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ARTICLE

Second-Order Confidence in Supervaluationism

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Abstract

Recently, Wilcox (JGPS 51: 65–87, 2020) argued against the so-called *wide interval view* and in favor of the *principle of indifference* as the correct response to unspecific evidence. Embedded in a formal model of the beliefs of an agent, the former presupposes imprecise probabilities and the latter numerically precise degrees of belief. His argument is illustrated by a thought experiment that comes with a fundamental intuition. According to Wilcox, the *wide interval view* is incompatible with this intuition and, thus, undermined. In contrast, I show that the intuition behind the thought experiment is, in fact, compatible with the *wide interval view* if it is embedded into a specific conception of imprecise probabilities as model of belief. This conception is an extension of a framework which I call *modified supervaluationism* (MSV) and which I recently presented elsewhere (Karge 2021, 175–191). To accommodate the thought experiment's fundamental intuition, it introduces a notion of *second-order beliefs*.

Keywords Imprecise probabilities · Supervaluationism · Wide interval view

1 Introduction

Recently, Wilcox (2020) argued against the so-called *wide interval view* and in favor of the *principle of indifference* as the correct response to unspecific evidence. Embedded in a formal model of the beliefs of an agent, the former presupposes imprecise probabilities and the latter numerically precise degrees of belief. His argument is illustrated by a thought experiment that comes with a fundamental intuition. According to Wilcox, the *wide interval view* is incompatible with this intuition and, thus, undermined. Since it depends on imprecise probabilities whereas *the principle of indifference* is based on precise ones, this can be seen as an argument against imprecise probabilities as a model of belief as a whole. In contrast, I show that the intuition behind the thought experiment is, in fact, compatible with the *wide interval view* if it is embedded into a specific conception of imprecise probabilities as model of belief. This conception is an extension of a framework which I

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call *modified supervaluationism* (MSV) and which I recently introduced elsewhere (Karge 2021).

For this purpose, I first introduce some key concepts of Wilcox' argument and imprecise probabilities in Sect. 2. Subsequently, in Sect. 3, I discuss *modified supervaluationism*. Finally, I illustrate in Sect. 4 how MSV can be extended such that it reconciles the intuition behind Wilcox' thought experiment with the wide interval view.

2 Preliminaries

This section outlines the central concepts of this paper. More precisely, I define both competing views: the principle of indifference and numerically precise degrees of belief as well as the wide interval view and imprecise degrees of belief. In the final part of this section, I discuss Wilcox' argument against the wide interval view.

2.1 Belief and Degrees of Belief

I start by defining precise and imprecise degrees of belief. On one account, called orthodox Bayesianism, we can represent an agent's belief with a single probability function. That is, a function which assigns to each proposition, the object of belief, a real number between 0 and 1 that reflects the agent's confidence in that proposition. More formally, a probability function can be defined as follows:

Definition 1 (Probability Function) A probability function Pr is a function $Pr : 2^{\Omega} \rightarrow \mathbb{R}$, satisfying the probability axioms (Mahtani 2019).

Ω is taken to be a set of possible worlds, or states of affairs (Mahtani 2019). A proposition then simply is a subset of Ω. We then take its powerset 2^{Ω} to describe all possible propositions over the set of possible worlds. The real number assigned to a proposition is called the agent's *degree of belief* in that proposition.

Assume, however, an agent has to evaluate a proposition such as *global sea level will rise at least 1.5 ms until the year 2100 above the level of 2000*. What precise probability should she assign to that proposition? Since orthodox Bayesianism represents an agent's belief with a single probability function, such a precise value has to be given (Rinard 2015). Considering propositions of this type, it seems highly implausible to represent belief states with a single probability function (Schoenfield 2012).

On an alternative account, degrees of belief can be defined based on imprecise probabilities. One way to construe imprecise probabilities is the following:

Definition 2 (Imprecise Probabilities) Imprecise probabilities are sets of probability functions (Bradley and Steele 2014).

A specific set of probability functions is called the agent's *representor* $\mathcal{P}$ (Bradley and Steele 2014). In order to facilitate discussing the set of values the representor assigns to a specific proposition, we can define the *imprecise degree of belief* in a proposition as follows:

Definition 3 (Imprecise Degree of Belief) An agent's imprecise degree of belief in a proposition H is represented by a function, $\mathcal{P}(H)$, with $\mathcal{P} = \{Pr(H) : Pr \in \mathcal{P}\}$ (Bradley 2015).

Note, however, that this definition comes with a little abuse of notation (Bradley 2015) and that it can be argued that it does not adequately represent an agent's imprecise degree of belief (Bradley 2019a). In our setting, the *imprecise degree of belief* serves as a more convenient way to compare an agent's confidence in given propositions.

Still, the imprecise degree of belief can be illustrated by the following example: Let A be the proposition that global sea level will rise at least 1.5 ms until the year 2100 above the level of 2000. Assume, our agent is 60–80% confident that this will be the case. Thus, we can represent the agent's imprecise degree of belief in A with: $\mathcal{P}(A) = [0.6, 0.8]$.

