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BRIEF REPORT

# Ozawa's Intersubjectivity Theorem as Objection to QBism Individual Agent Perspective

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## Abstract

QBism's foundational statement that "the outcome of a measurement of an observable is personal" is in the straight contraversion with Ozawa's Intersubjectivity Theorem (OIT). The latter (proven within the quantum formalism) states that two observers, agents within the QBism terminology, performing joint measurements of the same observable $A$ on a system $S$ in the state $\psi$ should get the same outcome $A = x$. In Ozawa's terminology, this outcome is intersubjective and it can't be treated as personal. This is the strong objection to QBism which can't survive without updating its principles. The essential aspect in understanding of the OIT-impact on QBism's foundations takes the notion of quantum observable. This paper comprises the complementary discussion highlighting the difference between the accurate, von Neumann, and inaccurate, noisy, quantum observables which are represented by PVMs and POVMs respectively. Moreover, we discuss the OIT-impact on the Copenhagen interpretation of quantum mechanics.

**Keywords** QBism · Ozawa intersubjectivity theorem · Quantum instruments · Measurement process · Copenhagen interpretation · Bohr · Schrödinger

## 1 Introduction

In this paper I move ahead my critical analysis of QBism's foundations (see, e.g., [1–4] for QBism basics). This paper, as well as my two previous articles [5, 6], straightly critiques the individual agent perspective on measurement's outcomes. My previous appraisal convinced QBists to specify the level of agent's individuality. In contrast to the general subjective probability theory, the class of agents should be restricted, at least to agents who were educated in basics of quantum theory. So, Ivan who lives in a Siberian village, a busy hunter, can't be treated as a QBism's agent.

Now I have an intention to offense QBism by using Ozawa's Intersubjectivity Theorem (OIT) [7]. Qbism's statement that "the outcome of a measurement of an observable is personal" is in the straight contraversion with OIT. This theorem is not so widely known and one

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of the present paper's intention is the theorem's advertizement. OIT states that two observers, agents within the QBism terminology, performing joint measurements of the same observable $A$ on a system $S$ in the state $\psi$ should register the same outcome $A = x$ with probability one. Hence, the outcome is intersubjective [7], and it's unnatural to consider outcomes of quantum observations as agent's personal experiences.

OIT is proven within the quantum formalism, it is the rigorous mathematical statement. But, as many theorems having the quantum foundational impact, its interpretation is not straightforward. The analysis of the OIT-impact onto QBism is coupled to the foundations of quantum measurement theory and especially the notion of quantum observable. Therefore, this paper comprises the complementary discussion, highlighting the difference between the accurate, von Neuman, and inaccurate, noisy, quantum observables, mathematically represented by projection valued measures (PVMs) and positive operator valued measures (POVMs), respectively. QIT is about the agents who are able to perform the joint accurate measurements. For such agents, measurement's outcome loses its personalization, in favour of intersubjectivity.

The conclusion of our analysis is that QBism should update its ideology by taking in consideration OIT (see Section 6). Thus, I am in line with the criticism of QBism presented in article [7]. However, I depart from its conclusion that OIT contradicts to the Copenhagen interpretation; in contrast, OIT peacefully coexist with this interpretation. It is relevant to recall here that QBism fundamentally differs from the Copenhagen interpretation [2].

This is the good place to note that relational quantum mechanics (RQM) [8] also confronted the problem of intersubjectivity. This problem stimulated reconsideration of QQM's foundations and completion of RQM by the following postulate [9]:

6. "Internally consistent descriptions: In a scenario where ***F*** measures ***S,*** and ***W*** also measures ***S*** in the same basis, and ***W*** then interacts with ***F*** to "check the reading" of a pointer variable (i.e., by measuring ***F*** in the appropriate "pointer basis"), the two values found are in agreement."

I think that similar reconsideration of QBism's foundations should be performed and may be QBism's version of RQM's postulate 6 can be formulated.

