarded as axioms) is the coordination, understood as correlation between concepts (mathematical ones, mainly) and objects, completed.[41]

But this is where Schlick and Reichenbach parted ways in 1920. The account I have sketched above seems consistent with Reichenbach's notion of coordination (in 1920) but not with Schlick's interpretation. Recall that Schlick talks of coordination to previously constituted objects and dismisses Reichenbach's talk to the effect that one of the sides of the coordination is constituted through the selfsame coordination which, as we have just seen, is essential for the Kantian notion of objectivity. But without such a resource, I claim, the idea of uniqueness of coordination loses its substantive role and is introduced as just something already presupposed; and this is complemented with a limited way of understanding Schlick's notion of convention that reduces to Duhemian holism (see Sect. 5). In contrast, Reichenbach's original idea, inspired by Kant's scheme even if critical of it, is effective insofar as a certain coordination constitutes the object which is always subjected to eventual future empirical correction. Coordination and constitutivity are then, in Reichenbach's original proposal, two sides of the same operation that results in objectivity in the physical sciences. This fully justifies Reichenbach's talk of coordinating principles as constitutive principles.

A remarkable consequence of this understanding of coordination in physical science (or cognitive coordination as Reichenbach sometimes calls it) is that it diminishes the gap between mathematical and physical knowledge. Kant's stand in this respect can be summarized as follows: as much in mathematics as in physics, what makes knowledge possible is the existence of synthetic a priori judgements which, as discussed above, are so by the intervention of intuition. The difference between the two contexts is attributed to the different origin of the intuition side: sensible in the physical case, pure in mathematics. Again, I intend to leave out the psychological connotations; without them, what Kant is saying is that, in both contexts, the mathematical and the physical, the same procedure of conceptual determination takes place: construction on the empirical or pure manifold of intuition. This makes mathematical and physical knowledge of the same kind and opens the door to explaining the applicability of mathematical concepts in experience.

Knowing of Reichenbach's rejection of the role of intuition, is this a genuine option when it comes to interpreting his notion of coordination? From an abstract interpretive point of view, it would seem so. Coordination as constitution works as much in the physical realm as in the mathematical. What is constituted in the mathematical context is mathematical objectivity (not empirical objectivity) but, one might say, the process is the same in both contexts. The difference now is that, in pure mathematics the uniqueness of the coordination is regarded as being somehow prescribed top-down (from the conceptual to the intuitive side): not from the eventually inexhaustible source of empirical contents, but from the given axioms, which can, in principle, be arbitrarily chosen so long as they are consistent. In this sense, one might say, coordination can be reduced to the consistency of the axioms even though it can, nonetheless, be seen as given by construction on intuition.

[41] The idea of something like uniqueness being at the core of the Kantian programme can be found in, and therefore read as part of, some neo-Kantian interpretations of Kant. As an example of this, see this fragment of a letter from Cassirer to Schlick quoted in Stump (2015).

I would like to consider as a priori valid in a rigorous sense only the idea of 'unity of nature', that is the lawfulness of experience in general, or put in a brief formulation: the uniqueness (Eindeutigkeit) of coordination.

This makes the connection with convention more transparent in the mathematical case and it is linked to the problem of the arbitrariness of coordination that I commented on above. With respect to the historical adequacy of interpreting Reichenbach's coordination in this fashion, things are not so clear. One possibility would be to understand that Reichenbach's explicit rejection of intuition might just affect the psychological dimension of the term. On the other hand, one might more controversially argue that what Reichenbach ended up doing in his axiomatization of relativity cannot be understood without bringing some form of intuition into the picture.

