

# Climate policy and optimal public debt

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## Abstract

Employing a two-period model with an environmental externality, this paper investigates the relation between emission taxation and the optimal level of public debt. The central insight is that the effect of emission taxation on optimal borrowing is ambiguous and may lead to lower or higher optimal debt. In the context of climate change, we even show that the counterintuitive result of a higher optimal debt level is likely in the short-run and possibly also in the long-run, a result that provides a novel rationale for public borrowing. Our basic arguments turn out to be robust against several generalization.

**Keywords** Adaptation  $\cdot$  Environmental externality  $\cdot$  Public debt  $\cdot$  Climate policy  $\cdot$  Tax smoothing

JEL Classification  $H23 \cdot H63 \cdot Q54 \cdot Q58$ 

## **1** Introduction

The substantial social and economic costs of environmental degradation arising from climate change have been thoroughly detailed, among others, by Tol (2002a, b) and Stern (2008). Governments worldwide therefore have set ambitious goals to reduce or completely curb carbon emissions as required by the Paris agreement which aims at keeping global warming below 2 degrees (United Nations, 2015). Market-based

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instruments like emission taxes or permits are known to be the most efficient way to reduce carbon emissions (De Mooij et al., 2012). Moreover, they have the added benefit of generating substantial revenues for governments. For example, the European emission trading system alone was projected to generate  $\notin$  46.8 bn in 2022 (European Commission, 2021), while Jakob et al. (2016) find that a global carbon price increasing from US\$ 12 in 2015 to US\$ 175 in 2030 would generate average annual revenues of US\$ 1.6 tn. The huge fiscal potential may provide national governments the incentive to use revenues from carbon pricing in order to reduce their high levels of public debt. This is of particular relevance, as many countries still struggle with the sustainability of public finance in the wake of the Great Recession and the Covid-19 pandemic.

The present paper challenges the use of public revenues from emission pricing for reducing public debt. We investigate the relation between emission taxation and public debt, addressing the following normative research question. When a tax is implemented not only to satisfy public spending requirements, but also to lower emissions and internalize the associated environmental externality, will it provide a justification for decreasing or increasing the optimal level of public debt? At first glance, it may seem intuitive that additional revenues from emission taxation should be employed to reduce public debt. But the central insight from our analysis is that the effect of emission taxation is ambiguous and may lead to lower or higher optimal debt levels. In the context of climate change, we even argue that the counterintuitive result of a higher optimal debt level may be plausible in the short-run and possibly also in the long-run, a result that provides a novel rationale for public borrowing.

In order to derive these insights, we develop a model based on the canonical tax-smoothing framework of Barro (1979, 1989). We consider a two-period variant of Barro's approach and explicitly introduce an environmental externality into the analysis. More precisely, a representative household is supposed to consume two goods in each period, one of which pollutes the environment. Emissions create pollution and thereby cause environmental damages. A benevolent government levies an emission tax on the polluting good which serves two purposes. First, it internalizes the environmental externality and, second, it provides funds for fulfilling exogenously given public spending requirements in the two periods. Moreover, in the first period, deficit spending is possible via public debt that matures in the second period.

As a benchmark, we first replicate the tax-smoothing result in the absence of environmental damages. In order to minimize the present value of the excess burden, optimal tax rates and revenues are then constant and optimal public debt is nonzero only if spending requirements vary over time. Against this benchmark, we show that the optimal tax and debt policy deviates from tax-smoothing when environmental externalities are present. The impact of emission taxation on the optimal budget balance turns out to be contingent on two factors: First, the development of cumulative marginal damages over time, i.e., whether first- or second-period emissions are more harmful and, second, the location of the tax rates on the Laffer curve, i.e., whether a higher tax rate increases or decreases tax revenues. Depending on the respective realization of each factor, we identify four different regimes which can either mandate public debt or savings. For *increasing* cumulative marginal damages and tax rates on the *increasing* side of the Laffer curve (regime A) or *decreasing* cumulative marginal damages and tax rates on the *decreasing* side of the Laffer curve (regime B), optimal debt becomes positive even if public spending requirements are constant over time. In contrast, public saving is optimal if cumulative marginal damages are *increasing* and tax rates are on the *decreasing* side of the Laffer curve (regime C) or cumulative marginal damages are *decreasing* and tax rates are on the *increasing* side of the Laffer curve (regime D).

In order to explain the intuition of these results, consider first regime A. With increasing cumulative marginal damages, the need for internalizing the environmental externality is stronger in the second period than in the first period. Therefore, the optimal second-period emission tax rate exceeds the optimal first-period emission tax rates. Since under regime A both emission tax rates are located on the increasing part of the Laffer curve, we obtain higher tax revenues in the second period than in the first period. Hence, a positive optimal debt level in the first period is employed to shift some of the tax revenues through time, even if spending requirement are the same in both periods. Under regime B, declining cumulative marginal damages imply decreasing optimal tax rates, indeed, but both tax rates are now on the decreasing side of the Laffer curve. Thus, optimal tax revenues are still higher in the second period and and we obtain a positive optimal debt level. Optimal public savings under regimes C and D can be explained by analogous arguments.

In order to highlight the meaning of the results from our theoretical analysis, we discuss them against the background of greenhouse gas emissions and climate change. Cumulative marginal damages are then represented by the Social Cost of Carbon (SCC). It turns out that distinguishing between the short-, mid- and long-run is useful. In the short-run, say until 2035, the SCC is most likely to be increasing (EPA, 2022), and we will argue that emission tax rates on the increasing side of the Laffer curve are plausible in this short time period. Hence, we obtain the counterintuitive positive effect on optimal public debt under regime A. Beyond the shortrun, the tax base erosion effect has to be taken into account, because carbon pricing and taxation aim at largely removing greenhouse gas emissions. In the mid-run, e.g., until 2070, we consider that this effect may lead to an intermediate regime with increasing tax rates on different sides of the Laffer curve and higher tax revenues in the first period. The optimal budget policy may then feature public savings. However, for the long-run perspective after 2070, there is first evidence that the SCC may decline again (Kornek et al., 2021). Provided the tax base erosion effect does not completely eliminate greenhouse gas emissions, for instance, because carbon capture technologies are available, we may end up in regime B that also implies the counterintuitive positive effect on optimal debt.

We consider several extensions to examine the adaptability of the baseline model and verify our findings in more general settings. First, a richer framework taking into account the debt burden caused by interest payments or distortionary effects of public debt is considered. The debt burden of interest payments affects the quantitive level of optimal public debt, but leaves the normative impact of emission taxation on optimal debt completely intact. The applicability to the case of climate change is even improved, since a positive discount rate is explicitly taken into account. Analyzing the debt burden caused by distortionary effects of debt is more complicated, but we can qualitatively show that the counterintuitive positive effect on optimal debt becomes more likely, as an increase in public debt has the direct positive effect of reducing consumption and emissions. Second, for the case that maximal revenues from emission taxation are not sufficient to meet the government's spending requirements, we introduce an additional distortionary tax. This tax turns out to be subject to the standard tax-smoothing principle, while taxation of the externality still follows the same optimal rules established in the baseline model. Third, we consider investments in adaptation as an endogenous channel of public spending. Upfront first-period adaptation improves the economy's ability to cope with pollution in the second period. We then identify a direct positive effect of adaptation on optimal public debt, since adaptation investment should be distributed over both periods, as well as a positive or negative indirect effect on public debt, since adaptation moves the model closer to a framework without externality and, thereby, mitigates the deviation from the tax-smoothing principle identified in the baseline model. However, the positive direct effects turns out to dominate the indirect effect, so our counterintuitive result of an increasing optimal debt level becomes more likely.

Our paper contributes to the literature in two ways. First, we add an additional dimension to the discussion on the fiscal implications of climate policy. A longstanding strand of this literature analyzes the double dividend according to which the revenues from emission taxes can be used to reduce other distortionary taxes (e.g., see Bovenberg and De Mooij, 1994; Proost and Van Regemorter, 1995; Parry, 1995; Goulder, 1995). A related topic is discussed in the study by Franks et al. (2017) who employ a dynamic general equilibrium model to investigate whether emission taxation attains a higher welfare level than taxation of mobile capital. However, none of these contributions examines the link between emission taxation and optimal public debt. Second, our paper introduces the issue of emission taxation into the literature on public debt. The existing literature can basically be divided into positive studies explaining the accumulation of public debt, like the political economy models by, e.g., Persson and Svensson (1989), Tabellini and Alesina (1990) and Woo (2003), and normative studies investigating optimal public debt, like the tax-smoothing theory of Barro (1979, 1989). Recently, a growing number of contributions have started to address the interactions between public debt and climate policy. Fodha and Seegmuller (2014) examine the welfare effect of an environmental abatement policy which may either be funded by tax revenues or public debt in a dynamic model finding that pollution abatement should not be conducted at the costs of increased debt when the capital stock is low. In a model without emission taxation, Catalano et al. (2020) investigate the impact of fiscal policy on public adaptation investments in a multi-country setup, where they show that early debt-funded adaptation spending has a long-run beneficial effect despite its initially detrimental impact on the debt-to-GDP ratio. Zenios (2022) shows that exposure to climate change can affect fiscal stability by lowering GDP growth. Andersen et al. (2020) focus on the intergenerational distributional effects of costly public abatement and how taxation and debt can be employed to reach an intergenerational Pareto improvement. Finally, Kellner (2023) reexamines the strategic role of public debt under reelection uncertainty when public spending is associated with pollution. While these papers also investigate the link between debt and environmental issues, none of them takes into

account the dual role of emission taxation as a means of financing public spending and correcting externalities.