Right away we initiate with the mathematical formulation of OIT and its proof. We set out to make the presentation very shortly (see [7] for details). The indirect measurement scheme is the heart of OIT. We go ahead with the recollection of the notion of quantum observable, namely, Hermitian operator or PVM, and generalized quantum observable (POVM) and the indirect measurements scheme for their generation.

## 2 Quantum Observables vs. Generalized Quantum Observables

In quantum mechanics' axiomatics, von Neumann [10] introduced quantum observables as Hermitian operators acting in complex Hilbert space $\mathcal{H}$, the state space of a system.[1] The spectral decomposition is the essential part in this framework.

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[1] Why did he select the Hermitian operators for mathematical representation of observables in quantum theory? Moreover, he considered only such observables as the genuine quantum observables. I guess that he followed Schrödinger's quantization rule for the position and momentum observables which are realized by Hermitian operators in $L_2$-space. This rule implies that each classical observable given by the real-valued function $A = A(q, p)$ on the phase space is represented as a Hermitian operator in $L_2$-space.

We restrict considerations to observables represented by the operators with totally discrete spectra $X \subset \mathbb{R}$. Here

$$A = \sum_x x E_A(x), \tag{1}$$

where $E_A(x)$ is projection on the eigensubspace corresponding to the eigenvalue $x$; these projectors form the resolution of unity:

$$I = \sum_x E_A(x). \tag{2}$$

The Born rule determines the probabilities of the outcomes of measurements for a system $\mathcal{S}$ in the state $\psi$,

$$P(A = x|\psi) = \langle\psi|E_A(x)|\psi\rangle. \tag{3}$$

Later generalized quantum observables were invented. Such observables are represented by POVMs. We restrict considerations to POVMs with a discrete domain of definition $X$. POVM is a map $x \to \Pi(x)$ : for each $x \in X$, $\Pi(x)$ is a positive contractive self-adjoint operator (i.e., $0 \leq \Pi(x) \leq I$) (called an *effect*), and effects form the resolution of unity

$$\sum_x \Pi(x) = I. \tag{4}$$

This map defines an operator valued measure on algebra of all subsets of set $X$. For $O \subset X$,

$$\Pi(O) = \sum_{x \in O} \Pi(x).$$

The condition (4) is the operator-measure counterpart of the condition normalization by 1 for usual probability measures.

POVM $\Pi$ represents statistics of measurements for observable $A$ with the following generalization of the Born's rule:

$$P(\Pi = x|\psi) = \langle\psi|\Pi(x)|\psi\rangle. \tag{5}$$

We remark that equality (4) implies that

$$\sum_x P(A = x|\psi) = 1.$$

Any quantum observable $A$ can also be represented as POVM of the special type – PVM $E_A = (E_A(x))$.

Quantum observables given by PVMs were interpreted by von Neumann [10] as describing *accurate measurements*. And generalized observables given by POVMs which are not PVMs are interpreted as representing *inaccurate measurements*. In von Neumann's [10], the notion of measurement's precision was not completely formalized. Only recently the consistent formalization of this notion was presented in [12].

We shall keep firmly the expression "quantum observable" for observable axiomatically introduced by von Neumann [10] and represented by PVMs and the expression "generalized quantum observable" for POVMs.

## 3 Generalized Quantum Observables from the Indirect Measurement Scheme

The indirect measurement scheme involves the following components

- the states spaces $\mathcal{H}$ and $\mathcal{K}$ of the systems $\mathcal{S}$ and the apparatus $\mathcal{M}$ for measurement of some observable $A$;
- the evolution operator $U = U(t)$ representing the interaction-dynamics for the system $\mathcal{S} + \mathcal{M}$;
- the meter observable $M$ giving outputs of the pointer of the apparatus $\mathcal{M}$.