An additional interpretation of the agent's representor will be central to a later part of this paper. The idea is to understand the representor as a credal committee where every probability function in that committee represents the opinion of one of its members (Bradley and Steele 2014). The opinions of its members then reflect the beliefs of an agent (Bradley 2019b).

2.2 The Two Competing Views

One way to motivate imprecise degrees of belief as model of belief is to argue that they are the correct response to the type of evidence agents typically receive. Since this evidence often is imprecise or incomplete, the agent's representation of belief should reflect this uncertainty (Joyce 2010). According to Joyce, instead of assigning precise probability values to propositions based on such evidence the agent's degree of belief ought to reflect the unspecific nature of the evidence by considering all values that are not excluded by the evidence (Joyce 2005).

Wilcox, on the other hand, argues in favor of a different approach to handle incomplete information: the principle of indifference. More precisely, Wilcox argues for a restricted variant of this principle which he defines as follows:

Definition 4 (Restricted Principle of Indifference) The restricted principle of indifference demands that in evidentially symmetric cases of total stochastic ignorance where there is a uniquely correct partition of finitely many possible outcomes, the agent has to assign equal and precise probabilities to each possible outcome (Wilcox 2020).

But what is meant by *evidentially symmetric*, *total stochastic ignorance* and *uniquely correct partition*? Wilcox understands these notions as follows:

Definition 5 (Evidential Symmetry) Evidential symmetry describes "cases where our evidence bearing on the possible outcomes does not support one outcome more than any other" (Wilcox 2020).

More specifically, this includes cases where the agent's evidence doesn't provide information on the objective chances of the possible outcomes (Wilcox 2020). This is accounted for by the following notion:

Definition 6 (Total Stochastic Ignorance) Total stochastic ignorance means "situations where one lacks any evidence about relevant objective chances that bear on what one's credences should be" (Wilcox 2020).

Finally, a clear definition of *Uniquely Correct Partition* is not given. However, he gives an example to intuitively motivate it: Assume, there is a prize behind one of three doors. The uniquely correct partition then is the following one:

Partition$_1$: {the prize is behind door 1, the prize is behind door 2, the prize is behind door 3}.
An alternative, but not intended, partition can be described as follows:
Partition$_2$: {the prize is behind door 1, the prize is behind door x} (Wilcox 2020).

The *uniquely correct partition condition* is meant to guarantee that each probability value that is assigned to an event via the principle of indifference does not depend on the particular description of the experiment. For instance, in the above example, the principle of indifference would demand to assign each event in *Partition*$_1$ a probability of 1/3 whereas each event in *Partition*$_2$ would be assigned a probability of 1/2.

That being said, we clarified the view that Wilcox argues for. Now, lets specify the view that he argues against. That is, the wide interval view:

Definition 7 (Wide Interval View) "[T]he wide interval view is the norm that in evidentially symmetric cases of total stochastic ignorances, one's credence should be maximally non-committal about the relevant outcomes. In a sense, then, one's credal state should be spread over all of the possible probability values in the interval [0, 1]" (Wilcox 2020).

With that, the wide interval view corresponds to Joyce's idea to consider all possible probability values that are not excluded by the evidence.

2.3 Wilcox' Argument

After having introduced the formal framework underlying Wilcox' argument, I now outline the parts of his argument that are relevant to our discussion. The argument can be structured as follows:

(1) He constructs a thought experiment;
(2) he then extracts the fundamental intuition behind that experiment;
(3) subsequently, he argues that the wide interval view is incompatible with this intuition.

(1) Let's start by describing the thought experiment: Suppose, we have two urns: *urn 1* and *urn 2*. These urns are presented to an agent who receives as information that each ball in urn 1 is either black or white and that each ball in urn 2 is of one of 10 colors, including black and white. Moreover, the agent does not know the ratio of balls and colors. That is, she has no information on how many balls of each color the urns contain. It is possible, for instance, that all balls from urn 1 are black and that half of the balls from urn 2 are blue and the other half orange (Wilcox 2020).

Next, a ball is drawn from each urn. Let's call the ball drawn from urn 1 *ball 1* and the one from urn 2 *ball 2*. The agent gets now asked whether or not she is more confident that ball 1 is black than that ball 2 is black.

(2) With that, Wilcox states the fundamental intuition behind the thought experiment:
Fundamental Intuition. An agent should be more confident that ball 1 is of a certain color than ball 2 is since ball 1 can only be black or white whereas ball 2 can be of one of 10 colors (Wilcox 2020).

(3) Furthermore, assume that this intuition is correct. Wilcox argues that the *wide interval view* is incompatible with this intuition. More precisely, the wide interval view demands the agent to have a maximally uncommitted imprecise degree of belief in both the proposition that ball 1 is of a certain color as well as that ball 2 is of a certain color. That is, the agent should have as imprecise degree of belief in both propositions the interval [0, 1] since the ratio of the balls and colors is unknown (Wilcox 2020). By assigning both propositions the same interval of possible values, the agent has no reason to believe that ball 1 being of a specific color is more likely than ball 2 (Wilcox 2020). Hence, according to the wide interval view, the agent cannot be more confident that ball 1 is of a certain color than ball 2 because her degree of belief is maximally uncommitted regarding both balls. Thus, it is incompatible with the intuition behind the thought experiment.