The subsequent analysis is organized as follows. In Sect. 2, we introduce the baseline framework. In Sect. 3, we derive optimal tax and debt regimes. In Sect. 4, we discuss the extensions. The final section concludes the paper.

## 2 Model

To identify the basic mechanisms, we deliberately choose the simplest model possible. This is suitable, since our main aim is to show that the effect of emission taxation on optimal debt is ambiguous and to highlight the importance of this ambiguity by identifying conditions under which the counterintuitive debt-increasing effect of emission taxation occurs. In Sect. 4, we introduce richer modeling assumptions and show that our basic arguments still hold.

## 2.1 Private sector

We consider an economy with a representative household that lives for two periods. The first period can be understood as the present, whereas the second period stands for the future. In period t = 1, 2 the household consumes a composite good Y in quantity  $y_t$  and a polluting good X in quantity  $x_t$ . The household's utility in period t is given by the quasi-linear function

$$u_t = y_t + V(x_t),\tag{1}$$

with V' > 0 and V'' < 0. In each period, the household receives an exogenous endowment normalized to one. Goods *Y* and *X* can be produced from the endowment by a one-to-one-technology, so prices of both goods are equal to one. Good *Y* is untaxed, whereas good *X* is taxed by a unit tax with tax rate  $\tau_t$  in period *t*. The household may receive a lump-sum transfer  $z_t$  from the government in period *t*. The private budget constraint in period *t* is

$$y_t + (1 + \tau_t)x_t = 1 + z_t.$$
 (2)

Tax rates and lump-sum transfers are taken as given by the household.<sup>1</sup>

The household chooses consumption in order to maximize the present value of its utility given by  $w = u_1 + u_2$ , where the discount rate is normalized to zero. Inserting (2) into (1), the maximization problem reads

$$\max_{\{x_t\}_{t=1,2}} w = \sum_{t=1,2} \left\{ V(x_t) + 1 + z_t - (1+\tau_t)x_t \right\}.$$

<sup>&</sup>lt;sup>1</sup> We initially ignore private savings of the household, but relax this assumption in Sect. 4.

The household is also affected by the environmental damage which we introduce below. We ignore the damage in the above problem, since the household takes it as given. The first-order condition with respect to  $x_t$  is

$$V'(x_t) = 1 + \tau_t, \qquad t = 1, 2.$$
 (3)

According to (3), the household's optimal consumption of good X in period t is a function of the tax rate in period t. Formally, Eq. (3) implies  $x_t = X(\tau_t)$  with  $X'(\tau_t) = 1/V'' < 0$  and  $X''(\tau) = -V'''/V''^3 \ge 0.^2$ 

#### 2.2 Government

In addition to taxing good X, the government may raise revenues in the first period through issuing public debt b which has to be repaid in the second period. Since the discount rate of the household has been normalized to zero, we also set the interest rate equal to zero, implying zero interest payments in the second period.<sup>3</sup> Public policy pursues two goals. First, revenues from taxation and debt are used to finance public spending requirements in both periods. In the basic model, we follow the taxsmoothing literature and assume exogenously given spending requirements  $g_1 \ge 0$ and  $g_2 \ge 0$  in the two periods.<sup>4</sup> Second, the government uses taxation in order to internalize the pollution externality caused by private consumption of good X. In period 1, this externality is reflected by the damage function  $D_1(x_1)$  with  $D'_1 > 0$  and  $D_1'' \ge 0$ . In period 2, the damage function reads  $D_2(x_2 + \gamma x_1)$  with  $D_2' > 0$ ,  $D_2'' \ge 0$ and  $\gamma > 0$ . The parameter  $\gamma$  allows differentiating between flow pollution ( $\gamma = 0$ ) and stock pollution ( $\gamma > 0$ ). Notice that we make a distinction between the functional form of first- and second-period damages, i.e., the damage function  $D_1$  may be different from the damage function  $D_2$ . For climate change, this assumption can be motivated by, e.g., tipping points which considerably change the structure of damages once the global temperature exceeds a certain threshold, even if the extent of pollution returns to lower levels in later periods. Moreover, differences in the damage functions may also implicitly reflect discounting of future damages, which does not explicitly show up in our basic model, since we have normalized the discount rate to zero.5

Formally, the government's welfare maximization problem can be stated as

$$\max_{\{b,\tau_t,z_t\}_{t=1,2}} w = \sum_{t=1,2} \left\{ V[X(\tau_t)] + 1 + z_t - (1 + \tau_t)X(\tau_t) \right\} - D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1)],$$
(4)

<sup>&</sup>lt;sup>2</sup> We implicitly assume  $V''' \ge 0$ , which is satisfied, e.g., if V is quadratic or if V is monotone and has monotone derivatives. In the latter case, V''' > 0 is implied by V' > 0 and V'' < 0.

<sup>&</sup>lt;sup>3</sup> An endogenous interest rate that may differ from zero is considered in Sect. 4.

<sup>&</sup>lt;sup>4</sup> In Sect. 4, we extend the basic model in order to account for endogenous public spending.

<sup>&</sup>lt;sup>5</sup> This will become obvious in Sect. 4, where we show that our basic arguments may also hold for a uniform convex damage function  $D_1 \equiv D_2 \equiv D$  with D'' > 0 and a nonzero discount rate.

subject to

$$\tau_1 X(\tau_1) + b = g_1 + z_1, \quad \tau_2 X(\tau_2) - b = g_2 + z_2, \tag{5}$$

$$z_1 \ge 0, \qquad z_2 \ge 0. \tag{6}$$

According to (4), the government maximizes the present value of the household's utility net of environmental damages, taking into account the public budget constraints given in (5) and the household's consumption reactions determined by  $x_t = X(\tau_t)$ . Moreover, due to (6), we restrict the policy space to nonnegative lumpsum transfers. The reason is that we follow the tax-smoothing literature and focus on the case where the government has to use distortionary taxation in order to meet its spending requirements. If we would allow for negative transfers, the government would have an incentive to use these transfers in order to finance the spending requirements in a non-distortionary way. Notice that we nevertheless need the transfers since in our framework, in contrast to the previous tax-smoothing literature, tax revenues may exceed the spending requirements due to the government's second goal of internalizing the pollution externality. Hence, the transfers exist in order to redistribute back potential excess revenues from the emission tax in a nondistortionary way. As shown below, (6) will be binding—and  $z_1$  and  $z_2$  will vanish in the (most realistic) case where emission tax revenues are not sufficient to finance the public spending requirements.

The solution to the government's welfare maximization problem (4)–(6) can be characterized with the help of the Lagrangian

$$\begin{split} L &= \sum_{t=1,2} \left\{ V[X(\tau_t)] + 1 + z_t - (1 + \tau_t)X(\tau_t) \right\} \\ &- D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1)] \\ &+ \lambda_1[\tau_1 X(\tau_1) + b - g_1 - z_1] + \lambda_2[\tau_2 X(\tau_2) - b - g_2 - z_2], \end{split}$$

where  $\lambda_1$  and  $\lambda_2$  are the Lagrange multipliers associated with the public budget constraints in (5). The Kuhn–Tucker first-order conditions read

$$L_b = \lambda_1 - \lambda_2 = 0, \tag{7}$$

$$L_{\tau_1} = -X(\tau_1) - \left\{ D'_1[X(\tau_1)] + \gamma D'_2[X(\tau_2) + \gamma X(\tau_1)] \right\} X'(\tau_1) + \lambda_1 \Big[ X(\tau_1) + \tau_1 X'(\tau_1) \Big] = 0,$$
(8)

$$L_{\tau_2} = -X(\tau_2) - D'_2[X(\tau_2) + \gamma X(\tau_1)]X'(\tau_2) + \lambda_2 \Big[X(\tau_2) + \tau_2 X'(\tau_2)\Big] = 0,$$
<sup>(9)</sup>

$$L_{\lambda_1} = \tau_1 X(\tau_1) + b - g_1 - z_1 = 0, \tag{10}$$

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$$L_{\lambda_2} = \tau_2 X(\tau_2) - b - g_2 - z_2 = 0, \tag{11}$$

and the slackness conditions are

$$L_{z_1} = 1 - \lambda_1 \le 0, \quad z_1 \ge 0, \quad z_1 L_{z_1} = 0, \tag{12}$$

$$L_{z_2} = 1 - \lambda_2 \le 0, \quad z_2 \ge 0, \quad z_2 L_{z_2} = 0,$$
 (13)

where in (8) and (9) we used (3). For the second-order conditions to be satisfied, the determinant |H| of the bordered Hessian needs to be negative. We determine |H| in Appendix A.1 and will verify |H| < 0 in all relevant cases.

