Journal for General Philosophy of Science (2024) 55:59–78
https://doi.org/10.1007/s10838-023-09653-6

ARTICLE

Kant's Crucial Contribution to Euler Diagrams

Jens Lemanski[1,2]

Accepted: 21 April 2023 / Published online: 17 August 2023

Abstract
Logic diagrams have been increasingly studied and applied for a few decades, not only in logic, but also in many other fields of science. The history of logic diagrams is an important subject, as many current systems and applications of logic diagrams are based on historical predecessors. While traditional histories of logic diagrams cite pioneers such as Leibniz, Euler, Venn, and Peirce, it is not widely known that Kant and the early Kantians in Germany and England played a crucial role in popularising Euler(-type) diagrams. In this paper, the role of the Kantians in the late eighteenth and early nineteenth centuries will be analysed in more detail. It shows that diagrams (or intuition in general) were a highly contentious topic that depend on the philosophical attitude and went beyond logic to touch on issues of physics, metaphysics, linguistics and, above all, mathematics.

Keywords Visual representation and reasoning · Logic diagrams · History of science · Metaphysics · Kant · Euler

1 Introduction

It is a commonly held belief that the history of logic is characterised by misunderstandings and subsequent rediscoveries (Peckhaus 1997). This is also the case when it comes to the so-called 'golden age of logic diagrams', which has been studied by many authors in recent years (Moktefi and Shin 2012; Englebretsen 2019; Bhattacharjee et al. 2022). These studies have corrected several factual errors and misunderstandings that were uncritically propagated in many accounts of the history of logic in the 20th century. For example, the purely syntactic difference between Euler and Venn diagrams was often not taken into account. Furthermore, the misinformation that the diagrams named after Euler were also first invented by him or that they were known in advance only to Leibniz. Moreover, it was pointed out that it was not only British logicians who paved the way from Euler to Venn.

✉ Jens Lemanski
jenslemanski@uni-muenster.de

1 Philosophisches Seminar, Westfälische Wilhelms-Universität, Münzstr. 10, 48143 Münster, NRW, Germany

2 Arbeitsstelle Kulturwissenschaftliche Grundlagen, KSW, FernUniversität in Hagen, Universitätsstr. 33, 58097 Hagen, NRW, Germany

However, the history of eighteenth- and nineteenth-centuries logic diagrams has often been reduced to a few prominent names.

If one takes a closer look at the familiar names in this historiography of logic diagrams, three things stand out in particular: (1) Leibniz and his logic diagrams are given special prominence; (2) Euler and his diagrams were particularly relevant because, since they are named after him, he also popularised them; (3) Kant and Kantians are mentioned almost nowhere and therefore play no role in this history. At first glance, these three points may not seem unusual. After all, in many of the major 20th century histories of logic, Leibniz is highly regarded for his contributions to modern logic, while Kant is often dismissed, and Euler is primarily recognised for having paved the way for John Venn and his successors (Lu-Adler 2018, 1ff.).

In this paper, I would like to clear up these three often mentioned prejudices and correct them as follows: (1) In his published writings, Leibniz and then especially his successors in the eighteenth century argued strongly against diagrams. (2) Because of this discrediting of diagrams by the Leibnizians, Euler's diagrams were almost forgotten, and (3) it was Kant and the early Kantians who made Euler diagrams popular. If we can summarise by saying that without Kant, research into Euler diagrams would have been delayed even further in the 19th century, the question arises as to why there are such different views on the diagrams. At the end of the paper, I will argue that the evaluation of diagrams depends on the philosophical school to which one belongs.

To support this main thesis, I will proceed in several steps: In Sect. 2, I will give a brief overview of the research on Euler-type diagrams before Euler and some indications will be given of how strongly Leibnizians argued against diagrams. Section 3 will present the history of Euler's diagrams, with a special focus on the problems that the reception of Euler's logic had in the late eighteenth century. In Sect. 4, I will then present the main thesis, namely that Kant and his followers popularised Euler's logic diagrams in the first place. In support of this thesis, I will sketch two histories of development in Sect. 5, namely the reception of Kant's Euler diagrams in English and German-speaking countries until 1820. In the last section I will try to give some general conclusions about the importance of this discussion for the philosophy of science. Beyond these three corrections of history, an answer should also be given to the question why the respective philosophers and logicians had and have such different views on logic diagrams.

In my presentation, I will adhere to two principles. Firstly, I will refrain from making any value judgements regarding the various logic diagrams. Secondly, I will limit my sources to those that had a direct impact on the prevailing opinions of the time. For instance, Leibniz's manuscripts on logic diagrams will not be discussed in Sects. 2, 3, 4 and 5, as they were not known during the eighteenth and nineteenth centuries and only began to influence the history of logic in the 20th century.

2 Euler-Type Diagrams

In many short accounts of the history of logic diagrams, it is common to find references that attribute the origin of so-called 'Euler diagrams' to Euler himself. Furthermore, studies on diagrams before and after Euler often present a selection of great names intended to emphasise the significance of these diagrams, e.g. Leibniz, Euler, Venn, Peirce. Nowadays, however, informed researchers know that there were already Euler diagrams before Euler (and Leibniz) and that not all diagrams between Euler and Venn were of one type.

Diagrams before and after Euler, which do not correspond exactly to Euler but nevertheless have a certain similarity, are called 'Euler-type diagrams' or 'Eulerian diagrams' (Moktefi and Shin 2012). We call the original diagrams, which can also be found in Euler, 'Euler diagrams' or 'Euler's diagrams'.

Thanks to intensive research, however, we now know that there were already diagrams in the Middle Ages for which there were good reasons to classify them as Euler-type diagrams (Hodges 2023) and that in the early modern period (i.e. here: since the printing press) there were at least eleven authors before Euler who used Eulerian diagrams, even if not always in as differentiated a manner as Euler. According to Lemanski (2021, Ch. 2.2), these eleven authors of the early modern period can also be divided into two periods: The *first period* includes Juan Luis Vives, Nicolaus Reimarus Ursus, Bartholomäus Keckermann and Johann Heinrich Alsted. This period begins around 1515 and ends around 1615. The *second period* is dominated by two schools, the Weigel and the Weise schools: the Weigel school includes Erhard Weigel as the founder of the school and his pupils Johann Christoph Sturm and Gottfried Wilhelm Leibniz. Christian Weise and his pupils Samuel Großer and Johann Christian Lange belong to the Weise School. The second period begins around 1650 and ends around 1715. This determines the periods of Euler's diagrams before Euler. Several generations after the end of the second period, i.e. in the 1760s, a *third period* then sets in, which brings forth the original Euler diagrams and initially includes the authors Gottfried Ploucquet, Johann Heinrich Lambert and—of course—Leonhard Euler.

Of all the authors mentioned, only the diagrams by Leibniz and Weise were not known to the contemporary public. One can assume that they were known only to pupils or perhaps to acquaintances and certain pen pals. Until today, many logic diagrams are only in archives and have not yet been published, e.g. Euler's diagrams from the 1730s and 40s (Kobzar 2005).

It is noteworthy that almost all authors using Euler-type diagrams after the middle of the sixteenth century were Protestants who taught in north-central Europe. Most were critics of scholastic logic and the prevailing Aristotelian interpretation of logic. Moreover, all authors saw in the Euler- or Euler-type diagrams they used an application of geometry to logic: one group used diagrams for didactic reasons since the geometric shapes revealed a simple decision procedure for testing the validity of inferences (e.g. Vives, Weise). The second group intended to develop a logical calculus and saw in the geometric form an alternative to algebraic notation (e.g. Leibniz, Lange). A third group preferred geometric figures in logic, as they hoped for a stronger representation of real object relations in logic (e.g. Reimers, Weigel) (Lemanski 2021).

Each of the three periods is characterised by two features. Firstly, in each period the authors know at most the diagram users from their own period, but not possible previous diagram users. Secondly, the first diagram users in each period declare to have invented Euler-type diagrams, since they no longer know the diagrams of their predecessors.

The fact that early modern users of diagrams did not know the diagrams of their predecessors is a subject that has been speculated about in various researches and with regard to different periods. One plausible thesis as to why diagrams are forgotten and then reinvented comes not from research on the early modern period, but from research on the transitional period between the early modern and modern eras: in physics and mathematics, geometric figures, diagrams and pictures were stigmatised for a long time because they were said to tend to be misunderstood and prone to error. This period around 1880 is referred to in the philosophy of science and mathematics as the 'crisis in intuition' (Hahn 1980; Volkert 1986). Whether such a crisis can also be generally applied to phases of the eighteenth century, however, will not be discussed further here.

