International Journal of Theoretical Physics (2021) 60:3548–3565
https://doi.org/10.1007/s10773-021-04916-8

Effect on Jarlskog Determinant Above the GUT Scale Within Four Flavor Neutrino Framework

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Received: 2 June 2021 / Accepted: 22 July 2021 / Published online: 12 August 2021

Abstract

We study the Planck scale effects on Jarlskog determiant in the four flavor framework. On electroweak symmetry breaking, quantum gravitational effects lead to an effective SU(2) × U(1) invariant dimension-5 Lagrangian including neutrino and Higgs forces, which perturbed the neutrino mass term and produce an extra terms in the neutrino mass matrix. We consider that gravitational interaction is independent from flavor and compute the Jarlskog determiant due to Planck scale effects. In the case of leptonic sector, the strentgh of CP violation is measured by Jarlskog determiant. We applied our approach to study Jarlskog determinant in the four flavor neutrino mixing above the GUT scale.

Keywords Jarlskog determinant · CP violaton · Planck scale

1 Introduction

The evidence of a deficit of detected solar neutrinos [1] indicates that electron neutrinos must also participate in lepton mixing. Recently from the analysis of the Super-Kamiokande data [2], the normal (inverted) neutrino mass squared differences with 90% CL were found to be $1.9(1.7) \times 10^{-3} eV^2 \leq \Delta_{31} \leq 2.6(2.7) \times 10^{-3} eV^2$. The parameters of sterile neutrino (mass squared difference $\Delta_{41} = m_4^2 - m_i^2$ and the mixing angle $(\sin^2\theta_{14})$) are excluded by the joint results of Day Bay, MINOS and Bugey-3 data [3]. The latest best fit value of neutrino mass square difference and mixing angle from the analysis of global neutrino oscillation data are found to be [2] $\Delta_{41} = 1.7 eV^2 \, and \, \sin^2\theta_{14} = 0.019$. In search of neutrino oscillation, three different scale of neutrino mass-squared differences $\triangle m^2_{sun} \ll \triangle m^2_{atm} \ll \triangle m^2_{LSND}$ obtained by atmospheric and solar neutrino oscillation experiments and LSND collabration ,which required four netrinos with definite mass to explain these data. Super- Kamiokande Collaboration, Y. Fukuda et al. [4] experimenatal data was in good agreement with two flavor oscillation i.e.$\nu_\mu \rightarrow \nu_\epsilon$ and provided evidence for neutrino oscillation. Another Liquid Scientillator Neutrino Detector (LSND) also

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confirmed atmospheric neutrino oscillation $\nu_\mu \rightarrow \nu_\epsilon$ through muon neutrino flux from Π^+decay in flight (DIF) which has completely different backgrounds from the $\bar{\nu}_\mu \rightarrow \bar{\nu}_\epsilon$ DAR oscillation search . LSND experiment provided that both effect was due to neutrino oscillation by C. Athanassopoulos et al. [5]. All these neutrino oscillation data could be explained successfully by four neutrinos with definite mass by S.M. Bilenky et al. [6] .Thus four-neutrino models have been studied by many authors [10–13].

The assumption of the violation of CP and T symmetries are to be profound in the four-neutrino mixing models. The modern long oscillation length accelerator neutrino experiments are needed for the observation of leptonic side CP and T-violation. There are many ways to incorporated the non trivial CP violating complex phases in 4x4 lepton flavor mixing matrix. These relative phases and magnitude of the coupling consants are responsible for maximal CP violation. For real mixing matrix there is no CP symmetry violation, but in case of neutrino flavor mass mixing matrices some rephasing Jarlskog invariants are assigned for CP and T violation by C. Jarlskog [7] . Within the framework of four-neutrino mixing, the role of these Jarlskog invariants are very much crucial for the study of violation in CP and T.

This paper is organized as follows. In Section 2, we briefly describe the Jarlskog invariants which leads to an CP violation. In Section 3 we summarize the effect of planck scale for four neutrino mixing angle. We also investigate the effect of these planck scale on Jarlskog determinant in Section 4. Results and Conclusions are briefly discussed in Sections 5 and 6 respectively.

2 Jarlskog Determinant in Four Flavor Framework

CP violation arise as three or more generation [8, 9]. CP violation in neutrino oscillation is interesting because it relates directly to CP phase parameter in the mixing for n ¿ 3 degenerate neutrino. We can write down the compact formula for the difference of transition probability between conjugate channel.

$$\Delta P_{\alpha\beta} = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \tag{1}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - \sum_{i<j}\left[4Re(X^{ij}_{\alpha\beta})\sin^2\Delta_{ij} - 2Im(X^{ij}_{\alpha\beta})\sin 2\Delta_{ij}\right] \tag{2}$$

where,

$$(\alpha,\beta) = (s,e)\,,(\tau,s)\,,(e,\mu)$$

$$X^{ij}_{\alpha\beta} = \left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right)$$

$$\Delta_{ij} = \frac{\delta m^2_{ij}L}{4E}\ ,\ \delta m^2_{ij} = m^2_i - m^2_j \quad ,$$

L is the oscillation distance and E is the neutrino energy. The quantity $X^{ij}_{\alpha\beta}$ [14] are related to the Jarlskog invariant by [15]

$$J^{ij}_{\alpha\beta} \approx Im\left(X^{ij}_{\alpha\beta}\right) \tag{3}$$

The goal of ongoing experiments are the determination of the upper bound on the parameters of four neutrino mixing angles. In particularly, the observation of δ is quites interesting for the point of view that δ related to the origin of the matter in the universe. The determination of δ is the important goal of the experiments.

As we know that sterile neutrino do not interact with the weak interaction, they might be oscillate with active neutrinos. Once consider the sterile neutrino we are led to a more general (n + n´) × (n + n´) lepton flavor mixing matrix [16], defined as U in the chosen flavor basis. For the flavor mixing of one sterile neutrino (ν_s) and three active neutrinos (ν_e, ν_μ, ν_τ), the explicit form of U [10] can be written as

$$U = \begin{pmatrix} U_{s1} & U_{s2} & U_{s3} & U_{s4} \\ U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \end{pmatrix}, \tag{4}$$

The matrix U contains six mixing angles and three phase angles for Dirac neutrinos and for Majorana neutrinos, We required three additional phase angles for proper defining the oscillation matrix U . The rephasing invariants Jarlskog [7] of CP or T violation for both cases are as follows :

$$J_{\alpha\beta}^{ij} \approx Im\left(U_{\alpha i} U_{\beta j} U_{\alpha j}^{*} U_{\beta i}^{*}\right) \tag{5}$$

where, ($\alpha,\ \beta =$ s, e, μ, τ) and ($i = 1, 2, 3, 4$).