In the following section, I present a formal framework of imprecise degrees of belief that can be extended in such a way that it reconciles the wide interval view with the fundamental intuition.

3 Modified Supervaluationism

The framework I introduce in this section is called *Modified Supervaluationism* (MSV). The starting point is *supervaluationism*, originally, a semantic theory to characterize vagueness. Recently, however, it has been applied to imprecise credences. To see this, I first introduce standard supervaluationism. Next, I show how it can capture imprecise probabilities and the way MSV modifies the original framework. Finally, I sketch a more general motivation for introducing MSV.

3.1 Supervaluationism

To understand supervaluationism, consider a vague predicate such as *tall*. Vague predicates can be made more precise. To some, *being tall* means to be of at least 180 cm of height, to others that may be 185 cm. Each such cutoff point constitutes a precisification of that very predicate (Keefe 2008). To determine the truth value of a proposition that contains a vague predicate, supervaluationism demands complete agreement among the precisifications on that value (Varzi 2007). Put into supervaluationistic terms, *complete agreement* is understood as a proposition being either *determinately true* or *determinately false*.

According to standard supervaluationism, a proposition is *determinately true* if it is true according to all admissible precisifications. If a proposition is false according to all admissible precisifications, we call this proposition *determinately false*. Finally, supervaluationism allows for propositions to have no semantic value: In that case, we claim that if a proposition is true according to some, but not all, admissible precisifications then it is *indeterminate* whether it is true (Rinard 2015).

Subsequently, we can very naturally capture imprecise degrees of belief in this framework by specifying the meaning of *admissible precisficications*.

Definition 8 (Admissible Precisification) The admissible precisifications are the functions in an agent's representor.

As model of belief, supervaluationism in conjunction with imprecise probabilities needs to offer a sound concept of what it means for an agent to be more confident in one proposition than in another. Such a concept, which I call *comparative confidence*, can easily be defined based on the idea of propositions being *determinately true*:

Definition 9 (Comparative Confidence) Given two propositions A and B. If it holds according to all precisifications (probability functions) in the agent's representor that $Pr(A) > Pr(B)$ then it is *determinately true* that the agent is more confident in A than in B (Rinard 2015).

Consider the following example:

Example 1 (Comparative Confidence) Assume that H denotes the proposition *a particular coin comes up heads on its next toss*. Let H' be the proposition that it lands tails. Let $\mathcal{P}(H) = (0.5, 1]$ be the agent's representor for H. Then it is true according to all admissible precisifications that $Pr(H) > Pr(H')$ and it is thus determinately true that the agent is more confident in H than in H'.

3.2 Modified Supervaluationism

Next, I introduce modified supervaluationism. In MSV the notion of propositions being *determinately true* is replaced by propositions being *predominantly true*:

Definition 10 (Predominantly True) A proposition is predominantly true if it is true according to a relative majority of admissible precisifications (Karge 2021).

This can be illustrated as follows:

Example 2 (Predominantly True) Assume, there are ten admissible precisifications for the predicate *tall*. According to one of those the threshold for being tall is 170 cm, according to two precisifications it is 175 cm, according to three it is 185, and according to four precisifications the threshold lies at 180 cm. In this case, it is predominantly true that someone who is at least 180 cm tall is tall (Karge 2021).

As for standard supervaluationism, imprecise probabilities can easily be captured in MSV by taking as *admissible precisifications* the functions in the agent's representor. From this weaker notion of the truth of a proposition, we can derive a weaker notion of an agent being more confident in a proposition than in another. Let's call this *predominant confidence*.

For this, recall the idea that an agent's representor can be seen as a belief committee. Following this idea, I suggest to regard the representor as a voting committee where each member represents one precisification. Each member then votes for the proposition it conceives as the most probable. Following this idea, we can define *predominant confidence*:

Definition 11 (Predominant Confidence) Given two propositions A and B. An agent is predominantly more confident in proposition A than in proposition B if a greater proportion of members of the representor vote for proposition A than for proposition B.

Let me illustrate this idea by the following example:

Example 3 (Predominant Confidence) Assume H is the proposition that *a particular coin comes up heads on its next toss*. Let H' be the proposition that it lands tails. Suppose we have $\mathcal{P}(H) = [0.4, 1]$ as our agent's imprecise degree of belief. The members of the representor that represent the precisifications in the interval (0.5, 1] then vote for proposition H. The ones representing [0.4, 0.5), in turn, vote for proposition H'. Since a greater proportion of votes is in favor of H, the agent is predominantly more confident in proposition H.