However, it can be stated in purely factual terms that for about 60 years after the Thirty Years' War there were very many texts on Euler-type diagrams in Central Europe, but then for almost 50 years none were published at all. In detail, we do not find any published Euler-type diagram in the period between 1714 and 1758, but we do find a clear prevailing opinion about the significance of geometric figures: Leibniz had indeed made diagrams himself and worked with them, but they remained unknown until Couturat's edition in 1903 (Moktefi and Shin 2012). (The only exception seems to be the 1820 edition (Leibniz 1820) and some fragments in the 1890 edition of Gerhardt.) In Leibniz's writings, on the other hand, one finds many references to mathematics being non-intuitive, based only on the laws of thought and diagrams being misleading: "Geometers do not derive their proofs from diagrams" is one of the more famous quotes of Leibniz and many similar ones have been compiled in Hogan (2020). Instead of using figures, Leibniz based the entirety of mathematics on the innate laws of thought. "The great Foundation of Mathematics is the principle of Contradiction, or Identity," he wrote in his second letter to Clarke (Rodriguez-Pereyra 2018).

Wolff confirmed this view and the adherents of this rationalist and logicist position prevailed among the Protestants in Central Europe in the eighteenth century. Individual references to Leibniz's diagrammatic interest were not received or even dismissed as abstruse. Thus, from Leibniz's death onwards, a rationalist direction developed in Central Europe that was sceptical of any use of figures and diagrams in logic.

An example of this hostility to diagrams on the part of the Leibnizians can be seen in the debate between Daniel Friedrich Hoheisel and Andreas Rüdiger on the one hand and Christian Albrecht Körber on the other in the 1730s. As a follower of Locke, Rüdiger advocated in Central Europe the peripatetic principle that *nihil est in intellectu, quod non antea fuerit in sensibus*. Hoheisel, a follower of Rüdiger, criticised Wolffianism for underestimating the role of the senses and intuition in finding proofs in geometry. Already the subtitle of Körber's rebuttal argued that one could not prove 'solo oculorum usu', as Rüdiger and Hoheisel state (Körber 1731). Körber defended the then-dominant Leibniz-Wolff view that one only had to use the laws of thought in mathematics and the a priori definitions, axiomata, theoremata, corollaria and problemata as a basis. In the debate on intuition in the 1720s, Körber also had numerous comrades-in-arms, such as Volckmar Conrad Poppo, Jacob Wilhelm Michael Wasser or Friedrich Philipp Schlosser.

This example, which will be taken up several times later, may be one reason why no diagrams can be found between the years 1714 and 1758. As logic diagrams were understood as applications of geometry in logic and figures in geometry became suspect, this skepticism was transferred from mathematics to logic. To this day, the fact that Leibniz, who left so many diagrams of logic in his manuscripts, was responsible for the hostility to diagrams in the eighteenth century seems absurd to many researchers (if they even take into account that in the eighteenth century there was a very different image of Leibniz than today). As we shall see later, however, this paradoxical constellation is repeated to some extent in Hegelianism.

3 Euler Diagrams

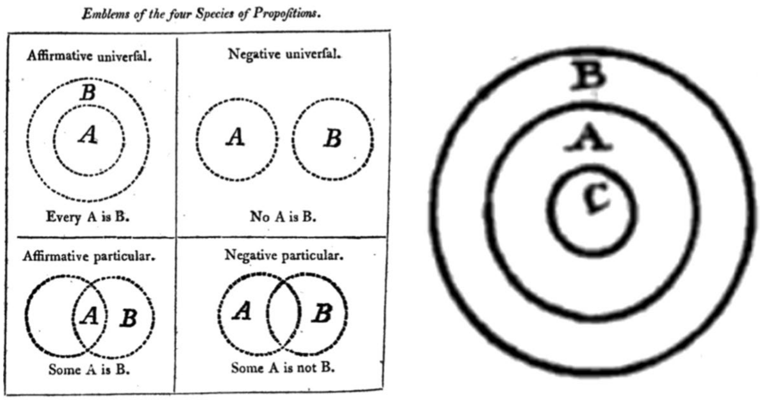
Euler's attention had already been drawn in his youth when in 1722 he was a respondent in a debate on whether or not a logical calculus or at least a simple logical decision procedure could be developed and whether this should be of algebraic or geometric form (Fellmann and Mikhajlov 1998, 14). Euler seemed to have quickly developed a preference for using geometric figures in logic. However, since such procedures were not tolerated in Central Europe by Leibnizians and Wolffians, he conceived his approach of Euler diagrams in the late 1730s and -40s in St. Petersburg (Kobzar 2005). During this time, Euler also wrote several treatises that strongly polemicised against Leibnizianism and Wolffianism. The most famous of these is *Gedancken von den Elementen der Cörper* (Thoughts on the Elements of Bodies), published anonymously in 1746, in which Euler harshly criticises the nonsensical nature of the monad theory and calls for the empiricisation of the natural sciences (Calinger 2016, Ch. 8.2). Among Leibnizians, the author soon became known and punished with criticism or contempt. Jean Henri Samuel Formey, Johann Christoph Gottsched, the aforementioned Körber and even Wolff strove partly to unite internally, partly to publicly discredit Euler on questions of logic, natural philosophy and metaphysics.

If one looks at the polemics in Euler's writings alone, it should quickly become clear that Euler saw in Leibniz and Wolff the most bitter arch-enemies, whose theses he considered to be false, immoral and even atheistic. The climax of Euler's attack on Leibniz-Wolff metaphysics then became public knowledge in the academy dispute of the 1750s between Samuel König and Voltaire on the one hand and Maupertuis and Euler on the other (Knobloch 1992; Pulte 1989). The thesis of many historians of logic that Leibniz inspired Euler to elaborate his logic diagrams is completely abstruse in view of the fact that Leibniz's diagrams were not known at all in the eighteenth century and that Euler polemicised strongly against almost all core theses of the Leibnizian doctrinal edifice. The only evidence that many historians keep bringing up is that there are simply no logic diagrams in the literature between the death of Leibniz and Euler's *Letters to a German Princess* dated to the year 1762 (Ploucquet's diagram of 1759 disregarded). But, as argued in Sect. 2, there are other reasons for this.

Rather, it seems correct that Euler may have been inspired by Bernoulli and the ongoing debate in Basel about formalising logic by employing either algebra or geometry. It was also this question about the possibility of a logical calculus that led Gottfried Ploucquet to try out both geometrical and algebraic approaches at the end of the 1750s. Johann Heinrich Lambert, who published similar ideas independently but almost simultaneously, clashed with Ploucquet in the 1760s over who had first developed the logical calculus (using geometric forms). Both believed to have published a similar invention to the infinitesimal calculus in the early 1760s and they soon compared themselves to Newton and Leibniz.

When Euler finally published his logic diagrams in the *Letters* in 1768, it was clear that Ploucquet and Lambert had both failed. A logical calculus, Bök wrote, was an idea akin to the philosopher's stone, the squaring of the circle or perpetual motion: it was impossible (Lemanski 2021, 214ff.). To what extent Euler had taken note of this is unknown; Kant, however, repeats Bök's then-popular phrase (Klein 1785, 42) almost verbatim in the often misinterpreted introductory passage in the second edition of the *Critique of Pure Reason*: Logic had not gone a step forward or back since Aristotle because Leibniz, Lambert and Ploucquet had been unsuccessful in their efforts. Even the Leibnizians admitted that the idea of a calculus of logic, which they attributed to Hobbes, had failed.

Euler's treatise on logic was therefore more modest and subject to two ideas. The first was to test which conclusion can follow from two premises and whether an inference in which premises and conclusion are given is valid (Bernhard 2008). Furthermore, Euler's logic was closely interwoven with his ideas on philosophy. "All this knowledge is acquired, only in so far as the objects make an impression on some one of our senses" (Euler 1802, 311) as Euler translates the empiricist principle of *nihil est in intellectu*. Later, just before



(a) Species of Propositions (b) Modus Barbara

Fig. 1 Euler diagrams taken from (Euler 1802, I, p. 398) and (Euler 1803, I, plate I)

his treatise on logic begins, he modifies Locke's empiricism into a philosophy of language based on the senses and which understands the generality of language as an abstraction from the individual objects that are only given in intuition (Euler 1802, 377).