Since U is Unitary matrix, Thus we get,

$$J_{\alpha\beta}^{ii} = J_{\alpha\alpha}^{ij} = J_{\beta\beta}^{ij} = J_{\alpha\beta}^{jj} = 0 \tag{6}$$

and

$$J_{\alpha\beta}^{ij} = -J_{\alpha\beta}^{ji} = J_{\beta\alpha}^{ij} = -J_{\beta\alpha}^{ji} \tag{7}$$

Hence we deals with four flavor framework by incorporating the sterile neutrino of eV range and the mixing of this sterile neutrino with three neutrinos is light. By adding one sterile neutrinos [11], there is an increment in mixing angles and CP violating phases in the PMNS matrix $U_{4\times 4}$ which is given by,

$$U = R_{34} R_{24} R_{14} R_{23} R_{13} R_{12}, \tag{8}$$

where the matrices R_{ij} are rotations in ij space,

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} e^{-i\delta_{12}} & 0 & 0 \\ -s_{12} e^{i\delta_{12}} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad and \quad R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij} e^{i\delta_{ij}}$, $c_{ij} = \cos\theta_{ij}$.

Note that there are in total six CP-violating phase δ_{ij}. The explicit form of U is

$$U_{s1} = (c_{14} c_{13} c_{12}),$$

$$U_{s2} = (c_{14} c_{13} s_{12} e^{-i\delta_{12}}),$$

$$U_{s3} = (c_{14} s_{13} e^{-i\delta_{13}}),$$

$$U_{s4} = \left(s_{14} e^{-i\delta_{14}}\right),$$

$$U_{e1} = (-c_{24} c_{23} s_{12} e^{i\delta_{12}} - c_{24} s_{23} e^{-i\delta_{23}} s_{13} e^{i\delta_{13}} c_{12} - s_{24} e^{-i\delta_{24}} s_{14} e^{i\delta_{14}} c_{13} c_{12}),$$

$$U_{e2} = (c_{24}c_{23}c_{12} - c_{24}s_{23}e^{-i\delta_{23}}s_{13}e^{i\delta_{13}}s_{12}e^{-i\delta_{12}} - s_{24}e^{-i\delta_{24}}s_{14}e^{i\delta_{14}}c_{13}s_{12}e^{-i\delta_{12}}),$$

$$U_{e3} = (c_{13}c_{24}s_{23}e^{-i\delta_{23}} - s_{24}e^{-i\delta_{24}}s_{14}e^{i\delta_{14}}s_{13}e^{-i\delta_{13}}),$$

$$U_{e4} = (c_{14}s_{24}e^{-i\delta_{24}}),$$

$$\begin{aligned} U_{\mu 1} = {} & (c_{34}s_{23}e^{i\delta_{23}}s_{12}e^{i\delta_{12}} - c_{34}c_{23}s_{13}e^{i\delta_{13}}c_{12} + s_{34}e^{-i\delta_{34}}s_{24}e^{i\delta_{24}}c_{23}s_{12}e^{i\delta_{12}} \\ & + s_{34}e^{-i\delta_{34}}s_{24}e^{i\delta_{24}}s_{23}e^{-i\delta_{23}}s_{13}e^{i\delta_{13}}c_{12} \\ & - s_{34}e^{-i\delta_{34}}c_{24}s_{14}e^{i\delta_{14}}c_{13}c_{12}), \end{aligned}$$

$$\begin{aligned} U_{\mu 2} = {} & (-c_{34}s_{23}e^{i\delta_{23}}c_{12} - c_{34}c_{23}s_{13}e^{-i\delta_{13}}s_{12}e^{i\delta_{12}} - s_{34}e^{-i\delta_{34}}s_{24}e^{i\delta_{24}}c_{23}c_{12} \\ & + s_{34}e^{-i\delta_{34}}s_{24}e^{i\delta_{24}}s_{23}e^{-i\delta_{23}}s_{13}e^{i\delta_{13}}s_{12}e^{-i\delta_{12}} \\ & - s_{34}e^{-i\delta_{34}}c_{24}s_{14}e^{i\delta_{14}}c_{13}s_{12}e^{-i\delta_{12}}), \end{aligned}$$

$$U_{\mu 3} = (c_{34}c_{23}c_{13} - s_{34}e^{-i\delta_{34}}s_{24}e^{i\delta_{24}}s_{23}e^{-i\delta_{23}}c_{13} - s_{34}e^{-i\delta_{34}}c_{24}s_{14}e^{i\delta_{14}}s_{13}e^{-i\delta_{13}})$$

$$U_{\mu 4} = (s_{34}e^{-i\delta_{34}}c_{24}c_{14}),$$

$$\begin{aligned} U_{\tau 1} = {} & (s_{34}s_{12}s_{23} + s_{34}e^{i\delta_{34}}c_{23}s_{13}e^{i\delta_{14}}c_{12} + c_{34}s_{24}e^{i\delta_{24}}c_{23}s_{12}e^{i\delta_{12}} \\ & + c_{34}s_{24}e^{i\delta_{34}}s_{23}e^{-i\delta_{23}}s_{12}e^{i\delta_{12}}c_{12} \\ & - c_{34}c_{24}s_{14}e^{i\delta_{14}}c_{13}c_{12}), \end{aligned}$$

$$\begin{aligned} U_{\tau 2} = {} & (s_{34}e^{i\delta_{34}}c_{12}s_{23}e^{i\delta_{23}} + s_{34}e^{i\delta_{34}}c_{23}s_{13}e^{i\delta_{13}}s_{12}e^{-i\delta_{12}} - c_{34}s_{24}e^{i\delta_{24}}c_{23}c_{12} \\ & + c_{34}s_{24}e^{i\delta_{34}}s_{23}e^{-i\delta_{23}}s_{13}e^{i\delta_{13}}s_{12}e^{-i\delta_{12}} \\ & - c_{34}c_{24}s_{14}e^{i\delta_{14}}c_{13}s_{12}e^{-i\delta_{12}}), \end{aligned}$$

$$U_{\tau 3} = (-s_{34}e^{i\delta_{34}}c_{23}c_{13} - c_{34}s_{24}e^{i\delta_{24}}s_{23}e^{-i\delta_{23}}c_{13} - c_{34}c_{24}s_{14}e^{i\delta_{14}}s_{13}e^{-i\delta_{13}})$$

$$U_{\tau 4} = (c_{34}c_{24}c_{14}),$$

We get the analytical expression for $P(\alpha, \beta)$ using the usual form of the MNS matrix parametrization given by U,