## 3 Optimal tax and debt policy

To analyze the government's welfare maximum characterized by conditions (7)-(13), we first examine the public budget constraints (10) and (11). Adding both equations gives the government's intertemporal budget constraint

$$\tau_1 X(\tau_1) + \tau_2 X(\tau_2) = g_1 + g_2 + z_1 + z_2, \tag{14}$$

stating that the present value of tax revenues (LHS) has to equal the present value of public spending and transfers (RHS). Notice that for constant spending requirements and transfers the present values of emissions tax revenues on the LHS of (14) has to be constant, too. Thus, the question of optimal taxation in our model is the question of how to optimally distribute tax revenues over the two periods. Subtracting (11) from (10) yields

$$b = \frac{g_1 - g_2}{2} + \frac{\tau_2 X(\tau_2) - \tau_1 X(\tau_1)}{2} + \frac{z_1 - z_2}{2}.$$
 (15)

We will use (15) in order to determine the optimal level of public debt. The first term on the RHS shows the usual tax-smoothing motive for governmental borrowing: Public debt is used to equalize variations in exogenous public spending. In the absence of externalities, the previous tax-smoothing literature referred to in the Introduction has shown that this is the only motive for debt, since tax rates and tax revenues are constant over time in the optimum, rendering the second term on the RHS of (15) equal to zero.<sup>6</sup> In contrast, we will show below that tax revenues may vary over time in our analysis with environmental externalities. Thus, taxation can affect optimal debt policy also via the second term on the RHS of (15). Specifically, if tax revenues in the second period,  $\tau_2 X(\tau_2)$ , are larger than tax revenues in the first period,  $\tau_1 X(\tau_1)$ , then the second term on the RHS of (15) is positive, providing a novel rationale for public debt. For notational convenience, we denote tax revenues in period t

<sup>&</sup>lt;sup>6</sup> We here ignore the last term in (15), since—as stated above—in the most realistic case we have  $z_1 = z_2 = 0$ . On page 13, we briefly comment on the case with positive transfers  $z_1$  and  $z_2$ .

associated with tax rate  $\tau_t$  by the Laffer curve  $R(\tau_t) := \tau_t X(\tau_t)$  which is assumed to be twice continuously differentiable and satisfy  $R'(\tau) = X(\tau) + \tau X'(\tau) \ge 0$  if and only if  $\tau \le \overline{\tau}$  with  $\overline{\tau} > 0$  and  $R''(\tau) = 2X'(\tau) + \tau X''(\tau) < 0$ . Hence, the Laffer curve is inverted U-shaped with a unique maximum at the positive tax rate  $\overline{\tau}$ .<sup>7</sup>

Next, we rewrite the first-order conditions of welfare maximization in order to identify conditions under which tax rates and revenues differ across the two periods. From (7), we obtain  $\lambda_1 = \lambda_2 =: \lambda$ . Using this in (8) and (9) yields

$$\lambda = \frac{X(\tau_1) + \left\{ D'_1[X(\tau_1)] + \gamma D'_2[X(\tau_2) + \gamma X(\tau_1)] \right\} X'(\tau_1)}{X(\tau_1) + \tau_1 X'(\tau_1)},$$
(16)

$$\lambda = \frac{X(\tau_2) + D'_2[X(\tau_2) + \gamma X(\tau_1)]X'(\tau_2)}{X(\tau_2) + \tau_2 X'(\tau_2)},$$
(17)

Since  $\lambda \ge 1 > 0$  from the slackness conditions (12) and (13), the respective nominator and denominator on the RHS of (16) and (17) must have the same sign. As a benchmark, we start with the case where consumption does not cause damages and obtain the following result, proven in Appendix A.2.

**Proposition 1** . If  $D_1 \equiv D_2 \equiv 0$ , then the optimal policy is characterized by  $\lambda > 1$ ,  $z_1 = z_2 = 0$ ,  $\tau_1 = \tau_2 = \tau$  and  $b = (g_1 - g_2)/2$ , where  $\tau$  is implicitly determined by  $\tau X(\tau) = (g_1 + g_2)/2$  and lies on the increasing side of the Laffer curve  $R(\tau)$ .

Proposition 1 replicates the results from the previous tax-smoothing literature: If good *X* does not cause externalities, the only purpose of taxation is to meet the spending requirements. Since taxation is distortionary, the government chooses tax rates that minimize the excess burden. The minimum is obtained when tax rates and, thus, tax revenues are constant over time ( $\tau_1 = \tau_2 = \tau$  and  $R(\tau) = \tau X(\tau) = (g_1 + g_2)/2$ ). Due to the excess burden, the government will not generate more revenues than required for public spending, so transfers are zero ( $\lambda > 1$  and  $z_1 = z_2 = 0$ ). As a result, (15) reduces to  $b = (g_1 - g_2)/2$ , i.e., public debt or savings will only occur if the spending requirements are non-constant over time. More precisely, a strictly positive level of debt is optimal if spending is larger in period 1 than in period 2.

Having established the classical tax-smoothing benchmark, we can now turn to the case with externalities. The intertemporal sum of tax revenues,  $R(\tau_1) + R(\tau_2)$ , is still dictated by the exogenous sum  $g_1 + g_2$ . However, how much each period contributes to this sum, is now affected by the internalization objective which, in

<sup>&</sup>lt;sup>7</sup> In the basic model, we also implicitly assume that the maximal tax revenues from both periods are enough to cover the spending requirements, i.e.,  $2R(\bar{\tau}) > g_1 + g_2$ . In case of emission taxation, this might be viewed as unrealistic, since the maximum possible revenues from emission taxation are hardly enough to finance total governmental expenditures. However, if revenues from emission taxation are earmarked, then  $g_1$  and  $g_2$  may reflect only a (rather small) share of total public spending. Moreover, and more importantly, in Sect. 4 we show that our results also hold when some further costly taxation is available in addition to emission taxation.

turn, has ramifications for the optimal debt level. With  $D_1, D_2 \neq 0$ , (16) and (17) no longer imply  $\lambda > 1$  as in the case without externalities. For now,  $\lambda > 1$  is therefore assumed. We then obtain the following proposition that represents our main finding and is proven in Appendix A.3.

**Proposition 2**. If  $D_1, D_2 \neq 0$  and  $\lambda > 1$ , then the optimal policy is characterized by

- (i)  $z_1 = z_2 = 0$ .
- (ii)  $\tau_1 \leq \tau_2$  if and only if  $D'_1 + \gamma D'_2 \leq D'_2$ . (iii) If  $D'_1 + \gamma D'_2 < -x_1/X'_1$  and  $D'_2 < -x_2/X'_2$ , then  $\tau_1$  and  $\tau_2$  are both on the increasing side of the Laffer curve and  $\tau_1 \ge D'_1 + \gamma D'_2$  and  $\tau_2 > D'_2$ . Moreover,

$$b \stackrel{\geq}{\equiv} \frac{g_1 - g_2}{2} \quad \Leftrightarrow \quad D'_1 + \gamma D'_2 \stackrel{\leq}{\equiv} D'_2.$$

(iv) If  $D'_1 + \gamma D'_2 > -x_1/X'_1$  and  $D'_2 > -x_2/X'_2$ , then  $\tau_1$  and  $\tau_2$  are both on the decreasing side of the Laffer curve and  $\tau_1 < D'_1 + \gamma D'_2$  and  $\tau_2 < D'_2$ . Moreover,

$$b \stackrel{\geq}{\equiv} \frac{g_1 - g_2}{2} \quad \Leftrightarrow \quad D'_1 + \gamma D'_2 \stackrel{\geq}{\equiv} D'_2.$$

Parts (i) and (ii) of Proposition 2 characterize the optimal tax-transfer policy. Since  $\lambda > 1$  is presupposed in Proposition 2, the nonnegative constraints are binding and, thus, the optimal transfers are zero  $(z_1 = z_2 = 0)$ , as shown in part (i) of Proposition 2. Part (ii) shows that the relation between the optimal emission tax rates in the two periods follows the relation of these tax rates' Pigouvian levels, represented by the cumulative marginal damage that one unit of emissions causes over its lifetime. If the cumulative damage is lower for first-period than second-period emissions  $(D'_1 + \gamma D'_2 < D'_2)$ , then the optimal tax rate is lower in the first period than in the second period, and vice versa.

Interestingly, according to part (iii) and (iv) of Proposition 2, the optimal tax rates usually deviate from their Pigouvian level. The reason is that for  $\lambda > 1$  and  $z_1 = z_2 = 0$ , revenues from Pigouvian emission taxation would not be enough to meet the exogenous spending requirements. The direction of deviation depends on whether the optimal tax rates are on the increasing or decreasing side of the Laffer curve. If the cumulative marginal damages are relatively low  $(D'_1 + \gamma D'_2 < -x_1/X'_1)$ and  $D'_2 < -x_2/X'_2$ ), then both tax rates are still on the increasing side of the Laffer curve and set above their Pigouvian level in order to ensure higher tax revenues, as proven in part (iii) of Proposition 2. In contrast, if the cumulative marginal damages are relatively high  $(D'_1 + \gamma D'_2 > -x_1/X'_1)$  and  $D'_2 > -x_2/X'_2)$ , then the optimal tax rates move to the decreasing side of the Laffer curve and are set below their Pigouvian level in order to generate enough tax revenues, as shown in part (iv) of Proposition 2. Notice that the optimal tax rates may be on the decreasing side of the Laffer curve when externalities are taken into account, in contrast to the case without externalities considered in Proposition 1.