Euler's logic is the traditional syllogistic, which is built up compositionally from concepts, then judgements and finally inferences. A judgement consists of two different concepts, e.g. *A* and *B*, and both are represented by a circle. In the case of general affirmative judgements, one circle is completely contained in another (*aAB*); in the case of general negative judgements, both circles do not intersect at any point (*eAB*); in the case of particular judgements, two partially intersecting circles are drawn and, with the help of the position of the denoting letter, it is differentiated whether the intersection (affirmative: *iAB*) or the region outside the intersection is meant (negative: *oAB*). This results in four construction rules, which are illustrated in Fig. 1a: From left to right: top (*aAB*, *eAB*); bottom (*iAB*, *oAB*).

In order to prove which conclusion follows from which premises, one draws all possible geometric positions of the circles for the premises in a diagram and reads off whether the same positional relationship always results between the concept relations not mentioned in the premise. If this is the case, this judgement follows from the two premises. But if this is not the case, the conclusion does not follow from the premises. For example, 1b shows the premise *aAB* and the premise *aCA*. Now we see from the diagram 1b that according to the rules of construction for the two premises, the conclusion *aCB* necessarily follows, in short $aAB, aCA \vdash aCB$. This inference was called 'modus Barbara' in scholasticism.

Euler's treatise on logic was divided into two parts. In the first part, Euler wanted to find out which conclusion could be necessarily deduced at all, given two premises. We have shown this above as an example. In the second part, Euler used diagrams to find out whether an inference is valid or not. If only one diagram can be drawn with the help of the construction rules that is contrary or contradictory to the conclusion, then the inference is invalid. One has therefore called the first part the inference test and the second part the validity test (Bernhard 2008). The function of Euler's logic diagrams, however, was altogether more modest than the claim of Ploucquet or Lambert. Nevertheless, Euler's diagrams were published just at a time when the debate between Ploucquet and Lambert ended and when both logicians were considered to have failed in the race for a logic calculus in the prevailing public opinion.

At the time of the publication of his diagrams, Euler had a secure standing in St. Petersburg and did not need to fear reprisals. Nevertheless, it was Leibnizians who not only criticised but even censured his logic based on geometry. The Leibnizians certainly acknowledged Euler's mathematical achievements, but repeatedly remarked that he had no idea of logic and metaphysics. The result of this criticism by the prevailing rationalists of the time was purified editions of Euler's letters from the 1770s onwards: in new editions, they simply published the scientific treatises, but deleted the logic and metaphysics from the German translations without replacement.

The first German edition of the *Letters* was still published complete and without any comments by editors until 1773. However, reviews were quickly found pointing out that Euler's logic and metaphysics did not meet the standards of the academic philosophers (Hennings 1775, 634). In philosophy and logic textbooks, everything that did not belong to physics was hushed up from the 1770s onwards. Euler's second edition of the *Letters* in France, published in 1787, was already shortened. The second, then abridged and annotated German translation was provided by Friedrich Christian Kries, a student of Georg Christoph Lichtenberg. He explained that Euler's logic and metaphysics are "not of such general interest to the reader" and that one would have to comment too much to correct them (Euler 1792, IVf.). For this reason, they were omitted and the title changed to *Briefe über verschiedene Gegenstände aus der Naturlehre* (Letters on Various Objects from the Theory of Nature). Particularly among Leibnizians, the deletion of logic and metaphysics quickly met with approval: one reviewer judged that Euler had only written these letters on logic and metaphysics anyway "to contradict Leibniz and Wolff", but that he was "in a world that was completely foreign to him" (Anonymous 1793, 110). Another wrote that in the new edition "one had done very well to omit the letters concerning logic and metaphysics, and to restrict only to the theory of nature" (Mayer 1793, 558).

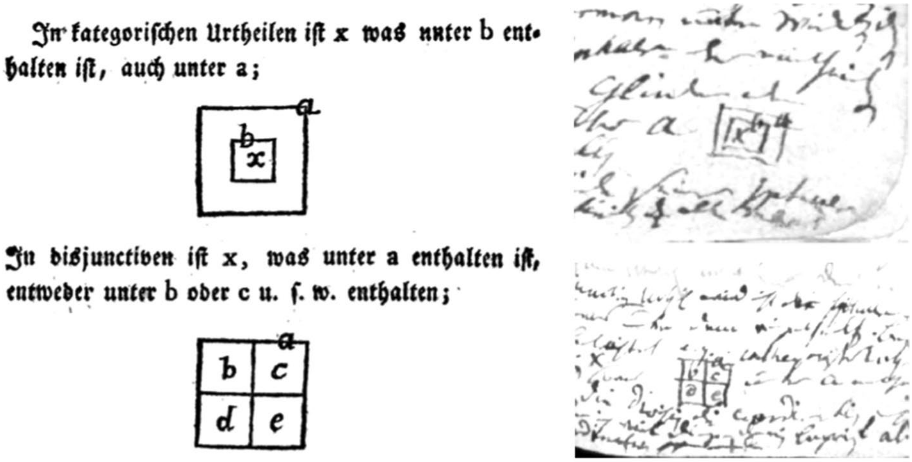
Euler's logic was thus ignored or forgotten from the 1770s onwards, just like the approaches of Lambert and Ploucquet.

4 Kant's Eulerian Diagrams

Given the situation described in Sect. 3, it is not surprising that Kant did not resort to Euler's logic for his courses and lectures, but used a textbook from the Leibniz-Wolff school as the basis of his logic teaching. After all, the Leibniz-Wolff logic was the standard at the time and alternative approaches were considered to have failed. As is well known, the script used by Kant for his logic lectures is Georg Friedrich Meier's *Auszüge aus der Vernunftlehre* (Extract from the Doctrine of Reason), which of course, like all Leibniz-Wolff logics, does not use logic diagrams but builds on the laws of thought.

Kant himself, however, knew Euler well. For example, he sent him his first publication and quoted natural philosophical positions from Euler's letters as early as 1770 in his dissertation (AA II 414 u. 419). In 1772, we already find a reference to Euler's logic diagrams in *Logik Philippi*. In Kant's personal interleaved edition of Meier's textbook, there are also numerous logic diagrams drawn by Kant himself, some of which are clearly linked to Euler. Relevant, for example, is Fig. 2, in which one can see at the top left that Kant closely follows Euler's diagrams on the doctrine of judgement (cf. Fig. 1a): Although *A* and *B* are exchanged in the *a*-proposition in comparison to Fig. 1a, Kant adopts Euler's peculiarity of

Fig. 2 Kant's Eulerian diagrams taken from I. Kant's Lectures on Logic. TÜR, Mscr 92, 94



(a) Jäsche Logic

(b) Kant's Handwritten Manuscripts

Fig. 3 Euler and Partition Diagrams taken from (**a**) (Kant et al. 1800, §29) and (**b**) Kant's Lectures, TÜR, Mscr 92, 94

using the position of the letter *A* to designate the segment in question in the case of particular judgements (*i*, *o*). We also see this in the lower row of Fig. 1a. On the right in Fig. 2 we also see the modus Barbara, which again only differs from Euler's Fig. 1 in that two designations are interchanged: Instead of *aAB*, *aCA* ⊢ *aCB* we find *aCB*, *aAC* ⊢ *aAB*.

It can thus be assumed that Kant regularly used diagrams in his logic lectures, most of which are inspired by Euler. Some, however, go beyond this and have not been found in any known logic before Euler. Insofar as these diagrams go beyond Euler but are clearly influenced by his method, we speak of Euler-type diagrams. Some of Kant's handwritten diagrams from Meier's logic textbook were later also included and published in the so-called *Jäsche Logic* (Kant et al. 1800). It should be noted, however, that many of the diagrams included by Jäsche are not Euler diagrams, but Euler-type diagrams. The most relevant of these is the diagram at the bottom in Fig. 3, which is a precursor of the partition diagrams then used from the 1820s onwards (Demey 2020, Sect. 4).

