How to Justify the Symmetrization Postulate in Quantum Mechanics

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Abstract

The aim of this paper is to reconstruct and correct one argument (known in the literature as the exchange degeneracy argument) in support of the symmetrization postulate in quantum mechanics. I identify the central premise of the argument as a thesis specifying a particular ontic (or epistemic) property of quantum superpositions. The precise form of this thesis depends on some underlying assumptions of a metaphysical character (concerning identifications of objects across possible worlds, or representations *de re*). I compare the exchange degeneracy argument with alternative formal arguments for the symmetrization postulate, and I discuss the role and meaning of labels in the symmetric/antisymmetric representations of the states of many particles.

Keywords Symmetrization postulate · Permutation invariance · Exchange degeneracy · Quantum superposition · Indiscernibility · Transworld identity

1 Introduction

Much of the recent discussions on the metaphysical consequences of quantum mechanics take their cue from the way the quantum formalism treats systems of many particles of the same type. The central philosophical question considered in this context is whether quantum objects can be discerned by their properties or relations, with the most prominent answer stating that particles belonging to a given category are completely indiscernible and therefore violate the Leibnizian Principle of the Identity of Indiscernibles (PII). Numerous strategies of defending PII have been considered, including the most popular strategy based on the notion of weak discernibility.¹ All these philosophical considerations take for

¹ Among the most-often quoted works that promulgate the view that PII is violated in QM are French and Redhead (1988), Redhead and Teller (1992), Butterfield (1993), Huggett (2003), French and Rickles (2003). The weak discernibility strategy of defending PII has been developed in Saunders (2003; 2006), Muller and Saunders (2008), Muller and Seevinck (2009), Muller (2011; 2015); for a critical overview see Bigaj (2015a). An alternative method of defending PII in quantum mechanics based on the so-called summing strategy is proposed in Friebe (2014).

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granted the well-known fact that systems of many particles of the same type are described in a permutation-invariant way. More specifically, the quantum-mechanical formalism adopts the symmetrization postulate (SP) which stipulates that the available states of many particles of the same type have to be either exclusively symmetric (bosons) or exclusively antisymmetric (fermions). However, surprisingly little attention has been paid by philosophers to how quantum mechanics justifies the symmetrization postulate. The present paper attempts to fill this lacuna.

Several philosophically salient questions regarding SP can be posed. Besides the already mentioned question of its justification, we may inquire whether SP is unique to quantum mechanics, or is it a symptom of a broader phenomenon that may also be present in one form or another in classical physics. Another important task is to evaluate precisely all the metaphysical and physical consequences of SP, such as the above-mentioned indiscernibility thesis and the ensuing violation of the PII, the status of quantum particles as non-individuals and so on. I will touch upon some of these issues in this paper, but the main focus will be on the justification problem.

Generally speaking, principles such as SP can be argued for in many different ways and on many levels. We will distinguish three main types of arguments that may be used here, ordered with respect to their closeness to the ‘bedrock level’ of empirical data. First, there are low-level empirical predictions that can be taken to corroborate SP. These are mostly empirically verifiable facts about the behavior of large ensembles of quantum objects (quantum statistics—Fermi–Dirac for fermions and Bose–Einstein for bosons).² Further from specific empirical data are high-level physical and methodological considerations, based on criteria such as simplicity, formal elegance, verificationism and the like.³ Finally, even higher-level metaphysical or semantical assumptions (regarding, for instance, identifications of objects across possible worlds), may turn out to be necessary for SP to be properly justified. In the current discussions I will largely ignore the first type of arguments, turning my attention instead to the high-level physical and metaphysical justifications.

The plan of the article is as follows. In Sect. 2 I will make some terminological distinctions regarding the relations between physical objects and their mathematical representations, and I will specify the domain of applicability for the symmetrization postulate. In addition to that I will attempt to clarify some confusing remarks about the notion of so-called “identical” particles and their nature that persist in physical textbooks. Section 3 will present the basic structure of the argument from exchange degeneracy that is proposed in the literature in support of the symmetrization postulate. I will identify the key premise of the argument that is usually not spelled out correctly, and I will indicate that there is a certain conceptual problem with the structure of the argument. In Sect. 4 I will bring some metaphysical considerations on modality *de re* and transworld identity to bear on this issue. The corrected argument from exchange degeneracy is presented in Sect. 5, where I also emphasize that what distinguishes the quantum case of permutation-invariance from the

² Another spectacular example of an empirically verified prediction of the symmetrization postulate is provided by experiments aimed to test the Conserved Current Hypothesis (see Sakurai and Napolitano 2011, 449–450 for details). The experiments, performed in the early 1980s, involved a sequence of beta and alpha decays, which at the end produced a pair of alpha particles. The theoretical predictions regarding the correlations between the directions of the particles created in the process were made both with and without the symmetrization of the wave function of the two alpha particles, and it turned out that only the former method produced a fit with the experimental data.

³ Arguments based on the considerations of symmetry, or general physical principles such as the equivalence of all coordinate systems, clearly belong to this category.

classical case is the reliance on the superposition principle. Section 6 contains an alternative argument for SP based on the distinction between pure and mixed states. In Sect. 7 I will briefly address the issue of how to properly interpret labels present in the formal representations of the physical states of the same-type particles.

2 Preliminary Terminological Distinctions

The discussions presented in this paper will in part concern the delicate matter of the relations between physical reality and its mathematical representations, so it is appropriate to make clear certain assumptions and distinctions pertaining to this subject. Let $\mathcal{P}$ be a domain of physical entities of some type (for instance physical states of quantum systems), and $\mathcal{M}$ a domain of mathematical objects that serve as representations of the elements of $\mathcal{P}$ ($\mathcal{M}$ can be the set of vectors in a Hilbert space). In order to speak about a particular representation, we have to introduce a mapping m connecting elements of $\mathcal{P}$ with elements of $\mathcal{M}$. The pair $(\mathcal{M}, m)$ can be called a representational framework—note that the same mathematical domain can occur in many distinct representational frameworks, depending on the selected mapping m . Generally speaking, mapping m does not have to be a function, let alone a one-to-one function. If two elements of $\mathcal{P}$ are mapped onto one and the same element of $\mathcal{M}$, we say that the representational framework $(\mathcal{M}, m)$ is *ambiguous*. If one element of $\mathcal{P}$ is mapped onto more than one element of $\mathcal{M}$, we say that the representational framework is *redundant*.⁴ In what follows we will assume that there is no ambiguity, hence m defines a function from $\mathcal{M}$ to $\mathcal{P}$. However, this function is not assumed to be one-to-one, thus redundancy is not excluded. A well-known case of redundancy present in quantum mechanics is caused by the fact that two vectors that differ by a phase represent the same physical state. This redundancy can be eliminated by changing the domain of mathematical objects from vectors to rays, but the resulting mathematical structure loses certain nice mathematical features (for instance the set of rays is no longer a vector space). In the subsequent discussions I will nevertheless continue using vectors (“kets”) as mathematical representations of physical states. Whenever I use the term “state”, I mean by it an element of the physical domain $\mathcal{P}$.

