Between Monoculture and Cultural Polarization: Agent-based Models of the Interplay of Social Influence and Cultural Diversity

Andreas Flache¹

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Abstract

Social influence is one of the most important processes in human social interaction. Very often in human social interaction, influence is assimilative in that individuals become more similar to others they interact with. Nevertheless, cultural differences continue to remain in many realms of human life, for example, in the form of technological boundaries. Research on social influence points to a range of possible reasons for persistent cultural diversity, but there is much less clarity about the interplay of various factors and conditions for cultural influence with fundamental processes of social interaction at the micro-level. In this article, I show how agent-based computational modeling can be used as an approach for unraveling the complex interplay between simple first principles of interpersonal social interaction and emergent societal outcomes. I give a brief overview illustrating some of the main approaches agent-based modelers have developed in recent decades to understand conditions and processes of the emergence of cultural diversity. Models will be discussed that generate mainly cultural consensus as long-term behavior, but also models that generate clustering of cultural attitudes in geographical or social space, and models that imply cultural polarization with sharp cultural boundaries between emergent factions. It will be discussed how model dynamics depend on further assumptions, for example about random events, or the scaling of cultural attitudes, and what are further developments in the literature, possible future directions and challenges for the application of computational agent-based modeling in archeological research on cultural boundaries.

Keywords Social influence · Cultural diversity · Computational modeling · Agent-based models · Opinion dynamics · Polarization · Technological boundaries

✉ Andreas Flache
a.flache@rug.nl; <http://www.gmw.rug.nl/~flache/>

¹ Department of Sociology/Interuniversity Center for Social Science Theory and Methodology (ICS), University of Groningen, Groningen, The Netherlands

Introduction

Social influence is one of the most important processes in human social interaction. Social influence can occur in many different ways. A particularly prominent form is assimilative influence in interpersonal encounters. Often individuals modify their opinions, beliefs, or behavior towards becoming more similar to those of others they interact with. Assimilative influence can occur for many different reasons, for example, because an individual is persuaded by convincing arguments put forward in a discussion (Myers 1982); people may follow the lead of those deemed more knowledgeable in situations of uncertainty (Bikhchandani et al. 1992), or due to social pressure to conform with group norms (Festinger et al. 1950; Wood 2000). Empirical studies from different domains of society illustrate how social influence reduces differences between people, as has been found in experiments on conformity (Asch 1956), research on small group behavior (Sherif and Sherif 1979), innovation diffusion (Rogers 1995), or interaction in online social networks (Bond et al. 2012).

Despite the prominence of assimilative social influence in human social interaction, cultural diversity is a persistent phenomenon. This would not be difficult to explain if there were no social interactions across the boundaries of different cultural regions. However, most humans do not live in disconnected networks. Migration, communication across large geographic distances, and global trade are no new phenomena; they have existed for a long period in human history. Yet, cultural differences have remained and continue to remain in many realms of human life. Archeologists, in particular, have highlighted the existence of “technological boundaries” (Stark 1998) that separate spatially neighboring regions in which different people persist having different technologies for performing the same crafts, although contact and interaction between them would make the spreading of techniques possible. Roux (Roux 2013) demonstrated this for the failure of the use of the potter’s wheel to spread in the Mediterranean for a long time, despite its superior technological properties. Further field studies (Roux et al. 2017) showed that technological boundaries can even arise and be persistent “where different social groups live in close geographical proximity and use different ceramic techniques for making utilitarian vessels ... [although] ... the boundary ceramic techniques present advantages over each other” (p. 321). The question why cultural differentiation persists may have become even more pressing in the recent decades, which are characterized by more spatial mobility and ever more powerful communication technologies for cultural contact between widely disparate cultural and geographical regions around the world. This has led observers to ask whether social and cultural differences at the global scale will survive increasing exposure to outside influences (e.g., Greig 2002; Friedman 2005).

Examples for the persistence of cultural differentiation even in our contemporary world abound. They can be found at the meso-level scale of multinational organizations (Early and Mosakowski 2000), but also at the macro-scale of large societies with modern communication technologies. For instance, Mark (1998, 2003) showed how in the US society preferences for a particular music style cluster among those who are similar in age or education, and clearly differ between those groups. Similarly, Glaeser and Ward (2006) concluded from their analysis of the regional distribution of political and social views in USA that “the extent and permanence of cultural divisions across space is one of America’s most remarkable features” (119–120). Theorists of cultural

differentiation (Bourdieu 1984) have long emphasized the persistence of boundaries between societal groups, like between an upperclass “high-brow” culture and a lower class “low brow” culture, despite interaction between members of different classes that would allow for assimilative social influence across subgroup boundaries.

The co-existence of persistent cultural diversity as a macro-level social phenomenon with assimilative influence as a micro-level social process led the political scientist Robert Axelrod to wonder in a highly influential paper in 1997 “If people tend to become more alike in their beliefs, attitudes, and behavior when they interact, why do not all such differences eventually disappear?” (Axelrod 1997, p. 203). There are many possible answers to this question and Axelrod did, of course, not believe that real social-influence dynamics are exclusively driven by repeated encounters in which any two individuals become more alike every time they interact. Answers can be sought in social-structural conditions like “small-world” networks (Watts 1999) in which relations are clustered within cultural groups and sparse between different cultures. Answers may also be found in the micro processes of social influence. Social influence is not always assimilative. Individuals may also reject outside influences, for example because their cultural preferences can be deeply entrenched and rooted in central aspects of their social identities. Or individuals sometimes even seek to become more different from those they interact with (*e.g.*, Mazen and Leventhal 1972), for example, because they perceive others as members of negatively evaluated outgroups.

While research on social influence points to a range of possible reasons for persistent cultural diversity, there is much less clarity about how the many factors and mechanisms that could affect cultural diversity in the real world interact with fundamental processes of social interaction at the micro-level. This was the main inspiration for Axelrod’s question. Axelrod took the perspective of agent-based computational modeling (*e.g.*, Epstein 1999; Hedström and Manzo 2015; Macy et al. 2011; Macy and Flache 2009; Smith and Conrey 2007), an approach that puts emphasis upon unraveling the complex interplay between simple “first principles” (Mark 1998) of interpersonal social interaction and emergent societal outcomes that often can be unexpected and unintended from individuals’ point of view. Agent-based computational modeling (ABCM) is closely linked to methodological individualism and the social-mechanism approach to social theory (Coleman 1990; Hedström 2005; Manzo 2014), which views macro-phenomena (like the pattern of local convergence and global diversity observed in many cultural phenomena) as the result of the actions of the individuals that constitute society at the micro-level. ABCM in particular implements the rules of behavior and interaction in computer code, and conducts systematic computational experiments to derive how societal outcomes could result from these micro-level processes.