The upper diagram of Fig. 3 again shows the modus Barbara, which, however, has the peculiarity that the terminus minor *x* is only represented as a letter without any circle, quasi

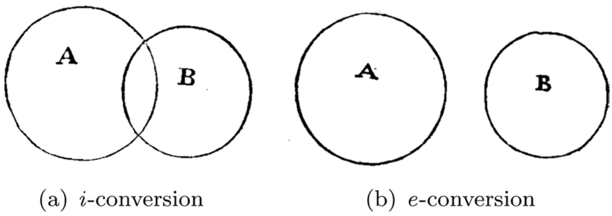


Fig. 4 Euler diagrams taken from (Kiesewetter 1791, 150, 152)

like the indication of a certain point in the diagram. Kant wrote: "In categorical judgments, *x* is what is contained under *b*, and likewise under *a*" (Kant 1819, 151). The lower diagram, on the other hand, is more interesting, as it indicates a disjunctive judgement and thus goes beyond traditional syllogistic. Kant commented on this diagram as follows: "In disjunctive ones *x*, contained under *a*, is contained under either *b*, or *c*, and so on" (Kant 1819, 152).

Since we cannot represent such inferences with Euler's logic, but the kinship of the diagrams is recognisable, Kant not only received Euler, but also tried to extend his diagrams. We can see from the example of Fig. 3b that the diagrams in the *Jäsche Logic* are not a later invention by Jäsche, but originate from Kant's own notes. It is interesting to note that many of the self-drawn diagrams (e.g. Fig. 4b) were created around 1790. In addition, it should be noted that Kant had to deal increasingly with geometric figures during these years, as this was the peak of the famous Kant-Eberhard dispute in geometry (Lemanski 2021, Sect. 2.3). The debate revolved around not only the theoretical aspects of geometry but also the role of visual figures and intuition in providing evidence.

That this phase around 1790 is an important time and sets a scientific revolution in motion is also clear from the fact that the textbooks on logic (and geometry) significantly change after this time. As described in Sect. 2, there are no Eulerian diagrams between about 1714 and 1758. But then, with the logics of Ploucquet, Lambert and Euler in the 1760s, the third period of logic diagrams starts. Even after that, these alternative approaches are considered to have failed, and from the 1770s, Leibniz-Wolff logics continue to dominate.

Even when the first logics emerge in the late 1780s, enriching the Leibniz-Wolff logic with elements taken from the Kantian *Critique of Pure Reason*, logic diagrams or references to the same are not present. A good example of this is the *Grundriss der allgemeinen Logik und kritische Anfangsgründe der allgemeinen Metaphysik* (Outline of General Logic and Critical Principles of General Metaphysics) published in 1788 by the Hallens philosophy professor Ludwig Heinrich von Jakobs. Only in 1787, a short note on Lambert's line diagrams is given in Gotthilf Samuel Steinbart's second edition of *Anleitung des Verstandes zum regelmäßigen Selbstdenken* (Guideline for the mind to think for itself according to rules) (Steinbart 1787, 17).

Suddenly, however, the situation changed and logic diagrams became more and more widely known. Promoted by the Prussian minister Johann Christoph von Woellner, Johann Gottfried Kiesewetter read logic in Halle from the end of 1789, after detours via Berlin and again Königsberg. Starting in autumn 1788, Kiesewetter studied logic with Kant in Königsberg. It was apparently during this time that he first came into contact with Euler's diagrams. Kiesewetter became Kant's most influential student and, after his return from Königsberg, established logic diagrams in Halle, the former capital of Wolffianism.

His lectures in Berlin and Halle were well attended, but also well controlled by the Prussian authorities (AA XI, 112; Br. 394). In 1791, Kiesewetter's *Grundriß einer allgemeinen Logik nach Kantischen Grundsätzen* (Outline of a General Logic According to Kantian

Principles) was finally published. Kiesewetter wrote in the preface that he had been working on this textbook for several years, that Kant had read it and also improved it (Kiesewetter 1791, 5*). Kant was quite indignant, however, as he had the impression that Kiesewetter had simply written down large parts of his own lecture and published them under his name [AA XXIV, 958].

However, this snappishness on Kant's part, which lasted only a short time, may also have been the result of the fact that Kant was involved in a completely different dispute about diagrams at the time: the debate with Johann August Eberhard, Abraham Gotthelf Kästner and other Leibnizians was also about the question, already discussed in the Körber-Hoheisel dispute, of whether diagrams have an epistemic value in proof in geometry. In this context, Leibnizians held the innatist view based on the laws of thought, while Kant, Leonhard Reinhold and others argued that mental or empirical figures would play a central role in geometry (Onof and Schulting 2014). Kant, whose transcendental philosophy is also based on uniting the quarrelling schools of rationalism and empiricism by combining a priori and aposteriori elements, realised during these years that he would have to prevail against the criticism of both schools.

Anyway, Kiesewetter's logic (§23) and other books on logic contain Euler diagrams, which do not even begin to represent Euler's entire method, but nevertheless draw attention to the explanatory power of logic diagrams. For example, Fig. 4a, b show Euler diagrams, but only to facilitate the conversion rules, i.e. $iAB \vdash iBA$ (4a) and $eAB \vdash eBA$ (4b).

With Kiesewetter's lectures on logic and the accompanying compendia, a new reception of the logic diagrams suddenly spread around Halle from the 1790s onwards, especially among logicians who came from the Leibniz-Wolff school but who nevertheless engaged critically or constructively with Kant's ideas. These include the *Institvtiones logicae et metaphysicae* (1792) by the Jena philosophy professor Johann August Heinrich Ulrich, the *Analytik der Urtheile und Schlüsse* (1792) by the Hallens philosopher Johann Christoph Hoffbauer, the *Grundriß der Logik* (1793) by the Hallens philosopher and mathematician Johann Gebhard Maaß, the *Versuch einer Geschichte der Logik* (1794) by the Hallens private scholar Wilhelm Ludwig Gottlob von Eberstein, and finally the comprehensive *Encyclopädisches Wörterbuch der kritischen Philosophie* (1799ff.) by the Halle-born Kantian George Samuel Albert Mellin.

Although Mellin's contribution to logic became obsolete soon after the publication of the *Jäsche Logic*, in which some logic diagrams were published in a form authorised by Kant, the first three volumes of Mellin's dictionary form one of the most important documents for the logic of early Kantianism. For it is especially in volume 2, which was published before the *Jäsche Logic*, that one can see how Kant's formal logic was imagined at the time. In the article 'Elementarlogik' (Elementary Logic) (Mellin 1799, 264–271) it becomes clear that one can extract and interpret some information about general or formal logic from Kant's writings published up to 1799, but that Kiesewetter's writings represent the authoritative information on Kant's logic (Mellin 1799, 271).

In the thirty-page article 'Figur' (Figure) Mellin goes even further (Mellin 1799, 581–611). Here, he connects Kant's *Critique of Pure Reason* with the *The False Subtlety* and explains the ideas with the explicit help of Lambert's and Euler's diagrams. This shows that logic diagrams were already regarded as an elementary component of Kantian logics before the *Jäsche Logic*. For example, Fig. 5 shows the representation of the modus Barbara with Lambertian line diagrams on the left (labelled Fig. 30) and the same mode with Eulerian circle diagrams on the right (labelled Fig. 31).

Mellin's approach remains typical of the logic textbooks between 1790 and about 1820: Euler and Lambert are particularly appreciated by Kantians, radical Leibnizians or

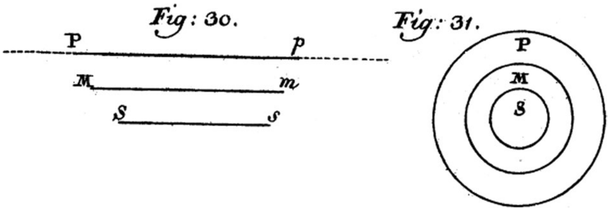


Fig. 5 Lambert and Euler diagrams taken from the appendix of Mellin (1799)

Wolffians continue to reject all logic diagrams, and progressive Leibnizians are most likely to refer to Ploucquet and possibly Lambert for logic diagrams, but never to Euler. It was only after Kant's death that it became obvious that Kantianism had overcome the empiricist and rationalist doctrines and that a return to the Leibniz-Wolffian programme also seemed unthinkable. How dominant Kantian logic is in the nineteenth century becomes clear, for example, from the fact that Moritz Wilhelm Drobisch only very cautiously revives the Leibnizian idea of a calculus in his dissertation in 1827 and Gottlob Frege still justifies at the beginning of his *Begriffsschrift* for why he initially reduces the Kantian forms of judgement to hypothetical judgements (Peckhaus 1997).