The symmetrization postulate in quantum mechanics is supposed to be applied to the mathematical representations of the states of many particles of the same type (e.g. many electrons, many photons, neutrinos, etc.). A bit confusingly, particles of the same type are often referred to in the physical literature as “identical”. This is to a certain extent justified by the usage of the word “identical” in ordinary language, where it is typically taken to mean “indistinguishable”, or “looking alike” (as in “identical twins”).⁵ However, philosophers almost unanimously reserve the word “identity” to denote numerical identity (being one and the same object). For that reason I will refrain from using the term “identical particles”, replacing it with the longer phrase “particles of the same type”. Even more confusing is the way some textbooks explain the alleged central feature of particles of the same type, namely their “indistinguishability”. For instance, Asher Peres writes (2003, 126):

A quantum system may include several subsystems of identical nature, which are physically *indistinguishable*. Any test performed on the quantum system treats all

⁴ I borrow these terminological distinctions from Jeremy Butterfield.

⁵ Thanks to James Ladyman for pointing this out.

these subsystems in the same way, and is indifferent to a permutation of the labels that we attribute to the identical subsystems for computational purposes.

The quoted passage seems to suggest that for instance two electrons cannot possess any properties that would differentiate between them by means of physical tests. And yet a couple of lines later Peres says something to the contrary.

Note that it *is* meaningful to ask questions about the electron closest to the nucleus, or about the most distant one—but *not* about the electron labelled 1 or 2.

The (relational) property of being the closest to the nucleus seems to differentiate the two electrons (as it is possessed by one of them and not the other), even though we somehow cannot use this property to attach labels to the electrons (in the tensor product formalism). We will return to the issue of the connection between labels and physical properties later. For now we will move on to an alternative and slightly less baffling characteristic of systems of the same type taken from another well-known textbook (Cohen-Tannoudji et al. 1978, 1371)⁶:

Two particles are said to be identical if all their intrinsic properties (mass, spin, charge, etc.) are exactly the same: no experiment can distinguish one from the other.

While it is still unclear why experiments cannot distinguish between, let us say, the electron closer to the nucleus and the electron further from the nucleus, at least we now know that “identical” particles (i.e. particles of the same type) are defined as those that share a particular subcategory of their properties, namely the so-called intrinsic properties. In the current context the term “intrinsic property” refers to properties that are state-independent, and not to non-relational properties (i.e. properties that do not involve other objects). In other words, an intrinsic property of a particle does not change over time, and therefore can be used as a defining characteristic of the type that the particle belongs to.

Having defined what we mean by particles of the same type, we can now formulate the symmetrization postulate whose justification we are seeking. Limiting ourselves to the case of two-particle systems, we can adopt the following formulation:

- (SP) For any system consisting of two particles 1 and 2 of the same type (and therefore sharing all their state-independent properties), their joint state is represented either exclusively by vectors $|\psi(1, 2)\rangle$ that are symmetric with respect to permutations (i.e. $|\psi(1, 2)\rangle = |\psi(2, 1)\rangle$), or exclusively by vectors that are antisymmetric with respect to permutations ($|\psi(1, 2)\rangle = -|\psi(2, 1)\rangle$)

In the next section we will try to reconstruct a particular argument for SP given in the literature.

⁶ Adding to the confusion are remarks that are meant to explain specific quantum–mechanical reasons for the “indistinguishability” that Peres speaks about. For example Jun John Sakurai and Jim Napolitano in their textbook write “in quantum mechanics, however, identical particles are truly indistinguishable. This is because we cannot specify more than a complete set of commuting observables for each of the particles; in particular, we cannot label the particle by coloring it blue” (Sakurai and Napolitano 2011, 446). Peres continues in a similar vein “This issue [i.e. indistinguishability—TB] does not occur in classical physics, because classical objects have an inexhaustible set of attributes, and therefore are always distinguishable” (Peres 2003, 126). I have to admit that the connection between the existence of an inexhaustible set of attributes and the guaranteed possibility of distinguishing objects using those attributes eludes me.

3 The Exchange Degeneracy Argument: First Approach

One of the main textbook arguments given in support of the symmetrization postulate is the so-called argument from exchange degeneracy. In this section I will analyze in detail its original logical structure, as laid out for instance in Cohen-Tannoudji et al. (1978, 1375–1377). I will identify some gaps present in the argument, and will try to fill them in this and subsequent sections.

The starting point of the argument is the assumption that we have selected a system containing two particles of the same type, and that we have performed a preparatory measurement of the same observable on each of them, finding two incompatible results (e.g. one particle revealed its z -spin as having the value “up”, while the other revealed the value “down”). After this measurement, each particle taken separately would be characterized by a pure state corresponding to a particular value of the measured parameter. Thus, in the first approximation, we may be tempted to assign to the composite system a state that is the direct product of two orthogonal states: $|u\rangle \otimes |v\rangle$, where $\langle u|v\rangle = 0$. However, we can immediately notice that we might have chosen an alternative mathematical representation of the same physical state using the “permuted” ket $|v\rangle \otimes |u\rangle$, as no physical importance is attached to the choice which particle’s states are described in which Hilbert space. Yet kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$ are not only distinct but even orthogonal vectors in the tensor-product Hilbert space $\mathcal{H} \otimes \mathcal{H}$. Thus we have a clear case of representational redundancy here that we spoke about earlier: two distinct mathematical objects represent the same physical situation.⁷

However, the situation is arguably much more serious than a mere case of representation redundancy. The argument proceeds to show that the existence of two kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$ representing the same physical state leads to a contradiction. We can observe first that these two vectors span a two-dimensional subspace ε_{uv} consisting of all kets of the form $\alpha |u\rangle \otimes |v\rangle + \beta |v\rangle \otimes |u\rangle$, where α, β —any numbers. Next, Cohen-Tannoudji et al. invoke the superposition principle in order to argue that all vectors $\alpha |u\rangle \otimes |v\rangle + \beta |v\rangle \otimes |u\rangle$ should represent the same state as the state referred to by both product kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$ (p. 1376). But it may be proven that there is a legitimate empirical procedure that could distinguish between states represented by various vectors of the form $\alpha |u\rangle \otimes |v\rangle + \beta |v\rangle \otimes |u\rangle$. In order to do that, let us consider a Hermitian operator A acting in a one-particle Hilbert space $\mathcal{H}$, whose eigenvectors are orthogonal kets $\frac{1}{\sqrt{2}}(|u\rangle + |v\rangle)$ and $\frac{1}{\sqrt{2}}(|u\rangle - |v\rangle)$. It is easy to calculate that when we perform a measurement of the same observable A on each particle whose joint state is represented by the superposition $\alpha |u\rangle \otimes |v\rangle + \beta |v\rangle \otimes |u\rangle$, the probability of finding both particles in state $\frac{1}{\sqrt{2}}(|u\rangle + |v\rangle)$ will equal $|\frac{1}{2}(\alpha + \beta)|^2$. Hence this probability will be generally different for different values of α and β . It looks like some elements of the subspace ε_{uv} can be differentiated by means of physically meaningful experiments, and therefore they cannot possibly represent the same physical state.

⁷ The orthogonality of kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$ seems to suggest, according to the usual quantum-mechanical algorithm, that the probability of finding the system in the state represented by one ket is zero when the state is described by the other ket. Thus it is more than redundancy that we are facing here: two permuted states seem to be strongly incompatible with one another. However, this incompatibility does not lead to any observable effect that may be discovered in real experiments. Later in this section we will see how representational redundancy coupled with an additional assumption regarding superpositions can lead to differences that could be subject to experimental verification.