ABCM is a formal approach that forces the modeler to express theoretical assumptions with mathematical precision and deduce their consequences with a formalized method, computer simulation. Formal derivation of model implications is particularly useful when the macro-level consequences of interdependent individual actions cannot simply be inferred from aggregating individual actions. I will present in this paper a number of models that show how cultural diversity or even sharp cultural boundaries can arise although individual agents in the model follow decision rules in which neither diversity nor polarization are goals they strive to bring about. ABCM is a powerful tool to analyze such complex micro-macro interactions, but it is not the only formal method

of theory development. ABCM can be seen as taking a middle ground between non-formal theorizing and purely mathematical approaches, like game-theory or differential equation models. A strength of purely mathematical approaches is the possibility to deduce general results that hold independently of specific initial conditions or parameter values of a model; but often, this is only possible when modelers greatly simplify in their mathematical formulations complexities of the substantive theories and the empirical context they want to address. An example is the original Kermack and McKendrick (1927) SIR model of infectious disease spreading that describes an epidemic as a system of three-coupled nonlinear ordinary differential equations. This model allows to derive a general relationship between its parameters on which it depends whether a disease can cause an epidemic outbreak. However, some strong simplifications in this model are the reduction of individuals to representative actors assigned to classes of “susceptibles,” “infectives,” and “removed,” the assumption that these agents randomly mix, or the assumption that actors move from one class to another based on probabilities that do not depend on further individual characteristics, local context, or network structures in which they are embedded. Analytical solutions like those available for the SIR model often become impossible or at least impractical to obtain once more realistic assumptions are introduced, such as heterogeneous network structures, non-linear decision rules at the individual level, decisions based on heterogeneous local contexts, and, importantly, feedback loops that can create self-reinforcing dynamics in which small events at the local level can have large and unanticipated consequences at a larger scale. ABCM does not offer general analytical results for models comprising these complexities, but it provides a formal apparatus that allows to nevertheless rigorously deduce model outcomes with systematic computational experiments, even when analytical solutions cannot be obtained or have not yet been found. Scholars in a range of disciplines are therefore increasingly interested in ABCM as tool to elaborate the link between micro-processes and macro-outcomes in explanations of social phenomena, a development that has left its mark also in the increasing interest of archeologists in ABCM as a tool of research (Cegielski and Rogers 2016; Wurzer et al. 2015).

In this article, I aim to give a brief overview illustrating some of the main approaches that agent-based modelers have developed in recent decades to understand the conditions and processes of the emergence of cultural diversity. A large literature has evolved addressing this topic, often not explicitly addressing cultural diversity, but instead processes of the dynamics of opinions, or social influence in general. Following Axelrod, I adopt in the following the strong simplification that “culture is taken to be what social influence influences” (Axelrod 1997, p. 207) and use the concepts of cultural attitudes, opinions, culture, and cultural diversity as generic terms referring to individual attributes subject to social influence, as well as to their distribution in a population. An advantage of this definition is that it is applicable to very different forms of cultural attitudes, like words people use in language, technologies used for certain forms of work, fashion choices or choices for particular brands of smartphones or computers. This should not obscure that culture is a highly complex concept¹ and that different conceptualizations of culture may also require different formal representations

¹ Axelrod notes that already Kroeber and Kluckhohn (1952) compiled a list of “more than a hundred definitions of culture” (1997, p. 206).

in ABCM of cultural influence, with different implications. One example is the difference between continuous and nominal cultural traits. In early formal models of social influence (Abelson 1964; DeGroot 1974; French 1956; Harary 1959; Lehrer 1975), actors adopt a position on a continuous spectrum representing their “opinion” on some issue, for example, their stance on what the appropriate age of first marriage would be. Other modelers assumed instead that opinions represent a selection from a menu of distinct options available in a cultural domain, like a persons’ favorite political party, music band, or movie genre (Axelrod 1997; Nowak et al. 1990). In examples presented in this paper, I will show that the assumed scale of the influence dimension can alter model predictions decisively in some cases. At the same time, many fundamental results of models employing substantively similar assumptions about the process of influence hold for both nominal and continuous representations of cultural attitudes (Flache et al. 2017).

In the context of a special issue on the diffusion of innovations, it is important to note that social influence as conceived in ABCM models of cultural diversity is different from diffusion or contagion processes (e.g., Valente 1996) that often are also used to model the spreading of innovations. Contagion is often modeled as a process that is unidirectional, drawing on models similar to the SIR model. For example, a person who is unaware of the existence of a new technology for pottery can learn about this technology in encounters with others who have already adopted it, but the non-adopter cannot make adopters forget their already acquired knowledge of the technology. Models of the diffusion of innovations also can include the possibility that adopters cease to use an innovation and that this also affects adoption decisions of network neighbors (Karsai et al. 2014). However, models of social influence move one step further in assuming a bi-directional process in which the new cultural attitude adopted after an interaction can be a new recombination of the attitudes or behaviors that the interacting individuals held prior to their encounter. This view can be seen as similar to that taken by models that describe the creation of a new cultural product (like a movie) as recombination of already existing products (Gravino et al., 2016). In fact, models, very similar to those discussed in this paper, have been applied for modeling the diffusion of innovations, taking into account that attitudes towards adoptions are shaped in a bi-directional process of mutual influence between adopters and non-adopters (e.g., Deffuant et al., 2005).

It is impossible to do justice to all ABCM contributions to modeling cultural diversity or social influence dynamics in this paper. Instead, I will highlight some of the main approaches that in my view represent important streams of thought; and I will concentrate on simple models and illustrative examples of their dynamics and core results. However, this should not obscure that many papers in the field contain more sophisticated models and theoretical analyses than those presented here and—sometimes—also empirical applications of such models. The literature contains especially also models that combine various of the approaches presented here in one model, and add theoretical assumptions about the cognitive and psychological mechanisms that trigger different micro-processes of social influence in social interactions (e.g., Duggins 2017; Mäs et al. 2010). The interested reader can find overviews over parts of this literature for example in (Bianchi and Squazzoni 2015; Castellano et al. 2009; Flache et al. 2017; Mäs et al. 2014; Sobkowicz 2009).

The remainder of the paper is organized as follows. First I will discuss early models of assimilative influence that mainly highlighted the emergence of consensus (also denoted monoculture in the following), but also can generate diversity under specific assumptions. Subsequently I focus on models combining homophily and assimilative social influence, which offer a possible explanation of the emergence of global diversity and local clustering of cultural attitudes. This is followed by a discussion of models that add to assimilative influence a further possibility - simultaneous differentiation from negatively evaluated sources of influence, a micro-process that Roux and her co-authors (2017) have proposed as explanation of the persistence of technological boundaries. The article finishes with a brief discussion of the state of the art and future directions for the field.

Models of Assimilative Influence: the Emergence of Monoculture

Fundamental Assumptions and Results

Drawing on evidence from experimental studies and field research in small group behavior, psychologist Robert Abelson proposed in 1964 (Abelson 1964) the following assumption as basic building block for the earliest generation of formal models of cultural influence (Abelson 1964; DeGroot 1974; French 1956; Harary 1959; Lehrer 1975). If two members of a group interact, “each member of the group changes his attitude position towards the other by some constant fraction of the ‘distance’ between them” (Abelson 1964, p. 143). Computer simulations were not readily available in the time that these models were developed. Instead, their authors aimed to simplify theoretical assumptions such that model behavior could be analyzed with purely mathematical means. While models of assimilative influence have been formally elaborated in various different ways, many of them share as core assumptions that cultural attitudes are represented as numbers on a continuous scale, and assimilative influence is then implemented as some form of moving one’s own opinion towards a weighted average of the opinions of all actors with whom an individual is connected in a social network (Friedkin and Johnsen 1990, 2011). Network links are assumed to remain unchanged over time in these models, but they are weighted. Weights scale the strength of social influence one actor exerts upon another and can be seen as capturing structural differences in, for example, persuasiveness, frequency of interaction, or power between actors.