Although comprehensible on an intuitive level, the above example raises the question of how to determine a majority of votes based on uncountably many probability functions. In case there are only finitely many precisifications, the application of MSV is easy enough. When allowing for infinitely many precisifications, however, we can no longer determine the proportion of precisifications in favor of a proposition by counting. Although the development of a rigorous framework and a surrounding philosophical discussion for the infinite case must be left to future work, I will briefly illustrate one way to realize the application of MSV to infinitely many precisifications. One way to accomplish this is to take into account that, in our setting, admissible precisifications are confined to the unit interval [0, 1]. The standard way to measure the length of an interval is to apply the *Lebesque Measure*. For any closed, $[a, b]$, open, (a, b), or half open, $(a, b]$ or $[a, b)$, interval it holds that its Lebesque measure is of length $l = b - a$. Applying the Lebesque measure to MSV, we can determine the proportion of votes by measuring the length of the corresponding interval. For the above example, we receive $l(H) = 0.5$ as length of the interval representing the votes in favor of H as well as $l(H') = 0.1$ for the those voting in favor of H'. Thus, the agent is predominantly more confident in proposition H.

3.2.1 Motivation and Summary of MSV

A major motivation for the introduction of MSV is of decision-theoretical nature. In 2010, Adam Elga illustrated that standard approaches to capture imprecise probability epistemologically fail to be manifested in decision-making (Elga 2010). As a response, more elaborated accounts of imprecise credences have been developed. Subsequently, however, it has been shown that those accounts that solve Elga's problem fail in another decision-theoretical scenario which is often times used in order to motivate imprecise credences in the first place: the Ellsberg Paradox (Bradley 2019). MSV, in turn, manages to solve both the Ellsberg paradox as well as Elga's problem (Karge 2021).

In a nutshell, supervaluationism is a theory of vagueness that can easily be applied to imprecise degrees of belief. Moreover, modified supervaluationism alters the original account by only demanding predominant truth instead of determinate truth. From this, a weaker notion of comparative confidence can be derived.

4 Modified Supervaluationism and Wilcox' Argument

Before analyzing Wilcox' argument based on MSV, it is crucial to distinguish *objective chances* from *epistemic possibilities*. As Wilcox mentions himself, one could criticize his thought experiment by arguing that it is, in fact, not an experiment under total stochastic ignorance (Wilcox 2020). The agent receiving as information that balls from urn 2 could potentially be of more colors could be seen as evidence about the objective chances of drawing a ball of a specific color. Wilcox, however, rejects this idea and argues that this information only is relevant evidence when it comes to the *epistemic possibilities*.

4.1 Objective Chances and Epistemic Possibilities

By an *epistemically possible* proposition Wilcox means that, given the agent's knowledge, she cannot rule out that proposition. For instance, the agent cannot rule out that all balls in urn 2 are magenta (Wilcox 2020). *Objective chances*, on the other hand, describe evidence about the actual proportion of balls of different colors in the urn example (Wilcox 2020).

In order to further clarify the distinction between *objective chances* and *epistemic possibilities* as well as prepare the analysis of Wilcox' urn experiment based on MSV, I briefly discuss two types of uncertainty in urn experiments that correspond to the here made distinction of objectives chances and epistemic possibilities.

4.1.1 Urn Experiments and Epistemic Possibilities

In urn experiments with some number of $1, \dots, n$ balls and $1, \dots, m$ many different colors, we can generally distinguish two types of uncertainty. The first type of uncertainty is of objective nature and concerns information on the identity of the drawn ball. The second type of uncertainty is of subjective nature and concerns the color composition of the urn itself (Machina 2011). In our setting, this second type of uncertainty is reflected by epistemic possibilities. Sometimes, these types of uncertainty are mixed in a particular urn experiment. This is the case in the well-known *Ellsberg Urn* where we assume that a given urn contains 90 balls of which 30 balls are known be red whereas we only know of each of the remaining 60 balls that they are either black or yellow (Machina 2011). To best reflect these two types of uncertainties, one can use an *orthogonal representation* of the urn experiment. In this representation, we label each individual ball by assigning it a serial number and then represent its respective color. In the Ellsberg urn, we can assume that we label the red balls as $ball_1, \dots, ball_{30}$. The color of the remaining balls $ball_{31}, \dots, ball_{90}$, however, is unknown. For these remaining balls there are 2^{60} different states to consider (Machina 2011). To illustrate this, one can consider a simplified Ellsberg urn with only three balls where one is known to be red:

In this example, the serial number i of the drawn ball refers to the objective uncertainty of the urn experiment (i.e. the red ball 1 is drawn) and the event whether, for instance, ball 2 is black concerns the subjective uncertainty and depends on whether the states (BB) or (BY) obtain (Machina 2011) (Fig. 1).