Here the quantum observables $A$ and $M$ can be represented as PVMs, $E_A = (E_A(x))$, $E_M = (E_M(x))$, where $E_A(x)$, $E_M(x)$ are projections in Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$ respectively. It is assumed that the compound system's evolution is driven by the Schrödinger equation, so the evolution operator is unitary.

Formally, an *indirect measurement model* for an observable $A$, introduced in [11] as a "measuring process", is a quadruple

$$(\mathcal{K}, |\xi\rangle, U, M)$$

where $|\xi\rangle \in \mathcal{K}$ represents the apparatus state.

We explore the Heisenberg picture. To describe meter's evolution, we represent it in the state space of the compound system, i.e., as $I \otimes M$. The meter observable evolves as

$$M(t) = U^\star(t)(I \otimes M)U(t). \tag{6}$$

By the Born rule

$$P(M(t) = x|\psi\xi) = \langle\psi\xi|E_{M(t)}(x)|\psi\xi\rangle. \tag{7}$$

This is the probability distribution for the outputs of measurements done by the apparatus and given by the meter. In principle, one can ignore the representation of the measurement process as the system-apparatus interaction and operate solely with system's states. In this picture one proceeds with generalized observables given by POVMs. The meter observable generates the POVM $\Pi = (\Pi(x))$

$$\Pi(x) = \langle\xi|E_{M(T)}(x)|\xi\rangle, \tag{8}$$

where $T$ is the time needed to complete the experiment.

The probability distribution of the generalized observable given by a POVM is determined by (5).

Generally the probability distribution generated by a measurement process does not coincide with the probability distribution of the quantum observable $A$ for which this process was constructed, i.e., generally

$$P(\Pi = x|\psi) = \langle\psi|\Pi(x)|\psi\rangle \neq P(A = x|\psi) = \langle\psi|E_A(x)|\psi\rangle, \tag{9}$$

We remark that, as was proven by Ozawa [11], any generalized observable (POVM) can be generated via the indirect measurement scheme. Typically one operates solely with generalized observables by ignoring the indirect measurement scheme. This simplifies considerations, but it can lead to misunderstanding of the foundations the quantum measurement theory.

## 4 Probability Reproducibility Condition

**Definition** *A measurement process* $(\mathcal{K}, |\xi\rangle, U, M)$ *reproduces the probability distribution for quantum observable A (accurate von Neumann observable) if*

$$P(A = x|\psi) = P(M(T) = x|\psi\xi). \tag{10}$$

In this case

$$\langle \psi\xi | E_{M(T)}(x) | \psi\xi \rangle = \langle \psi | E(x) | \psi \rangle. \tag{11}$$

or

$$\langle \psi | \Pi(x) | \psi \rangle = \langle \psi | E(x) | \psi \rangle, \tag{12}$$

and hence,

$$\Pi(x) = E(x),$$

**Proposition** *Probability reproducibility condition for a measurement process is equivalent to the representation of the corresponding generalized observable by the PVM $E_A$ of measured quantum observable $A$.*

## 5 Intersubjectivity of Outcomes of Quantum Observables

Following [7], consider two remote observers $O_1$ and $O_2$ who perform joint measurements on a system $\mathcal{S}$, in mathematical terms it means that the meter quantum observables of the corresponding measurement processes commute,

$$[M_1(t), M_2(t)] = 0.$$

Here each apparatus has its own state space, i.e., $\mathcal{K} = \mathcal{K}_1 \otimes \mathcal{K}_2$. We call such measurements local. In this situation the joint probability distribution is well defined

$$P(M_1(t) = x, M_2(t) = y | \psi\xi_1\xi_2) = \langle \psi\xi_1\xi_2 | E_{M_1(t)}(x) E_{M_2(t)}(y) | \psi\xi_1\xi_2 \rangle \tag{13}$$

Suppose that both observers perform the accurate measurements of the quantum observable $A$ given by PVM $E_A = (E_A(x))$. Then the corresponding POVMs $\Pi_j, j = 1, 2,$ coincide with $E_A$ :

$$\Pi_1(x) = \Pi_2(x) = E_A(x). \tag{14}$$

This equality implies:

**Theorem** (OIT [7]) *Two observers performing the joint local and probability reproducible measurements of the same quantum observable A on the system $\mathcal{S}$ should get the same outcome with probability 1:*

$$P(M_1(T) = x, M_2(T) = y | \psi\xi_1\xi_2) = \delta(x - y) P(E = x | \psi) = \delta(x - y) \| E(x)\psi \|^2. \tag{15}$$

## 6 Intersubjectivity Challenges QBism

We start with the following citation of Fuchs and Schack [2]:

"The fundamental primitive of QBism is the concept of experience. According to QBism, quantum mechanics is a theory that any agent can use to evaluate her expectations for the content of her personal experience."

See also [13]: "In QBism, a measurement is an action an agent takes to elicit an experience. The measurement outcome is the experience so elicited. The measurement outcome is thus personal to the agent who takes the measurement action."

However, OIT implies that, for accurate local observables, measurement's outcome is intersubjective which is the strong objection to QBism. There is nothing concerning personal experiences and QBists should response to this objection.

My suggestion (see also [15]) is to proceed not with individual agents and their personal experiences, but with the *universal agent* **(in terminology of Brukner** [14]**, "hypothetical agent")** and consider measurement outcomes as the experiences of this agent (up to [16]). I remark that consideration of universal agents is common in general theory of decision making. However, for QBists, such solution seems to be unacceptable, since it would destroy consistency of the QBism's private agency perspective. QBism rejects even the possibility two agents sharing of the experience [13]: "... quantum theory provides a calculus for gambling on each agent's own experiences - it doesn't give anything else than that. It certainly doesn't give one agent the ability to conceptually pierce the other agent's personal experience."

The OIT-objection to QBism is foundationally interesting and generates the discussion on the notion of quantum observable. Due to efforts of Helström, Holevo, and Ozawa [17–20], [11], generalized quantum observables which are mathematically represented by POVMs became one of the basic tools of quantum information theory. Nowadays the special role of accurate observables represented by PVMs is not emphasized. In particular, the notion of observables in QBism is identified with generalized quantum observable given by POVM. However, the clash between QBism and OIT stimulates highlighting of the accurate PVM- as the genuine quantum observables, and treating the generalized quantum observables which are not accurate POVM as imprecise and noisy ones. Of course, it is a well known fact, but the clash between OIT and QBism is good occasion to emphasize this difference.

*What does this difference between accurate PVM and noisy POVM observables mean for QBism?*

I have the following picture of the situation. OIT holds only for the accurate PVM-observables; for generalized quantum observables, it can be violated and generally it is impossible to assign the same value for measurements' outcomes for observers $O_1$ and $O_2$. Thus, QBism ideology of the personal experiences of observers (agents) can still be kept for such generalized observables. But, where does individuality come from? *The personal experiences come from noise!* So, different observers performing inaccurate measurements are coupled to different noisy environments. This is just my personal view on consequences of IOT for QBism.

In conclusion, QBism might response to the OIT-challenge by considering the universal agent who is able to perform accurate measurements; another possibility is to proceed without referring to the universal agent, but then individuality of experience is due to noise generated in the process of measurement. Since this noise is generated by physical processes in the measurement apparatus, this is merely the "personal experience of the apparatus". Of course, it can be treated as the personal experience of the observer performing this measurement, but this treatment loses the flavor of subjectivity.

## 7 Intersubjectivity and Copenhagen Interpretation

We start the discussion with following important citation [21][2]:

"Bohr's interpretation, in any of its versions, will be distinguished in this study from "the Copenhagen interpretation," because there is no single such interpretation, as even Bohr has changed his a few times. It is more suitable to speak, as Heisenberg did [23], of "the Copenhagen spirit of quantum theory" or, as a handier shorthand, "the spirit of Copenhagen," referring to certain common features of a group of interpretations, which may be different in their other features."