A central result of the literature on these early models is that convergence to global consensus is inevitable in a connected (or what Abelson called a “compact”) network—in which there are no components that are entirely disconnected from outside influences. At the same time, analysis of these models has also highlighted that even when networks are connected, some level of local convergence alongside global diversity can be maintained at least in the medium term, for example, when networks exhibit strong local clustering of relations.

An Illustrative Implementation: Assimilative Influence on a Ring Lattice

For illustration, I show the dynamics of a simple model of assimilative influence in a locally clustered network. To illustrate that the core-results from the literature are robust against small variations in the assumptions, the model I will use here deviates from classic averaging models in some aspects. At the same time, my model retains the defining features of the classic models of assimilative influence, opinions are represented on a continuous scale and influence is assimilative in Abelson's sense. That is, if influence occurs, individuals who interact shift towards the position of the source of influence in the attitude space by some constant fraction of the prior distance between their attitudes. Finally, the connections in the social network that define who can interact with whom and how strong influence is between a particular pair of individuals, are constant over time.

I begin with a description of the highly simplified representation of a spatially clustered network that I use in the simulations. Following the approach proposed by Watts and Strogatz (Watts 1999; Watts and Strogatz 1998), individual actors are arranged on a circle and are connected only with those other actors in the population whose location is within a certain number of steps left or right to their own location on the circle.² Figure 1 illustrates this idealized local interaction structure, implemented as a ring lattice.

In most of the classic averaging models of assimilative influence, it is assumed that, if influence occurs, an agent moves towards the weighted average of the cultural attitudes of all other agents with whom she is connected. The model presented here reduces the elementary action in which influence occurs to an encounter between only two connected individuals, only one of whom changes her attitude after the encounter. More precisely, the influence dynamic consists of a sequence of events in which at every time point exactly one randomly chosen population member i can update her attitude and does so by selecting at random one of her spatial neighbors j (see Fig. 1) to interact with. If i and j interact, then i modifies her attitude to move closer towards the opinion of j . Equation [1] below describes the rule for attitude change.³ To simplify the exposition, I assume here that cultural attitudes are constrained to the range between 0 and 1 ($0 \leq o_{i,t} \leq 1$) throughout.

$$o_{i,t+1} = o_{it} + \Delta o_{it} = o_{it} + \mu w_{ij} (o_{jt} - o_{it}) \quad (1)$$

For simplicity, differences in influence weights have been reduced here to whether two agents are connected or not. Thus, in Eq. [1], $w_{ij} = 1$ if i is linked with j in the network;

² The circular structure obviously is unrealistic as representation of a geographical spatial dimension. In real spatial networks, some actors can reside on the boundaries of a geographic space where they have less geographic neighbors than those in the interior, and some may be more central in the network than others. The circular structure is here chosen to concentrate on the fundamental properties of the influence process itself. This notwithstanding, there can be important effects of network structure itself on the degree of opinion diversity and time of convergence to consensus. Effects of network structure on opinion dynamics in models similar to the one used here are for example analyzed in Social Influence Network Theory (e.g., Friedkin & Johnsen 1990, 2011).

³ The assumption of a random sequence of pairwise interactions is adapted in a somewhat modified way from a prominent model proposed by (Deffuant et al. 2000).

Fig. 1 Ring lattice network for 10 agents, with interaction radius of 2: every agent is connected with neighbors maximally two steps to the left and to the right of own location on circle.

otherwise, $w_{ij} = 0$. The parameter μ ($0 < \mu \leq 1$) defines the rate of opinion convergence and can be used to smoothen opinion dynamics.

For an illustrative simulation of this model of assimilative influence, I increased the population size to $N = 100$, and assumed a somewhat wider range of local interaction than shown in Fig. 1. In the simulation, the interaction radius is extended to 5 steps. That is, an agent is connected with five neighbors to the left and five neighbors to the right of her location on the circle, thus being able to interact with 10% of the members of the total population. To illustrate how the dynamics of the model generate convergence on a monoculture, the population starts with random initial attitudes drawn from a uniform distribution.

Figure 2 shows the convergence towards monoculture that is characteristic for models of assimilative influence if the network is connected. The left graph (2A)

Fig. 2 Example of convergence towards monoculture under the assimilative influence model ($N = 100$, initial attitude uniform random, interaction range = 5, $\mu = 0.5$, 10,000 events)

illustrates how after about 2000 simulation events all opinion trajectories of individuals have moved close towards the mean value of the initial distribution, around an attitude of about 0.5. The right graph (2B) highlights the corresponding emergence of a unimodal frequency distribution with almost exclusive concentration of attitudes at the center of the scale after 10,000 simulation events.

The reason for the convergence shown in Fig. 2 is that under assimilative influence, interactions always imply a shift of an individual towards the attitude of the source of influence, but never away from it or towards a position more extreme than that held by the source. Thus, even the individuals with the most extreme attitudes in the initial distribution (closest to zero or one) can never pull others towards more extreme positions than their own. At the same time, those with the most extreme attitudes will sooner or later interact with more moderate others who move their attitudes closer to the center of the attitude range. This occurs on both sides of the opinion spectrum such that the range of attitudes present in the population gradually narrows down until eventually all differences disappear.

While the dynamics converge towards monoculture in a connected network, some level of local attitude clustering can persist at least temporarily. Figure 3 illustrates this with showing how cultural attitudes vary over time across the spatial dimension, represented by a walk around the circle on which agents are located. Figure 3A shows the initial random distribution in which no spatial clustering can be discerned whatsoever. However, after about 1000 events some local convergence can be observed. Approximately from position 40 to 45, and from position 75 to 90 on the circle local clusters formed with an attitude of about 0.6, while local convergence on an attitude of 0.4 has occurred approximately between positions 50 and 60 and 25 to 40. In yet other regions of the circle, there is still more local random variations in attitudes or attitudes shift gradually across space. Figure 3C demonstrates how this tendency towards local clustering has disappeared and given way towards almost perfect monoculture with approximately $\phi = 0.5$ after 10,000 simulation events. The reason for the temporary phase of local attitude clustering is the local clustering of network relations, in combination with the initial random distribution. If in some regions of the network agents happen to agree somewhat more among each other by random chance, unanimity can develop faster in these regions around the local average attitude. Only after

Fig. 3 Temporarily stable spatial clustering of opinions under assimilative influence ($N = 100$, initial opinion uniform random, interaction range = 5, $\mu = 0.5$)

sufficient time, all agents have interacted often enough to assure that every individual has been influenced at least indirectly by all initial attitudes present in the population such that eventual convergence towards the initial average attitude is the most likely long-term outcome. Temporary local clustering and eventual convergence in connected networks is a result that is not restricted to the specific updating mechanism used here, but has also been shown for the “classical model” (Hegselmann and Krause 2002) of assimilative influence that employs simultaneous opinion averaging by all agents (see for example some cases discussed by French, 1956). Further studies also demonstrated effects of network topology. For example, using a very similar model of assimilative influence, Flache and Macy (2011b) added a small number of random connections that turned a local interaction structure into a small-world network (Watts 1999; Watts and Strogatz 1998). They found that monoculture obtained considerably faster.