It is worth stressing that measuring the two-particle observable $A \otimes A$ is realizable in practice, as this observable is represented by a symmetric (permutation-invariant) Hermitian operator, and therefore does not require from us that we identify particles as bearing specific labels. In contrast to that, no symmetric operators exist that could differentiate between the original vector $|u\rangle \otimes |v\rangle$ and its permuted form $|v\rangle \otimes |u\rangle$. These vectors (or, more accurately, the corresponding states) could be distinguished only with the help of asymmetric tensor products of operators, for instance $A \otimes I$, where I is the identity operator. However, the standard commentary to this type of differentiability is that observables represented by such operators are unphysical, since in order to perform an appropriate measurement we would have to identify particles by their labels.⁸ We will return to this argument shortly. For now we should acknowledge the fact that the possibility of differentiating between various elements of the subspace ε_{uv} , revealed in the above argument does not rely on the identification of individual particles by their labels.

There is one crucial step in the argument reconstructed above that needs to be clarified. The authors mention the name of the principle of superposition, but what do they mean by that? Typically, the principle of superposition in quantum mechanics is interpreted as stating that any linear combination of two (or more) vectors represents a possible physical state (cf. Hughes 1989, 92). However, in the context of the argument from exchange degeneracy we clearly need a stronger principle. We need an assurance that if we superpose two distinct vectors which nevertheless represent the same state, the resulting mathematical object will represent the very same state. Thus I suggest introducing an interpretational principle which I will refer to as Ontic Conservativeness of Superpositions (OCS):

(OCS) If vectors $|\varphi\rangle$ and $|\psi\rangle$ represent the same physical state, then any linear combination $\alpha|\varphi\rangle + \beta|\psi\rangle$ represents the very same state

This seems like a plausible principle, given that the quantum-mechanical rule of thumb is to interpret the squared coefficients $|\alpha|^2$ and $|\beta|^2$ (assuming, of course, that $|\alpha|^2 + |\beta|^2 = 1$, and that the vectors $|\varphi\rangle$ and $|\psi\rangle$ are orthogonal) as representing the probabilities that the system can be found in states described, respectively, by $|\varphi\rangle$ or by $|\psi\rangle$. If $|\varphi\rangle$ and $|\psi\rangle$ denote the same physical state s , the probability that the system will be found in s equals 1, which apparently justifies OCS. And yet, as we have seen, adopting the unconditional validity of OCS spells trouble in the case of representational redundancy brought about by the existence of permuted kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$.

The argument from exchange degeneracy establishes that the assumption that all elements of subspace ε_{uv} should be admitted as representations of possible states violates the OCS principle. However, the argument, as it stands, does not give us a positive solution of the problem of the violation of OCS, let alone pick the specific solution based on the Symmetrization Postulate. Still, some approaches to the issue of representational redundancy are clearly excluded by the aforementioned argument. For instance, in classical physics we encounter a similar problem in that when we consider the phase space for two indistinguishable classical particles, any physical state of these particles that is not singular can be represented by any of the two distinct points: $\langle q_1, p_1; q_2, p_2 \rangle$ and $\langle q_2, p_2; q_1, p_1 \rangle$, where

⁸ As Peres writes “one may toy with the idea of devising “personalized” tests which would be sensitive to individual electrons—and it is indeed easy to write down vector bases corresponding to such tests—but this fantasy cannot be materialized in the laboratory” (Peres 2003, 126).

q_i is the position of the i -th particle and p_i its momentum.⁹ The standard treatment of this type of degeneracy is to identify the permuted points (to consider the reduced phase space with the permutation group “quotiented out”). But in the quantum case this method will not work. Given the assumed identification of the kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$, any vector of the form $\alpha |u\rangle \otimes |v\rangle + \beta |v\rangle \otimes |u\rangle$ should represent the same state, and yet—as we have seen—legitimate measurement procedures distinguish between various numerical values of α and β .¹⁰

Thus, the only way to avoid the contradiction seems to select exactly one vector from ϵ_{uv} that will represent the state originally represented by the kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$. It remains to be seen that this vector should be either symmetric or antisymmetric (we will return to this issue in Sect. 5). For now, however, we will focus on a different possible strategy of dealing with the challenge from exchange degeneracy. Perhaps we were too hasty in applying OCS to the states represented by permuted kets $|u\rangle \otimes |v\rangle$ and $|v\rangle \otimes |u\rangle$, because maybe these kets can be interpreted as referring to ontologically distinct physical states after all. In order to clarify this, we will have to delve deeper into the notion of permutation of particles.

4 Three Interpretations of Permutation

Let us go back to the original description of the empirical situation in which we have prepared two particles of the same kind in respective states $|u\rangle$ and $|v\rangle$. To facilitate further discussion let us use labels “1” and “2” in order to make a formal distinction between particles (these labels obviously reflect the order in which the tensor product of two states is taken). The original state of the composite system can be written now as $|u(1)\rangle \otimes |v(2)\rangle$. A natural interpretation of label “1” is that it refers to the particle which in the current physical situation has been prepared in state $|u\rangle$. However, we are not equating label “1” with the description “the particle that is in state $|u\rangle$ ”. Labels are supposed to pick out the same object regardless of which state it is in; descriptions can only help in singling out the object in question by (contingently) providing us with a link between the label and the corresponding object.

Our task now is to figure out an interpretation of the permuted vector $|v(1)\rangle \otimes |u(2)\rangle$. We should first distinguish between two possible interpretations of permutations: *passive* and *active*.¹¹ Under the passive interpretation a permutation amounts to a mere change in labels without any real change in the physical situation (in order to describe this change we would obviously have to refer to labels themselves, not to objects to which the labels are attached). We simply assume that the particle prepared in state $|u\rangle$ will be labeled 2, while label “1” will be reserved for the particle in state $|v\rangle$. Given that by assumption we do not change anything in the physical state of the system, the kets $|u(1)\rangle \otimes |v(2)\rangle$ and

⁹ The same applies in the case when the particles are distinguishable, but their identifying properties are dynamically irrelevant. See Saunders (2015, 176ff) for a discussion of the classical case of permutation invariance.

¹⁰ A more formal reason why the classical solution does not work in the quantum case is that the operation of ket addition is not invariant with respect to permutations, i.e. the result of summing two vectors representing the same equivalence class depends on which vector from the class we select.

¹¹ This distinction is obviously modelled on the analogous concepts of passive and active coordinate transformations in the context of General Relativity and the famous hole argument (see Norton 2015).

$|v(1)\rangle \otimes |u(2)\rangle$ must represent the same physical state. However, each ket functions in a different representational framework, and for that reason they should not be considered as being elements of the same Hilbert space. To clarify this point further, let us recall that in order to represent a given physical situation using a particular mathematical object we need two elements: a domain of appropriate mathematical objects $\mathcal{M}$ (e.g. vectors in a Hilbert space) and an interpretational rule (mapping) m connecting these objects to their physical counterparts. The same mathematical domain $\mathcal{M}$ combined with a different interpretational rule m' gives rise to a different representational framework. Since, under the passive interpretation, vectors $|u(1)\rangle \otimes |v(2)\rangle$ and $|v(1)\rangle \otimes |u(2)\rangle$ belong to two different representational frameworks, it does not make much physical sense to add them, even though mathematically such a procedure is perfectly admissible. Consequently, if we insist to consider the sum $\alpha |u(1)\rangle \otimes |v(2)\rangle + \beta |v(1)\rangle \otimes |u(2)\rangle$ as representing a particular physical state (as is required in the argument from exchange degeneracy), the second summand cannot be taken as resulting from the first by applying to it the permutation in the passive sense. The exchange degeneracy argument presupposes the active interpretation of permutations, not the passive one.

