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CORRECTION

Correction to: On reconstructing parts of quantum theory from two related maximal conceptual variables. (Intern. J. Theor. Phys. 61, 69, 2022.)

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A serious misformulation in my article is corrected.

The main result of this article, Theorem 1 of Section VI, point (ii) (page 7 of 19), should read:

Theorem 1 *Consider a situation where there are two maximal accessible conceptual variables θ and ξ in the mind of an actor or in the joint minds of a communicating group of actors. Make the following assumptions*:

(i) *On one of these variables, θ, there can be defined transitive group actions G with a trivial isotropy group and with a left-invariant measure ρ on the space Ω_θ.*
(ii) *There exists a unitary multi-dimensional representation $U(\cdot)$ of the group behind the group actions G defined on θ such that the coherent states $U(\mathrm{g})|\theta_0\rangle$ are in one-to-one correspondence with the values of g and hence with the values of θ.*
(iii) *The two maximal accessible variables θ and ξ can both be seen as functions of an underlying inaccessible variable $\phi \in \Omega_\phi$. There is a transformation k acting on Ω_ϕ such that $\xi(\phi) = \theta(k\phi)$.*

Then there exists a Hilbert space $\mathcal{H}$ connected to the situation, and to every accessible conceptual variable there can be associated a symmetric operator on $\mathcal{H}$.

The important change is that in point (ii) the representation $U(\cdot)$ is not assumed to be irreducible. It is crucial that this is not assumed. This is the whole point of the proof on the pages 9-12 (see for instance Lemma 1 on page 10). In this proof, the representation may

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be reducible or irreducible. The important point is that it is multi-dimensional, so that the given coherent states can be in one-to-one correspondence with the group elements.

For the applications of Theorem 1 is is crucial that reducibility is permitted. When the representation $U(\cdot)$ is not assumed to be irreducible, it is possible to show that in very many cases the assumption (ii) can be automatically fullfilled. Consider here the case where the maximal variable θ takes n different values, so that the Hilbert space $\mathcal{H}$ has n dimensions:

In this case, let G be the cyclic group acting on the distinct values $u_1,\ldots,u_n$ of θ, that is, the group generated by the element g_0 such that $g_0 u_i = u_{i+1}$ for $i = 1,\ldots, n-1$ and $g_0 u_n = u_1$.

This is an Abelian group, which only has one-dimensional irreducible representations. However, we can define $U(\cdot)$ as taking values as diagonal unitary $n \times n$ matrices with different complex nth roots of the identity on the diagonal. For the specific matrix $U(g_0)$, take these nth roots in their natural order, and then let every element of G be mapped into the diagonal matrices $U(\cdot)$ by the corresponding cyclical permutation.

It is easy to see then that the coherent states $U(\mathrm{g})|\theta_0\rangle$ are in one-to-one correspondence with the group elements $g \in G$ when $|\theta_0\rangle$ is a unit vector with one element equal to 1 and the others zero. Also, G is transitive on its range and has a trivial isotropy group. Thus the only assumption of Theorem 1 that is left to verify, is the assumption that ξ can be found as a related variable to θ, that is, the existence of an inaccessible variable ϕ and a transformation k in the corresponding space Ω_ϕ such that $\xi(\phi) = \theta(k\phi)$.

In the special case where $\theta = \theta^a$ and $\xi = \theta^b$ are two different components of a spin/angular momentum vector, the last requirement can be satisfied by letting ϕ be the projection of the full spin/angular momentum vector upon the plane spanned by the two components, and taking k to be the 180° rotation around the midline between the two component directions.

In this special case, we can let the group K be the group of rotations in the above plane. Then $\theta(\cdot)$ is permissible with respect to this group, where the notion of permissibility is defined in the article. (Note that k does not belong to K in this case, but this is not necessary.)

Further developments, generalizing this special case, are under consideration now.

Also, this Theorem 1 should have the following addition: The case $\phi = (\theta,\xi)$ with k just exchanging θ and ξ, should be explicitly excluded. The proof of Lemma 1 does not work for this case. If this case had been allowed, all maximal accessible variables would then by definition have been related.

Finally, I take this opportunity to correct two misprints on page 10 in the proof of Theorem 1:

On line 7, replace $g\,j\psi(\phi) = (g\theta(k\phi), \theta(\phi))$ by $g\,j\psi(\phi) = (g\theta(k\phi), g\theta(\phi))$.

On line 8, replace $h\,j\psi(\phi) = (\xi(\phi), h\xi(k^{-1}\phi))$ by $h\,j\psi(\phi) = (h\xi(\phi), h\xi(k^{-1}\phi))$.