Fig. 1 Orthogonal Representation of the simplified Ellsberg Urn (Machina 2011)

serial number of drawn ball				
3	black	yellow	black	yellow
2	black	black	yellow	yellow
1	red	red	red	red
	BB	*BY*	*YB*	*YY*

composition of the urn

On a more general level, urns where no information on objective chances is given are also referred to as *ambiguous urns* (Epstein 1999). In Wilcox' thought experiment we encounter an ambiguous urn as only information regarding the subjective type of uncertainty described here is provided. For instance, if the simplified Ellsberg urn were purely ambiguous we would have to go through all possible color compositions for the first ball as well. In the following, I will adopt the orthogonal representation for ambiguous urns in order to analyze Wilcox' thought experiment on a combinatoric level. This not only allows to more precisely reflect the subjective uncertainty and epistemic possibilities but also for the framework presented here to be flexible enough to handle urn experiments involving mixed types of uncertainty. Note, however, that this account adds an additional assumption to Wilcox' framework that is not necessarily made originally. Namely, that the balls in the urn experiment are physically distinguishable. In the orthogonal representation, as described above, this is achieved through serial labels for the individual balls.

4.1.2 Epistemic Possibilities and the Wide Interval View

Following Wilcox, I assume that it is correct that the wide interval view with imprecise degrees of belief in their standard variant cannot capture epistemic possibilities. The reason for that is the very idea that the probability values over the unit interval ought to be distributed across all values that are not excluded by the agent's evidence (Joyce 2005). Since evidence typically only excludes values regarding the objective chances of events, it does not concern mere possibilities. Still, a defender of imprecise credences could argue that the wide interval view generally is the correct response to evidence that concerns objective chance, but that it is not sufficiently fine-grained when it comes to epistemic possibilities. To make this distinction more clear with respect to Wilcox' thought experiment, let us compare his setting to the Ellsberg urn. Given that, in the Ellsberg urn, we have information on the proportion of red balls, distributing the probability values for the proposition that the next ball drawn is blue across all values not excluded by the evidence yields $\mathcal{P}(Ball_{Blue}) = [0, 2/3]$. As is standard in analyzing the Ellsberg urn based on imprecise credences, the imprecise degree of belief in its interval representation is derived by taking the convex set over all possible distributions of the agent's representor (Steele 2007; Bradley 2019a). This is still the case if the number of balls in the urn is known to the agent. Since in Wilcox' thought experiment the agent has no information on the actual proportion of balls, the agent's imprecise degree is maximally uncommitted for any fixed number of balls by the same analysis. That is, even though each possible color composition (i.e. the epistemic possibilities) in the orthogonal representation constitutes objective probabilities for specific balls being drawn when a fixed number of balls is given, these probabilities are not accounted for in the imprecise degree of belief of an agent. In the next section, a more fine-grained imprecise degree of belief is developed by taking into account the contribution of each epistemic possibility through counting.

Granting that the thought experiment does, in fact, rely on epistemic possibilities and not on evidence regarding objective chances, it is still questionable whether a model of belief should be capable of capturing mere epistemic possibilities. More precisely, one could argue that epistemic possibilities are no form of evidence and that an agent's belief should only reflect the evidence she has.

Nonetheless, if one decides that a correct model of belief should be capable of capturing epistemic possibilities, the wide interval view with imprecise probabilities falls short. Thus, in the following, I present an extension of the MSV framework that allows to reconcile the wide interval view with epistemic possibilities. This extension can then be seen as a feature that can be added to MSV in case epistemic possibilities need to be accounted for. It operates on what can be called second-order beliefs and its objective is to accommodate two seemingly conflicting issues:

1. The beliefs of an agent in the thought experiment are maximally uncommitted and can be represented by the interval [0, 1] as imprecise degree of belief in the relevant propositions.
2. Even though the agent represents the belief in ball 1 and 2 being of a specific color by the same imprecise degree of belief, we can reasonably state that the agent is more confident in ball 1 being of a specific color than ball 2.

4.2 Second-Order Beliefs

In this section, I show how MSV can be extended in order to capture epistemic possibilities, and, with that, Wilcox' thought experiment. This extension is based on what can be called *second-order beliefs*.

Originally, supervaluationism only considers *admissible precisification*. For second-order beliefs, I want to take into consideration all *possible precisifications*:

Definition 12 (Possible Precisification) In evidentially symmetric cases of total stochastic ignorance where there is a uniquely correct partition of finitely many possible outcomes, the *possible precisifications* are the combinatoric possibilities of all events for all specific values that can occur in that situation.

Since Wilcox' thought experiment is such a situation, I regard as possible precisifications all combinatoric possibilities of how a number of balls and possible colors can be distributed based on the orthogonal representation of ambiguous urns.

Next, I present a method to evaluate the possible precisifications of a specific situation based on MSV. To begin with, observe that possible precisifications constitute probability functions. Each probability function, in turn, is a voter in a voting committee. This idea can be illustrated by Wilcox' thought experiment:

Consider the following two propositions: Proposition 1: The next ball drawn from urn 1 is black. Proposition 2: The next ball drawn from urn 2 is black. Now, possible precisifications constitute probability functions since every possible precisification assigns a precise probability to proposition 1 and proposition 2. Take for example the possible precisification that all balls in urn 1 are white. This possible precisification then assigns a probability of 0 to proposition 1. Moreover, it can happen that several possible precisifications assign the same probability value to the same proposition. In that case, the voter in the voting committee representing those precise values receives as many votes as possible precisifications it represents. For instance, in the simplified Ellsberg urn example, this is the case for the (BY) and (YB) precisification as they assign the same probability to a drawn ball being black. Once we know all the probability values and numbers of votes, we can let the committee vote on the propositions in question. If one proposition receives a higher percentage of the total votes than another proposition, we say that the agent is predominantly more confident in the former proposition.