Incidentally, observe that under the passive interpretation of permutation the previously considered asymmetric observables of the form $A \otimes I$ (where I is the identity operator) are perfectly acceptable and physically meaningful. The corresponding physical procedure is as follows: select the particle prepared in state $|u\rangle$ (for instance select the electron that passed through the Stern–Gerlach apparatus in the upward direction), and then perform on this particle a measurement corresponding to A while leaving the other particle alone. However, note that if we wanted to apply the same experiment to the case when the system's state is described by the permuted vector $|v(1)\rangle \otimes |u(2)\rangle$, we would have to represent the measured observable with the help of the permuted operator $I \otimes A$, since vector $|v(1)\rangle \otimes |u(2)\rangle$ functions in a different representational framework, and we have to be consistent. So, obviously and predictably, measuring the physical observable corresponding to the operator $A \otimes I$ (in the first framework) cannot distinguish between the states represented by the vectors $|u(1)\rangle \otimes |v(2)\rangle$ and $|v(1)\rangle \otimes |u(2)\rangle$.

The active interpretation of permutations, on the other hand, does not presuppose any change in the representational framework. Instead, we are supposed to consider an alternative physical situation in which particles are literally “swapped”. That is, the particle labeled 1, which in the original scenario was prepared in state $|u\rangle$, is now assumed to be in state $|v\rangle$, while the particle labeled 2 takes on state $|u\rangle$ instead of $|v\rangle$. However, this broad description calls for further explanations. What does it mean that the “same” particle receives a physical characteristic different from the one possessed actually? What are the meaning and criteria of sameness here? By considering questions like these we are entering the realm of the metaphysics of modality, in particular the problem of modality *de re*. There are two major approaches to the question of how to interpret counterfactual statements regarding a particular object: one propounded by Saul Kripke, the other one by David Lewis.¹² According to Kripke, the answer to this question is rather straightforward: a counterfactual statement attributing a property P to an actual object a describes a possible scenario in which a possesses property P . It has to be stressed that this scenario involves an object that is *numerically identical* with the actual object a , where numerical identity is not in any way reducible to qualitative properties. Kripke insists that characterizations of

¹² These interpretations are presented and discussed in Kripke (1980) and Lewis (1986).

possible situations (possible worlds) do not have to be given in purely qualitative terms. It is perfectly acceptable to include in such characterizations reference to individual objects (“rigid designators” in Kripke’s terminology). This happens, for instance, when we say “Let’s imagine that Donald Trump lost the 2016 election”. According to Kripke, what we describe here is a world in which Donald Trump himself, and not someone satisfying some qualitative descriptions of Trump, lost the election.¹³

In contrast to that, Lewis subscribes to the view that possible worlds ought to be described exclusively in qualitative language. In order to evaluate a modal statement regarding a particular individual, we have to identify its *counterpart* using the criterion of qualitative similarity. Lewis rejects one variant of haecceitism, according to which there may be two possible worlds which do not differ qualitatively but do differ with respect to their representations *de re*. In other words, for Lewis representations *de re* supervene on the totality of qualitative facts.¹⁴

Adopting the Kripke-style approach first we should conclude that vectors $|u(1)\rangle \otimes |v(2)\rangle$ and $|v(1)\rangle \otimes |u(2)\rangle$ represent distinct but qualitatively (and thus empirically) indistinguishable physical situations. The indistinguishability stems from the fact that particles labeled 1 and 2 do not have any identifying qualitative features other than their contingently occupied states. In spite of that it is still assumed that the states of affairs represented in statements “Particle 1 occupies state $|u\rangle$ ” and “Particle 2 occupies state $|u\rangle$ ” are numerically distinct. For that reason we will need an assumption slightly stronger than OCS in order to reconstruct the argument from exchange degeneracy. Namely, we ought to assume the following general principle regarding the meaning of superpositions (let’s call it Epistemic Conservativeness of Superpositions—ECS):

(ECS) If vectors $|\varphi\rangle$ and $|\psi\rangle$ represent physical states that are *empirically indistinguishable*, then any linear combination $\alpha|\varphi\rangle + \beta|\psi\rangle$ represents a state that is also *empirically indistinguishable* from them

Arguably, principle OCS does not logically entail ECS, as it is conceivable even with OCS being true that the ontological distinctness between the indistinguishable states represented by $|\varphi\rangle$ and $|\psi\rangle$ could give rise to some empirically discernible differences when superposing these states. On the other hand, because identity logically implies empirical indistinguishability, from the ECS principle it follows that superposing two vectors that represent the same state creates a state indistinguishable from the original one. This contention is strictly speaking weaker than principle OCS (we may call it Ontic-Epistemic Conservativeness of Superpositions—OECS), as OCS stipulates that in this case the superposed vector represents the very same state, but OECS is entirely sufficient for the exchange degeneracy argument to go through.

Under the Lewisian approach we have to decide first what qualitative features should be used in order to identify particles labeled 1 and 2 in the counterfactual scenario described by the permuted vector. It is most natural to assume that the criterion of transworld

¹³ However, we have to remember that in some contexts descriptions can act as rigid designators. We will ignore this complication here.

¹⁴ The distinction between the three interpretations of permutations: passive, the active interpretation Kripkean style and the active interpretation Lewisian style, roughly corresponds to my earlier distinction between exchange of labels, exchange of haecceities, and exchange of essences, as introduced in Bigaj (2015b).

identity should be based on the intrinsic, or state-independent properties of particles. In other words, the properties that we will use to identify for instance a particular electron in a counterfactual scenario will be its mass, charge, total spin, etc. The permuted vector $lv(1) \otimes lu(2)$ will characterize a situation in which a particle possessing all the intrinsic properties of the system identified in the actual world as bearing label 1 will occupy state lv , while another particle with all the intrinsic properties of system 2 will now be in state lu . But given that all particles of the same type have identical intrinsic properties, this is just a redescription of the very same situation as the one represented by the original vector $lu(1) \otimes lv(2)$. In other words, vectors $lu(1) \otimes lv(2)$ and $lv(1) \otimes lu(2)$ correspond to the same qualitative description “one electron in state lu , another in state lv ”, and this description should be associated with the same physical situation in order to avoid postulating worlds distinguishable by their haecceitistic differences only. It should be clear that in this case the appropriate assumption needed in the exchange degeneracy argument ought to be the original principle OCS.

5 The Exchange Degeneracy Argument Reconsidered

It turns out that we can recover the original argument from exchange degeneracy under both available interpretations of modality *de re*, and consequently, of permutations of particles. However, the puzzle with this argument that we have identified earlier remains unsolved. The argument clearly establishes that some vectors in the subspace ε_{uv} spanned by $lu(1) \otimes lv(2)$ and $lv(1) \otimes lu(2)$ represent empirically distinguishable states, if they represent anything at all. How, then, can we choose one vector out of ε_{uv} in order to stand for the whole subspace? And why does it have to be a vector that is either symmetric or antisymmetric?

Fortunately, we already have all the ingredients necessary to reformulate the argument so that it can properly support the symmetrization postulate in the form of the superselection rule that chooses either symmetric or antisymmetric vectors as the only representations of physical states of same-type particles. The focal point of this reconstruction will be the principle OCS (or ECS, if we decide to follow the Kripke-style account of permutations). I claim that the argument from exchange degeneracy should aim at saving OCS (ECS), and it should proceed to show that the only acceptable way to keep this principle is to limit the available states to either symmetric or antisymmetric ones. We have already established that in order to avoid the conflict with OCS (ECS), at most one (up to the multiplicative constant) vector from ε_{uv} can represent an admissible state, since any two non-parallel vectors span the entire subspace, hence if any two non-parallel vectors are available, so is the entire subspace ε_{uv} . It only remains to argue that this unique ‘representative’ vector needs to possess the required type of symmetry.