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = {} & \delta_{\alpha\beta} - [4\{Re(X^{12}_{\alpha\beta})\sin^2\Delta_{12} + Re(X^{13}_{\alpha\beta})\sin^2\Delta_{13} + Re(X^{14}_{\alpha\beta})\sin^2\Delta_{14} \\ & + Re(X^{23}_{\alpha\beta})\sin^2\Delta_{23} + Re(X^{24}_{\alpha\beta})\sin^2\Delta_{24} + Re(X^{34}_{\alpha\beta})\sin^2\Delta_{34}\} \\ & - 2\{J^{12}_{\alpha\beta}\sin 2\Delta_{12} + J^{13}_{\alpha\beta}\sin 2\Delta_{13} + J^{14}_{\alpha\beta}\sin 2\Delta_{14} \\ & + J^{23}_{\alpha\beta}\sin 2\Delta_{23} + J^{24}_{\alpha\beta}\sin 2\Delta_{24} + J^{34}_{\alpha\beta}\sin 2\Delta_{34}\}] \end{aligned} \quad (9)$$

We get nine independent [24] $J^{ij}_{\alpha\beta}$, whose magnitudes depends only upon three of the six CP-violating phases or their combinations in a specific parametrization of U. The nine jarlskog $J^{ij}_{\alpha\beta}$and six mixing angles for four flavour mixing are define using the matrix U in (4) are,

$$\begin{aligned}
J^{13}_{se} &= Im\left(U_{s1}U_{e3}U^*_{s3}U^*_{e1}\right)\\
J^{24}_{se} &= Im\left(U_{s2}U_{e4}U^*_{s4}U^*_{e2}\right)\\
J^{34}_{se} &= Im\left(U_{s3}U_{e4}U^*_{s4}U^*_{e3}\right)\\
J^{13}_{\tau s} &= Im\left(U_{\tau 1}U_{s3}U^*_{\tau 3}U^*_{s1}\right)\\
J^{14}_{\tau s} &= Im\left(U_{\tau 1}U_{s4}U^*_{\tau 4}U^*_{s1}\right)\\
J^{34}_{\tau s} &= Im\left(U_{\tau 3}U_{s4}U^*_{\tau 4}U^*_{s3}\right)\\
J^{23}_{e\mu} &= Im\left(U_{e2}U_{\mu 3}U^*_{e3}U^*_{\mu 2}\right)\\
J^{24}_{e\mu} &= Im\left(U_{e2}U_{\mu 4}U^*_{e4}U^*_{\mu 2}\right)\\
J^{34}_{e\mu} &= Im\left(U_{e3}U_{\mu 4}U^*_{e4}U^*_{\mu 3}\right)
\end{aligned} \tag{10}$$

And,

$$\sin^2\theta_{14} = |\mathrm{U_{s4}}|^2 \tag{11}$$

$$\sin^2\theta_{24} = \frac{|\mathrm{U_{e4}}|^2}{1-|\mathrm{U_{s4}}|^2} \tag{12}$$

$$\sin^2\theta_{34} = \frac{\left|\mathrm{U}_{\mu 4}\right|^2}{1-|\mathrm{U_{s4}}|^2-|\mathrm{U_{e4}}|^2} \tag{13}$$

$$\sin^2\theta_{13} = \frac{|\mathrm{U_{s3}}|^2}{1-|\mathrm{U_{s4}}|^2} \tag{14}$$

$$\sin^2\theta_{12} = \frac{|\mathrm{U_{s2}}|^2}{1-|\mathrm{U_{s4}}|^2-|\mathrm{U_{s3}}|^2} \tag{15}$$

$$\sin^2\theta_{23} = \frac{|\mathrm{U_{e3}}|^2\left(1-|\mathrm{U_{s4}}|^2\right)-\left(|\mathrm{U_{s4}}|^2\,|\mathrm{U_{e4}}|^2\right)}{1-|\mathrm{U_{s4}}|^2-|\mathrm{U_{e4}}|^2} + \frac{|U_{s1}U_{e1}+U_{s2}U_{e2}|^2\left(1-|U_{s4}|^2\right)}{\left(1-|U_{s4}|^2-|U_{s3}|^2\right)\left(1-|U_{s4}|^2-|U_{e4}|^2\right)} \tag{16}$$

Equation (10) is very much crucial to study the role of violation in CP and T within the framework of four-neutrino mixing. B. S. Koranga et. al. [17–19, 22, 23] did some analysis on CP and T violation within two or three flavor framework for different parameterization and above GUT scale.

3 Four Neutrino Mixing Angles Above the GUT Scale

Neutrino mass square differences and mixing angles above the GUT scale are developed in paper [21]. A neutrino mass matrix M is given by

$$\mathbf{M} = U^*diag(M_i)U^\dagger, \tag{17}$$

where, $U_{\alpha i}$ is the usual mixing matrix and M_i, the neutrino masses is generated by Grand unified theory. Most of the parameter related to neutrino oscillation are known, the major expectation is given by the mixing elements U.

In term of the above mixing angles, the mixing matrix is

$$U = diag(e^{if1}, e^{if2}, e^{if3}, e^{if4})R(\theta_{34})R(\theta_{24})\Delta R(\theta_{14})\Delta^* R(\theta_{23})R(\theta_{13})R(\theta_{12})diag(e^{i\alpha}, e^{i\beta}, e^{i\gamma}, 1). \quad (18)$$

The matrix $\Delta = diag(e^{\frac{i\delta}{2}}, 1, 1, e^{\frac{-i\delta}{2}})$ contains the Dirac phase. This leads to CP violation in neutrino oscillation α, β and γ are the so called Majorana phases, which effects the neutrinoless double beta decay. f_1, f_2, f_3 and f_4 are usually absorbed as a part of the definition of the charge lepton field. Planck scale effects will add other contribution to the mass matrix that gives the new mixing matrix can be written as [20, 21]

$$\begin{aligned} U' &= U(1 + i\delta\theta), \\ &= \begin{pmatrix} U_{s1} & U_{s2} & U_{s3} & U_{s4} \\ U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \end{pmatrix} \\ &+ i \begin{pmatrix} \sum_i U_{si}\delta\theta_{i1} & \sum_i U_{si}\delta\theta_{i2} & \sum_i U_{si}\delta\theta_{i3} & \sum_i U_{si}\delta\theta_{i4} \\ \sum_i U_{ei}\delta\theta_{i1} & \sum_i U_{ei}\delta\theta_{i2} & \sum_i U_{ei}\delta\theta_{i3} & \sum_i U_{ei}\delta\theta_{i4} \\ \sum_i U_{\mu i}\delta\theta_{i1} & \sum_i U_{\mu i}\delta\theta_{i2} & \sum_i U_{\mu i}\delta\theta_{i3} & \sum_i U_{\mu i}\delta\theta_{i4} \\ \sum_i U_{\tau i}\delta\theta_{i1} & \sum_i U_{\tau i}\delta\theta_{i2} & \sum_i U_{\tau i}\delta\theta_{i3} & \sum_i U_{\tau i}\delta\theta_{i4} \end{pmatrix} \end{aligned} \quad (19)$$