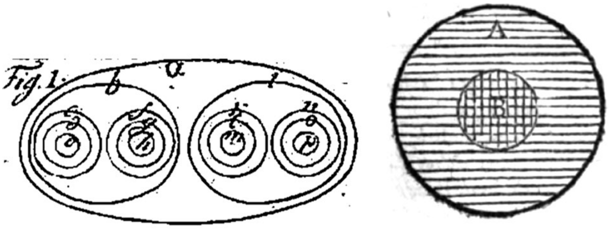
5 Eulerian Diagrams in the Early Nineteenth Century

Ernst Schröder had claimed that since Leonhard Euler's *Letters to a German Princess*, diagrams have probably been used or at least referred to in all works on logic (Schröder 1890, 155). That this judgement is exaggerated was proven in Sect. 3 by the fact that it was only with Kant and Kiesewetter that logic diagrams found their way into the logics of the so-called 'long 19th century' from the 1790s onwards. At least the logic textbooks between 1770 and 1790 are free of Euler(-type) diagrams. John Venn's generalised calculation that about every second book on logic in the nineteenth century contains Euler diagrams seems more plausible (Venn 1894, 110). However, the role of Kant and Kantians is never emphasised in these historical accounts, sometimes not even mentioned.

I aim to demonstrate that Kantians played a crucial role in popularising Euler diagrams during the nineteenth century. To support this claim, I will investigate two lines of tradition in detail up to 1820 and provide a few general remarks. On the one hand, I am concerned with the reception of Euler diagrams in Germany and then in England, i.e. also the two languages that Venn preferred to include in his bibliography of logic. Particularly in the latter presentation, some references to the reception in other countries are also included. However, a more detailed presentation would require a separate study. Since the period after 1820 would also require its own investigation, this section is limited to the period of the so-called 'first generation of Kantians'.

5.1 Eulerian Diagrams in the German-Speaking World

It is certain that many nineteenth-century philosophers in the direct succession of Kant used logic diagrams in their teaching, even if they did not always print these diagrams in their own textbooks. A good example of this is Gottlob Ernst 'Aenesidemus' Schulze, who does not illustrate logic diagrams in his *Grundsätze der allgemeinen Logik* (Principles of General Logic) of 1802, but who demonstrably uses them in his lectures (D'Alfonso 2018).



(a) n-term Diagrams (b) Shaded Circles

Fig. 6 Eulerian diagrams taken from Krause (1803, I, 211) and Krug (1806, 45)

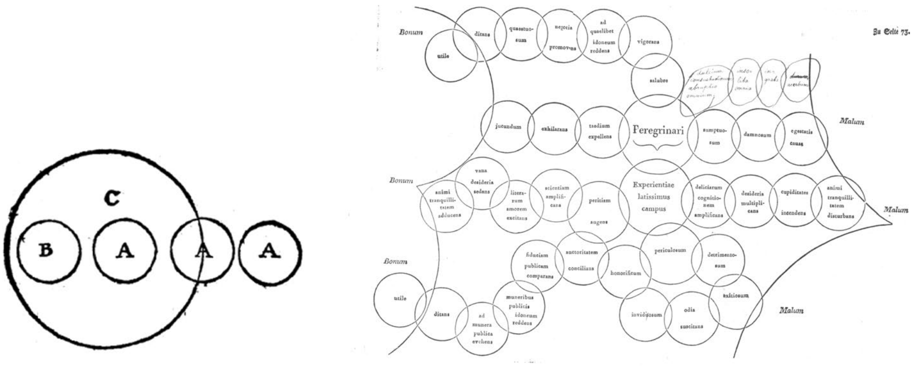
The situation is different, however, with Karl Christian Friedrich Krause, for example, who based his lectures in Jena on the *Grundriss der historischen Logik* (Outline of Historical Logic, 1803). In this, with the help of the Aristotelian rules of syllogistics, an attempt is made to inductively find all the basic figures for valid syllogisms. In doing so, Krause goes far beyond both Kantian and Eulerian logic, since, for example, he first presented subordinated concepts with *n*-terms in the third period of Euler(-type) diagrams, as can be seen in Fig. 6a. (However, many of Krause's really interesting logic diagrams and theses were not published until the late 1820s.)

In 1803, Krause does not use such spectacular diagrams for inferences, but for the representation of the relationship of concepts to each other. In Fig. 6a we see 15 circles labelled from *a* to *p*. Krause thus illustrates the extent to which concepts can be subordinated, associated and separated in *n*-levels (Krause 1803, 211): *a* and *b*, *a* and *c*, *a* and *d* etc. are subordinated in level 1 (because they are not separated from each other by a circle); *c* and *k* as well as *f* and *u* are separated and in each case in level 2 to *a* (because they are each separated from *a* by one circle). From this diagram, but also from Krause's theory of categories, it can be seen that Kant and Euler are both models for the approach, even if differences and innovations can also be found here.

Wilhelm Traugott Krug, who came from the University of Jena and later succeeded Kant in Königsberg, also uses Euler diagrams and Lambertian line diagrams in parallel juxtaposition in his *Denklehre oder Logik* from 1806. For the first time in this period, we find (cross-)hatched or shaded circles as in Fig. 6b, which approximate the idea of the empty set or impossibility. The circumferences of the circles in Fig. 6b express that all *B* is in *A* (*aBA*) and some *A* is *B* (*iBA*). The hatching, however, indicates the negation of these circles, so that it is true that if no *A*, then necessarily no *B*, but that if no *B*, then possibly no *A*. Krug follows Euler and Lambert as far as possible in his logic, which is generally based on Kant.

Jakob Friedrich Fries, who, like Krause, had studied with Fichte in Jena, used several Euler diagrams to illustrate syllogisms in his *System der Logik* from 1811. Here, for the first time, the probably not very advantageous idea appears of drawing in particular judgements not as in Euler, but by repeating circles. Figure 7a is unfortunately not explained precisely. It can be assumed that Fries wants to discuss a misconstructed inference here, indicating the indeterminate relationship between *A* and *C* by repeating *A*.[1]

[1] Fries writes the example next to the diagram (Fries 1811, 216): "All violets are flowers. Some violets are not fragrant. Some flowers are not fragrant." The position of the premises does not correspond to the modus Bocardo and the subject-predicate position does not correspond to the modus Baroco.



(a) Repeating A-Diagram (b) n-Term Graph

Fig. 7 Eulerian diagrams taken from (Fries 1811, 216) and (Schopenhauer 2010, §9)

Finally, in 1819, Arthur Schopenhauer, who had also completed his doctorate in Jena and who worked closely with Krause for a while, came up with the idea of finding the basic topological forms of two circles in order to establish the basic diagrams for a logic (Moktefi 2020). In doing so, Schopenhauer tried to combine the approach of the 1760s with the topological ideas of his mathematics teachers Bernhard Friedrich Thibaut and Franz Ferdinand Schweins (Lemanski 2022). Schopenhauer applied Euler, Eulerian and also partition diagrams to the philosophy of mathematics, philosophy of language and eristic in his main work *Die Welt als Wille und Vorstellung* (The World as Will and Representation) (§9) and especially in his lectures of the early 1820s (Demey 2020). Thus, these diagrams were received also in areas that were no longer connected with logic. The example in Fig. 7b shows an *n*-term diagram from eristic in which numerous circles are mapped for partial judgements that can be interpreted as a directed graph (Bhattacharjee and Lemanski 2022).

It is particularly striking that Schopenhauer goes one step further than his predecessors. He sees in the logic diagrams not only a method of logic imposed by transcendental philosophy, but his lectures of the 1820s justifies the whole of logic and mathematics through the diagrams. Transcendental deduction can be improved by the logic diagrams and these forms of intuition also help with the problem of justification, since logic and mathematics cannot prove themselves (Schopenhauer 2022, 244). Thus, for him, intuition becomes a way out of the trilemma of infinite regress, dogmatism and the vicious circle. This tendency to see in logic diagrams more than just a didactic or heuristic tool will be perpetuated in philosophy and mathematics after the 1820s before the crisis in intuition.

What all these writings mentioned up to 1820 have in common is that they are strongly oriented towards Kant, even if many try to surpass the Kantian approach with their own ideas. Some of these ideas are an important part of the development of Venn diagrams (Moktefi and Lemanski 2022). On the other hand, there was no strong orientation towards Euler's logic: although Krause represents the most comprehensive approach, his method is not suitable for proving the conclusions of premises and the validity of syllogisms. A proper logical method of Euler diagrams for testing inferences, as it is understood among experts today, had not been established in the German-speaking world even by Kant. But nevertheless, logic diagrams according to Euler became more and more popular. Almost all logicians mentioned here up to the 1820s found followers who further elaborated their Kantian-Eulerian method.