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[2] Arkady Plotnitsky presented this viewpoint at the second Växjö conference in 2002 (see [22]).

It seems that one of such common features is the statement that measurements' outcomes cannot be treated as the objective properties of a system $\mathcal{S}$. They are results of the complex process of interaction of a system and an apparatus, see Bohr [24]:

"This crucial point ... implies the impossibility of any sharp separation between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear. In fact, the individuality of the typical quantum effects finds its proper expression in the circumstance that any attempt of subdividing the phenomena will demand a change in the experimental arrangement introducing new possibilities of interaction between objects and measuring instruments which in principle cannot be controlled. Consequently, evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena exhausts the possible information about the objects."

The indirect measurement scheme matches perfectly with the Copenhagen interpretation. Therefore it is surprising that OIT contradicts to it. The clash between OIT and the the Copenhagen interpretation was highlighted in the conclusion section of OIT-article [7]:

"Schrödinger [25] argued that a measurement does not ascertain the pre-existing value of the observable and is only required to be repeatable. Since the inception of quantum mechanics, this view has long been supported as one of the fundamental tenets of quantum mechanics. In contrast, we have shown that any probability reproducible measurement indeed ascertains the value that the observable has, whether the repeatability is satisfied or not."

I disagree with the author of [7]. The seed of this misunderstanding is in ignoring the two level structure of physical theories, ontic and epistemic [26–28]. The former is about reality as it is and the latter is about knowledge about reality. Bohr and Schrödinger wrote about the ontic reality, about impossibility to assign to quantum systems preexisting values and here "preexisting" is the synonym for "objective", "ontic". But OIT is not about such values, it is about epistemic reality, reality of knowledge about the possible outcome of measurement.

Hence, in my opinion *OIT can peacefully coexist with the Copenhagen interpretation.*

But, as was stressed, OIT is a challenge for QBism which operates at the epistemic level of scientific description of quantum phenomena. This is the good place to recall that QBism should be sharply separated from the Copenhagen interpretation, see again Fuchs and Schack [2]:

"According to QBism, quantum mechanics can be applied to any physical system. QBism treats all physical systems in the same way, including atoms, beam splitters, Stern-Gerlach magnets, preparation devices, measurement apparatuses, all the way to living beings and other agents. In this, QBism differs crucially from various versions of the Copenhagen interpretation."

## 8 Concluding Remark

QBism is often presented as Bayesian, subjective probability interpretation, of quantum mechanics. However, the Bayesian viewpoint on quantum probabilities is not coupled solely to QBism. I think that quantum Bayesianism preliminaries were already presented by Schrödinger in his "Cat Paradox" paper [25], see, e.g.,

"It ($\psi$-function) is now the means for predicting probability of measurement results. In it is embodied the momentarily-attained sum of theoretically based future expectation, somewhat as laid down in a catalog. It is the relation- and -determinacy-bridge between measurements and measurements ...”

“For each measurement one is required to ascribe to the $\psi$-function (= the prediction-catalog) a characteristic, quite sudden change, which depends on the measurement result obtained, and so cannot be foreseen ...”

Similar interpretation of the wave function is explored in works of M. D’Ariano (see, e.g., [29]) and my papers (see, e.g., [32]) within the Växjö interpretation of quantum mechanics [30, 31]. D’Ariano uses subjective probabilities and I use statistical ones.[3]

As was pointed out a few times, the essence of QBism is in treatment of measurements’ outcomes as the personal experiences of observers and OIT is the objection for this key-point of QBism (and not for the Bayesian approach to quantum probabilities).

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## Declarations

**Competing interests** The authors declare no competing interests.

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[3] By the the Växjö interpretation the quantum formalism is treated as a special calculus of probability update based on the quantum analog of the formula of total probability – the classical formula perturbed by the interference term.

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