Where $\delta\theta$ is a hermition matrix that is first order in μ [20, 21]. The first order mass square difference $\Delta M_{ij}^2 = M_i^2 - M_j^2$, get modified [20, 21] as

$$\Delta M_{ij}^{'2} = \Delta M_{ij}^2 + 2(M_i Re(m_{ii}) - M_j Re(m_{jj})), \quad (20)$$

and

$$m = \mu U^t \lambda U, \quad (21)$$

where

$$\mu = \frac{v^2}{M_{pl}} = 2.5 \times 10^{-6} eV.$$

The change in the elements of the mixing matrix, which we parametrized by $\delta\theta$ [20, 21], is given by

$$\delta\theta_{ij} = \frac{iRe(m_{ij})(M_i + M_j) - Im(m_{ij})(M_i - M_j)}{\Delta M_{ij}^{'2}}. \quad (22)$$

The above equation determine only the off diagonal elements of matrix $\delta\theta_{ij}$. The diagonal element of $\delta\theta_{ij}$ can be set to zero by phase invariance.

Using (19), we can calculate neutrino mixing angle due to Planck scale effects [31],

$$\sin^2\theta'_{14} = \left|\mathrm{U}'_{s4}\right|^2, \quad (23)$$

$$\sin^2\theta'_{24} = \frac{\left|\mathrm{U}'_{e4}\right|^2}{1 - \left|\mathrm{U}'_{s4}\right|^2}, \quad (24)$$

$$\sin^2\theta'_{34} = \frac{\left|U'_{\mu4}\right|^2}{1-\left|U'_{s4}\right|^2-\left|U'_{s4}\right|^2}, \tag{25}$$

$$\sin^2\theta'_{13} = \frac{\left|U'_{s3}\right|^2}{1-\left|U'_{s4}\right|^2}, \tag{26}$$

$$\sin^2\theta'_{12} = \frac{\left|U'_{s2}\right|^2}{1-\left|U'_{s4}\right|^2-\left|U'_{s3}\right|^2}, \tag{27}$$

$$\sin^2\theta'_{23} = \frac{\left|U'_{e3}\right|^2\left(1-\left|U'_{s4}\right|^2\right)-\left(\left|U'_{s4}\right|^2\left|U'_{e4}\right|^2\right)}{1-\left|U'_{s4}\right|^2-\left|U'_{e4}\right|^2}$$

$$+\frac{\left|U'_{s1}U'_{e1}+U'_{s2}U'_{e2}\right|^2\left(1-\left|U'_{s4}\right|^2\right)}{\left(1-\left|U'_{s4}\right|^2-\left|U'_{s3}\right|^2\right)\left(1-\left|U'_{s4}\right|^2-\left|U'_{e4}\right|^2\right)}. \tag{28}$$

where,

$$U'_{\alpha1} = U_{\alpha1}+\sum_i U_{\alpha i}\left(\frac{-Re(m_{i1})(M_i+M_1)-iIm(m_{i1})(M_i-M_1)}{M_i^2-M_1^2+2(M_iRe(m_{ii})-M_1Re(m_{11})}.\right),$$

$$U'_{\alpha2} = U_{\alpha2}+\sum_i U_{\alpha i}\left(\frac{-Re(m_{i2})(M_i+M_2)-iIm(m_{i2})(M_i-M_2)}{M_i^2-M_2^2+2(M_iRe(m_{ii})-M_4Re(m_{22})}.\right),$$

$$U'_{\alpha3} = U_{\alpha3}+\sum_i U_{\alpha i}\left(\frac{-Re(m_{i3})(M_i+M_3)-iIm(m_{i3})(M_i-M_3)}{M_i^2-M_3^2+2(M_iRe(m_{ii})-M_4Re(m_{33})}.\right),$$

$$U'_{\alpha4} = U_{\alpha4}+\sum_i U_{\alpha i}\left(\frac{-Re(m_{i4})(M_i+M_4)-iIm(m_{i4})(M_i-M_4)}{M_i^2-M_4^2+2(M_iRe(m_{ii})-M_4Re(m_{44})}.\right), \quad \alpha = s, e, \mu, \tau$$

4 Jarlskog Above the GUT Scale

Normal order or inverted order neutrino mass spectrum crucially depends on the neutrino mixing angles define in (23–28) and plank scale correction has very subtle effect on these spectrums. Thus, we consider a degenerate neutrino spectrum and take the common neutrino mass to 2 eV, which is the upper limit from the tritium decay experiment [25]. Let us compute new nine Jarlskog determiant due to new mixing due to Planck scale effects. which is simply given by replacing the matrix U by perturbed matrix U',