Fig. 8 Eulerian diagrams taken from (Nitsch 1796, p. 122)

Only Hegelianism, which emerged in the 1820s, followed Leibnizianism insofar as logic diagrams were strictly forbidden in this school (Pluder 2022). As we know today, however, Hegel himself used logic diagrams (Harris 1983). Perhaps also due to the fact that Leibniz and Hegel themselves did not adhere to the prohibitions they propagated in their writings, Kantianism and with it the Eulerian method became more and more prevalent in the German-speaking world of the nineteenth century, until finally the crisis in intuition mentioned in Sect. 2 set in around 1880.

5.2 Eulerian Diagrams in the English-Speaking World

According to the information presented so far, one might think that the reception of Euler diagrams goes back only indirectly to Kant and that rather Kiesewetter is much more significant. For the German-speaking world, this is certainly true for the 1790s. Also in the Scandinavian countries, the *Lärobok i logiken, af J. G. C. C. Kiesewetter*, published in six editions between 1806 and 1835 in the Swedish translation by the Finn Johan Wilhelm Tuderus, is dominant.

The reception of Euler's diagrams in the English-speaking world, however, went directly via Kant, and thus without taking a diversion via Kiesewetter. The first Kantian to initiate the reception of Kant in England is decisive for this. This is undoubtedly Friedrich August Nitsch, who also studied with Kant in Königsberg around 1790 and also taught there. Nitsch came to London at the end of 1792 and taught Kantian philosophy there between 1794 and 1796.

The result of this teaching activity was the book entitled *A General and Introductory View of Professor Kant's Principles*. It is undisputed in research that this was the first book in the English-speaking world to give a comprehensive and good overview of Kant's philosophy known to date. In the chapter on reason, Nitsch also compiled several principles of logic. Principle 81 dealt with inferences and also gave the relationship between intuition and inference, which Nitsch described with the help of Euler-type diagrams: "All men are mortal; the great Locke was a man; therefore the great Locke was mortal" (Nitsch 1796, 121f.) was represented by three conceptual spheres of different sizes, corresponding to the extensionality of the concept: 'Mortal', 'Man' and 'Locke'. The relation of these three concepts (left in Fig. 8) was described in the inference (right in Fig. 8), in which both premises and the conclusion are represented. They are *a*-propositions for Euler diagrams according to Fig. 1.

Two authors who attended Nitsch's lectures and were strongly influenced by the diagrams are Thomas Wirgman and Henry James Richter. Neither were professional philosophers, but they shaped the early reception of Kant in England. Both were intensively interested in Kant's formal logic and, as can be seen from their tracts and pictures, especially in the diagrammatic representation of Kant's philosophy.

Wirgman published his first articles on Kant including several logic diagrams in 1812 in the *Encylopedia Londinensis*. An extended version of the article entitled 'logic' is dated 15 October 1813 and was published in the same encyclopaedia in 1815. With 30 pages in small type, this version is the size of a small booklet and even has its own synoptic index. The article contains several Euler-type diagrams that correspond to

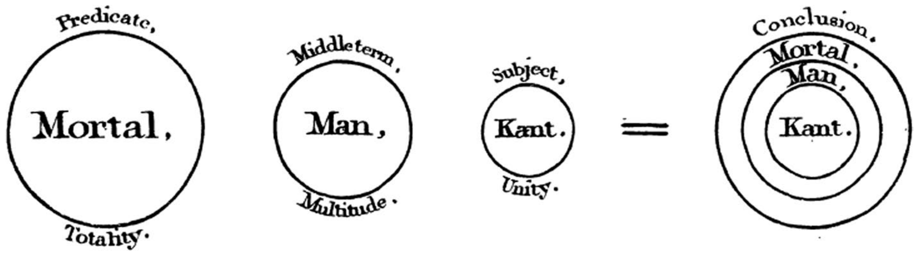


Fig. 9 Eulerian diagrams taken from (Wirgman 1815, table)

Fig. 10 Eulerian diagrams taken from (Wirgman 1815, frontispiece)

the *Jäsche Logic* (Fig. 3: (Wirgman 1815, 22f.) or were adopted from Nitsch (Fig. 8 ≃ Fig. 9) or a diagrammatic interpretation of the Kantian theory of categories. The latter point is also indicated in Fig. 9. Here, Wirgman not only shows a modus Barbara, but also identifies the circles with the three categories of the Kantian category table: 'Totality' contains 'Multitude', and 'Multitude' contains 'Unity'.

Nitsch's two students, i.e. Wirgman and Richter, held the view that the relationship of Kant's doctrine of categories and judgements could be well represented with Euler-type diagrams. Circular figures reminiscent of the modus Barbara appear particularly frequently, partly because Kant advocated the *nota notae* as the supreme principle in the *False Subtlety*, which can be exemplified especially by the perfect modus Barbara.

Richter gives a relevant interpretation in his painting entitled ‘Philosophy’, which is used as a frontispiece in the *Encyclopedia Londinensis*. As we see in Fig. 10, Philosophy is a woman sitting on a throne and holding a staff on which is written *Critique of Pure Reason*. On the ground in front of her is written “Königsberg A.D. 1781”, and on her head are four circles with the name K A N T. Above this, the quadruple is repeated in several ways. A three- or fourfold halo indicates the faculties of the human mind in the form of the modus Barbara: Sense (Space, Time), Understanding and Reason. Within the circle for Understanding are the four categories that Wirgman also indicates in the *Encyclopaedia* (Wirgman 1815, 20): Quantity, Quality and Modality are each again indicated in the form of a modus Barbara, Relation, however, by three *e*-propositions (as in Figs. 1, 2 or 5).

In this way, Wirgman and Richter attempt to point out that the compositional structure of the faculties of the human mind as given in the *Critique of Pure Reason* is oriented towards the organisation of Kant’s general logic, which in turn refers to Aristotle (Santozki 2006). *Reason* (outer halo of Fig. 10) deals with inferences and contains *understanding* (middle halo) which deals with the fundamental judgements and categories since inferences are built up from these. But the understanding also contains the *senses* (inner halo), since otherwise concepts would be empty, as Kant’s famous quote says. This containment relation is expressed by Euler diagrams for *a*-propositions.

Since the four pairs of three categories are treated by Kant as judgements in the section on understanding, they are also in the middle halo. If the categories or the corresponding judgements are structured in a transitive way, which Kant himself points out several times, they are represented as modus Barbara, as can be seen by unity, multitude and totality in Fig. 9. But if the categories are organised by an anti-symmetrical relation, like substance and accident, they are represented as Euler diagrams for *e*-propositions. This interpretation of Kant, which starts with Nitsch, is certainly more complex and requires a more precise interpretation elsewhere. It should be noted that Wirgman further elaborated this approach in several books in the 1820s and -30 s, which were published in English, French, and German. Even though Wirgman was an idiosyncratic scholar, he was particularly influential in the English-speaking world.

The works by Wirgman and Richter exemplify the diagrammatic orientation of the early English reception of Kant. In 1819, John Richardson’s translation of the *Jäsche Logic*, while often incorrect, further spread the influence of the diagrammatic approach in the English-speaking world (Kant 1819). In sum, it can be said that even in the following decades, most English-speaking logicians who used Eulerian or Euler diagrams were strongly influenced by Kant and the early English-language reception of Kant. The most prominent example here is certainly William Stirling Hamilton of Preston, whose *Lectures on Logic* had a strong influence on English-language logic up to John Venn.

However, one can also see from the examples given that the visual representation was more important to the early British Kantians than the testing of inferences with the help of Euler’s method. Although Figs. 8 and 9 each show a modus Barbara, the majority of the diagrams relate more to the representation of the Kantian categories and further doctrines.

6 Conclusion

In the introduction it was announced that three points would be corrected that are usually associated with the history of the so-called ‘golden age of logic diagrams’. In summary, these three points can be corrected as follows: (1) Leibniz’s published writings, and in particular the school of Leibnizians and Wolffians, were counterproductive to the development of logic diagrams in the eighteenth and nineteenth centuries. (Of course, this says nothing about the value of his diagrams in the manuscripts today, but in the history of diagrams Leibniz and his followers emerge as disruptive factors.) (2) Euler and Kant fought intensively against the rationalists' hostility to diagrams in this period, but Euler's diagrams were still censored by Leibnizians in the late eighteenth century. (3) Kant and his early students made Eulerian diagrams known in Central Europe. The period around and after 1790 is crucial for the diagrammatic revolution to break through. This is where the so-called third period of Euler-type diagrams really begins.