8 Reichenbach's Project of Axiomatizing Relativity

In 1924, Reichenbach published *The axiomatization of the theory of relativity* (Reichenbach 1924). In that work and in the context of physical theories, Reichenbach distinguishes between definitions and axioms. Definitions, he says, have a different nature in physical and mathematical theories: while mathematical definitions just clarify the meaning of a concept by means of other concepts, physical definitions "coordinate a mathematical definition to a 'piece of reality'" (Reichenbach 1924, 8). They are also called coordinative or real definitions by Reichenbach. Since it is true that definitions are 'arbitrary forms of thought, capable of neither empirical confirmation or refutation',[42] in contrast to axioms which, according to Reichenbach 'are empirical assertions capable of verification', through the notion of a *coordinative* definition Reichenbach understands a kind of explicit definition or concrete definition in which a (previously defined) concept is directly connected to some physical object. Nonetheless, he stresses that although definitions have the form of free stipulations, their arbitrariness is constrained: they must fulfil the requirement of uniqueness of coordination, but 'whether they fulfil these demands is not solely a matter of form, but depends upon the validity of the axioms'. Alternatively, as he says in *The Philosophy of Space and Time* (Reichenbach 1928, 14):

> "Since the concepts are interconnected by testable relations, the coordination may be verified as true or false, if the requirement of uniqueness is added, i.e., the rule that the same concept must always denote the same object."

In other words, one can think of definitions as links from concepts (themselves defined by axioms) to physical objects. But eventually, what the object being linked *is* depends on what the constrains established by the axioms are. As an example of a coordinative definition, Reichenbach mentions the definition of a unit of length or of simultaneity by means of light in Special Relativity. In these examples one can see, Reichenbach says, "the duality of coordinative definitions" (Reichenbach 1928, 15). Physical concepts are defined by means of other concepts through axioms, but also by direct reference to physical things. This is the key to understanding the notion of coordinative definitions.

Coordinative definitions are, therefore, distinguished from mathematical conceptual definitions: while mathematical concepts are defined only in terms of other concepts, physical concepts also require the coordination of a mathematical definition to a 'piece of reality'. In spite of this difference, in both types of definition there is a conventional element. In the case of the coordinative definitions this is, again, constrained by the uniqueness of

[42] Reichenbach (1921).

coordination. Reichenbach, alluding to Poincaré and Einstein, links the notions of coordination and convention in physical theories (Reichenbach 1928, 14):

> "The view that every spatial and temporal metric presupposes coordinative definitions has been generally accepted and is known as conventionalism".

What concept of convention is at work here? As Reichenbach coins the notion of coordinative definitions after the letter exchange with Schlick and his subsequent conventionalist shift, one might think that the notion of convention involved here is one that negates the constitutive character of coordination. But is this what we should infer from the structure of the axiomatization that he is proposing? This arguably does not seem to be the case. What we find in the previously quoted precisions and in Reichenbach's realization of the axiomatization of relativity by means of the production of his 'light' and 'matter axioms', is something similar to Poincaré's and Schlick's original notion of geometry and convention[43] (with the light principle, encoded in the light axioms, playing the same role for chronongeometry as the principle of free mobility plays for spatial geometry).

This claim is sustained by the notion of coordination that is operative in Reichenbach's axiomatization. Coordination, even in this definitional fashion, does not seem to be established with respect to a previously constituted object. This is apparent, for instance, when Reichenbach claims that "the physical thing that is coordinated is not an immediate perceptual experience but must be constructed from such an experience by mean of an interpretation" (Reichenbach 1924, 8). In the coordination, there is an original designation of a given empirical object as that to be coordinated to a mathematical concept, but the actual coordinated objects are only constituted through the axioms. Reichenbach first constructs a light geometry starting with the characterization of idealized physical processes that he calls 'signals'. The behaviour of such signals is prescribed through the formulation of different axioms. All this belongs to the context of implicit mathematical conceptual definitions. Through a number of coordinative definitions, Reichenbach makes contact with the phenomenon of light and produces the concept of simultaneity. After this, through the definition of an inertial system, he is able to derive the Lorentz transformations. Nevertheless, for the task to be completed, in the framework of SR, the theory must state that the light geometry constructed by the light axioms is surveyed by material rods and clocks; this is ensured by the matter axioms.