$$\begin{aligned}
J_{se}^{13'} &= Im\left(\left(U_{s1}+i\sum_i U_{si}\delta\theta_{i1}\right)\left(U_{e3}+i\sum_i U_{ei}\delta\theta_{i3}\right)\left(U_{s3}^*-i\sum_i U_{ei}^*\delta\theta_{i3}^*\right)\left(U_{e1}^*-i\sum_i U_{ei}^*\delta\theta_{i1}^*\right)\right)\\
J_{se}^{24'} &= Im\left(\left(U_{s2}+i\sum_i U_{si}\delta\theta_{i2}\right)\left(U_{e4}+i\sum_i U_{ei}\delta\theta_{i4}\right)\left(U_{s4}^*-i\sum_i U_{ei}^*\delta\theta_{i4}^*\right)\left(U_{e2}^*-i\sum_i U_{ei}^*\delta\theta_{i2}^*\right)\right)\\
J_{se}^{34'} &= Im\left(\left(U_{s3}+i\sum_i U_{si}\delta\theta_{i3}\right)\left(U_{e4}+i\sum_i U_{ei}\delta\theta_{i4}\right)\left(U_{s4}^*-i\sum_i U_{ei}^*\delta\theta_{i4}^*\right)\left(U_{e3}^*-i\sum_i U_{ei}^*\delta\theta_{i3}^*\right)\right)\\
J_{\tau s}^{13'} &= Im\left(\left(U_{\tau 1}+i\sum_i U_{\tau i}\delta\theta_{i1}\right)\left(U_{s3}+i\sum_i U_{ei}\delta\theta_{i3}\right)\left(U_{\tau 3}^*-i\sum_i U_{\tau i}^*\delta\theta_{i1}^*\right)\left(U_{s1}^*-i\sum_i U_{si}^*\delta\theta_{i1}^*\right)\right)\\
J_{\tau s}^{14'} &= Im\left(\left(U_{\tau 1}+i\sum_i U_{\tau i}\delta\theta_{i1}\right)\left(U_{s4}+i\sum_i U_{ei}\delta\theta_{i4}\right)\left(U_{\tau 4}^*-i\sum_i U_{\tau i}^*\delta\theta_{i4}^*\right)\left(U_{s1}^*-i\sum_i U_{si}^*\delta\theta_{i1}^*\right)\right)\\
J_{\tau s}^{34'} &= Im\left(\left(U_{\tau 3}+i\sum_i U_{\tau i}\delta\theta_{i3}\right)\left(U_{s4}+i\sum_i U_{ei}\delta\theta_{i4}\right)\left(U_{\tau 4}^*-i\sum_i U_{\tau i}^*\delta\theta_{i4}^*\right)\left(U_{s3}^*-i\sum_i U_{si}^*\delta\theta_{i3}^*\right)\right)\\
J_{e\mu}^{23'} &= Im\left(\left(U_{e2}+i\sum_i U_{ei}\delta\theta_{i2}\right)\left(U_{\mu 3}+i\sum_i U_{\mu i}\delta\theta_{i3}\right)\left(U_{e3}^*-i\sum_i U_{ei}^*\delta\theta_{i3}^*\right)\left(U_{\mu 2}^*-i\sum_i U_{\mu i}^*\delta\theta_{i2}^*\right)\right)\\
J_{e\mu}^{24'} &= Im\left(\left(U_{e2}+i\sum_i U_{ei}\delta\theta_{i2}\right)\left(U_{\mu 4}+i\sum_i U_{\mu i}\delta\theta_{i4}\right)\left(U_{e4}^*-i\sum_i U_{ei}^*\delta\theta_{i4}^*\right)\left(U_{\mu 2}^*-i\sum_i U_{\mu i}^*\delta\theta_{i2}^*\right)\right)\\
J_{e\mu}^{34'} &= Im\left(\left(U_{e3}+i\sum_i U_{ei}\delta\theta_{i3}\right)\left(U_{\mu 4}+i\sum_i U_{\mu i}\delta\theta_{i4}\right)\left(U_{e4}^*-i\sum_i U_{ei}^*\delta\theta_{i4}^*\right)\left(U_{\mu 3}^*-i\sum_i U_{\mu i}^*\delta\theta_{i3}^*\right)\right) \quad (29)
\end{aligned}$$

In term of mixing angle and Dirac phases, we can write Jarlskog determinant $J_{\alpha\beta}^{ij'}$ above the GUT scale are,

$$\begin{aligned}
J_{se}^{13'} =& \frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{14}\cos\theta'_{23}\sin\theta'_{13}\sin\theta_y\\
&-\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{23}\cos\theta'_{14}\cos^2\theta'_{24}\sin\theta_z\\
&+\frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos^2\theta'_{12}\cos\theta'_{14}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right) \quad (30)
\end{aligned}$$

$$\begin{aligned}
J_{se}^{24'} =& \frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{13}\cos\theta'_{14}\cos\theta'_{23}\sin\theta_y\\
&+\frac{1}{4}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin^2\theta'_{12}\cos\theta'_{14}\sin\theta'_{24}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right) \quad (31)
\end{aligned}$$

$$J_{se}^{34'} = \frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{14}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right) \quad (32)$$

$$\begin{aligned}
J_{\tau s}^{13'} =& \frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{34}\cos^2\theta'_{12}\cos\theta'_{14}\cos\theta'_{23}\cos\theta'_{24}\sin\theta_x\\
&+\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{14}\cos\theta'_{23}\cos^2\theta'_{34}\sin\theta_y\\
&+\frac{1}{8}\left(\cos^2\theta'_{34}\sin^2\theta'_{24}-\sin^2\theta'_{34}\right)\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{23}\cos\theta'_{13}\cos^2\theta'_{14}\sin\theta_z\\
&-\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{34}\cos\theta'_{13}\cos^2\theta'_{14}\cos^2\theta'_{23}\sin\theta'_{24}\sin\left(\theta_x-\theta_y\right)\\
&-\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{34}\cos\theta'_{14}\cos\theta'_{24}\sin\theta'_{13}\sin\theta'_{23}\sin\left(\theta_x+\theta_z\right)\\
&+\frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos^2\theta'_{12}\cos\theta'_{14}\cos^2\theta'_{34}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right)\\
&-\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{34}\cos^2\theta'_{14}\cos\theta'_{13}\sin^2\theta'_{23}\sin\theta'_{24}\sin\left(\theta_x-\theta_y+\theta_z\right) \quad (33)
\end{aligned}$$

$$\begin{aligned}J_{\tau s}^{14'} &= -\frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{34}\cos^2\theta'_{12}\cos\theta'_{14}\cos\theta'_{23}\cos\theta'_{24}\sin\theta_x \\ &-\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{24}\cos^2\theta'_{14}\cos\theta'_{23}\cos^2\theta'_{34}\sin\theta_y \\ &-\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{14}\sin 2\theta'_{34}\cos\theta'_{13}\cos\theta'_{14}\cos\theta'_{24}\sin\theta'_{23}\sin(\theta_x+\theta_z) \\ &-\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{24}\cos^2\theta'_{14}\cos\theta'_{12}\cos^2\theta'_{34}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right) \quad (34)\end{aligned}$$

$$\begin{aligned}J_{\tau s}^{34'} &= \frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{34}\cos\theta'_{14}\cos\theta'_{23}\cos\theta'_{24}\sin\theta_x \\ &+\frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{14}\cos^2\theta'_{34}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right) \quad (35)\end{aligned}$$