Alternative Models of Assimilative Influence

Models of assimilative influence exhibit an inexorable tendency towards monoculture if networks are connected and attitudes are represented in a continuous spectrum. Especially the latter assumption was relaxed by modelers who thought of cultural attitudes as discrete choice options, like the choice of a preferred music style from a distinct set of options. With discrete cultural attitudes, social influence cannot always result in a shift towards a compromise position in between the prior attitudes of a source and a target of influence. Instead, models have been proposed in which agents imitate the attitudes of a local majority (Galam 2002), sometimes with influence of network neighbors weighted by their individual “social impact” (Nowak et al. 1990). Another well-known model is the “voter model” (Holley and Liggett 1975; see further Castellano et al. 2009) in which agents copy the binary opinion of one of their network contacts. These models can generate metastable or, in some cases, even long-run stable spatial clustering of attitudes, especially when interaction is constrained to local neighborhoods. A further way to reconcile persistent diversity with assimilative influence is to assume that agents hold initial views that represent their individual interests, or their entrenched beliefs, and thus exert a constant “pull” towards an agent’s initial attitude that competes against the pressures from external sources of influence (Friedkin 1990, 2011). Such approaches offer possible explanations for persistent cultural diversity while retaining assimilative influence as the fundamental micro-level process. However, they do not take into account the possibility that assimilative social influence may sometimes not occur at all, even when people are connected in a social network. This possibility has been explored by the class of models to which I turn next.

Models Combining Homophily and Assimilative Social Influence

Fundamental Assumptions and Results

Building on previous work by Carley (1991) and by Nowak, Szamrej, and Latané (Nowak et al. 1990), Axelrod (Axelrod 1997) proposed an agent-based model of

“cultural dissemination” in which he combined assimilative cultural influence with “homophily,” the principle that “likes attract.” More specifically, Axelrod assumed that whether social influence occurs between connected individuals and how strongly they influence each other, depends on individuals’ similarity. One justification of this assumption is that individuals are cognitively more receptive to new information if the source of influence has more shared ideas and values with the recipient so that new information can be better connected to existing views (Mark 1998). Homophily has been found to be one of the most prevalent regularities of social life in the sense that people more likely interact and communicate with similar others (Lazarsfeld and Merton 1954; McPherson et al. 2001; Wimmer and Lewis 2010). Homophily may also be caused by structural patterns of social interaction that systematically sort similar people into similar “foci” (Feld 1982) where they meet and interact, like schools, neighborhoods, or workplaces; but the roots of homophily can also be emotional in that more similar people like and trust each other more (Byrne 1971) and, therefore, are more open to influence from each other.

Axelrod’s model became highly influential in the literature on ABCM of cultural diversity, because it highlighted a simple process through which diversity could emerge and persist despite connected networks. Axelrod added to earlier models that the likelihood of interaction between two neighbors could vary over time, depending on their similarity. More specifically, if two agents are too dissimilar, they will not interact and thus no social influence occurs between them. Homophily thus can generate a self-reinforcing dynamic in which similarity strengthens influence and influence leads to greater similarity. Similarly to what we have seen for continuous models of assimilative influence (Fig. 3), there can be from a random start an emergent local consensus in regions with initially high similarity. But convergence on a local consensus also implies divergence from the cultural profiles that develop in neighboring regions of the network. Once the members of two cultural regions have become too dissimilar to influence one another, their cultures evolve along divergent paths. This model accounts for both tendencies evident in cultural evolution—the relentless assimilation of cultural minorities and the emergence of global diversity between emergent cultural regions.

A Simple Example Based on Axelrod’s Original Model

In the following, I show the dynamics of this model with a simple example. Following Axelrod, I adopt a two-dimensional spatial framework in which a population of N agents is distributed over a bounded (non-toroidal) lattice, where each cell is occupied by a single agent. Most implementations of this model in the literature use a regular lattice structure, with squared cells arranged like on a checkerboard. I deviate here from this approach and use instead an irregular cellular grid structure (Flache and Hegselmann 2001), based on a so-called Voronoi diagram. This is to demonstrate that basic features of the dynamic known from earlier literature are robust across different spatial implementations. Regular spatial grids can be seen as an unrealistic representation of a real interaction structure, because all agents with the same distance to the boundary of a cellular space have the same number of neighbors and structural position vis a vis other network members. Irregular cellular grids based on Voronoi-diagrams impose instead some random variation in number of neighbors and also some variation in the structural position of agents in the interaction network (Okabe et al. 1992), while

retaining the assumption that cells are arranged in a two-dimensional space without empty space between them. Despite these differences, all results presented in this paper using an irregular spatial grid replicate on a qualitative levels findings obtained with otherwise equivalent models that use regular spatial grids.

Like in the simple model of assimilative influence discussed above, the degree to which interaction is locally constrained can be varied by the interaction radius around an agent's local position. In an irregular cellular grid, the interaction radius is implemented as the largest number of consecutive cells that are to be passed on the shortest path from a focal agent to a neighbor with whom interaction is possible. In my implementation, two cells are neighbors if they share a common border. Figure 4 shows the application of this approach to the two-dimensional irregular lattice.

Axelrod defined the cultural profile of an agent as a vector of F features. On any feature f , an agent has a trait q represented by an integer value in the range $q = \{0 \dots Q-1\}$, where Q is the number of possible traits on that feature. A simple illustrative example is a cultural space containing three features, "music," "religion," and "politics" in which per feature, agents can choose one out of two possible traits ($F=3$, $Q=2$). For music, for example, the choice might be between "classic" and "rock-pop". For religion, it could be between "Christianity" or "Islam" and for politics "Liberal" or "Conservative."

The similarity between the cultural profiles of two agents defines the likelihood that they will interact, given a chance. At every time point, one of the agents in the population is randomly chosen and gets the possibility to interact with one of her randomly chosen neighbors. The implementation of homophily is as follows. The focal agent chooses to actually interact with the neighbor with a probability p that is equal to the proportion of all F features on which they have identical traits. Consider the following situation based on our previous example. Agent A holds the profile {classic, christian, liberal}, and agent B {classic, islam, conservative}. If A could choose whether to interact with B, A would interact with a probability of one third, because they agree on only one out of three features (their common preference for classic music). If interaction takes place, this will lead to social influence. The implementation of social influence is that the initiating agent selects at random one of the features on which they do not agree yet, and then copies the trait of the source of influence on that feature. In our example, if A chooses to interact with B, she could choose to copy B's trait either on religion or on politics, resulting in her new cultural profile of either {classic, islam, liberal} or {classic, christian, conservative}, both with equal probability. Notice that this implies in particular that if two agents already agree on all features, they will always interact but this interaction has no effect on their cultural attitudes.

Fig. 4 Cells belonging to the neighborhood of the agent in the center in an irregular lattice, radii 1 and 3

Most importantly for the model dynamics, if two neighboring agents have no traits in common, they will not interact and thus no longer influence each other.

Figure 5 depicts prototypical dynamics of this model for a setting similar to one that Axelrod found to be conducive to high levels of diversity. I use a population of $N = 100$ agents located in the cells of a bounded spatial lattice, with interaction radius = 1 (see Fig. 4). The cultural space consists of $F = 5$ cultural features with $Q = 15$ traits per feature. A central question in much of the literature is whether and under what conditions persistent cultural clustering can arise from an initial situation of anomie, in which there is no relation between individuals' cultural profiles whatsoever. To address this question, I chose as initial condition a random cultural profile in which for all agents each feature is assigned a random trait q from a uniform distribution.