As possible precisifications, and probability functions more generally, can assign the same value to a given proposition, the agent's imprecise degree of belief is slightly altered in order to allow for duplicates, i.e. multisets, of probability values. This allows to count and succinctly represent the number of votes derived from the possible precisifications. That is, it is convenient to represent all assigned probabilities by the representor and their corresponding number of votes in a similar way as the agent's imprecise degree of belief. Let's call this *Multiset Degree of Belief*:

Definition 13 (Multiset Degree of Belief) Let A be some proposition. Suppose, $a, \dots, n$ denote probability values for A derived from the agents representor and $x, y, z \in \mathbb{N}$ denote values for the number of votes. The *multiset degree of belief* is then given by: $\mathcal{E}(A) = \{a_x, b_y, \dots, n_z\}$.

Having defined the multiset degree of belief, we can specify the idea of an agent being more confident in some proposition than another based on epistemic possibilities. For this, I define what I call *second-order confidence*:

Definition 14 (Second-Order Confidence) If an agent's imprecise degrees of belief in two propositions A and B are equal and if the agent is predominantly more confident in proposition A based on the multiset degree of belief, we say that the agent is more *second-order confident* in A.

4.3 Second Order Beliefs and Wilcox' Thought Experiment

In the following, I argue that an agent is more second-order confident in proposition 1 than proposition 2. This is done in two steps: First, I illustrate how this can be done for two (computationally less demanding) variants of Wilcox' experiment with fewer possible colors and few balls; second, I generalize these results to any number of possible colors and balls.

The first part is achieved in four steps: (i) I first state all possible precisifications as well as (ii) compute their corresponding probability function and (iii) the number of votes for each member of the voting committee. Finally, (iv) I determine the proportion of votes for each proposition. In order to compare variants of the thought experiment with different numbers of balls and possible colors more easily, I make the following two assumptions: (i) Urn 1 and urn 2 always contain the same number of balls. (ii) The intuition behind Wilcox' experiment also holds for urns where less than 10 possible colors can be realized as long as urn 2 contains balls of more possible colors than urn 1. Thus, in each variant, urn 1 contains balls that are either black or white and urn 2 contains balls that are either black or white or of at least one additional possible color.

Before starting the analysis, two aspects have to be discussed in more detail: First, it has to be determined how the members of the voting committee do, in fact, vote. As stated earlier, each possible precisification constitutes a specific probability function that assigns a probability value to an event. If that probability value is greater than 0.5, that is, if the event is more likely to occur than not according to that function, then the member representing this function votes for the event in question. Second, it is important to highlight that the combinatoric analysis follows the orthogonal representation of ambiguous urns given that Wilcox' urn experiment is based only on the subjective type of uncertainty discussed earlier.

4.3.1 The Urn Experiment

Now we are ready to state a few examples that illustrate Wilcox' thought experiment and the underlying epistemic possibilities:

Proposition 1 with 3 balls. For an urn with three balls of two possible colors there are eigth possible precisifications. According to one possible precisification, all balls are white. According to three, there is exactly one black ball. These precisifications constitute the probability function which assigns proposition 1 a probability of 1/3. Since there is exactly one black ball in the urn according to this precisification, the probability of drawing a black ball from three balls in total is 1/3. The member of the voting committee representing this probability function receives three votes. Furthermore, there are three possible precisifications for there being exactly two black balls. The corresponding probability is 2/3 and the voting member receives three votes. Finally, according to one precisification, all balls are black. This yields the following multiset degree of belief: $\mathcal{E}(\textit{Proposition 1}) = \{0_1, 1/3_3, 2/3_3, 1_1\}$.

In this example, proposition 1 receives four votes in total. Three from $2/3_3$ and one from 1_1. Thus, proposition 1 receives 50% of the total votes.

Proposition 1 with 6 balls. For six balls with two colors, there are 64 possible precisifications. According to one, all balls are white. According to six, there are exactly five black balls or only one. There are 15 precisifications where exactly two or four balls are black and 20 precisifications where three are black. Combined with their corresponding probabilities, we get: $\mathcal{E}(\textit{Proposition 1}) = \{0_1, 1/6_6, 2/6_{15}, 3/6_{20}, 4/6_{15}, 5/6_6, 1/6_1\}$.

Here, proposition 1 receives 22 votes. It is important to note that the member representing $3/6_{20}$ refrains from voting because the function it represents assigns a probability of 1/2 to this event. 22 votes are 34% of the total votes.