From a formal point of view, it is rather straightforward to observe that selecting for this role any vector from ε_{uv} that is neither symmetric nor antisymmetric causes serious problems. Naturally, we would want to continue using the permutation operators as admissible mathematical objects representing *prima facie* legitimate physical procedures (swapping particles). However, if we assume that a vector $lv(1, 2)$ from ε_{uv} which is neither symmetric nor antisymmetric represents an available physical state of two particles, the permuted vector $lv(2, 1)$ is no longer admissible. Hence the permutation P_{12} takes us outside the set of accessible states, and therefore is not a well-defined operation on this set. This problem affects not only permutations but also other seemingly admissible operators. For instance,

if we decided to admit the product state $|u(1)\rangle \otimes |v(2)\rangle$ as the stand-in for the subspace ε_{uv} , then the symmetric projection operator $|\varphi\rangle\langle\varphi| \otimes |\psi\rangle\langle\psi| + |\psi\rangle\langle\psi| \otimes |\varphi\rangle\langle\varphi|$, where $|\varphi\rangle = \frac{1}{\sqrt{2}}(|u\rangle + |v\rangle)$ and $|\psi\rangle = \frac{1}{\sqrt{2}}(|u\rangle - |v\rangle)$, will take us outside the set of admissible vectors when applied to $|u(1)\rangle \otimes |v(2)\rangle$.¹⁵ And yet this projector represents a possible outcome of a legitimate measurement procedure.

The symmetric and antisymmetric sectors of the total tensor product Hilbert space have the nice property of being invariant under the action of all symmetric operators and the permutation operators. This gives us a strong reason to adopt SP as the best way to avoid the violation of OCS (ECS). So the argument from exchange degeneracy in its corrected version proceeds as follows. We start with the assumption that both vectors $|u(1)\rangle \otimes |v(2)\rangle$ and $|v(1)\rangle \otimes |u(2)\rangle$ represent admissible physical states of two same-type particles. Then we show that this assumption violates one of the two principles regarding the operation of superposition: either OCS or ECS (depending on our preferred metaphysical conception of modality *de re* and consequently on our interpretation of permutations). This leads to the conclusion that only one vector out of the subspace ε_{uv} can be admitted as representing the physical state initially associated with the kets $|u(1)\rangle \otimes |v(2)\rangle$ and $|v(1)\rangle \otimes |u(2)\rangle$. Finally, we argue that only restrictions to symmetric or antisymmetric vectors respect the condition of invariance under the action of physically meaningful operators, including the permutation operators.

It is worth stressing that this argument does not rely *exclusively* on the phenomenon of representational redundancy, as is commonly (and mistakenly) claimed. The main purpose of adopting the symmetrization postulate is not to merely eliminate the permutation-based redundancy (as we have seen, there are many alternative ways to achieve just that), but to make sure that there will be no interpretational inconsistency brought about by this redundancy in the mathematical formalism of quantum mechanics. The inconsistency revealed in the argument from exchange degeneracy manifests itself in the fact that various kets created by superposing vectors differing only with respect to the permutation of particles, may nevertheless represent empirically distinct states of affairs, if they represent anything at all. To bring this inconsistency to the surface, we have introduced the interpretational principle OCS, which makes it explicit that we should not be able to create a new physical state out of a combination of distinct mathematical objects that redundantly represent one and the same physical state. In the quantum case, as opposed to the classical case (which lacks the counterpart of quantum superpositions), the deeper reason for eliminating the permutation redundancy is to save OCS. The restriction of available states to either the symmetric or antisymmetric sectors achieves that objective, and in addition is the only solution that makes the sets of available states invariant under the action of admissible operators.

6 Alternative Arguments for SP

The argument from exchange degeneracy, as interpreted in the previous section, is by no means the only high-level justification of the symmetrization postulate one can propose. As a possible alternative, I would like to briefly discuss a formal argument due to Adam Caulton (private communication). I believe this argument might be preferred by

¹⁵ Similar projectors can be produced for other vectors in ε_{uv} which are neither symmetric nor antisymmetric.

many theoretical physicists due to its focus on some mathematical aspects of the quantum description of systems of many particles. The starting point of the reasoning is the assumption that the algebra of operators $\mathcal{A}$ representing physical properties of systems of two particles of the same type should only contain permutation-invariant (i.e. symmetric) operators. We have already touched upon this assumption in previous sections—more on that will follow. Now, it is rather straightforward to show that if A is one of the symmetric operators in $\mathcal{A}$, and $|\psi_A\rangle$ and $|\psi_S\rangle$ are, respectively, antisymmetric and symmetric states of two particles, then the cross-element $\langle\psi_S|A|\psi_A\rangle$ will be equal 0.

This can be shown as follows (P_{12} denotes the operation of permutation, which is its own inverse):

$$\begin{aligned}\langle\psi_S|A|\psi_A\rangle &= \langle\psi_S|P_{12}AP_{12}|\psi_A\rangle \text{ (from the symmetry of } A\text{)} \\ &= \langle P_{12}\psi_S|A|P_{12}\psi_A\rangle = -\langle\psi_S|A|\psi_A\rangle \text{ (from the symmetry properties of } |\psi_A\rangle \text{ and } |\psi_S\rangle\text{)}.\end{aligned}$$

Suppose now, contrary to what the symmetrization postulate stipulates, that a two-particle system can occupy a state $|\psi\rangle$ which is neither symmetric nor antisymmetric. As is well known, such a state can always be expressed as a superposition of $|\psi_A\rangle$ and $|\psi_S\rangle$ with non-vanishing coefficients: $|\psi\rangle = a|\psi_A\rangle + b|\psi_S\rangle$.¹⁶ When we calculate the expectation value for any symmetric operator A in state $|\psi\rangle$, the above-proven fact implies that the result will be as follows: $\langle\psi|A|\psi\rangle = |a|^2\langle\psi_A|A|\psi_A\rangle + |b|^2\langle\psi_S|A|\psi_S\rangle$. The exact same result is obtainable when we assume that the state of the system is not pure but mixed: $|a|^2|\psi_A\rangle\langle\psi_A| + |b|^2|\psi_S\rangle\langle\psi_S|$. Consequently, the pure state $|\psi\rangle$ is empirically indistinguishable from a mixture of two pure states $|\psi_A\rangle$ and $|\psi_S\rangle$.

Assuming the ignorance interpretation of mixed states, we may try to use the above-proved fact as a direct argument in favor of the symmetrization postulate. If occupying the mixed state $|a|^2|\psi_A\rangle\langle\psi_A| + |b|^2|\psi_S\rangle\langle\psi_S|$ really implies that the system is in one of the pure states $|\psi_A\rangle$ or $|\psi_S\rangle$, and if the superposition $|\psi\rangle = a|\psi_A\rangle + b|\psi_S\rangle$ is empirically indistinguishable from the mixed state, we may be tempted to conclude that each time we attribute the pure state $|\psi\rangle = a|\psi_A\rangle + b|\psi_S\rangle$ to a system of two particles, in fact the system occupies one of the two permutation-invariant states: $|\psi_A\rangle$ or $|\psi_S\rangle$. However, this is a very bad argument. Firstly, empirical indistinguishability is not the same as identity. Even if states $|a|^2|\psi_A\rangle\langle\psi_A| + |b|^2|\psi_S\rangle\langle\psi_S|$ and $a|\psi_A\rangle + b|\psi_S\rangle$ cannot be told apart by means of legitimate experimental procedures, this does not imply that ontologically we are dealing with the same state of affairs here. Secondly, the ignorance interpretation of mixtures is all but refuted. There are two major problems with it: the existence of an infinite number of alternative decompositions of a given mixed state, and the fact that reduced states of entangled systems are mixed (for details see Hughes 1989, 144 and 150).