The simulation run shown in Fig. 5 illustrates how after about 20,000 simulation steps spatially consecutive “cultural zones” (Axelrod 1997) have developed in which agents share the same trait on feature 1. Agents within different zones may agree on other traits though, such that further interaction across the boundaries of these zones is still possible and the cultural distribution can further change. Sooner or later, however, the feedback loop of increased similarity and increased probability of interaction between initially more similar neighbors assures that their traits align across all features in the cultural space, producing an equilibrium outcome like the one depicted in the rightmost graph in Fig. 5. In this situation, agents with different colors disagree not only on feature 1, but also on all other features. “Cultural regions” (Axelrod 1997) of consecutive cells have formed within which consensus is perfect and between which interaction is impossible. Averaged across 10 independent realizations, a simulation of this model generates 14.4 distinct cultural regions in equilibrium, while initially each of the 100 population members had her own unique cultural profile. This reflects the pattern of local consensus and global cultural diversity often observed in the empirical world, which Axelrod sought to explain.

Extensions of Axelrod's Model in the Literature

Already, the original study of this model by Axelrod showed that the outcome of local consensus and global diversity only occurs within a specific window of conditions. These include a sufficiently small range of interaction, not too many features, an intermediate number of traits, and a medium-sized population. Nevertheless, Axelrod's seminal contribution has sparked a large follow-up literature addressing a wide range of extensions, modifications, and analytical treatments (for an overview, see *e.g.*,

Fig. 5 Dynamics of typical model run of Axelrod's model ($N = 100$, interaction radius = 1, $F = 5$, $Q = 15$). Color of a cell represents trait on feature 1

Castellano et al. 2009). Studies addressed, for example, the role of mass media (González-Avella et al. 2007; Shibanaï et al. 2001), globalization, modeled as increasing spatial range of interaction (Greig 2002), or geographical boundaries (Parisi et al. 2003) on the dynamics of cultural diversity. In the context of empirical research on political opinion formation, modified versions of the model were developed to understand persistent diversity in political discussion networks of US citizens (Huckfeldt et al. 2004). While much of this work suggested that the model's ability to generate robust cultural diversity generalizes to more complex versions, follow-up work also has highlighted that cultural diversity may be a highly fragile outcome that critically hinges upon some empirically implausible assumptions.

One problem follow-up research identified in particular is the sensitivity of cultural differentiation to random deviations. Axelrod's model assumes that interaction and thus influence is impossible if agents disagree on all features. However, in the real world, agents may occasionally be exposed to other sources of influence than their network neighbors, or may make errors in perceiving each other's traits or similarity, even when distinct cultural regions have crystallized out. Addressing these sources of error, further studies (e.g., Klemm et al. 2003; De Sanctis and Galla 2009) showed how a small probability of random perturbation of cultural traits undermines the emergence of distinct cultural regions and makes monoculture a highly likely outcome. One way how diversity can be preserved is to extend homophily towards "network homophily" (Centola et al. 2007) such that agents can choose to structurally disconnect from dissimilar neighbors so that even "erroneous" interactions are precluded (similar to people who move into a different neighborhood to avoid contact with old neighbors, because they have become too different from them). Flache and Macy (2011a) suggested another reason why cultural clustering could be robust despite random errors. They replaced the bilateral interpersonal interaction Axelrod's model assumes with multilateral social interaction in which an agent can be influenced by more than one neighbor at a time. Like in models of assimilative influence with local majority voting, they assumed that agents consider the aggregate distribution of traits in their entire neighborhood, but unlike those earlier models, they combined this with homophily. The likelihood that a specific neighbor is taken into account for assessing the local majority depends on the similarity to that neighbor. More recently, Ulloa (Ulloa et al. 2016) further extended this model to include conformity pressures exerted by social institutions, like pressures from a family to which one belongs, which discourages deviation of their members from those parts of a cultural profile that represent the family identity.

Interpersonal *versus* Social Influence: the Problem of Random Deviations

To illustrate the problem of random deviations and the effects of multilateral social influence, I present in the following a version of the model in which a small probability is added that agents interact with neighbors with zero cultural overlap. Figure 6 demonstrates the difference in the dynamics that follow for this situation from the interpersonal influence model of Axelrod (1997) and Flache and Macy's (Flache and Macy 2011a) model of social influence. The figure shows prototypical runs of both models for a set of conditions chosen such that cultural diversity can impossibly be sustained by Axelrod's original model. With $N = 1024$, the population is about ten times larger than in the simulations depicted in Fig. 5. Moreover, I increased the range

Fig. 6 Effect of social vs interpersonal influence on cultural diversity in Axelrod model. ($N = 32 \times 32$, irregular grid structure, neighborhood radius = 3, $F = 5$, $Q = 15$, interaction error 1%). Color of a cell represents trait on feature 1

of interaction to 3 (see Fig. 4) and added a probability of 1% in every step that an agent interacts with a neighbor even if their cultural similarity was 0. These changes were implemented in both of the models shown in Fig. 6.

Following the implementation of Flache and Macy (2011a), the social influence model (bottom row in Fig. 6) differs from the interpersonal influence model (top row) as follows. The agent that is chosen for an interaction forms a subset from her spatial neighbors that contains those neighbors whose traits are taken into account in the current interaction. Each spatial neighbor is selected into this subset with the same probability applied in Axelrod's original interpersonal influence model, given by the proportion of features on which the focal agent and the neighbor have the same trait. Having formed the subset, the focal agent selects at random one of the features on which there is no consensus yet among the neighbors in the subset. Finally, the focal agent copies on this feature the trait that has the largest number of followers in the subset, randomizing between traits if a tie between features occurs. There is one exception from this rule. If no other trait receives more support than the trait currently held by the focal agent, no change will occur. This model implements homophily in the same way than Axelrod did, assuming that more similar neighbors are more likely to be influential. However, influence is now no longer imposed exclusively by one neighbor at a time, but by the local majority in the subset of influential neighbors. Figure 6 shows how this change affects cultural diversity.

Figure 6 demonstrates how cultural regions both form faster and persist much longer if cultural influence is based on multilateral social influence rather than bilateral interpersonal influence. Initially, all traits are chosen at random, but influence leads in both models towards the emergence of locally more homogenous cultural regions

after some time. Once cultural regions start to crystallize out, it is much less likely under multilateral influence that an agent deviates from a locally emergent cultural norm. In the original Axelrod model, one “erroneous” interaction with an outside source of influence can suffice to let an agent adopt a cultural trait that is different from that of her local cultural region. Under multilateral influence, the outside influence will most likely be overruled by the simultaneous influences of compliant neighbors who are both more numerous and more similar to the focal agent. As Fig. 6 shows, this implies that under multilateral influence clearly distinct clusters can still persist after 10^{10} simulation events. At the same time, in the long run, there can be no other equilibrium result than monoculture also for multilateral influence. The 1% chance of an interaction error in every event makes sure that sooner or later events will occur in which agents on the border between clusters happen to be influenced by a majority of outgroup members and happen to spread the outside traits they acquired in subsequent interactions within group members. Yet, the probability that this happens is so small that the long-run outcome of monoculture is far beyond the reach of the simulation time that was practically available for this experiment.