$$\begin{aligned}J_{e\mu}^{23'} &= -\frac{1}{4}(\cos^2\theta'_{12}\cos^2\theta'_{23}\sin^2\theta'_{24}-\cos^2\theta'_{12}\cos^2\theta'_{24}\sin^2\theta'_{23}+\sin^2\theta'_{12}\sin^2\theta'_{14}\sin^2\theta'_{24} \\ &+\sin^2\theta'_{12}\sin^2\theta'_{23})\sin 2\theta'_{13}\sin 2\theta'_{34}\sin\theta'_{14}\cos\theta'_{23}\cos\theta'_{24}\sin\theta_x \\ &+\frac{1}{4}(\cos^2\theta'_{13}\cos^2\theta'_{34}\sin^2\theta'_{23}-\cos^2\theta'_{23}\cos^2\theta'_{34}\sin^2\theta'_{13}+\sin^2\theta'_{13}\sin^2\theta'_{14}\sin^2\theta'_{34} \\ &-\sin^2\theta'_{34}\sin^2\theta'_{23})\sin 2\theta'_{12}\sin 2\theta'_{24}\sin\theta'_{14}\cos\theta'_{13}\cos\theta'_{23}\cos\theta'_{24}\sin\theta_y \\ &+\frac{1}{8}(\cos^2\theta'_{24}\cos^2\theta'_{34}-\cos^2\theta'_{24}\sin^2\theta'_{14}\sin^2\theta'_{34}-\cos^2\theta'_{34}\sin^2\theta'_{14}\sin^2\theta'_{24} \\ &+\sin^2\theta'_{14}\sin^2\theta'_{24}\sin^2\theta'_{34})\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{23}\cos\theta'_{13}\cos\theta'_{34}\sin\theta_z \\ &+\frac{1}{4}\left(\sin^2\theta'_{34}\cos^2\theta'_{13}-\cos^2\theta'_{24}\sin^2\theta'_{13}\right)\sin 2\theta'_{12}\sin 2\theta'_{34} \\ &\cos^2\theta'_{23}\sin\theta'_{13}\sin^2\theta'_{14}\sin\theta'_{24}\sin\left(\theta_x-\theta_y\right) \\ &+\frac{1}{4}(\cos^2\theta'_{23}-\sin^2\theta'_{24}\cos^2\theta'_{13}-\cos^2\theta'_{24}\sin^2\theta'_{13}+\sin^2\theta'_{13}\sin^2\theta'_{14}\sin^2\theta'_{24}) \\ &\sin 2\theta'_{12}\sin 2\theta'_{34}\cos\theta'_{13}\cos\theta'_{24}\sin\theta'_{23}\sin(\theta_x+\theta_z) \\ &-\frac{1}{4}(\cos^2\theta'_{12}\cos^2\theta'_{23}\cos^2\theta'_{34}-\cos^2\theta'_{12}\cos^2\theta'_{23}\sin^2\theta'_{34}-\cos^2\theta'_{23}\cos^2\theta'_{34}\sin^2\theta'_{12} \\ &+\sin^2\theta'_{12}\sin^2\theta'_{14}\sin^2\theta'_{34}-\sin^2\theta'_{12}\sin^2\theta'_{23}\sin^2\theta'_{34})\sin 2\theta'_{13}\sin 2\theta'_{24}\sin\theta'_{14}\sin\theta'_{23}\sin\left(\theta_y-\theta_z\right) \\ &+\frac{1}{4}(\cos^2\theta'_{12}\cos^2\theta'_{13}\cos^2\theta'_{24}-\cos^2\theta'_{12}\cos^2\theta'_{24}\sin^2\theta'_{13}\sin^2\theta'_{14}-\cos^2\theta'_{13}\cos^2\theta'_{24}\sin^2\theta'_{12}\sin^2\theta'_{14} \\ &+\cos^2\theta'_{24}\sin^2\theta'_{12}\sin^2\theta'_{13}\sin^2\theta'_{14})\sin 2\theta'_{23}\sin 2\theta'_{34}\sin\theta'_{23}\sin\left(\theta_x-\theta_y+\theta_z\right) \\ &-\frac{1}{4}(\cos^2\theta'_{13}\cos^2\theta'_{24}\sin^2\theta'_{23}-\cos^2\theta'_{13}\sin^2\theta'_{23}\sin^2\theta'_{24}\sin^2\theta'_{14}-\cos^2\theta'_{24}\sin^2\theta'_{23}\sin^2\theta'_{13}\sin^2\theta'_{14}) \\ &\sin 2\theta'_{12}\sin 2\theta'_{34}\sin\theta'_{13}\sin\theta'_{24}\sin\left(\theta_x-\theta_y+2\theta_z\right) \\ &-\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{24}\sin 2\theta'_{34}\cos\theta'_{13}\cos\theta'_{24}\sin^2\theta'_{14}\sin\left(\theta_x+\theta_y\right) \\ &+\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{23}\sin 2\theta'_{34}\sin\theta'_{13}\cos\theta'_{24}\sin\theta'_{14}\sin(\theta_x-\theta_z) \\ &+\frac{1}{8}\left(\cos^2\theta'_{34}-\sin^2\theta'_{34}\right)\sin 2\theta'_{24}\sin 2\theta'_{23}\sin 2\theta'_{34}\sin\theta'_{13}\cos\theta'_{12}\sin\theta'_{24}\sin\theta'_{14}\sin\theta'_{23}\sin\left(\theta_y-2\theta_z\right) \\ &-\frac{1}{8}\left(\cos^2\theta'_{12}-\sin^2\theta'_{12}\right)\sin 2\theta'_{13}\sin 2\theta'_{23}\sin 2\theta'_{34}\sin^2\theta'_{24}\cos\theta'_{24}\sin\theta'_{14}\sin\theta'_{23}\sin\left(\theta_x-2\theta_y+2\theta_z\right) \\ &-\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{23}\sin 2\theta'_{34}\cos\theta'_{12}\cos\theta'_{23}\sin\theta'_{24}\cos\theta'_{34}\sin\theta'_{14}\sin\theta'_{34}\sin\left(\theta_x-2\theta_y+\theta_z\right) \\ &+\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{24}\sin 2\theta'_{34}\sin\theta'_{13}\sin\theta'_{14}\sin^2\theta'_{23}\sin\theta'_{24}\sin\left(\theta_x-2\theta_y+3\theta_z\right) \quad (36)\end{aligned}$$

Table 1 Some value of Jarlskog Determinant above GUT scale for various value of Majorana phases. Input value of mixing angles $\theta_{13} = 10^o, \theta_{23} = 45^o, \theta_{12} = 34^o, \theta_{34} = 18.5^o, \theta_{24} = 4^o, \theta_{14} = 3.6^o$