An alternative is to use the above argument as a *reductio* for the assumption that systems of “indistinguishable” particles can occupy states which are neither symmetric nor antisymmetric. If we insist that pure states should never be empirically indistinguishable from mixed states, we have no choice but to adopt the full symmetrization postulate which allows only one type of states (symmetric or antisymmetric) for a given quantum system. Thus the argument sketched above does support SP. It is an open question (and one that

¹⁶ To be accurate, the existence of a decomposition of any arbitrary vector into the symmetric and antisymmetric components is guaranteed only in the case of two particles. When the number of particles is greater than 2, the total tensor-product space is no longer the direct sum of its symmetric and antisymmetric sections, since there are other sectors with higher types of symmetry (so-called parastatistics). Still, this fact does not affect the argument above. I am grateful to Fred Muller for pointing this out to me.

we do not have to answer definitively) which of the arguments—the exchange degeneracy argument or the pure/mixed states argument—is stronger or more compelling. I have two reasons to prefer the former, but I admit they are not decisive. First, I lack strong “meta-physical” or “physical” intuitions regarding the assumption that pure states should never behave as mixtures (of course there is a story to tell here about interference effects and the like, but it is rather intricate, and not at all “high-level”). In contrast to that, the principles OCS and ECS have a clear and intuitive meaning even for someone with a rudimentary knowledge of the quantum–mechanical formalism. Once we grasp the basic meaning of the notion of quantum superposition, it should become quite obvious that we cannot create a new state out of a superposition of identical states, even if these states happen to be represented by different mathematical objects.

The second reason for my preference is that the current argument relies on an additional premise in the form of the symmetrization postulate with respect to operators which is conspicuously absent from the exchange degeneracy argument. This postulate seems to be on the same conceptual level as the principle SP applied to states. In fact, it is often claimed that both operator- and state-restricted symmetrization postulates stem from the same source, i.e. the assumption that permuted states are empirically indistinguishable. If by empirical indistinguishability we understand the condition that the expectation values for any physically meaningful observable A should be the same in permuted states, i.e. $\langle \psi | A | \psi \rangle = \langle P_{12} \psi | A | P_{12} \psi \rangle$ for any $|\psi\rangle$,¹⁷ then we can immediately observe that this condition can be satisfied either by restricting the states $|\psi\rangle$ to symmetric or antisymmetric states ($P_{12}|\psi\rangle = \pm |\psi\rangle$), or by restricting operators to symmetric ones ($P_{12} A P_{12} = A$), or both. Caulton’s argument for SP described above establishes that if we decide to place the symmetrization restriction on observables rather than states, we should limit the states to either symmetric or antisymmetric anyway, since otherwise we will not have a clear distinction between pure and mixed states. So perhaps there is no need to rely on the operator-restricted symmetrization postulate after all—if we reject it, we have to symmetrize/antisymmetrize the available states in order to satisfy the condition of empirical indistinguishability; if we accept it, the pure/mixed states argument shows that the restriction on the states is necessary as well.

Still, the situation is far from being cut and dried. The condition $\langle \psi | A | \psi \rangle = \langle P_{12} \psi | A | P_{12} \psi \rangle$ with the universal quantifier binding the variable A presupposes that all operators represent meaningful physical observables (quantities). This is by no means an uncontroversial assumption (see van Fraassen 1991, 396ff for a discussion). Thus it is at least theoretically possible that we could satisfy the above equation for a limited class of operators without the full symmetrization postulate applied to states $|\psi\rangle$. Given this possibility, however remote, the exchange degeneracy argument presents itself as a viable alternative in an attempt to justify the symmetrization postulate at a high level of abstraction, due to its relatively meager and intuitive assumptions.

As we have seen, the symmetrization postulate can be supported by various high-level methodological and physical assumptions. Thus, strictly speaking, it should not be seen

¹⁷ However, let us observe that this formal expression of the condition of empirical indistinguishability is adequate only under the active interpretation of permutations. If we adopt the passive interpretation as described in Sect. 4, then one and the same operator A will represent distinct physical observables when applied to ψ and $P_{12}\psi$. In that case the proper formulation of the condition should contain the permuted operator $P_{12} A P_{12}$ in the right-hand side part of the equation, to reflect the fact that we have changed the representational framework. With this correction in place the condition is satisfied trivially and with no need for any restriction on states or operators.

as a postulate that we adopt without further justification (other than low-level empirical support). We do not have to *postulate* SP in the hope that the theory containing it will produce predictions that agree with experimental data. We may *derive* it from apparently more fundamental and more universal principles, such as the principle stating that the operation of superposition should satisfy the condition of conservativeness. Given that SP plays a pivotal part in modern discussions on naturalistic metaphysics, the possibility of deriving it from more fundamental assumptions may throw new light on the exact source of the radical metaphysical shift brought about by the arrival of quantum mechanics.

7 The Meaning of Labels in Symmetric/Antisymmetric States

This is not a place to discuss all the consequences that SP is believed to have with respect to the philosophical issues such as the violation of PII, the failure of identity and individuality, and so on, as this would require a separate article or even an entire book. However, at the end of this survey we may take advantage of the suggestions made in Sect. 4 regarding the meaning and reference of labels, in order to shed some light on the interpretation of symmetric and antisymmetric states of many particles. Let us focus our attention on the following, most typical fermionic state (most of what we will say about it can also be said about bosonic, i.e. symmetric states):

$$\frac{1}{\sqrt{2}}(|u(1)\rangle|v(2)\rangle - |v(1)\rangle|u(2)\rangle). \quad (1)$$

The main question we should ask now is how to interpret labels “1” and “2” present in this formula (how to fix their reference). As we recall, one way to do that is to use a particular description in terms of measurable properties in order to contingently fix the reference of both labels. Unfortunately, this method fails in the current situation, since there is no description in terms of physical observables that would differentiate between particles 1 and 2 (the state (1) is permutation-invariant by design). Another option is to interpret state (1) as representing a possible situation, and to use some criterion of transworld identity in order to “transfer” the reference of labels “1” and “2” from another world where they have been fixed. This strategy may initially seem to be promising, as it is often claimed that state (1) arises as a result of the antisymmetrization of the product state $|u(1)\rangle \otimes |v(2)\rangle$. Hence it may be tempting to use this non-symmetric product state as a way to fix the reference of labels “1” and “2” in some world w_0 , and then to stipulate that labels used in formula (1) are supposed to refer to the same objects (whether in the Kripkean or Lewisian sense of “sameness”) in a different, counterfactual scenario. To put it differently, state (1) would be interpreted as describing a possible situation in which particles 1 and 2, discernible by their physical properties in world w_0 , become entangled with one another in a way which prevents their discernibility. If we add to that that the world w_1 in which particles 1 and 2 happen to occupy state (1) is our world, we can derive from this the inevitable conclusion that terms “1” and “2” refer to particles that cannot be *actually* told apart by any physically meaningful procedure.