Continuous Cultural Features and the Fragility of Cultural Diversity

Multilateral influence can stabilize cultural clusters against external pressures to deviate from the norm, but it fails to counter another source of fragility. Axelrod modified the framework of early models of assimilative influence not only by adding homophily, but also with replacing a continuous attitude space by a set of discrete cultural options between which agents have to choose. With this, he precluded the possibility that the attitude that results from influence is different from the prior attitudes of both interacting individuals. I imported this possibility into Axelrod’s framework, retaining the assumptions of homophily and multilateral influence. Drawing on earlier work (Flache et al. 2006), I implemented a version of the model in which agents can adopt on every feature every attitude value between 0 and 1. Like in early models of assimilative influence, social influence is furthermore implemented as weighted averaging.⁴ More precisely, an agent who is selected to update her cultural profile randomly selects a feature, and then moves her cultural attitude on that feature towards the weighted average of the attitude held by all neighbors (multilateral influence). Applied to modeling of the diffusion of innovations, this reflects the view that new technologies are not just assumed to spread, but also can be modified and changed in the process of social interaction between adherents of different technologies. Homophily, finally, enters the equation *via* computation of the influence weights. The more similar are the cultural profiles of two agents averaged across all features, the higher is the influence weight. Figure 7 demonstrates how under this dynamic initial diversity collapses into monoculture, very similar to the dynamics that we found for models of assimilative influence (see Fig. 2).

The dynamic results in monoculture although homophily could in principle prevent this. Like in Axelrod’s original model, two neighboring agents cannot influence each other if they are maximally different on all cultural features. However,

⁴ In the implementation this was approximated with assuming $Q = 10,000$ equidistant trait values in $[0,1]$ and rounding the result of the weighted averaging procedure to the nearest trait value on that scale.

Fig. 7 Collapse of diversity with continuous scaling of opinion dimensions and weighted averaging in model with multilateral social influence ($N = 32 \times 32$, neighborhood radius = 3, $F = 2$, $Q = 10,000$, no noise). Color of a cell represents trait on feature 1

with continuous features, it is highly unlikely that this happens from a random start. Only if an agent adopts on all features an attitude at one extreme end of the attitude scale and all neighbors' attitudes are located at the opposite extreme, the agent cannot be influenced. Isolation from influence is thus much less likely than in Axelrod's model, in which the likelihood of influence already drops to zero if agents have different traits on all features, regardless of the extent of their disagreement within a feature (Flache et al. 2006).

One way of maintaining the possibility of cultural clustering even with continuous features and weighted averaging is to impose threshold levels for a minimal degree of agreement that is needed before agents can influence each other (Flache et al. 2006; De Sanctis and Galla 2009). This is very similar to the mechanism implemented in models of "bounded confidence" (Deffuant et al. 2000; Hegselmann and Krause 2002), which show how cultural clustering can arise if the thresholds for influence are neither too prohibitive nor too low. Bounded confidence models have become a hugely influential class of ABCM models of social influence (for an overview see for example Lorenz 2007) that has helped to reconcile cultural clustering with weighted averaging in continuous attitude spaces and has in particular been used to understand dynamics of the diffusion of innovations (Deffuant et al. 2005). These models also have been applied to understand processes of collective extremization of a population, and how these are influenced by the topology of interaction networks (Amblard and Deffuant 2004). At the same time, under bounded confidence assumptions, cultural clustering can also be sensitive to some forms of noise (Mäs et al. 2010; Pineda et al. 2009) and model dynamics are constrained by an important property inherited from models of assimilative influence. Neither in bounded confidence models, nor in models of assimilative influence, a population can ever adopt more extreme attitudes than those of its initially most extreme members. This makes it difficult to explain processes of bi-polarization of opinions in which a group falls apart into increasingly opposed subgroups with more and more extreme views (Gentzkow 2016), or situations in which a group as a whole shifts towards a consensus more extreme than held by any of its members at the outset (Moscovici and Doise 1994; Myers 1982). In search for ways to understand such group processes from simple first principles, ABCM researchers began to explore differentiation as a further extension of the micro-processes of social interaction.

Models Combining Assimilation and Differentiation

Fundamental Assumptions and Results

Assimilative influence and homophily correspond to assimilation with and attraction to those who are similar. However, assimilation and homophily also have negative counterparts—differentiation from and xenophobia towards those who are different. This is suggested by theories of fundamental psychological processes in the formation of attitudes, like Heider’s balance theory (1946) or Festinger’s theory of cognitive dissonance (1957). In essence, both theories suggest that individuals experience tension if they agree with a negatively evaluated source of influence. One way to resolve this tension is differentiation, changing one’s attitude as to become more dissimilar to the source. Tension can also occur when an individual disagrees with a positively evaluated source. In this case, the tension can be resolved with assimilation, shifting one’s attitude closer to the source. In the context of technological boundaries, Roux and her co-authors (2017) relate negative influence to the wider social structure in which interactions are embedded, arguing that negative influence may explain resistance against the adoption of new techniques, when “technological boundaries overlap social boundaries, strengthened then by greater differentiation between groups” (p. 334).

In search for models that can capture emergent polarization, researchers have combined both assimilation and differentiation as micro-processes of social influence (Flache and Mäs 2008a,b; Huet et al. 2008; Jager and Amblard 2005; Macy et al. 2003; Mark 2003). Many of these models implement the interplay of both processes in a simple way. If two agents hold sufficiently similar cultural attitudes they evaluate each other positively and influence will be assimilative. However, if their cultural attitudes differ too much, differentiation prevails. As many studies showed, this simple approach can generate both convergence to monoculture or bi-polarization of the population into two increasingly antagonist factions, where the outcome depends on the exact initial conditions.

An Illustrative Implementation: Assimilation and Differentiation on a Ring Lattice

To illustrate the dynamics of combined assimilation and differentiation with a simple model, I adapt the model of assimilative influence given in Eq. [1]. Equation 2 shows how only one thing has changed in the model. This time, the influence weight w_{ij} is not fixed, but it changes over time depending on the degree of agreement between i and j . If the distance between their cultural attitudes exceeds 0.5, half of the maximal distance possible, the influence weight will be negative and i will shift her attitude away from j ’s attitude. If the distance is smaller than the threshold level of 0.5, the influence weight is positive and influence will be assimilative. With this model, influence can result in attitudes outside of the interval boundaries of 0 and 1. In this case, the resulting attitude of agent i is set to the interval boundary that was crossed by the attitude shift.

$$o_{i,t+1} = o_{it} + \Delta o_{it} = o_{it} + \mu(1-2|o_{jt}-o_{it}|)(o_{jt}-o_{it}) \quad (2)$$

For an illustrative simulation of this model, I adopt the same initial conditions used for generating Fig. 2 ($N = 100$, circular network with interaction range of 5, initial uniform random attitude distribution). This means that the weight computation of Eq. [2] only

applies when i and j are structurally connected in the circular network; otherwise, $w_{ij} = 0$. Figure 8 shows how the model dynamics gradually generate bi-polarization from a random start.