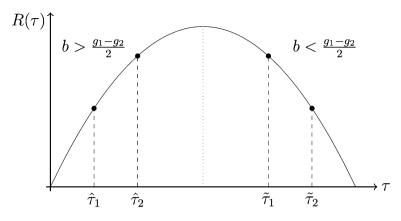


Fig. 1 The Laffer curve and optimal public debt

Parts (iii) and (iv) of Proposition 2 also characterize optimal debt. Since both tax rates usually differ from each other when externalities are taken into account, optimal public debt b deviates from the tax-smoothing level  $(g_1 - g_2)/2$  established in Proposition 1. Figure 1 illustrates this point and shows that the direction of deviation also crucially depends on the location of the optimal tax rates on the Laffer curve. The figure assumes that the cumulative marginal damage is lower for first-period than second-period emissions  $(D'_1 + \gamma D'_2 < D'_2)$ , so the emission tax rate is smaller in period 1 than in period 2 ( $\tau_1 < \tau_2$ ) due to part (ii) of Proposition 2. If cumulative marginal damages are relatively low  $(D'_1 + \gamma D'_2 < -x_1/X'_1 \text{ and } D'_2 < -x_2/X'_2)$ , part (iii) of Proposition applies and the optimal tax rates are located on the increasing side of the Laffer curve as represented by  $(\hat{\tau}_1, \hat{\tau}_2)$ . In this case, second-period tax revenues are higher than first-period tax revenues, which increases optimal public debt above the tax-smoothing level, i.e.,  $b > (g_1 - g_2)/2$ . In contrast, if cumulative marginal damages are relatively high  $(D'_1 + \gamma D'_2 > -x_1/X'_1 \text{ and } D'_2 > -x_2/X'_2)$ , part (iv) of Proposition 2 holds and optimal tax rates are represented by  $(\tilde{\tau}_1, \tilde{\tau}_2)$  on the decreasing side of the Laffer curve. Consequently, tax revenues are higher in the first period, implying a negative effect on optimal debt, i.e.,  $b < (g_1 - g_2)/2$ . If the cumulative marginal damages are decreasing over time  $(D'_1 + \gamma D'_2 > D'_2)$ , then  $\tau_1 > \tau_2$  and all effects on optimal public debt displayed in Fig. 1 are inverted, i.e., we find an incentive to decrease debt on the increasing side of the Laffer curve and an incentive to increase debt on the decreasing side.

To sum up, how optimal debt is affected by externalities depends on two factors: First, the evolution of cumulative marginal damages and, second, the tax rates' location on the Laffer curve. We obtain a debt-increasing effect if cumulative marginal damages are *increasing* and optimal tax rates are on the *increasing* side of the Laffer curve (regime A) or the cumulative marginal damages are *decreasing* and the optimal tax rates are on the *decreasing* side of the Laffer curve (regime B). A negative effect on optimal debt occurs if cumulative marginal damages are *increasing* and tax rates are on the *decreasing* side of the Laffer curve (regime C) or cumulative marginal damages are *decreasing* and the optimal tax rates are on the *increasing*  side of the Laffer curve (regime D). We compute numerical examples to ensure that a non-empty set of parameters exists which meets the conditions of each of the four regimes.<sup>8</sup>

By applying our analysis to the current problem of climate change, we may further illustrate the relevance of our results, in particular of regimes A and B as well as of an intermediate regime with tax rates on different sides of the Laffer curve. The latter regime has not yet been considered, but can easily be discussed with the help of Fig. 1. Notice first that in case of climate change the cumulative marginal damages  $D'_1 + \gamma D'_2$  and  $D'_2$  reflect the Social Costs of Carbon (SCC) of current and future greenhouse gas emissions, respectively. Our line of reasoning then depends on the time horizon considered. Let us start with a short-run view on global warming, say from now on to the year 2035. During this period, the EPA (2022) finds consistently increasing SCC. Moreover, if the SCC are still sufficiently low during the next years, the optimal tax rates are likely to be located on the increasing side of the Laffer curve.<sup>9</sup> Hence, in combination with increasing SCC we obtain regime A and a positive effect of the global warming externality on optimal public debt.

When we turn to a mid-run view, say up to the year 2070, we have to take into account that the main aim of climate policy is to mostly eliminate greenhouse gas emissions. Hence, an emission tax like a carbon tax will eventually erode its own tax base. In terms of our formal model, this implies that at least the second-period tax rate may be located on the decreasing side of the Laffer curve, close to the intersection with the horizontal axis. We may then obtain a negative effect of global warming on optimal public debt. For instance, if the optimal first-period tax rate is given by  $\hat{\tau}_1$  and the tax base erosion effect of emission taxation moves the optimal second-period tax rate to the right of  $\tilde{\tau}_2$ , then tax revenues will be higher in period 1 than in period 2 and optimal public debt is lower than without the global warming externalities. This regime with tax rates on different side of the Laffer curve is not included in Proposition 2, but it easily follows from our discussion of Fig. 1.

Finally, let us turn to the long-run, say the time period after 2070. The tax base erosion effect of climate policy will then still prevail, but over a very long time period the SCC may become decreasing. First evidence for such an effect is provided by Kornek et al. (2021), who take into account local inequality and transfers and show that in the USA, for instance, the SCC starts to fall after 2065. If they fall below their initial level, our formal analysis implies a higher emission tax rate in period 1 than in period 2. The consequences for optimal public debt then depend on the extent of the tax base erosion effect. If a greenhouse gas tax completely eliminates emissions, tax revenues are zero in both periods (optimal tax rates are at or above the intersection of the Laffer curve with the horizontal axis in Fig. 1) and the optimal level of public debt is the same as without the global warming externality. However, if optimal taxation still leaves room for a strictly positive emission level,

<sup>&</sup>lt;sup>8</sup> We relegate details into a supplementary appendix that can be obtained upon request.

<sup>&</sup>lt;sup>9</sup> To the best of our knowledge, there are no studies estimating Laffer curves of emission taxes. Trabandt and Uhlig (2011) find labor/capital taxes on the increasing side of the Laffer curve, providing plausibility that the same may be true for emission taxes, at least in the short-run.

for instance, because carbon capture technologies allow to compensate  $CO_2$  emissions, we may end up in a situation like regime B mentioned above, where the optimal emission tax rate is higher in period 1 than in period 2, both tax rates are located at the decreasing side of the Laffer curve and the global warming externality exerts a positive effect on public debt, even in the long-run.

Before turning to the extensions of our baseline model in the next section, a short remark on non-binding nonnegativity constraints is in order. If we assume  $\lambda = 1$ , in contrast to Proposition 2, the slackness conditions (12) and (13) imply that the transfers  $z_1$  and  $z_2$  may become positive. Abstracting from differences in first- and secondperiod transfers, we exclude public borrowing motives stemming from intergenerational redistribution issues. It can then be shown that all our results from Proposition 2 remain unchanged, except for  $z_1 = z_2 > 0$ ,  $\tau_1 = D'_1 + \gamma D'_2$  and  $\tau_2 = D'_2$ . Intuitively, in this case the revenues from Pigouvian emission taxation are more than enough to meet the public spending requirements and the excess revenues are redistributed back to the household by the transfers  $z_1$  and  $z_2$ . Admittedly, such a case seems to be less relevant and, therefore, we relegate the formal analysis to a supplementary appendix which can be obtained from the authors upon request.

### 4 Extensions

Our baseline model employs a highly stylized framework to identify the mechanism linking externalities and optimal debt. We now introduce several extensions to show that the basic effects also persist under richer assumptions.

#### 4.1 Further distortionary taxation

We start by taking into account a further distortionary source of tax revenues. So far, we implicitly assumed that revenues from emission taxation are sufficient to fund the intertemporal sum of public spending. This may be appropriate when we think of ear-marking emission tax revenues for financing 'mini-budgets' (see footnote 7). However, if  $g_1 + g_2$  represents a broader concept of public spending, it is hardly to be expected for taxation of a single externality to provide the required funds. Instead, an additional source of tax revenues is needed. Thus, we follow the canonical tax-smoothing approach (Barro, 1979) and consider a second distortionary tax raising revenues  $q_t > 0$  in period t. The tax is associated with costs  $C(q_t)$ , where C' > 0 and C'' > 0, which may reflect collection costs or, in a stylized way, the deadweight loss of the tax.

This extension can easily be incorporated into the analysis of the baseline model, if we define  $z_t := -q_t$ . Hence,  $z_t$  no longer reflects possible excess revenues from emission taxation, but the household's payments for the second distortionary tax. The household's optimal decisions at the margin remain unaffected and consumption of the polluting good is still  $x_t = X(\tau_t)$ . The government's welfare maximization problem also remains largely unchanged, except for replacing  $z_t$  by  $-q_t$  in (4) and (5), subtracting the distortion costs  $C(q_1) + C(q_2)$  from welfare in (4) and ignoring the nonnegativity constraints (6). The first-order conditions are given by (7)–(11) with  $z_t = -q_t$  and

$$L_{q_1} = -1 + \lambda_1 - C'(q_1) = 0, \quad L_{q_2} = -1 + \lambda_2 - C'(q_2) = 0.$$
(18)

The conditions in (18) replace the slackness conditions (12) and (13). Notice that we implicitly assume  $2R(\bar{\tau}) < g_1 + g_2$  (otherwise the maximum emission tax revenues are enough to meet the spending requirements), so we directly obtain the interior solution  $q_1, q_2 > 0$ . Together with  $\lambda_1 = \lambda_2$ , this implies

$$C'(q_1) = C'(q_2) \quad \Rightarrow \quad q_1 = q_2. \tag{19}$$

Hence, with respect to the second distortionary tax the standard tax-smoothing result holds, i.e., in order to minimize the deadweight loss of the tax, it is optimal to levy a constant tax over time. Moreover, costly taxation via  $q_t$  is debt-neutral. Since the first-order conditions (7)–(11) as well as (14) and (15) are unchanged, except of replacing  $z_t$  by  $-q_t$ , the results established in Sect. 3 are also true in the presence of a second distortionary tax.