Unfortunately, this strategy has one important downside: it namely presupposes that there is a possible world in which the state of two fermions of the same type can be written

as a product of two orthogonal kets.¹⁸ However, the symmetrization postulate does not express a contingent truth that characterizes our world only, but is supposed to be a law of nature. If we disregard law-breaking worlds as being too far from the actual world, then we have no choice but to assume that in relevant possible worlds all states of many particles of the same type are either symmetric or antisymmetric. Consequently, the method of fixing the referents of labels by transferring them from the worlds in which particles are described by non-symmetric product vectors fails. What can we do instead? Can we not simply use formula (1) as a means of picking referents for labels “1” and “2”? This would amount to the stipulation that “1” (or “2”) refers to *any* of the two particles that (contingently) occupy the antisymmetric state (1). I am doubtful that this procedure can be argued to be legitimate. Recall the debate between two philosophers A and B in Max Black’s classic piece on the identity of indiscernibles (Black 1952). After B introduced the famous example of a universe containing two identical iron spheres, A tries to respond to the challenge by starting “consider one of the spheres, *a*...”, to which B immediately retorts “How can I, since there is no way of telling them apart?”. And he (or she, since no gender of the disputants is revealed) continues “I don’t know how to identify one of two spheres supposed to be alone in space and so symmetrically placed with respect to each other that neither has any quality or character the other does not also have” (p. 156).¹⁹

Notably, philosopher B admits the possibility of using distinct labels to pick out individual spheres in a counterfactual scenario in which the symmetry of the universe is broken. S/he says “if something were to happen to introduce a change into my universe, so that an observer entered and could see the two spheres, one of them could have a name” (p. 157). But B (or Black himself) does not consider the option to use this counterfactual scenario as a springboard for introducing names in the symmetric case via the notion of transworld identity. This is what, as I suggest, can be done as follows. Relative to the world in which an observer stands closer to one sphere than to the other, and names the first one *a* while the second *b*, we may introduce possible worlds without the observer, in which the *same* spheres are present. According to Kripke, there will be two such worlds, differing only in the assignments of names to individual spheres, whereas for Lewis only one such world exists. It goes without saying that in these worlds the two spheres are not discernible by their properties (intrinsic or extrinsic), but this does not block the possibility to use proper names *a* and *b* to describe this situation. However, in the quantum case even this thin method of introducing labels via a counterfactual scenario is not available, as I have explained above. Thus an interpretation of states like (1) remains an open problem.

On the other hand, there is still a simple way to refer to individual quantum systems in terms of legitimate experimental procedures and their results. If I perform a Stern–Gerlach experiment and end up with two electrons, one deflected up, the other down, it seems perfectly admissible to refer to the first as the spin-up electron, and the second as the spin-down. But how can this be reconciled with the symmetrization postulate and the necessity of using permutation-invariant descriptions of the states of many particles? Here a simple example may help us realize that even using a language that is fully permutation-invariant

¹⁸ There is yet another problem with this strategy that affects fermionic but not bosonic states. As it turns out, vectors identical to (1) can be obtained by antisymmetrizing any product state consisting of two orthogonal kets spanning the same subspace as $|u\rangle$ and $|v\rangle$. Thus it is not determined which of these product states to choose in order to fix the reference of labels “1” and “2”.

¹⁹ In order not to be accused of committing a historical inaccuracy here, we should note that Black’s discussion predates the analysis of direct reference in terms of rigid designators given by Kripke. Thanks to Cord Friebe for pressing me on that point.

we may still individuate objects. Suppose that we have two beach balls, one of which is red and the other yellow.²⁰ Obviously, we can use the (incompatible) descriptions “the red ball”, “the yellow ball” in order to label these balls, for instance calling the first one “Fred” and the second one “Max”. But the language in which we introduce these names is clearly not permutation invariant. The sentence “Fred is red”, true in our world, turns false in the world in which the balls have been swapped.²¹ However, the disjunctive sentence “Either Fred is red or Max is red” is permutation invariant, since its truth value remains unchanged under the permutation of the objects. Similarly permutation-invariant is the statement “Either Fred is red and Max is yellow or Max is yellow and Fred is red”. But the last expression arguably differentiates between the two objects, since it logically entails that there is a ball which is red and there is a ball which is yellow. This existential claim individuates objects in the following sense: it implies that there are (at least) two numerically distinct objects that possess different and incompatible properties (in short: objects that are qualitatively discernible).

The aforementioned exercise may seem trivial, since we knew all along that the balls were discernible by their colors. But the point of this example was to show that individuation is possible even if we have at our disposal only permutation-invariant expressions. This observation can be carried over to the not-so-trivial quantum case. Suppose, then, that a composite system of two electrons has been assigned the antisymmetric spin-state of the form (1). Using labels “1” and “2” we can now formulate statements “Electron 1 (2) has spin up (down)”. The standard way to express these statements in the quantum-mechanical formalism is with the help of the projection operators of the form $P_{\uparrow} \otimes I$, $I \otimes P_{\uparrow}$ (where I is the identity operator) respectively for particles number 1 and number 2. A given sentence is true if the state of the system is an eigenstate of the corresponding projector with eigenvalue 1 (if the expectation value for this projector equals one). It is easy to observe that state (1) is not an eigenstate for the non-symmetric operators of the above kind (and, moreover, the expectation values for both operators are the same), and this fact is the basis for the commonly accepted conclusion that individual particles 1 and 2 cannot be differentiated by their spin-states. However, let us follow the beach balls example and consider the permutation-invariant statement “Either electron 1 has spin up and electron 2 has spin down, or electron 1 has spin down and electron 2 has spin up”. According to the standard quantum-mechanical rules, a disjunctive proposition of that sort is expressible with the help of the sum of projectors $P_{\uparrow} \otimes P_{\downarrow} + P_{\downarrow} \otimes P_{\uparrow}$.²² Now it is straightforward to notice that the state (1) with $|u\rangle$ and $|v\rangle$ replaced by $|\uparrow\rangle$ and $|\downarrow\rangle$ is an eigenvector for this operator with the corresponding value equal 1.

There is a clearly visible tension between the two formal results quoted in the above paragraph. It looks as though we had a true disjunction of two statements, each of which is not true. Some philosophers believe that this calls for a change in our logic, but this

²⁰ This is a slight modification of the example used by Saunders (2013, 2015, 170–173).

²¹ Keep in mind that we do not equate the name “Fred” with the description “the red ball”—we only use the latter to fix reference of the former in our world. In a counterfactual scenario in which the balls have been swapped, Fred is yellow. This is unconditionally true under the Kripkean analysis of modality *de re*, and true under the Lewisian analysis if we assume that Fred and Max have distinct essential properties.

²² This interpretation of disjunctive statements is implicitly assumed e.g. in Ghirardi et al. (2002, 74ff). See also a discussion in Bigaj (2015b, 193–195).

strategy fell from favor a long time ago.²³ Alternatively, we may take a second look at the assumed interpretation of the statements supposedly attributing properties to individual particles. The projection operators $P_1 \otimes I, I \otimes P_1$ representing these statements are clearly not permutation-invariant, and therefore are suspicious in the light of the inevitable symmetrization postulate. This may prompt us to abandon the assumption that labels “1” and “2” used in the formula (1) play any referential role at all. The task of individuating the components of the composite system falls on the permutation-invariant disjunctive statements, exactly as in the case of beach balls. The only (major) difference between these cases is that in the quantum case it is impossible to introduce new labels referring (in our world) to objects satisfying the descriptions “has spin up”, “has spin down”, since this would lead to the conclusion that the joint state is the direct product of two states, which violates the Symmetrization Postulate and has incorrect empirical consequences. Thus it is not the separate descriptions that pick out the individual electrons, but the entire permutation-invariant disjunction expressible with the help of the sums of projectors.