The preceding quick overview of Reichenbach's axiomatization then suggests what seems to be a perfect realization of Poincaré's notion of convention and of the idea of constitution through coordination. Consequently, it is also exemplary of the constitutive role of coordination principles which, officially, Reichenbach abandons with the apparent incorporation of Schlick's notion of convention. A key aspect of this triad formed by the notions of coordination, constitution and convention is the aforementioned dual role of coordinative definitions that, due to the previously undefined status of the coordinated object and their dependence on the validity of the axioms, do in fact play a constitutive role. A coordinative definition is not just a rigid designation, but neither it is simply an implicit definition given by axioms; this is the tension that underlies the determination of empirical concepts coordinated in this way. Dependence on the axioms is what provides the connection with the Hemholtz–Lie theorem and permits us to establish a link between coordination and invariances, which produces the spatio-temporal symmetries of the theory. Coordination with

[43] See my previous discussion in Sects. 3 and 4.

the real empirical processes reveals itself in the approximate character of the theories, providing their empirical connection and leaving open the possibility of their eventual rejection. The structures produced through such coordination are not only constitutive but also a priori for empirical contents, insofar as our instrumental capacities are within the range of approximation that is presupposed in the actual coordination.[44] So, it does not seem plausible to think that at this stage Reichenbach has, de facto, abandoned a priori (relativized) constitutive principles.

Without intending to claim that this was Reichenbach's intention, which would require a more profound textual analysis, one can detect a tension in the *Axiomatization* that leads to the following claim: the structure of the axiomatization is compatible with an understanding of coordination that has not abandoned completely its constitutive role. This would be in line with a more complex understanding of the notion of convention on behalf of Reichenbach: from the plane notion that he seems to attribute to Schlick in the correspondence, and that collapses to wholistic underdetermination, to an idea in which convention and coordination combine to define the physical object. I belief that here are the seeds for a proper realization of the relativized a priori programme, something that will have to be developed elsewhere.[45]

Furthermore, as suggested in previous sections, this exposition has the virtue of somehow unifying the accounts of objectivity in mathematics and physics, even though in the former there is no proper coordination. How is this so? As stressed above, it is due to the fact that the constitutive roles played by axioms is analogous in both contexts: they consist of constructions of concepts on a manifold that, generally, one might call intuition. In fact, they can be seen as the production of a certain determinate form of intuition.

In pure geometry—that is, mathematics considered independently from its eventual application to experience—such determination is not, when considered in this way, constrained by anything other than the mutual consistency of axioms. But due to the fact that the constructive procedure is the same in the physical context, through coordination, the mathematical structures can be applied to experience. In reality, this is the way to consider things if one assumes that the two sides of intuition and concept are initially separated, but not the most illuminating one. Complementarily, and more coherently, one can start from the synthesis given in experience, derive that these two sides are involved in the notion of empirical objectivity, and conceive of pure mathematics as an abstract description of the rules of the intuitive part, when leaving aside the empirical connections of its principles

[44] This is a mark of the central role that the notion of approximation plays in Reichenbach's account of coordination and convention. I concur with an anonymous reviewer in the impression that the present paper does not fully address this issue. Let me try here to, at least partially, mitigate this defect. A first indication of the importance of the notion of approximation is found, as mentioned above, in the place that the principle of general induction plays in *The Theory of Relativity and a priori Knowledge*. But this is later translated in the role that approximations play once conventions, as coordinative definitions, are incorporated by Reichenbach. Much more should be said about this, specially for the project of rehabilitating a coordination scheme for some contemporary discussions. (Padovani 2011) and (Reichenbach 1928) offer a very thorough account of the role of the notion of approximation for the characterization of coordination and stress how the notion of probability is essential to understand Reichenbach's account of the application of mathematics to physics encoded differently at different stages by the notion of coordination.

[45] Padovani (2011) also questions the aspect of the received view that regards the conventional shift as implying the abandonment of any sense of constitution. In Padovani's view, one can distinguish different layers of constitutivity and the basic one, expressed by the principle of probability and the principle of genidentity, does still work as such in the *Axiomatic*. On the other hand, Padovani (2021) also stresses the limitations of reading Reichenbach as a conventionalist.

(such as free mobility or the light principles). This presentation can then be taken as an indication of the plausibility of a post-Kantian characterization of the notion of intuition that is operative in mathematical and physical knowledge. I leave a full development of this for a different occasion.