#### 4.2 Debt burden

So far we ignored the debt burden caused by interest payments or by distortionary effects of public debt. In this section, we disregard the second distortionary tax again and extent our baseline model by a capital market that endogenously determines the interest rate. For the simplest model capturing the debt burden, assume the household's utility is no longer given by (1), but by  $u_1 = W(y_1) + V(x_1)$  with  $W'' \le 0 < W'$  in period 1 and  $u_2 = y_2 + V(x_2)$  in period 2. The present value of utility is  $w = u_1 + u_2/(1 + \rho)$ , where  $\rho > 0$  is a given discount rate. Instead of (2), the private budget constraint reads  $y_1 + (1 + \tau_1)x_1 + s = 1 + z_1$  in period 1 and  $y_2 + (1 + \tau_2)x_2 = 1 + z_2 + (1 + r)s$  in period 2, where *s* stands for the household's savings. The capital market equilibrium condition reads s = b and determines the interest rate *r*.

The household maximizes the present value of its utility with respect to consumption and savings, taking as given tax rates, transfers and the interest rate. In Appendix A.4, we derive the first-order conditions and show that these conditions together with the capital market equilibrium condition determine the equilibrium first-period consumption and the equilibrium interest rate as functions of the tax rate, transfer and public debt in the first period. Formally, we obtain  $x_1 = X^1(b, \tau_1, z_1)$  and  $r = R(b, \tau_1, z_1)$ with derivatives

$$X_b^1 = -X_z^1 = -\frac{(1+\tau_1)W''}{\Delta} \le 0, \quad X_\tau^1 = \frac{W' - x_1(1+\tau_1)W''}{\Delta} < 0, \tag{20}$$

$$R_{b} = -R_{z} = -\frac{(1+\rho)V_{1}^{\prime\prime}W^{\prime\prime}}{\Delta} \ge 0, \quad R_{\tau} = -\frac{(1+\rho)[V_{1}^{\prime} + x_{1}V_{1}^{\prime\prime}]W^{\prime\prime}}{\Delta} \gtrless 0, \quad (21)$$

where  $\Delta := (1 + \tau_1)^2 W'' + V_1'' < 0$  and  $V_1'' := V''(x_1)$ . Equilibrium consumption in period 2 is  $x_2 = X(\tau_2)$  as in the baseline model. Equations (20) and (21) show that

debt may now cause distortionary effects on consumption  $(X_b^1 < 0)$  and the interest rate  $(R_b > 0)$ , if W'' < 0. For W' = 1 and W'' = 0, these distortionary effects disappear and we obtain the same first-period consumption  $x_1 = X(\tau_1)$  as in the baseline model, even though savings are still endogenous.

The government maximizes the present value of the household's utility with respect to tax rates, transfers and debt, taking into account environmental damages, the capital market equilibrium (including the reaction of the household and the interest rate on policy changes), the nonnegativity constraints for the transfers and the public budget constraints. The latter are now given by  $\tau_1 X^1(b, \tau_1, z_1) + b = g_1 + z_1$  in period 1 and  $\tau_2 X(\tau_2) = g_2 + z_2 + [1 + R(b, \tau_1, z_1)]b$  in period 2. Hence, beside the above-mentioned distortionary effects of public debt on consumption and the capital market, the model extension captures a second potential source of the debt burden, i.e., interest payments  $R(b, \tau_1, z_1)b$  by the government in period 2. The first-order conditions of the welfare maximum are derived in Appendix A.4.

In order to disentangle the implications of the two sources of the debt burden, we start with the special case W' = 1 and W'' = 0. As shown in the discussion of (20) and (21), the distortionary effects of public debt then disappear and only the debt burden of interest payments remains. Appendix A.4 shows that the equilibrium interest rate is fixed to  $r = \rho$ . In the welfare maximum without environmental externality  $(D_1 \equiv D_2 \equiv 0)$ , the tax-smoothing result from Proposition 1 still holds, except that optimal tax revenues are  $R(\tau) = \tau X(\tau) = [(1 + \rho)g_1 + g_2]/(2 + \rho)$  and optimal debt amounts to  $b = (g_1 - g_2)/(2 + \rho)$ . Hence, the debt burden caused by interest payments leads to an adjustment of optimal tax revenues and debt by the discount rate,<sup>10</sup> yet the tax-smoothing role of public debt prevails. The results derived in Proposition 2 for the case with environmental damages  $(D_1, D_2 \neq 0)$ remain largely unchanged, too. Only the tax-smoothing level of debt again turns into  $b = (g_1 - g_2)/(2 + \rho)$  and the cumulative marginal damage of first-period emissions now amounts to  $D'_1 + \gamma (1 + \rho)^{-1} D'_2$  instead of  $D'_1 + \gamma D'_2$ . Hence, we can derive the same four regimes A-D as in the discussion of Proposition 2 and Fig. 1. Notice that the application of the results to climate change is even improved, since we may now obtain increasing SCC, that is  $D'_1 + \gamma (1 + \rho)^{-1} D'_2 < D'_2$ , even if  $\gamma$  is close to one (which seems plausible for many greenhouse gases, at least in the short- and mid-run) and damages in both periods are represented by the same convex function  $D_1 \equiv D_2 \equiv D$  with D'' > 0. In sum, taking into account the debt burden of interest payments leaves our main results from the basic model unchanged and even improves their applicability to the current problem of climate change.

Things are more complicated, if  $W'' \neq 0$ . Under this condition, public debt causes the additional debt burden of distorting consumption and the interest rate. It is then no longer possible to confirm Proposition 1 and 2. However, the main mechanism

<sup>&</sup>lt;sup>10</sup> More precisely, we have sign  $\{dR/d\rho\} = -\text{sign }\{db/d\rho\} = \text{sign }\{g_1 - g_2\}$ . For  $g_1 > g_2$ , optimal debt is positive (b > 0) and a higher interest rate  $r = \rho$  makes public borrowing more costly, so it is optimal to reduce debt *b* and increase tax revenues  $R(\tau)$ . For  $g_1 < g_2$ , optimal debt is negative (b < 0) and represents public savings. A higher interest rate  $r = \rho$  then implies higher interest receipts for the government and, thus, lower public savings *b* and tax revenues  $R(\tau)$ .

identified in the baseline model is still present and we can, at least qualitatively, show that the case for the counterintuitive positive effect of externalities on optimal debt (regimes A and B in the previous analysis) becomes more likely. To see this, rewrite from Appendix A.4 the first-order condition (A13) of welfare maximization with respect to debt as

$$\lambda_1 - (1+r)\lambda_2 + \left(\frac{1}{1+\rho} - \lambda_2\right)bR_b + \left[\lambda_1\tau_1 - \left(D_1' + \frac{\gamma}{1+\rho}D_2'\right)\right]X_b^1 = 0.$$

The expression  $\lambda_1 - (1 + r)\lambda_2$  on the LHS of this condition represents the taxsmoothing role of public debt, as in the baseline model. It shows that the basic mechanism identified in our previous analysis prevails. The second expression on the LHS containing the derivate  $R_b > 0$  reflects the effect of debt on private interest receipts, represented by the term  $1/(1 + \rho)$ , and public interest payments, represented by the multiplier  $\lambda_2$ , caused by an increase in the interest rate. The third expression on the LHS containing the derivative  $X_b^1 < 0$  shows that an increase in debt lowers private consumption in period 1 and, therefore, reduces first-period tax revenues, represented by the term  $\lambda_1 \tau_1$ , and environmental damages, represented by the SCC term  $D'_1 + \gamma(1 + \rho)^{-1}D'_2$ . All the debt effects containing the expressions  $R_b$ and  $X_b^1$  are new in comparison to the model without distortionary effects of public debt. But only the effect  $-[D'_1 + \gamma(1 + \rho)^{-1}D'_2]X_b^1 > 0$  is absent, if we ignore environmental externalities. Thus, when we introduce externalities into a world with distortionary effects of public debt, then debt causes an additional positive welfare effect and the counterintuitive regimes A and B become more likely.

#### 4.3 Adaptation to climate change

So far, we assumed that public spending is exogenously given. Many constituencies levying emission taxes earmark the revenues (at least partially) for green spending, e.g., the European emission trading system currently recommends that half of the revenues should be recycled for green spending (European Commission, 2020). Thus, we now extend the baseline framework by allowing endogenous public adaptation funded by emission taxation. Formally, we assume that the government can invest *a* in the first period, funding the adaptation technology T(a) with T'(a) > 0 and  $T''(a) \le 0$ . Adaptation reduces pollution damages in the second period, so the second-period damage function changes to  $D_2[x_2 + \gamma x_1, T(a)]$  with  $D_{2,X} := \partial D_2 / \partial (x_2 + \gamma x_1) > 0$ ,  $D_{2,XX} := \partial^2 D_2 / \partial (x_2 + \gamma x_1)^2 \ge 0$ ,  $D_{2,T} := \partial D_2 / \partial T < 0$  and  $D_{2,TT} := \partial^2 D_2 / \partial T^2 \ge 0$ . We suppose that the nonnegativity constraints are binding and, thus,  $z_1 = z_2 = 0$  from the outset.