Proposition 2 with 3 balls and 3 colors. There are 27 possible precisifications in total. Suppose, the balls are either black, white, or blue. According to eight precisifications, there is no black ball. This is the case when all balls are either white or blue, for instance. This precisification assigns proposition 2 a probability of zero and receives eight votes. Moreover, there are twelve precisifications where we have exactly one black ball, six precisifications where there are exactly two black balls and one precisification where all balls are black. This gives us: $\mathcal{E}(\textit{Proposition 2}) = \{0_8, 1/3_{12}, 2/3_6, 1_1\}$. In this scenario, proposition 2 receives seven out of the total 27 votes. That is 26%.

Proposition 2 with 6 balls and 6 colors. For this scenario, there exists 46,656 possible precisifications. According to 15,625 there is no black ball and according to 18,750 there is exactly one. Moreover, there are 9375 precisifications where we have two black balls, 2500 where there are three, 375 where we have four black balls, 30 for exactly five and one where all balls are black. This yields the following multiset degree of belief:

$\mathcal{E}(Proposition\ 2) = \{0_{15625}, 1/6_{18750}, 2/6_{9375}, 3/6_{2500}, 4/6_{375}, 5/6_{30}, 1/6_{1}\}$. In this final example, proposition 2 receives 406 votes, or, 0.9%.

To sum up, proposition 1 with three and six balls receives 50% and 34% respectively. Whereas proposition 2 with the same number of balls only receives 26% and 0.6% of the total votes. For these examples, it is clear that a greater proportion of members of the multiset degree of belief vote for proposition 1. With that, based on these examples, the agent is predominantly more confident in proposition 1 than proposition 2. Moreover, according to the wide interval view, the agent assigns the same imprecise degree of belief to both propositions. That is, $\mathcal{P}(Proposition\,1) = [0, 1]$ and $\mathcal{P}(Proposition\,2) = [0, 1]$. Thus, the agent is more second-order confident in proposition 1 than proposition 2 for those specific values.

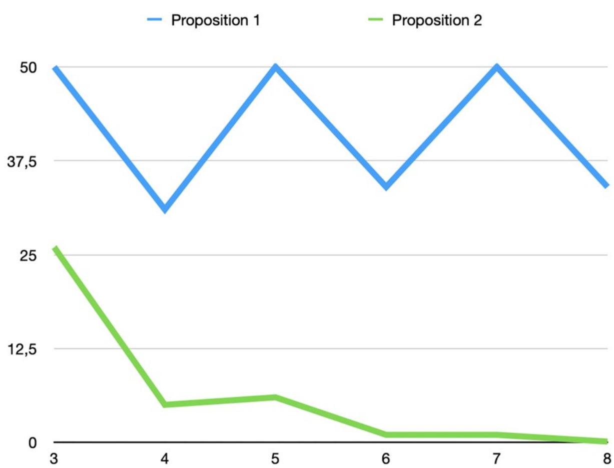
Additionally, this can be illustrated graphically for further examples. Figure 2 displays the share of votes depending on the number of balls and colors in the thought experiment. The x-axis is the number of balls from three to eight. For proposition 1 that implies two colors for any number of balls and for proposition 2 the number of colors equals the number of balls. The y-axis is the share of votes each proposition receives from the multiset degree of belief for the particular value.

From the graph it is clear that the proportion of votes for proposition 1 oscillates between roughly 1/3 and 50% depending on whether we have an even or uneven number of balls. When it comes to proposition 2, however, we can see that the share of votes starts at approximately 25% for three balls, falls to 5% for four balls and drops to 1% for only six balls. This illustrates that for all possible precisifications represented here, the agent is predominantly more confident in proposition 1 than in proposition 2.

Finally, this result can further be generalized to any number of possible precisifications: *Generalization*. For any number of balls and number of colors, an agent is more second-order confident in proposition 1 than proposition 2 in case urn 2 contains balls of possibly more colors than urn 1. In particular, this is the case for Wilcox' thought experiment. To see this, consider the following line of reasoning:

Let n be the number of balls and m be the number of colors. The total number of possible combinations then is m^n. For proposition 1, we have $m = 2$. Let k be the number of balls of a specific color. Taking the binomial coefficient, we compute the number of possibilities as follows:

Fig. 2 Share of votes depending of the number of colors and balls



$$\binom{n}{k} \tag{1}$$

For instance, the number of possibilities that exactly two balls are black from an urn that contains three balls which could be either black or white is:

$$\binom{3}{2} = 3 \tag{2}$$

It is important to note that the binomial coefficient is symmetric with regard to k and $n - k$. More formally:

$$\binom{n}{k} = \binom{n}{n-k} \tag{3}$$

With that, we also get:

$$\binom{3}{1} = 3 \tag{4}$$

It is for this reason that for an uneven number of balls, 50% of the total votes are in favor of proposition 1 whereas for an even number of balls the member of the voting committee representing $Pr\,(\textit{Proposition 1}) = 50\%$ refrains from voting.

When it comes to proposition 2, we cannot directly apply the binomial coefficient since for $m > 2$ the distribution of possible combinations is not symmetric. To see this, assume we wanted to compute the number of possibilities for there being exactly three black balls. Since we now have more than two possible colors, the remaining balls are not necessarily of the same color. Thus, the distribution of the colors of the remaining balls has to be computed independently and multiplied with the binomial coefficient.