9 Conclusions

Reichenbach's use of the notion of coordination oscillates between two poles: the, to a certain extent, Schlickean extreme in which coordination is conceived as a conventional application of a merely formal mathematical axiomatic system to physical phenomena, and a more Kantian one in which principles of coordination are regarded as the expression of the constitution of physical objects. We can associate different ways of understanding the nature of mathematical theories and different notions of convention with these two interpretive extremes. All this makes Reichenbach's notion of coordination, and its relations to the notions of constitution and convention, very rich but difficult to grasp. My impression is that much weight has been given to Reichenbach's conventionalist shift, presupposing a certain notion of convention, and perhaps not enough attention to some aspects of these notions that can open up fruitful interpretive avenues. My aim in this paper has been to stress the constitutive dimension of the notion of coordination and its links with some Kantian themes in order to provide a better understanding of the mutual relation between such concepts and, with it, of the nature of Reichenbach's adoption of conventionalist language.

It seems uncontroversial to claim that Schlick and Reichenbach, at the start of the 1920s, were not dealing with exactly the same notion of coordination. While Schlick saw examples of coordination to be between formal systems and pre-defined objects, Reichenbach originally stressed the undetermined character of the physical objects coordinated with mathematical concepts. This obviously has an effect on their different attitudes towards convention and constitution: only the latter conception of coordination seems compatible with a substantive notion of the constitution of objects. Less obvious, though, is how ideas relative to the definition of mathematical concepts interact with the notion of coordination. In principle, the working perspective on the determination of the referents of mathematical concepts is that provided by the idea, inspired by Hilbert's axiomatization of geometry, of implicit definitions. Based on that, the logical positivists founded their rejection of the intuitive character of mathematical knowledge and of any significance of synthetic a priori judgements. But such an interpretation is not innocent; stressing the merely conceptual determination of mathematical objects drives us towards an understanding of the systems of axioms as mere games with symbols and, in this sense, undoubtedly analytical. This is coherent with Schlick's conception: coordination is not constitutive and just indicates the relation between uninterpreted systems of axioms and physical experience. Nonetheless, as I have noted herein, Schlick's standpoint presupposes that the concepts defined in this way can be coordinated to experience, crudely put, that experience fits the axiomatically determined mathematical structures. The notion of convention completes the picture by finishing the determination of the coordination. This is what motivates Schlick's reaction to Reichenbach's 'constitutive principles': they are conventional definitions.

The situation is more nuanced and more difficult for Reichenbach. If he accepts, as it seems that he does, the standard conception of mathematical systems of axioms, with the consequent rejection of intuition and the synthetic a priori, then the idea of constitution as coordination suffers. This is the driving force that would lead him to accept a conventionalist stance. Nonetheless, as I have tried to show, Reichenbach can be read as not having completely abandoned his original reading of coordination, which involves pushing the notions of convention and conceptual determination into line, in the sense of making them compatible with the idea of the constitution of objects.

This has important effects on the interpretation of Reichenbach's position and the evaluation of the viability of his constitutive principles as a version of the relativized a priori. In relation to Reichenbach's position, I have defended the notion that it is important to make the connection of his idea of constitution by coordination with the Kantian notion of objectivity clearer. This involves revising the usual evaluation of the debates about the role of intuition in mathematical and empirical knowledge. I have undertaken part of this work in Sects. 6 and 7.

A second aspect to consider has to do with the differences between Schlick and Reichenbach in relation to conventionalism. Friedman is very convincing when he shows that what, perhaps controversially, would be, according to him, Schlick's rendition of Poincaréan conventionalism is inaccurate. I have argued that Reichenbach's interpretation of conventionalism is closer to the original, and this makes it more compatible with the idea of constitution. From this point of view, Reichenbach's conventionalist shift would be either less substantive or more incoherent than Friedman thinks.

These slight reconsiderations of Reichenbach's position have a major impact on the evaluation of his original project for revitalizing Kant's a priori by means of constitutive principles. Irrespective of what Reichenbach ended up thinking of his project, the voices that declare it inadequate might be too quick in inferring the inviability of the notion of constitution through coordination from the difficulties and obscurity of the original formulation. Its connection with the Helmholtz–Lie theorem through the properly understood notion of convention makes it conceivable that we could work out the details of how the constitution of objects works.

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