The household's utility maximization remains completely unchanged in this extension. The government additionally chooses adaptation investment *a* taking into account adaptation expenditures in the first-period budget constraint and the effects of *a* on second-period damages. In Appendix A.5, we show that imposing the Inada conditions on T(a) ensures a > 0 in the welfare optimum. Thus, in order to figure out the effects of endogenizing public spending, we may run a comparative static

analysis increasing a from zero to a strictly positive level. From the governmental budget constraints, it follows

$$b = \frac{g_1 - g_2}{2} + \frac{a}{2} + \frac{R(\tau_2) - R(\tau_1)}{2}.$$
 (22)

Differentiating (22) with respect to *a*, taking into account that the optimal tax rates will change if the government chooses a positive investment a > 0, yields

$$\frac{\mathrm{d}b}{\mathrm{d}a} = \frac{1}{2} + \frac{1}{2} \left[ \frac{\mathrm{d}R(\tau_2)}{\mathrm{d}a} - \frac{\mathrm{d}R(\tau_1)}{\mathrm{d}a} \right] \quad \text{with} \quad \frac{\mathrm{d}R(\tau_t)}{\mathrm{d}a} = R'(\tau_t) \frac{\mathrm{d}\tau_t}{\mathrm{d}a}.$$
 (23)

According to the first term on the RHS of (23), adaptation creates a direct positive effect on optimal debt, since debt is used to distribute the investment costs equally across periods. The second bracketed term on the RHS of (23) shows an additional indirect effect via changes in the optimal tax revenues that are caused by changes in the corresponding optimal tax rates.

In general, we cannot sign the sum of the direct and indirect effect. However, clear-cut results are obtained under a linear-quadratic specification of the model. Suppose  $V(x_t) = (1 + \alpha)x_t - \beta x_t^2/2$ ,  $D_1(x_1) = \delta_1 x_1$  and  $D_2[x_2 + \gamma x_1, T(\alpha)] = \delta_2[x_2 + \gamma x_1 - T(\alpha)]$  with  $\alpha, \beta, \delta_1, \delta_2 > 0$ .<sup>11</sup> For this model specification, Appendix A.5 shows  $\tau_1 \leq \tau_2$  if and only if  $\delta_1 + \gamma \delta_2 \leq \delta_2$ , so the optimal emission tax rate still remains higher in the period where emissions cause the higher cumulative marginal damage as in our basic model without adaptation. Moreover, we obtain  $dR(\tau_t)/da > 0$  for t = 1, 2. First-period investment in adaptation is therefore funded by increasing tax revenues in both periods. However, the extent of the revenue increase may differ between the two periods, with different consequences for the change in optimal debt. For taxes on the increasing side of the Laffer curve, it turns out

$$\frac{\mathrm{d}R(\tau_1)}{\mathrm{d}a} \begin{cases} > \\ = \\ < \end{cases} \frac{\mathrm{d}R(\tau_2)}{\mathrm{d}a} \text{ and } \frac{\mathrm{d}b}{\mathrm{d}a} \begin{cases} \in \left(0, \frac{1}{2}\right), \text{ if } \delta_1 + \gamma \delta_2 < \delta_2 \text{ (regime A)}, \\ = \frac{1}{2}, \quad \text{ if } \delta_1 + \gamma \delta_2 = \delta_2, \\ \in \left(\frac{1}{2}, 1\right), \text{ if } \delta_1 + \gamma \delta_2 > \delta_2 \text{ (regime B)}, \end{cases}$$
(24)

while tax rates on the decreasing side of the Laffer curve imply

<sup>&</sup>lt;sup>11</sup> The qualitative effects may also be replicated in a numerical model with quadratic damage functions. Details are available from the authors upon request.

$$\frac{\mathrm{d}R(\tau_1)}{\mathrm{d}a} \begin{cases} < \\ = \\ > \end{cases} \frac{\mathrm{d}R(\tau_2)}{\mathrm{d}a} \text{ and } \frac{\mathrm{d}b}{\mathrm{d}a} \begin{cases} \in \left(\frac{1}{2}, 1\right), \text{ if } \delta_1 + \gamma \delta_2 < \delta_2 \text{ (regime C)}, \\ = \frac{1}{2}, \quad \text{if } \delta_1 + \gamma \delta_2 = \delta_2, \\ \in \left(0, \frac{1}{2}\right), \text{ if } \delta_1 + \gamma \delta_2 > \delta_2 \text{ (regime D)}, \end{cases}$$
(25)

where regimes A–D correspond to regimes A–D in the baseline model. With respect to changes in tax revenues, (24) and (25) show that strictly positive adaptation levels always reduce the gap between first-period and second-period revenues. Consider regime A in order to illustrate (the discussion of the other regimes is perfectly analogous). Then, both tax rates are on the increasing side of the Laffer curve with a larger tax rate and tax revenues in period 2 than in period 1 as  $\delta_1 + \gamma \delta_2 < \delta_2$  implies  $\tau_1 < \tau_2$  and  $R(\tau_1) < R(\tau_2)$ . The first line in (24) shows that adaptation increases tax revenues in period 1 by more than in period 2 and, thus, narrows the gap between the optimal tax revenues in the two periods. Intuitively, the reason is that adaptation reduces the marginal environmental damages and, thereby, moves the model with externality closer to the model without externality. The deviation from the taxsmoothing principle, which was key in our basic model, is thus attenuated when we account for endogenous adaptation expenditures.

The consequences for optimal public debt depend on the regime under consideration. In regime A, for example, first-period tax revenues increase by more than second-period tax revenues, so the indirect effect of adaptation on optimal debt identified in (23) is negative and compensates the positive direct effect in (23). Consequently, the sum of both effects is below 1/2, as shown in the first line of (24). Under regime D, first-period tax revenues increase by less than second-period tax revenues, rendering the indirect effect of adaptation on optimal debt in (23) positive and complementing the direct effect in (23). The sum of both effects is then larger than 1/2, according to the third line in (24). The arguments in regimes B and C are analogous. Most importantly, however, (24) and (25) show that the total effect of adaptation on optimal debt is positive in all four regimes, i.e., the indirect effect never overcompensates the direct effect in (23), even if the indirect effect is negative. That is, endogenizing public spending from the baseline model and taking into account adaptation makes the counterintuitive positive effect of externalities on optimal debt either more pronounced, in regimes A and B where the effect is already positive in the absence of adaptation, or more likely, in regimes C and D where the effect is negative without adaptation. This is true even though adaptation attenuates the deviation of the optimal taxes from tax-smoothing.

## 5 Conclusion

In this paper, we introduce a taxable environmental externality into the standard taxsmoothing framework of optimal public debt. When the government levies an emission tax not only to raise funds for public expenditures, but also in order to internalize pollution damages, adhering to a balanced budget rule is no longer optimal, even if spending requirements are constant over time. Instead, running a surplus (negative deficit) is welfare maximizing either if the tax rates are on the decreasing side of the Laffer curve and cumulative marginal damages from pollution increase over time or if the tax rates are on the increasing side of the Laffer curve while marginal damages decrease over time. In contrast, running a strictly positive public deficit turns out to be optimal if the tax rates are on the increasing or if the tax rates are on the decreasing side of the Laffer curve and cumulative marginal damages are increasing or if the tax rates are on the decreasing side of the Laffer curve and cumulative marginal damages are decreasing. The latter two regimes appear most relevant in the context of climate change, at least in the short-and the long-run, interrupted by an intermediate case with tax rates on different sides of the Laffer curve and ambiguous effects of the climate change externality on optimal public debt in the mid-run. In extensions, we show that our results remain robust when a second distortionary tax, the debt burden or endogenous spending for adaptation are taken into account.

A fully dynamic interpretation of our results to the short-, mid- and long-run of global warming may be better investigated in a multi-period simulation model that encompasses other important aspects of climate change as, for example, explicit modeling of tipping points or carbon capture technologies. However, the aim of our analysis is to provide a stylized theoretical framework for identifying the basic mechanism at work. A more sophisticated and empirically calibrated model, based on overlapping-generations models similar to Chiroleu-Assouline and Fodha (2006) or Li and Lin (2021) or an extended DICE simulation model employed by van der Ploeg and Rezai (2019), for instance, is beyond the scope of our paper and left for future research.

## **Appendix A**

#### A.1 Determinant of the bordered Hessian of (7)-(11)

The bordered Hessian H of the system of equations (7)-(11) can be written as

$$H = \begin{pmatrix} L_{\lambda_{1}\lambda_{1}} & L_{\lambda_{1}\lambda_{2}} & L_{\lambda_{1}b} & L_{\lambda_{1}\tau_{1}} & L_{\lambda_{1}\tau_{2}} \\ L_{\lambda_{2}\lambda_{1}} & L_{\lambda_{2}\lambda_{2}} & L_{\lambda_{2}b} & L_{\lambda_{2}\tau_{1}} & L_{\lambda_{2}\tau_{2}} \\ L_{b\lambda_{1}} & L_{b\lambda_{2}} & L_{bb} & L_{b\tau_{1}} & L_{b\tau_{2}} \\ L_{\tau_{1}\lambda_{1}} & L_{\tau_{1}\lambda_{2}} & L_{\tau_{1}b} & L_{\tau_{1}\tau_{1}} & L_{\tau_{1}\tau_{2}} \\ L_{\tau_{2}\lambda_{1}} & L_{\tau_{2}\lambda_{2}} & L_{\tau_{2}b} & L_{\tau_{2}\tau_{1}} & L_{\tau_{2}\tau_{2}} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & x_{1} + \tau_{1}X'_{1} & 0 \\ 0 & 0 & -1 & 0 & x_{2} + \tau_{2}X'_{2} \\ 1 & -1 & 0 & 0 & 0 \\ x_{1} + \tau_{1}X'_{1} & 0 & 0 & L_{\tau_{1}\tau_{1}} & -\gamma D''_{2}X'_{1}X'_{2} \\ 0 & x_{2} + \tau_{2}X'_{2} & 0 & -\gamma D''_{2}X'_{1}X'_{2} & L_{\tau_{2}\tau_{2}} \end{pmatrix},$$
(A1)

with

$$\begin{split} L_{\tau_1\tau_1} &= -X_1' - (D_1' + \gamma D_2')X_1'' - (D_1'' + \gamma^2 D_2'')X_1'^2 \\ &+ \lambda_1(2X_1' + \tau_1 X_1''), \end{split} \tag{A2}$$

$$L_{\tau_2\tau_2} = -X_2' - D_2'X_2'' - D_2''X_2'^2 + \lambda_2(2X_2' + \tau_2X_2''), \tag{A3}$$

and  $x_t = X(\tau_t), X'_t := X'(\tau_t), X''_t := X''(\tau_t), D'_t := D'(x_t)$  and  $D''_t := D''(x_t)$ . Calculating the determinant of *H* with standard methods gives

$$|H| = (x_1 + \tau_1 X_1')^2 L_{\tau_2 \tau_2} + (x_2 + \tau_2 X_1')^2 L_{\tau_1 \tau_1} + 2\gamma (x_1 + \tau_1 X_1') (x_2 + \tau_2 X_2') D_2'' X_1' X_2'.$$
(A4)