After about 2000 simulation events the population begins to split into two approximately equally sized subsets, one shifting towards an attitude of 0, the other moving towards an attitude of one. After about 4000 events, this split has become almost perfect, and after 6000 events a stable situation has emerged in which all agents have adopted an attitude on one of the two extreme poles of the attitude range. Notice that with an initially uniform random distribution, an arbitrarily chosen pair of agents is more likely to have a positive influence weight than a negative one. However, if agents hold initially attitudes that are already quite extreme (close to 0 or 1 on the attitude scale), they are likely to have a negative relation with the neighbors they interact with. As a consequence, these agents will shift to even more extreme positions and “infect” more moderate other neighbors with their extremism. Next time such an extremist is picked as interaction partner by a moderate individual, if the moderate agrees sufficiently with the extremist to trigger assimilative influence he will be pulled towards the same pole of the attitude spectrum than the extremist. Alternatively, if the moderate disagrees enough to trigger differentiation, she will be “pushed” towards the opposite pole of the spectrum. This happens simultaneously in both subsets that start to form on opposite sides of the attitude spectrum, resulting in the cascade of increasing shifts towards extreme attitudes shown by Fig. 8A. Figure 8B illustrates the resulting increasing concentration of the frequency distribution on the two modes of 1 and 0. After about 6000 simulation events, the distribution has become perfectly bimodal.

The emergent bi-polarization shown by Fig. 8 does not necessarily mean that there is a corresponding spatial clustering of attitudes into two consecutive cultural regions. Figure 9 shows how in the final distribution some level of spatial clustering of attitudes arises, exemplified by several clusters of 5 to 10 of neighboring agents who adopt the same extreme attitude; but, there are also spatially isolated agents and very small clusters of two or three, with their neighbors on both sides holding an attitude opposite

Fig. 8 Example of bi-polarization ($N = 100$, initial opinion uniform random, interaction range = 5, $\mu = 0.5$, 10,000 events)

Fig. 9 Stable spatial clustering of opinions under assimilative influence and differentiation ($N=100$, initial opinion uniform random, interaction range = 5, $\mu=0.5$)

to their own. The reason for this pattern is again the interplay of assimilation and differentiation. Assimilation can shift groups of neighboring agents towards the same pole. However, if agents initially hold an attitude that strongly differs from that of their neighbors, these agents likely will push their neighbors to the opposite pole and simultaneously increasingly differentiate from them. In this way, single extremists can not only survive in between a local majority that is opposed to their extreme attitude, but they also contribute to radicalizing their neighbors.

Assimilation and Differentiation: a More Complex Example

The simulation shown in Figs. 8 and 9 used an extremely simple implementation of the simultaneous dynamics of assimilation and differentiation. As a final simulation experiment, I showed an implementation with a framework that is slightly more complex in order to demonstrate how this can generate richer spatial patterns and more cultural differentiation, while at the same time the fundamental dynamics of increasing bipolarization of the population is retained. I adopt the same population size ($N=1024$), two-dimensional spatial framework and opinion space that were already used for Fig. 7. Moreover, like in that model influence is simultaneously imposed by all neighbors. The main change relative to the model used for Fig. 7 is that now, like in the simple model described by Eq. [2], differentiation and xenophobia are added to the processes of assimilation and homophily. Technically, this is implemented such that agents move their attitudes towards the weighted average of the “pulls” and “pushes” imposed by assimilation towards or differentiation from all their neighbors in the spatial network. The weight of each single neighbor in forming this weighted average is computed based on the level of cultural disagreement, such that too much disagreement entails negative weights as in Eq. [2].

Figure 10 illustrates how the incorporation of differentiation as micro-process has radically changed the dynamics of cultural influence from the otherwise identical model of Fig. 7. While assimilative influence with homophily resulted in monoculture after about 50.000 simulation events, the model in which differentiation was added,

generates a dynamic in which the population self-organizes into spatial regions that are largely locally homogeneous, but strongly differ across space in terms of the locally prevalent attitude. After about 100,000 simulation events, the emergent pattern in attitudes on the first feature can be approximately described as a gradual shift of cultural attitudes when one moves from the lower left corner of the spatial grid across the diagonal to the upper right corner. On the whole, along this line attitudes shift from the extreme of $o = 0$ to the opposite pole ($o = 1$); but, the pattern is not that clear-cut. In particular, both the “black” zone in the lower left corner and the “white” zone in the upper right corner of the attitude space contain single “cultural deviants” who embrace the opposite pole. Like in the pattern shown by Fig. 9, these deviants “push” their local neighbors to differentiate from them into the opposite direction on the opinion scale. Over time, assimilation extends the areas in which extreme majorities form. After 1 million events, the middle region in which moderate attitudes survive (attitudes between about 0.1 and 0.9 in Fig. 10) has shrunk from both sides and after 100 million events there is only a small layer of moderates left. This layer is squeezed in between a large “white” extremist majority in the upper half of the spatial grid, and an opposed small “black” region in the lower left corner. However, at least temporarily, this dynamics exhibits what Duggins called “[s]trong diversity, defined as a smooth distribution of opinions along a continuous ideological spectrum” (2017, 1.4).

Discussion and Conclusion

The lack of cultural homogeneity in our social world raises the question how the omnipresent micro-process of assimilative social influence can be reconciled with the permanence and strength of cultural divisions that we often observe, for example, in the form of technological boundaries (e.g., Roux 2013) which inhibit the spreading of technologies between different cultures. The co-existence of cultural clustering at the macro-level with assimilative pressures in micro-social interactions is a paradigmatic example of a complex micro-macro interaction, a process in which a societal level outcome is neither necessarily intended nor beneficial from the perspective of the individuals whose actions bring it about. In this article, I focused on agent-based computational modeling (ABCM) as a new method of theoretically elaborating such micro-macro interactions. I have presented a number of computational models of

Fig. 10 Effect of continuous scaling of opinion dimension, multilateral influence and weighted averaging in cultural diversity model with both assimilation and differentiation ($N = 32 \times 32$, neighborhood radius 3, $F = 2$, $Q = 10,000$, no noise)

cultural influence dynamics that all endow their individual agents with assimilative influence in the sense that whenever two agents interact, a possible outcome is that their cultural attitudes become more similar. I have then demonstrated for some simple models how and under which conditions cultural heterogeneity can nevertheless result from the dynamics that ensue when multiple individuals interact based on this mechanism. Different models offer different possible explanations of cultural diversity. Broadly, these models combine in varying ways structural assumptions that constrain social interaction, most importantly local spatial clustering of social networks, and behavioral assumptions according to which influence is not always assimilative, but only under certain conditions. Especially, models add in various ways additional fundamental micro-processes like homophily, or differentiation, to the basic process of assimilative influence. Computational experiments show how such models point to different structural and behavioral conditions on which it depends whether social influence dynamics result in monoculture, cultural clustering, or even polarization between sharply antagonistic factions in cultural space.

Computational agent-based modeling is an approach that is by many seen as a highly promising tool for unraveling the complex interdependencies between individual behaviors and social outcomes. The simple models I have presented in this chapter illustrate this possibility, but leave at the same time many questions open as to how ABCM can be used in modeling cultural differentiation. In the following, five challenges and directions for future research are discussed, integration of further micro-processes known from empirical research on social influence, selection from the large variety of models the literature has produced, further analysis of macro-implications of models, empirically informed calibration of model assumptions and, finally, empirical validation of model implications.