α	β	γ	$J'_{se} \times 10^{-4}$	$J'_{e\mu} \times 10^{-4}$	$J'_{\tau s} \times 10^{-4}$
0^0	45^0	0^0	16.3268	-16.3216	0.1575
0^0	45^0	45^0	11.1673	-12.0438	-0.7514
0^0	45^0	90^0	15.9772	-16.226	0.1293
0^0	45^0	135^0	21.1395	-20.5047	1.0385
0^0	45^0	180^0	16.3268	-16.3216	0.1575
0^0	135^0	0^0	-16.3268	16.3216	-0.1575
0^0	135^0	90^0	-15.9772	16.226	-0.1293
0^0	135^0	180^0	-16.3268	16.3216	-0.1575
45^0	45^0	0^0	4.9008	-4.2205	0.8779
45^0	45^0	45^0	-0.0007	-0.0056	-0.0036
45^0	45^0	90^0	4.9861	-4.2947	0.8925
45^0	45^0	135^0	9.8869	-8.5091	1.7739
45^0	45^0	180^0	4.9008	-4.2205	0.8779
45^0	90^0	0^0	-11.4116	12.1015	0.7409
45^0	90^0	90^0	-10.9707	11.928	0.7845
45^0	90^0	180^0	-11.4116	12.1015	0.7409
45^0	135^0	0^0	-26.7413	27.5152	0.5718
45^0	135^0	45^0	-31.3015	31.6378	-0.2819
45^0	135^0	90^0	-25.9765	27.2563	0.6412
45^0	135^0	135^0	-21.4225	23.1361	1.4944
45^0	135^0	180^0	-26.7413	27.5152	0.5718
45^0	180^0	0^0	-10.4681	11.2299	0.7096
45^0	180^0	90^0	-10.0577	11.0699	0.7503
45^0	180^0	180^0	-10.4681	11.2299	0.7096
90^0	45^0	0^0	15.341	-15.4407	0.1486
90^0	45^0	90^0	15.0103	-15.3514	0.1219
90^0	45^0	180^0	15.341	-15.4407	0.1486
90^0	135^0	0^0	-15.341	15.4407	-0.1486
90^0	135^0	90^0	-15.0103	15.3514	-0.1219
90^0	135^0	180^0	-15.341	15.4407	-0.1486
135^0	45^0	0^0	26.7413	-27.5152	-0.5718
135^0	45^0	45^0	21.4225	-23.1361	-1.4944
135^0	45^0	90^0	25.9765	-27.2563	-0.6412
135^0	45^0	135^0	31.3015	-31.6378	-0.2819
135^0	45^0	180^0	26.7413	-27.5152	-0.5718
135^0	90^0	0^0	11.4116	-12.1015	-0.7409
135^0	90^0	90^0	10.9707	-11.928	-0.7845
135^0	90^0	180^0	11.4116	-12.1015	-0.7409
135^0	135^0	0^0	-4.9008	4.2205	-0.8779
135^0	135^0	45^0	-9.8868	8.5091	-1.7739
135^0	135^0	90^0	-4.9861	4.2947	-0.8925

Table 1 (continued)

α	β	γ	$J'_{se} \times 10^{-4}$	$J'_{e\mu} \times 10^{-4}$	$J'_{\tau s} \times 10^{-4}$
135^0	135^0	135^0	0.0007	0.0056	0.0036
135^0	135^0	180^0	-4.9008	4.2205	-0.8779
135^0	180^0	0^0	10.4681	-11.2299	-0.7096
135^0	180^0	90^0	10.0577	-11.699	-0.7503
135^0	180^0	180^0	10.4681	-11.2299	-0.7096

$$
\begin{aligned}
J_{e\mu}^{24'} &= \frac{1}{16}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\sin 2\theta'_{34}\cos\theta'_{14}\cos\theta'_{23}\sin\theta'_{14}\sin\theta'_{24}\sin\theta_x \\
&+\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{13}\cos\theta'_{14}\cos\theta'_{23}\sin^2\theta'_{34}\sin\theta_x \\
&-\frac{1}{4}\sin 2\theta'_{12}\sin 2\theta'_{24}\cos^2\theta'_{14}\cos^2\theta'_{23}\cos\theta'_{24}\cos\theta'_{34}\sin\theta'_{13}\sin\theta'_{34}\sin\left(\theta_x-\theta_y\right) \\
&+\frac{1}{16}\sin 2\theta'_{12}\sin 2\theta'_{14}\sin 2\theta'_{24}\sin 2\theta'_{34}\cos\theta'_{13}\sin\theta'_{23}\sin\theta'_{24}\sin\left(\theta_x+\theta_y\right) \\
&-\frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{14}\sin^2\theta'_{12}\sin\theta'_{23}\sin^2\theta'_{34}\sin\left(\theta_y-\theta_z\right) \\
&+\frac{1}{8}\sin 2\theta'_{12}\sin 2\theta'_{13}\sin 2\theta'_{34}\cos\theta'_{13}\cos^2\theta'_{14}\sin^2\theta'_{23}\sin\theta'_{24}\sin\left(\theta_x-\theta_y+2\theta_z\right) \\
&-\frac{1}{4}(\cos^2\theta'_{12}-\sin^2\theta'_{12}\sin^2\theta'_{13})\sin 2\theta'_{23}\sin 2\theta'_{24}\cos^2\theta'_{14}\cos\theta'_{34}\cos\theta'_{24}\sin\theta'_{34}\sin\left(\theta_x-\theta_y+\theta_z\right)
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
J_{e\mu}^{34'} &= -\frac{1}{16}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\sin 2\theta'_{34}\cos\theta'_{14}\cos\theta'_{23}\sin\theta'_{24}\sin\theta_x \\
&+\frac{1}{8}\sin 2\theta'_{13}\sin 2\theta'_{14}\sin 2\theta'_{24}\cos\theta'_{14}\sin\theta'_{23}\sin^2\theta'_{34}\sin\left(\theta_y-\theta_z\right) \\
&+\frac{1}{4}\sin 2\theta'_{23}\sin 2\theta'_{24}\cos^2\theta'_{13}\cos^2\theta'_{14}\cos\theta'_{24}\cos\theta'_{34}\sin\theta'_{34}\sin\left(\theta_x-\theta_y+\theta_z\right)
\end{aligned} \tag{38}
$$

where,

$$
\begin{aligned}
\theta_x &= \delta_{14}-\delta_{13}-\delta_{34} \\
\theta_y &= \delta_{14}-\delta_{12}-\delta_{24} \\
\theta_z &= \delta_{13}-\delta_{12}-\delta_{23}
\end{aligned} \tag{39}
$$

The values of six CP-violating phases δ_{ij} are between 0 and 2π.