#### A.2 Proof of Proposition 1

From  $D_1 \equiv D_2 \equiv 0$  and (16) and (17), we obtain

$$\lambda = \frac{X(\tau_1)}{[X(\tau_1) + \tau_1 X'(\tau_1)]}, \quad \lambda = \frac{X(\tau_2)}{[X(\tau_2) + \tau_2 X'(\tau_2)]}.$$
 (A5)

Since  $\lambda \ge 1 > 0$  and  $X(\tau_t) > 0$ , it follows  $X(\tau_t) + \tau_t X'(\tau_t) > 0$  for t = 1, 2. Hence, in each period the optimal tax rate is on the increasing side of the Laffer curve. As  $X'(\cdot) < 0$ , we have  $X(\tau_t) + \tau_t X'(\tau_t) < X(\tau_t)$  and therefore (A5) implies  $\lambda > 1$ and  $z_1 = z_2 = 0$  by (12) and (13). Moreover, (A5) shows that  $\tau_1$  and  $\tau_2$  are determined by the same equation implying  $\tau_1 = \tau_2 = \tau$ . Inserting into (14) gives  $\tau X(\tau) = (g_1 + g_2)/2$ . Finally, substituting  $\tau_1 = \tau_2 = \tau$  and  $z_1 = z_2 = 0$  into (15) yields  $b = (g_1 - g_2)/2$ .

#### A.3 Proof of Proposition 2

Part (i) immediately follows from  $\lambda > 1$ , (12) and (13). In order to prove part (ii), write (16) and (17) as

$$F(\tau_1) = G_1(\tau_1, \tau_2), \quad F(\tau_2) = G_2(\tau_2, \tau_1), \tag{A6}$$

with

$$F(\tau) := \tau - \frac{1 - \lambda}{\lambda} \frac{X(\tau)}{X'(\tau)},$$
  

$$F'(\tau) := 1 - \frac{1 - \lambda}{\lambda} \frac{[X'(\tau)]^2 - X(\tau)X''(\tau)}{[X'(\tau)]^2} \gtrless 0,$$
(A7)

and

$$G_{1}(\tau_{1},\tau_{2}) = \frac{D_{1}'[X(\tau_{1})] + \gamma D_{2}'[X(\tau_{2}) + \gamma X(\tau_{1})]}{\lambda},$$
  

$$\frac{\partial G_{1}(\tau_{1},\tau_{2})}{\partial \tau_{1}} = \frac{\left\{D_{1}''[\cdot] + \gamma^{2} D_{2}''[\cdot]\right\} X'(\tau_{1})}{\lambda} \le 0,$$
(A8)

$$G_{2}(\tau_{2},\tau_{1}) = \frac{D_{2}'[X(\tau_{2}) + \gamma X(\tau_{1})]}{\lambda},$$
  

$$\frac{\partial G_{2}(\tau_{2},\tau_{1})}{\partial \tau_{2}} = \frac{D_{2}''[\cdot]X'(\tau_{2})}{\lambda} \le 0$$
(A9)

where the signs of  $\partial G_1(\tau_1, \tau_2)/\partial \tau_1$  and  $\partial G_2(\tau_2, \tau_1)/\partial \tau_2$  follow from  $D''_t[\cdot] \ge 0$  and  $X'(\tau_t) < 0$ . Hence,  $G_1$  and  $G_2$  are non-increasing functions in  $\tau_1$  and  $\tau_2$ , respectively, while  $F(\tau)$  may be increasing or decreasing in  $\tau$ . Consider first the case where  $F(\tau)$  is increasing in  $\tau$ . This case is illustrated in Fig. 2. In the left panel of this figure, we consider the case where  $\tau_1$  and  $\tau_2$  are such that  $D'_1 + \gamma D'_2 > D'_2$  and, thus,  $G_1(\tau, \tau_2)$  lies above  $G_2(\tau, \tau_1)$ . It immediately follows that  $\tau_1 > \tau_2$ . In the right panel,  $\tau_1$  and  $\tau_2$  are such that  $D'_1 + \gamma D'_2 < D'_2$  and  $G_1(\tau, \tau_2)$  lies below  $G_2(\tau, \tau_1)$ . Hence, we obtain  $\tau_1 < \tau_2$ . If  $\tau_1$  and  $\tau_2$  are such that  $D'_1 + \gamma D'_2 = D'_2$ , then  $G_1(\tau, \tau_2)$  and  $G_2(\tau, \tau_1)$  are identical and we obtain  $\tau_1 = \tau_2$  (not displayed in Fig. 2). The same line of reasoning applies if the function  $F(\tau)$  is decreasing and steeper than  $G_1(\tau, \tau_2)$  and  $G_2(\tau, \tau_1)$ . In this case, it can be shown that  $L_{\tau_1\tau_1} > 0$  and  $L_{\tau_2\tau_2} > 0$  and, thus, the bordered Hessian is |H| > 0, i.e., the second-order conditions of the government's welfare maximization problem are violated.<sup>12</sup>

Next turn to part (iii). If  $D'_1 + \gamma D'_2 < -x_1/X'_1$  and  $D'_2 < -x_2/X'_2$ , (16) and (17) imply  $x_1 + \tau_1 X'(\tau_1) > 0$  and  $x_2 + \tau_2 X'(\tau_2) > 0$ , i.e., both tax rates are on the increasing side of the Laffer curve. Moreover, rearranging (16) and (17) in this case gives  $\tau_1 > D'_1 + \gamma D'_2$  and  $\tau_2 > D'_2$ . Taking into account part (ii) and that both tax rates are on the increasing side of the Laffer curve, we obtain  $\tau_1 X(\tau_1) \nleq \tau_2 X(\tau_2)$  if and only if  $D'_1 + \gamma D'_2 \rightleftharpoons D'_2$ . Using this together with  $z_1 = z_2 = 0$  in (15) proves the result with respect to optimal debt *b* in part (iii). The proof of part (iv) is perfectly analogous to that of part (iii).

<sup>&</sup>lt;sup>12</sup> We can rewrite (A3) as  $L_{r_2r_2} = (2\lambda_2 - 1)X'_2 - D''_2X''_2 - (D'_2 - \lambda_2\tau_2)X''_2$ . From (9) we obtain  $D'_2 - \lambda_2\tau_2 = (\lambda_2 - 1)X_2/X'_2$ . Inserting this expression into the second derivative of the Lagrangian gives  $L_{r_2r_2} = [(2\lambda_2 - 1)(X'_2)^2 - D''_2X'_2^3 + (1 - \lambda_2)X_2X''_2]/X'_2$ . If  $F(\tau)$  is decreasing and steeper than  $G_2$ , it is straightforward to show with the help of (A7) and (A9) that the bracket term in  $L_{r_2r_2}$  is negative and, thus,  $L_{r_2r_2} > 0$ . In the same way, we can show  $L_{r_1r_1} > 0$  if  $F(\tau)$  is decreasing and steeper than  $G_1$ . Using these signs in (A4) and taking into account that we focus on the case where both tax rates are on the same side of the Laffer curve, i.e., sign  $\{x_1 + \tau_1X'_1\} = \text{sign } \{x_2 + \tau_2X'_2\}$ , we obtain |H| > 0.

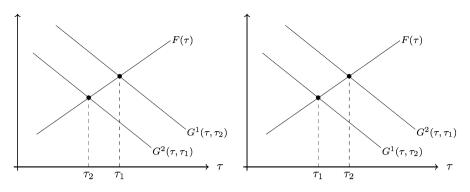


Fig. 2 Proof of Proposition 2 part (ii)

#### A.4 Model extension with debt burden

Inserting  $y_1$  and  $y_2$  from the private budget constraints into the utility function, the household's utility maximization problem can be written as

$$\begin{aligned} \max_{x_1, x_2, s} w &= W \Big[ 1 + z_1 - (1 + \tau_1) x_1 - s \Big] + V(x_1) \\ &+ \frac{1}{1 + \rho} \Big[ 1 + z_2 + (1 + r) s - (1 + \tau_2) x_2 + V(x_2) \Big]. \end{aligned}$$

The first-order conditions are

$$-W'\left[1+z_1-(1+\tau_1)x_1-s\right]+\frac{1+r}{1+\rho}=0,$$
(A10)

$$-(1+\tau_1)W'\Big[1+z_1-(1+\tau_1)x_1-s\Big]+V'(x_1)=0,$$
(A11)

$$-(1+\tau_2) + V'(x_2) = 0.$$
(A12)

Condition (A12) implies  $x = X(\tau_2)$ . Together with the capital market equilibrium condition s = b, conditions (A10) and (A11) determine  $x_1 = X^1(b, \tau_1, z_1)$  and  $r = R(b, \tau_1, z_1)$ . Inserting s = b into (A10) and (A11) and conducting a comparative static analysis, it is straightforward to prove (20) and (21).