The history of logic diagrams outlined here also allows for several philosophical conclusions. In summary, it can be said that the application and evaluation of logic diagrams depends on the basic philosophical position of the philosopher, logician or mathematician. This can be demonstrated not only by numerous quotations given in agreement by the authors listed in Sects. 2, 3, 4 and 5, but also by the correspondence between philosophical attitude and the use of diagrams in different periods of time. If the history of philosophy in the eighteenth and nineteenth centuries is divided into two major schools, namely empiricism and transcendental philosophy on the one hand and rationalism and logicism on the other, the following becomes clear:

(1) Empiricists and transcendental philosophers share an affinity for logic diagrams because they recognise in them intuitive proofs for rational relations or inferences. For them, diagrams have an epistemic value that can vouch for certainty in concrete proofs (Hoheisel, Euler) or that corresponds to the philosophical position and can concern the intuitive foundations of science (Kant, Schopenhauer). In this context, empiricists and transcendental philosophers differ above all with regard to the ontological status of the diagrams: While authors close to empiricism (such as Hoheisel, Rüdiger or Euler) see in diagrams an abstraction from empirical and sensual intuition, for transcendental philosophers (such as Kant, Kiesewetter or Krause) the mental image of a diagram is already sufficient for construction and verification. According to their principles, there is nothing in the mind that was not previously in the senses (empiricism), or else a mental structure must correlate with the senses, for otherwise it is empty (transcendental philosophy).

(2) Rationalists and logicists (such as Leibniz, Körber or Kästner) do not trust diagrams, but only the innate or internalised principles of thought. These principles are usually the laws of thought of logic that are always already present in the mind, which then radiate out into mathematics and its proofs, and the logical calculus is elevated to the status of the ideal. They thus oppose the empiricists' nihil est in intellectu with a nisi ipse intellectus. For them, the diagram has no epistemic value, but at most a heuristic or didactic function (Leibniz, Wolff), which, however, never goes so far that the diagram or the geometric figure can take the place of a proof in words or symbolic formulae. For rationalists, the diagram is in the best case only an aid that can usually be used in didactics or heuristics, or in the worst case even a deceptive sensualisation of an actually mental process (Eberhard, Kästner). The senses or sensualisations can deceive, but pure reason cannot. From the 1810s onwards, even the Hegelians took a radical position in this, since they regarded both symbolic logic and diagrams as arbitrary, static, external and thus 'dead modes' of representing the actually 'living' mental movement of the spirit. This direction not only differs strongly from empiricism and transcendental philosophy, but also breaks with the logicist approach.

The 'business cycle' of logic diagrams described in Sects. 2–5 goes hand in hand with the paradigm shift of the three schools of philosophy: After the Thirty Years' War, there was a second period of Eulerian diagrams that ended in the 1710s. After that time, the rationalists dominate in Central Europe. As in the Körber-Hoheisel dispute, they caused a recession in intuition-based diagrams. In the 1760s, a small expansion phase of logic diagrams begins with Ploucquet, Lambert and Euler, as the first two authors attempt to harmonise the ideal of the calculus advocated by rationalists with intuition-based visualisations supported by empiricists. Both fail, however, according to the prevailing opinion of the time and Euler's logic is demonised along with his other philosophical opinions, as Euler emerges as the antagonist of the dominant rationalists in natural philosophy and metaphysics. Only with Kant, who unites rationalist and empiricist approaches in transcendental philosophy, does a philosophical paradigm shift succeed. The generation of Leibnizians, who were still arguing with Kant, died out in the early nineteenth century and by the 1820s logic diagrams had established themselves as the formal-logical equivalent to transcendental philosophy. The 'golden age' or the third expansion phase in the modern history of logic diagrams is thus confined to the (long) nineteenth century.

Looking ahead, it is possible to follow this cycle and consider the late nineteenth century, marked by the crisis in intuition, as another period of recession, resulting in the fact that logic diagrams were not extensively studied in the twentieth century and did not gain widespread acceptance, with only a few exceptions. It was not until the 1990s that Sun-Joo Shin succeeded in harmonising the two different schools and their corresponding ideals (Shin 1994), as Ploucquet and Lambert had sought to do: she showed that logic diagrams can be given a precise syntax and semantics and are therefore also suitable for logical calculi. This approach was then quickly taken up in the cognitive sciences, in mathematics and artificial intelligence and is still expanding today (Shimojima 1996; Jamnik et al. 1999; Nakatsu 2010). Nowadays, the diagram-affine and diagram-phobic logicians and mathematicians can no longer be unproblematically understood as rationalists on the one hand and empiricists on the other. The division into diagram-affine and diagram-phobic logicians and mathematicians still works, however, as Shin herself describes in the Introduction to *The Logical Status of Diagrams*. Since if one takes into account Shin's own affinity to diagrams on the one hand and her criticism of the neologicists' hostility to diagrams on the other, one can say that the history of logic diagrams is not only a history of misunderstandings, but also of the reinvention of old arguments and philosophical positions.

Acknowledgements The author thanks the two anonymous reviewers and Amirouche Moktefi for valuable comments on the paper.

Funding Open Access funding enabled and organised by Projekt DEAL. The author is supported by the Fritz Thyssen Foundation (project: *History of Logic Diagrams in Kantianism*) and benefited from the ViCom-project *Gestures and Diagrams in Visual-Spatial Communication* funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)—(RE 2929/3-1).

Declarations

Ethical Standard There are no financial interests or ethical aspects to the paper. All funding is listed above.

References

Anonymous. 1793. Leonhard Eulers Briefe über Verschiedene Gegenstände aus der Naturlehre. In *Göttingische Anzeigen von Gelehrten Sachen 2*, 110–111.

Bernhard, Peter. 2008. Visualizations of the Square of Opposition. *Logica Universalis* 2: 31–41. https://doi.org/10.1007/s11787-007-0023-x.

Bhattacharjee, Reetu, and Jens Lemanski. 2022. Combing Graphs and Eulerian Diagrams in Eristic. In *Diagrams 2022: Diagrammatic Representation and Inference*, ed. Valeria Giardino, 1–16. Cham: Springer. https://doi.org/10.1007/978-3-031-15146-0_8.

Bhattacharjee, Reetu, Amirouche Moktefi, and Ahti-Veikko. Pietarinen. 2022. The Representation of Negative Terms with Euler Diagrams. In *Logic in Question: Talks from the Annual Sorbonne Logic Workshop (2011–2019)*, ed. Jean-Yves. Béziau, et al., 43–58. Cham: Springer International Publishing.

Calinger, Ronald S. 2016. *Leonhard Euler: Mathematical Genius in the Enlightenment*. Princeton and Oxford: Princeton University Press.

D'Alfonso, Matteo Vincenzo. 2018. Arthur Schopenhauer, Anmerkungen zu G. E. Schulzes Vorlesungen zur Logik (Göttingen 1811). In *I Castelli di Yale Online 6*, 191–246. https://doi.org/10.15160/2282-5460/1658.

Demey, Lorenz. 2020. From Euler Diagrams in Schopenhauer to Aristotelian Diagrams in Logical Geometry. In *Language, Logic, and Mathematics in Schopenhauer*, ed. Jens Lemanski, 181–205. Cham: Springer. https://doi.org/10.1007/978-3-030-33090-3_12.

Englebretsen, George. 2019. *Figuring It Out: Logic Diagrams*. Berlin, Boston: De Gruyter. https://doi.org/10.1515/9783110624458.

Euler, Leonhard. 1792. Briefe über verschiedene Gegenstände aus der Naturlehre. Translated by F. Kries. Leipzig: Dyck.

Euler, Leonhard. 1802. *Letters of Euler to a German Princess: Translated by Henry Hunter*, vol. 1. London: Murray and Highley.

Fellmann, Emil Alfred, and Gleb Konstantinovich Mikhajlov. 1998. Allgemeine Einleitung. In *Briefwechsel von Leonhard Euler mit Johann I Bernoulli und Niklaus I. Bernoulli: Commercium epistolicum vol. II*, ed. E.A. Fellmann and G.K. Mikhajlov, 7–28. Birkhäuser: Basel.

Fries, Jakob Friedrich. 1811. *System der Logik*. Heidelberg: Rohr und Zimmer.

Hahn, Hans. 1980. The Crisis in Intuition. In *Vienna Circle Collection: Empiricism, Logic and Mathematics*, vol. 13, ed. B. McGuinness, 73–102. Dordrecht: Vienna Circle Collection.