The foregoing considerations point towards a heterodox approach to the issue of how to individuate components of a composite quantum system, which denies that individual Hilbert spaces in the tensor product represent states of those components.²⁴ By adopting this approach we reject one of the main alleged consequences of SP, namely the Indiscernibility Thesis. But this is not to say that SP does not force us to abandon some other classical intuitions regarding the behavior and fundamental properties of quantum objects. One non-classical feature of the antisymmetric fermionic states of the form (1) derives from the fact, mentioned in ft. 18, that they can be rewritten in any orthogonal basis of the subspace spanned by vectors $|u\rangle$ and $|v\rangle$. This seems to imply that there are numerous alternative ways to individuate components of the composite system using incompatible properties, which leads to further complications in terms of a proliferation of distinct components of composite systems, a violation of the fundamental principles of mereology etc.²⁵ Another metaphysical issue stemming from the symmetrization postulate is the breakdown of the notion of diachronic identity (identity over time). This happens for instance in the case of interactions between particles of the same type, where the (anti)symmetrization of the total state leads to the impossibility of identification of particles before and after their interaction (cf. Cohen-Tannoudji et al. 1978, 1403–1405).²⁶ The symmetrization postulate has a profound impact on the metaphysical interpretation of quantum theory, and this is why it is so crucial to know what fundamental reasons we may have for adopting this principle.

²³ For a brief outline of so-called quantum logic see Hughes (1989, 178–217).

²⁴ An extensive analysis of this approach can be found in Caulton (2014; 2015). Similar ideas have been endorsed in Dieks and Lubberdink (2011; 2020), Saunders (2013; 2015) and Bigaj (2015b). Most recently, Leegwater and Muller (2020) have joined the ranks of those who believe that quantum particles of the same type can be at least momentarily discerned by their properties (in addition to that they offer a deep analysis of different variants of factorism, which broadly speaking is a view that the individuation of the components of a quantum system can be done by factorizing the global state space of the composite system). Perhaps the earliest record of the heterodox conception of how to individuate quantum particles is Andrea Lubberdink’s unpublished Master’s thesis (Lubberdink 1998).

²⁵ See discussions in Caulton (2015) and Bigaj (2016).

²⁶ I am discussing the case of particle scattering in great depth in my unpublished manuscript “Synchronic and diachronic identity for elementary particles”.

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References

- Bigaj, T. (2015a). Dissecting weak discernibility of quanta. *Studies in History and Philosophy of Modern Physics*, 50(2015), 43–53.
- Bigaj, T. (2015b). Exchanging quantum particles. In Bour, P. E., Heinzmann, G., Hodges, W. & Schroeder-Heister, P. (Eds.), *14th CLMPS 2011 proceedings, Philosophia Scientiae* (Vol. 19, pp. 185–198).
- Bigaj, T. (2016). On some troubles with the metaphysics of fermionic compositions. *Foundations of Physics*, 46(9), 1168–1184.
- Black, M. (1952). The identity of indiscernibles. *Mind*, 61(242), 153–164.
- Butterfield, J. (1993). Interpretation and identity in quantum theory. *Studies in History and Philosophy of Science*, 24, 443–476.
- Caulton, A. (2014). *Qualitative individuation in permutation-invariant quantum mechanics*. [arXiv:1409.0247v1](https://arxiv.org/abs/1409.0247v1) [quant-ph].
- Caulton, A. (2015). Is mereology empirical? Composition for fermions. In T. Bigaj & C. Wüthrich (Eds.), *Metaphysics in contemporary physics, Poznań studies in the philosophy of the sciences and the humanities* (pp. 293–321). Leiden: Brill/Rodopi.
- Cohen-Tannoudji, C., Diu, B., & Laloë, F. (1978). *Quantum mechanics* (Vol. 2). Hoboken: Wiley.
- Dieks, D., & Lubberdink, A. (2011). How classical particles emerge from the quantum world. *Foundations of Physics*, 41, 1051–1064.
- Dieks, D., & Lubberdink, A. (2020). Identical quantum particles as distinguishable objects. *Journal for General Philosophy of Science*. <https://doi.org/10.1007/s10838-020-09510-w>.
- French, S., & Redhead, M. (1988). Quantum physics and the identity of indiscernibles. *British Journal for the Philosophy of Science*, 39, 233–246.
- French, S., & Rickles, D. (2003). Understanding permutation symmetry. In K. Brading & E. Castellani (Eds.), *Symmetries in physics: Philosophical reflections* (pp. 212–238). Cambridge: Cambridge University Press.
- Friebe, C. (2014). Individuality, distinguishability and (non-)entanglement: A defense of Leibniz's principle. *Studies in History and Philosophy of Modern Physics*, 48, 89–98.
- Ghirardi, G., Marinatto, L., & Weber, T. (2002). Entanglement and properties of composite quantum systems: A conceptual and mathematical analysis. *Journal of Statistical Physics*, 108(112), 49–122.
- Huggett, N. (2003). Quartiles and the identity of indiscernibles. In W. K. Brading & E. Castellani (Eds.), *Symmetries in physics* (pp. 239–249). Cambridge: Cambridge University Press.
- Hughes, R. I. G. (1989). *The structure and interpretation of quantum mechanics*. Cambridge, MA: Harvard University Press.
- Kripke, S. (1980). *Naming and necessity*. Oxford: Blackwell.
- Leeewater, G., & Muller, F. A. (2020). The case against factorism: On the labels of $\otimes$ -factor hilbert-spaces of similar particles in quantum mechanics. *Journal for General Philosophy of Science*. <https://doi.org/10.1007/s10838-020-09514-6>.
- Lewis, D. (1986). *On the plurality of worlds*. Oxford: Blackwell.
- Lubberdink, A. (1998). *De individualiseerbaarheid van identieke deeltjes*. Master Thesis, Utrecht University.
- Muller, F. A. (2011). Withering away, weakly. *Synthese*, 180, 223–233.
- Muller, F. A. (2015). The rise of relationals. *Mind*, 124, 201–237.

- Muller, F. A., & Saunders, S. (2008). Discerning fermions. *British Journal for the Philosophy of Science*, 59, 499–548.
- Muller, F. A., & Seevinck, M. P. (2009). Discerning elementary particles. *Philosophy of Science*, 76, 179–200.
- Norton, J. (2015). The hole argument. In Zalta, E. N. (ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2015 Edition). Retrieved March 2019 from <https://plato.stanford.edu/archives/fall2015/entries/spacetime-holearg/>.
- Peres, A. (2003). *Quantum theory: Concepts and methods*. New York: Kluwer Academic Publishers.
- Redhead, M., & Teller, P. (1992). Quantum physics and the identity of the indiscernibles. *British Journal for the Philosophy of Science*, 43, 201–218.
- Sakurai, J. J., & Napolitano, J. (2011). *Modern quantum mechanics* (2nd ed.). San Francisco: Addison-Wesley.
- Saunders, S. (2003). Physics and Leibniz's principles. In K. Brading & E. Castellani (Eds.), *Symmetries in physics: Philosophical reflections* (pp. 289–307). Cambridge: Cambridge University Press.
- Saunders, S. (2006). Are quantum particles objects? *Analysis*, 66, 52–63.
- Saunders, S. (2013). Indistinguishability. In R. Batterman (Ed.), *Oxford handbook of philosophy of physics* (pp. 340–380). Oxford: Oxford University Press.
- Saunders, S. (2015). On the emergence of individuals in physics. In A. Guay & T. Pradeau (Eds.), *Individuals across Sciences* (pp. 165–190). Oxford: Oxford University Press.
- van Fraassen, B. (1991). *Quantum mechanics. An empiricist view*. Oxford: Clarendon Press.

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