The Lagrangian of the government's welfare maximization problem reads

$$\begin{split} & L = W \Big[ 1 + z_1 - (1 + \tau_1) X^1(b, \tau_1, z_1) - b \Big] + V \Big[ X^1(b, \tau_1, z_1) \Big] \\ & + \frac{1}{1 + \rho} \Big\{ 1 + z_2 + \Big[ 1 + R(b, \tau_1, z_1) \Big] b - (1 + \tau_2) X(\tau_2) + V \Big[ X(\tau_2) \Big] \Big\} \\ & - D_1 \Big[ X^1(b, \tau_1, z_1) \Big] - \frac{1}{1 + \rho} D_2 \Big[ X(\tau_2) + \gamma X^1(b, \tau_1, z_1) \Big] \\ & + \lambda_1 \Big\{ \tau_1 X^1(b, \tau_1, z_1) + b - g_1 - z_1 \Big\} \\ & + \lambda_2 \Big\{ \tau_2 X(\tau_2) - g_2 - z_2 - \Big[ 1 + R(b, \tau_1, z_1) \Big] b \Big\}. \end{split}$$

Differentiating and using the envelope theorem yields the first-order conditions

$$L_{b} = \lambda_{1} - (1+r)\lambda_{2} + \left(\frac{1}{1+\rho} - \lambda_{2}\right)bR_{b} + \left[\lambda_{1}\tau_{1} - \left(D_{1}' + \frac{\gamma}{1+\rho}D_{2}'\right)\right]X_{b}^{1} = 0.$$
(A13)

$$L_{\tau_{1}} = -x_{1}W' - \left(D'_{1} + \frac{\gamma}{1+\rho}D'_{2}\right)X_{\tau}^{1} + \left(\frac{1}{1+\rho} - \lambda_{2}\right)bR_{\tau} + \lambda_{1}\left[x_{1} + \tau_{1}X_{\tau}^{1}\right] = 0,$$
(A14)

$$L_{\tau_2} = \frac{1}{1+\rho} \left( -x_2 - D'_2 X'_2 \right) + \lambda_2 \left( x_2 + \tau_2 X'_2 \right) = 0, \tag{A15}$$

$$L_{\lambda_1} = \tau_1 x_1 + b - g_1 - z_1 = 0, \tag{A16}$$

$$L_{\lambda_2} = \tau_2 x_2 - g_2 - z_2 - (1+r)b = 0,$$
(A17)

as well as the slackness conditions

$$L_{z_1} = W' - \left(D'_1 + \frac{\gamma}{1+\rho}D'_2\right)X_z^1 + \left(\frac{1}{1+\rho} - \lambda_2\right)bR_z + \lambda_1 \left[\tau_1 X_z^1 - 1\right] \le 0,$$
(A18)

$$L_{z_2} = \frac{1}{1+\rho} - \lambda_2 \le 0, \quad z_1 \ge 0, \quad z_1 L_{z_1} = 0, \quad z_2 \ge 0, \quad z_2 L_{z_2} = 0.$$
(A19)

If W' = 1 and W'' = 0, conditions (A10) and (A11) imply  $r = \rho$  and  $x_1 = X(\tau_1)$ . From (20) and (21), we obtain  $X_b^1 = X_z^1 = R_b = R_z = R_\tau = 0$  and  $X_\tau^1 = 1/V_1'' = X_1'$ . Inserting this into (A13)–(A19) yields the same first-order and slackness conditions (7)–(13) as in the baseline model, except that we have to replace  $\lambda_2$  by  $(1 + \rho)\lambda_2$ ,  $D_1' + \gamma D_2'$  by  $D_1' + \gamma (1 + \rho)^{-1} D_2'$  and, in the second-period public budget constraints, *b* by (1 + r)b. Thus, the results derived in Propositions 1 and 2 do not change, which formally can be proven by the same steps as in Appendix A.2 and A.3 (remember the adjustments of optimal tax revenues, debt and first-period SCC mentioned in the text).

#### A.5 Model extension with endogenous adaptation

The Lagrangian of the welfare maximization problem with adaptation reads

$$L = \sum_{t=1,2} \left\{ V[X(\tau_t)] + 1 - (1 + \tau_t)X(\tau_t) \right\} - D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1), T(a)] + \lambda_1[\tau_1 X(\tau_1) + b - g_1 - a] + \lambda_2[\tau_2 X(\tau_2) - b - g_2].$$
(A20)

We obtain the first-order conditions

$$L_b = \lambda_1 - \lambda_2 = 0, \tag{A21}$$

$$L_{\tau_1} = -X(\tau_1) - \left\{ D'_1[X(\tau_1)] + \gamma D_{2,X}[X(\tau_2) + \gamma X(\tau_1), T(a)] \right\} X'(\tau_1) + \lambda_1 \left[ X(\tau_1) + \tau_1 X'(\tau_1) \right] = 0,$$
(A22)

$$L_{\tau_2} = -X(\tau_2) - D_{2,X}[X(\tau_2) + \gamma X(\tau_1), T(a)]X'(\tau_2) + \lambda_2 \Big[ X(\tau_2) + \tau_2 X'(\tau_2) \Big] = 0,$$
(A23)

$$L_{\lambda_1} = \tau_1 X(\tau_1) + b - g_1 - a = 0, \tag{A24}$$

$$L_{\lambda_2} = \tau_2 X(\tau_2) - b - g_2 = 0, \tag{A25}$$

as well as the slackness conditions for adaptation investments

$$L_a = -D_{2,T}[X(\tau_2) + \gamma X(\tau_1), T(a)]T'(a) - \lambda \le 0, \ a \ge 0, \ aL_a = 0.$$
(A26)

If T(a) satisfies the Inada condition  $\lim_{a\to 0} T'(a) = \infty$ , we obtain a > 0, since for  $a \to 0$  we have  $L_a \to \infty > 0$  and  $L_a \le 0$  is violated.

Equation (22) follows immediately by subtracting (A25) from (A24), taking into account  $R(\tau) = \tau X(\tau)$ . The relation  $\tau_1 \leq \tau_2$  if and only if  $\delta_1 + \gamma \delta_2 \leq \delta_2$ is proven by the same steps as in Proposition 2, since for this proof we only need (A22) and (A23) which do not depend on *a* under the linear-quadratic model specification. Notice that  $D'_1 + \gamma D_{2,X} = \delta_1 + \gamma \delta_2$  and  $D_{2,X} = \delta_2$ . In order to prove  $dR(\tau_i)/da = R'(\tau_i) d\tau_i/da > 0$  as well as (24) and (25), we can view (A21)–(A25) as a system of 5 equations that determines the five variables *b*,  $\tau_1, \tau_2, \lambda_1$  and  $\lambda_2$  as functions of *a*. For the utility function  $V(x_t) = (1 + \alpha)x_t - \beta x_t^2/2$ , we obtain  $X(\tau_t) = (\alpha - \tau_t)/\beta$ , so  $X'(\tau_t) = -1/\beta$  and  $X''(\tau_t) = 0$ . Totally differentiating (A21)–(A25) and using  $\lambda_1 = \lambda_2 = \lambda$  from (A21) yields the matrix equation

$$\begin{bmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & (1-2\lambda)/\beta & 0 & R'(\tau_1) & 0 \\ 0 & 0 & (1-2\lambda)/\beta & 0 & R'(\tau_2) \\ 1 & R'(\tau_1) & 0 & 0 & 0 \\ -1 & 0 & R'(\tau_2) & 0 & 0 \end{bmatrix} \begin{bmatrix} db \\ d\tau_1 \\ d\tau_2 \\ d\lambda_1 \\ d\lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} da.$$
(A27)

Applying Cramer's rule to (A27) yields

$$\frac{\mathrm{d}b}{\mathrm{d}a} = \frac{1}{1 + [R'(\tau_1)/R'(\tau_2)]^2},\tag{A28}$$

$$\frac{\mathrm{d}\tau_t}{\mathrm{d}a} = \frac{R'(\tau_t)}{R'(\tau_1)^2 + R'(\tau_2)^2}, \quad t = 1, 2.$$
(A29)

Equation (A29) implies

$$\frac{\mathrm{d}R(\tau_t)}{\mathrm{d}a} = R'(\tau_t) \frac{\mathrm{d}\tau_t}{\mathrm{d}a} = \frac{R'(\tau_t)^2}{R'(\tau_1)^2 + R'(\tau_2)^2} > 0, \quad t = 1, 2.$$
(A30)

In order to prove (24), suppose the tax rates are both on the increasing side of the Laffer curve. If  $\delta_1 + \gamma \delta_2 < \delta_2$ , then  $\tau_1 < \tau_2$  and  $R(\tau_1) < R(\tau_2)$ . Under the linear-quadratic specification, we have  $R(\tau_t) = \tau_t X(\tau_t) = (\alpha \tau_t - \tau_t^2)/\beta$ . It follows  $R''(\tau_t) = -2/\beta < 0$  and  $R'(\tau_1) > R'(\tau_2) > 0$ . By equations (A30), we obtain  $dR(\tau_1)/da > dR(\tau_2)/da$  and (A28) implies  $db/da \in (0, 1/2)$ . This completes the proof of regime A in (24). The proofs of regime D in (24) as well as of regimes B and C in (25) are completely analogous.

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#### Declarations

Conflict of interest None.

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