One important task for ABCM research of cultural differentiation is the exploration of further micro-processes known from empirical research on social influence. This is illustrated by a number of recent developments in the literature. The final modeling example shown in this paper illustrated how the combination of assimilation, homophily, xenophobia, and differentiation can generate both polarization and spatial differentiation of a population for settings that move beyond the simple ring lattice and one-dimensional cultural space used in earlier examples. However, this model is still relatively simple compared to more recent approaches. One example is the ICT model of Duggins (2017) that extends the array of behavioral processes discussed here with additional assumptions. It adds in particular conformity, modeled as a tendency to express “false” agreement with the perceived opinions of network neighbors, as well as commitment to extreme beliefs. Duggins’ simulations show how a combination of these processes can produce phenomena like “pluralistic ignorance,” a systematic discrepancy between true and expressed opinions in a population that potentially destabilizes an existing consensus, or persistent strong diversity. In line with models presented here, Duggins assumes differentiation as micro-process. Motivated by experiments that failed to support negative influence empirically Takács et al. 2016, another line of work points to a route to polarization without the need to assume any differentiation from negatively evaluated sources of influence. Building on persuasive argument theory (Myers 1982; Vinokur and Burnstein 1978), models proposed by Mäs and coauthors (Mäs et al. 2013; Mäs and Flache 2013) assume that agents with more similar opinions are also more likely to persuade each other with arguments supporting

their already prevailing opinion tendency. Their simulations demonstrate how this can foster extremization of opinions in different subgroups towards opposing poles of an opinion spectrum, if agents prefer interacting with similar others, based on the principle of homophily. Similar dynamics have also been derived from models of “biased assimilation” (Dandekar et al. 2013) in which agents are assumed to put more weight on those influences in the process of assimilation that are in line with their current opinion. As a third example, Huet and Deffuant (2010) proposed a model that combines bounded confidence dynamics with differentiation. They incorporated concepts of social identity theory, adding the distinction between agreement on a primary cultural dimension that defines whether agents perceive each other positively or negatively and a secondary dimension in which agents aim to align their agreement with these positive and negative evaluations.

Expanding the literature with further psychologically plausible models capable of generating cultural diversity is an important task, but it also makes it increasingly difficult for applied research to choose the best model for a given problem at hand. As will be discussed below, more careful theoretical comparison and empirical validation of competing models are needed to address this problem. However, as several reviews of the field have indicated (Sobkowicz 2009; Flache et al. 2017), this is still work in progress. Nevertheless, some heuristics can be proposed to help researchers assess under which conditions each of the model classes discussed here is more or less plausible to describe dynamics of cultural diversity. To begin with, a model with exclusively assimilative influence seems only plausible if social conditions are present under which actors are susceptible to influence from network neighbors, regardless of the level of disagreement about the opinion or behavior at stake. If populations are divided by sharp status distinctions, or strong antagonism or distrust between subgroups, it seems unlikely that influence can only be assimilative. In such a setting influence between members from the same subgroup may be assimilative, but homophily may prevent influence across subgroup boundaries because actors avoid contacts or are not open for outside influence in such encounters. Similarly, homophily seems a plausible assumption if structural conditions, like residential segregation or geographic boundaries, discourage interaction between dissimilar people. Finally, strong status distinctions, antagonism, or an emotional or social need for cultural distinctiveness (Brewer 1991) in interactions between individuals from different groups, seem plausible conditions under which the micro-level process of differentiation can occur (see also Roux et al. 2017). However, homophily may also interfere with differentiation, if actors prefer to avoid interactions in which they otherwise would feel pressed to increase their disagreement with others. This suggests that an additional precondition for differentiation as micro-process is that “negative” interactions cannot easily be avoided.

A further way to guide researchers in the selection of models is insight into empirically refutable predictions that different models make. It can sensitively depend on the exact social structural conditions assumed in a model and the precise way the micro-processes are specified, whether and under which conditions a dynamic generates monoculture, cultural clustering, strong diversity, anomie, or polarization into antagonistic factions. For example, drawing on models that combine assimilation and differentiation similar to the last model presented in this paper, several studies (Flache and Mäs 2008a, b; Flache and Macy 2011b) argued that more network contacts between groups with dissimilar opinions or strongly different

demographic characteristics can foster polarization. The reason is that more possibilities for encounters between dissimilar actors make it more likely that interactions occur in which differentiation prevails, triggering cascades of further differentiation in the process. Yet, other studies' drawing on social influence network theory (Friedkin and Johnsen 1990, 2011), bounded confidence models (Amblard and Deffuant 2004), or models of persuasive argument influence (Mäs et al. 2013; Mäs and Flache 2013) point to the opposite possibility. Broadly, in these models more interaction between different subgroups promotes consensus. Here, the prevailing effect of more network contacts is to introduce more channels for assimilative influence. Systematic comparison of model predictions for defined substantive questions, like the relation between network integration and polarization, is an important direction for future ABCM work on cultural differentiation which researchers are now beginning to explore (*e.g.*, Feliciani et al. 2017).

The theoretical developments discussed above are needed, but they are not enough. Theoretical work alone leaves many questions open as to how ABCM can be used in archeological research that eventually aims to link conceptual quantitative models to empirical data about the spreading and change of cultural artifacts and technologies in space and across social networks. An important challenge researchers face is how to calibrate both behavioral and structural assumptions of their models to available field data. One issue particularly pressing in the context of archeological research is the relation between the timescale of influence events in an ABCM to timescales in which empirically observable changes in cultural differentiation occur. Fortunately, a number of predictions of alternative models are distinguishable without having to know the precise duration of a process. For example, models with assimilation and differentiation imply that cultural differences between neighboring groups can increase over time, models with homophily suggest such differences first rapidly increase and then remain stable for a long time, and models with assimilative influence typically generate gradually decreasing differences. Yet, in many cases, model calibration is needed to narrow down the number of degrees of freedom a modeler has in specifying assumptions for a particular empirical realm of interest. Possible sources researchers can draw upon are, for example, increasingly available data about the structure of social networks that can be inferred from archeological data (Brughmans 2010), or data from laboratory experiments as well as field experiments about micro-dynamics of social influence under closely controlled or observed conditions (Chacoma and Zanette 2015; Takács et al. 2016). Moreover, further elaboration and improvement of assumptions about the micro-processes in models of social influence should not only be driven by empirical research, but also by further theoretical work that opens up the "black box" of the processes of assimilation and differentiation and derives them from deeper underlying psychological and cognitive principles (*e.g.*, Duggins 2017; Groeber et al. 2014).

Another challenge is that of model validation, testing predictions that ABCM generate against available data. The power of ABCM to conduct systematic theoretical experimentation can be used to generate competing and alternative macro-hypotheses about the patterns of spatial distribution, spatial spreading, or degree of diversity or homogeneity in cultural attitudes that follow from a particular set of models. This will help researchers to select between models based on available macro-level evidence, and generate new and fruitful research questions about the possible micro-processes that could have generated observable spatial structures in cultural attitudes. There are no

easy and general answers for how to move from highly abstract models that yield a “proof of principle” towards tools that can generate empirically testable explanations of archeological data. However, the promise ABCM holds for linking micro-assumptions to macro-outcomes in complex social dynamics has motivated many archeologists to start searching for possible solutions (Cegielski and Rogers 2016; Wurzer et al. 2015). This has already generated promising examples of applications of ABCM in search for explanations of persistent cultural diversity as well as other archeological phenomena. More progress lies ahead.

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