Harris, Henry Silton. 1983. *Hegel's Development: Night Thoughts (Jena 1801–1806)*, 2nd ed. Oxford: Clarendon. https://doi.org/10.1093/acprof:oso/9780198246541.001.0001.

Hennings, J.E. 1775. Leonhard Eulers Briefe an eine deutsche Prinzessin. In *Neue Philosophische Bibliothek 4*, 633–634.

Hodges, Wilfrid. 2023. A Correctness Proof for al-Barakāt's Logical Diagrams. In *The Review of Symbolic Logic* 16 (2): 369–384. https://doi.org/10.1017/S1755020321000332.

Hogan, Desmond. 2020. Kant and the Character of Mathematical Inference. In *Kant's Philosophy of Mathematics*, vol. 1, ed. Carl Posy and Ofra Rechter, 126–154. Cambridge: Cambridge University Press. https://doi.org/10.1017/9781107337596.007.

Jamnik, Mateja, Alan Bundy, and Ian Green. 1999. On Automating Diagrammatic Proofs of Arithmetic Arguments. *Journal of Logic, Language and Information* 8 (3): 297–321. https://doi.org/10.1023/A:1008323427489.

Kant, Emmanuel. 1819. *Logic*. Translated by John Richardson. London: Simpkin and Marshall.

Kant, Immanuel, and Gottlob Benjamin Jäsche. 1800. *Immanuel Kants Logik. Ein Handbuch zu Vorlesungen*. Königsberg: Nicolovius.

Kiesewetter, Johann Gottfried Carl Christian. 1791. *Grundriß einer reinen allgemeinen Logik nach kantischen Grundsätzen zum Gebrauch für Vorlesungen*. Berlin: Lagarde.

Klein, Anton. 1785. Leben und Bildnisse der Grosen Deutschen, Von verschiedenen Verfassern und Künstlern: Erster Band, Band 1. Mannhein: s.n.

Knobloch, Eberhard. 1992. La détermination Mathématique du Meilleur. In *Leibniz: Le Meilleur des Mondes (Studia Leibnitiana SH 21)*, ed. André Robinet and Albert Heinekamp, 47–65. Stuttgart: Steiner.

Kobzar, Vladimir Ivanovich. 2005. Euler's Elementary Logic (in Russian). *Logiko-Filosofskie Studii* 3: 130–152.

Körber, Christian Albrecht. 1731. Archimedes Defensvs Das ist Gründlicher Beweiß Daß das Theorema Archimedis [...] nicht solo oculorum usu [...] könne erfunden werden. Halle: Auf Kosten des Avtoris.

Krause, Karl Christian Friedrich. 1803. *Grundriss der Historischen Logik*. Jena, Leipzig: Gabler.

Krug, Wilhelm Traugott. 1806. *Denklehre oder Logik*. Königsberg: Goebbels, Unzer.

Leibniz. 1820. Leibniz über allgemeine Sprache: Stellen, welche hier das erstemal im Druck erscheinen. *Magazin für allgemeine Sprache* 3 (1): 112–163.

Lemanski, Jens. 2021. *World and Logic*. London: College Publications.

Lemanski, Jens. 2022. Schopenhauers Logikdiagramme in den Mathematiklehrbüchern Adolph Diesterwegs. *Siegener Beiträge Zur Geschichte Und Philosophie der Mathematik* 16: 97–127. https://doi.org/10.25819/ubsi/10256.

Lu-Adler, Huaping. 2018. *Kant and the Science of Logic: A Historical and Philosophical Reconstruction*. New York: Oxford University Press. https://doi.org/10.1093/oso/9780190907136.001.0001.

Mayer, J. T. 1793. Leonhard Eulers Briefe über verschiedene Gegenstände aus der Naturlehre. *Intelligenzblatt der Neuen Allgemeinen Deutschen Bibliothek* 21: 558-559.

Mellin, Georg Samuel Albert. 1799. *Encyclopädisches Wörterbuch der Kritischen Philosophie oder Versuch einer fasslichen und vollständigen Erklärung der in Kants kritischen und dogmatischen Schriften enthaltenen Begriffe und Sätze: Vol. II*. Jena, Leipzig: Frommanns Erben.

Moktefi, Amirouche. 2020. Schopenhauer's Eulerian Diagrams. In *Language, Logic, and Mathematics in Schopenhauer*, ed. Jens Lemanski, 111–127. Cham: Springer. https://doi.org/10.1007/978-3-030-33090-3_8.

Moktefi, Amirouche, and Jens Lemanski. 2022. On the Origin of Venn Diagrams. *Axiomathes* 32 (3): 887–900. https://doi.org/10.1007/s10516-022-09642-2.

Moktefi, Amirouche, and Shin Sun-Joo. 2012. A History of Logic Diagrams. In *Logic: A History of its Central Concepts*, ed. Dov M. Gabbay and John Woods, 611–682. Oxford: Elsevier.

Nakatsu, Robbie. 2010. *Diagrammatic Reasoning in AI*. Hoboken: Wiley. https://doi.org/10.1002/9780470400777.

Nitsch, Friedrich. 1796. A General and Introductory View of Professor Kant's Principles concerning Man, the World, and the Deity. London: Downs.

Onof, Christian, and Dennis Schulting. 2014. Kant, Kästner and the Distinction Between Metaphysical and Geometric Space. *Kantian Review* 19 (2): 285–304. https://doi.org/10.1017/S1369415414000065.

Peckhaus, Volker. 1997. Logik, Mathesis Universalis und Allgemeine Wissenschaft: Leibniz und die Wiederentdeckung der formalen Logik im 19. Jahrhundert. Berlin: Akademie.

Pluder, Valentin. 2022. The Limits of the Square: Hegel's Opposition to Diagrams in its Historical Context. In *The Exoteric Square of Opposition*, ed. I. Vandoulakis and J.-Y. Béziau, 123–139. Cham: Springer. https://doi.org/10.1007/978-3-030-90823-2_6.

Pulte, Helmut. 1989. *Das Prinzip der Kleinsten Wirkung und die Kraftkonzeption der Rationalen Mechanik: Eine Untersuchung zur Grundlegungsproblematik bei Leonhard Euler, Pierre Louis Moreau de Maupertuis und Joseph Louis Lagrange*. Stuttgart: Steiner.

Rodriguez-Pereyra, Gonzalo. 2018. The Principles of Contradiction, Sufficient Reason, and Identity of Indiscernibles. In *The Oxford Handbook of Leibniz*, ed. Maria Rosa Antognazza, 45–64. Oxford: Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199744725.013.002.

Santozki, Ulrike. 2006. *Die Bedeutung antiker Theorien für die Genese und Systematik von Kants Philosophie: Eine Analyse der drei Kritiken*. Berlin, Boston: De Gruyter. https://doi.org/10.1515/9783110882216.

Schopenhauer, Arthur. 2010. The World as Will and Representation. 2 vol., ed. by J. Norman, A. Welchman, and C. Janaway. Cambridge: Cambridge University Press.

Schopenhauer, Arthur. 2022. Vorlesung über die Gesamte Philosophie. Vol. I, ed. by Daniel Schubbe, Daniel Elon, and Judith Werntgen-Schmidt. Hamburg: Meiner.

Schröder, Ernst. 1890. *Vorlesungen über die Algebra der Logik (Exakte Logik)*, vol. I. Stuttgart: Teubner.

Shimojima, Atsushi. 1996. On the Efficacy of Representation (PhD thesis). Indiana.

Shin, Sun-Joo. 1994. *The Logical Status of Diagrams*. Cambridge: Cambridge University Press. https://doi.org/10.1017/CBO9780511574696.

Steinbart, Gotthilf Samuel. 1787. *Anleitung des Verstandes zum regelmäßigen Selbstdenken*, 2nd rev. ed. Züllichau: Frommanns Erben.

Venn, John. 1894. *Symbolic Logic*, 2nd ed. London: Macmillan and Co.

Volkert, Klaus Thomas. 1986. *Die Krise der Anschauung: Eine Studie zu formalen und heuristischen Verfahren in der Mathematik seit 1850*. Göttingen: Vandenhoeck & Ruprecht.

Wirgman, Thomas. 1815. Logic. In *Encylopaedia Londinensis or, Universal Dictionary of Arts, Sciences, and Literature*, ed. John Wilkes, 1–31. London: Ablard.