With that, we can compute the number of possibilities for a specific number of balls being of one color as follows:

$$\binom{n}{k} \times (m-1)^{(n-k)} \tag{5}$$

Assume, for instance, that we want to compute the number of possibilities for there being exactly two black balls in an urn that contains six balls of six possible colors (including black):

$$\binom{6}{2} \times (6-1)^{(6-2)} = 9375. \tag{6}$$

If this situation were symmetric, the number of possibilities for there being exactly four black balls would have to sum up to 9375 as well. However, there are only 375 such possibilities. This fact explains, moreover, why the share of votes is so small for proposition 2: Where the total number of combinations grows with m^n, the number of combinations for more than two colors grows primarily due to $(m-1)^{(n-k)}$.

Now, only those members of the voting committee vote for proposition 2 that represent probability functions such that $Pr\,(\textit{Proposition 2}) > 50\%$. That is the case for $k > \frac{n}{2}$. However, as illustrated, the number of possibilities for $k < \frac{n}{2}$ is obviously already significantly larger for small values for n. With that, we have for arbitrary values for m and n that the share of votes for proposition 2 is smaller than for proposition 1. Moreover, this was shown without using the assumptions made earlier to simplify the comparison of different versions of the thought experiment. Thus, the agent is predominantly more confident in proposition 1 than proposition 2 for all possible precisifications. Since, according to the wide interval view, the agent assigns the imprecise degree of belief to both propositions for any number of balls and colors, the agent is more second-order confident in proposition 1 than proposition 2.

With that, it was possible to construct a model of second-order confidence which captures epistemic possibilities based on MSV. Moreover, it allows to reconcile both seemingly conflicting issues: It assigns the same imprecise degree of belief to proposition 1 and proposition 2, but the agent is more (second-order) confident in the first proposition than in the latter.

5 Conclusion

In a nutshell, Wilcox argues that the wide interval view is incompatible with the fundamental intuition behind his thought experiment. This thought experiment relies on taking into account epistemic possibilities. In this paper, I showed that the wide interval view is, in fact, compatible with the fundamental intuition if it is embedded into modified supervaluationism. For this, a further feature has to be added to this framework. Namely, second-order confidence. Although it is questionable whether an agent's degrees of belief should reflect epistemic possibilities, it can be accounted for by this framework. This, I see as an advantage of MSV and the wide interval view: This view is not committed to epistemic possibilities but flexible enough to allow such an extension.

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References

Bradley, Seamus, and Katie Steele. 2014. Should Subjective Probabilities be Sharp. *Episteme* 11 (3): 277–289.

Bradley, Seamus. 2015. How to Choose Among Choice Functions. In *Proceedings of the Ninth International Symposium on Imprecise Probability: Theories and Applications*. https://www.sipta.org/isipta15/data/paper/9.pdf. Accessed 30 August 2021.

Bradley, Seamus. 2019a. Imprecise Probabilities. The Stanford Encyclopedia of Philosophy (Spring 2019 Edition). https://plato.stanford.edu/archives/spr2019/entries/imprecise-probabilities. Accessed 30 August 2021.

Bradley, Seamus. 2019b. A Counterexample to Three Imprecise Decision Theories. *Theoria* 85 (1): 18–30.

Elga, Adam. 2010. Subjective Probabilities Should be Sharp. *Philosophers' Imprint* 10.

Epstein, Larry G. 1999. A Definition of Uncertainty Aversion. *Review of Economic Studies* 66: 579–608.

Joyce, James M. 2005. How Probabilities Reflect Evidence. *Philosophical Perspectives* 19: 154–178.

Joyce, James M. 2010. A Defense of Imprecise Credences in Inference and Decision Making. *Philosophical Perspectives* 24: 281–323.

Karge, Jonas. 2021. A Modified Supervaluationist Framework for Decision-Making. *Logos & Episteme* 2: 175–191.

Keefe, Rosanna. 2008. Vagueness: Supervaluationism. *Philosophy Compass* 3 (2): 315–324.

Machina, Mark. 2011. Event-Separability in the Ellsberg urn. *Economic Theory* 48: 425–436.

Mahtani, Anna. 2019. Imprecise Probabilities. In *Open Handbook of Formal Epistemology*, ed. Richard Pettigrew and Jonathan Weisberg, 107–130. Ontario: PhilPapers Foundation.

Rinard, Susanna. 2015. A Decision Theory for Imprecise Probabilities. *Philosophers' Imprint* 15: 7.

Schoenfield, Miriam. 2012. Chilling out on epistemic rationality. *Philosophical Studies* 158: 197–219.

Steele, Katie. 2007. Distinguishing Indeterminate Belief from "Risk-Averse" Preferences. *Synthese* 158: 189–205.

Varzi, Achille C. 2007. Supervaluationism and its Logics. *Mind* 116: 633–676.

Wilcox, John. 2020. An Argument for the Principle of Indifference and Against the Wide Interval View. *Journal for General Philosophy of Science* 51: 65–87.