5 Numerical Results

Since, the role of Jarkslog invariant that is apear in the oscillation probability (see (2)) seems to be in additive manner as given in (9). Hence, we are numerically calculated the collective role of Jarlskog invariant of a particular oscillation is given by

$$
\begin{aligned}
J'_{se} &= J_{se}^{13'}+J_{se}^{24'}+J_{se}^{34'} \\
J'_{\tau s} &= J_{\tau s}^{13'}+J_{\tau s}^{14'}+J_{\tau s}^{34'} \\
J'_{e\mu} &= J_{e\mu}^{23'}+J_{e\mu}^{24'}+J_{e\mu}^{34'}
\end{aligned} \tag{40}
$$

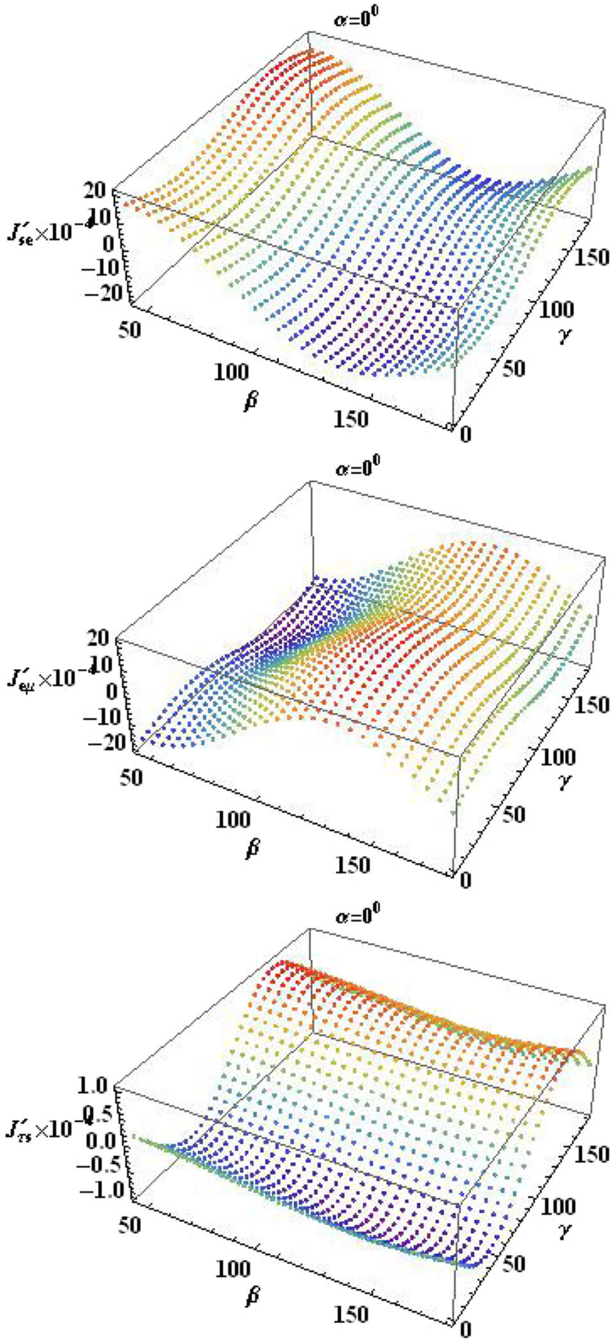

Fig. 1 For $\alpha = 0^0$, the variation of J'_{se} , $J'_{\tau s}$ *and* $J'_{e\mu}$ with β *and* γ. Input values are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\Delta_{41} = 1.7 eV^2$ and mixing angles $\theta_{13} = 10^o, \theta_{23} = 45^o, \theta_{12} = 34^o$,$\theta_{34} = 18.5^o, \theta_{24} = 4^o, \theta_{14} = 3.6^o$

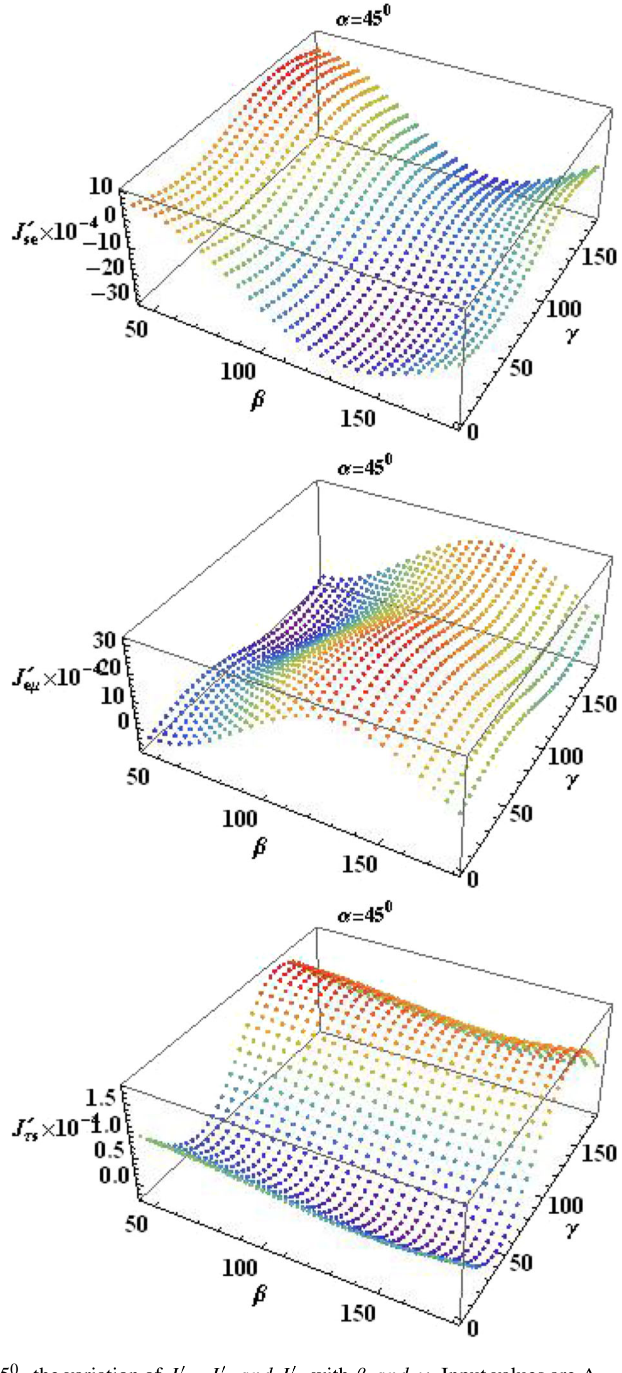

Fig. 2 For $\alpha = 45^0$, the variation of J'_{se} , $J'_{\tau s}$ and $J'_{e\mu}$ with β and γ. Input values are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\Delta_{41} = 1.7 eV^2$ and mixing angles $\theta_{13} = 10^o, \theta_{23} = 45^o, \theta_{12} = 34^o$,$\theta_{34} = 18.5^o, \theta_{24} = 4^o, \theta_{14} = 3.6^o$

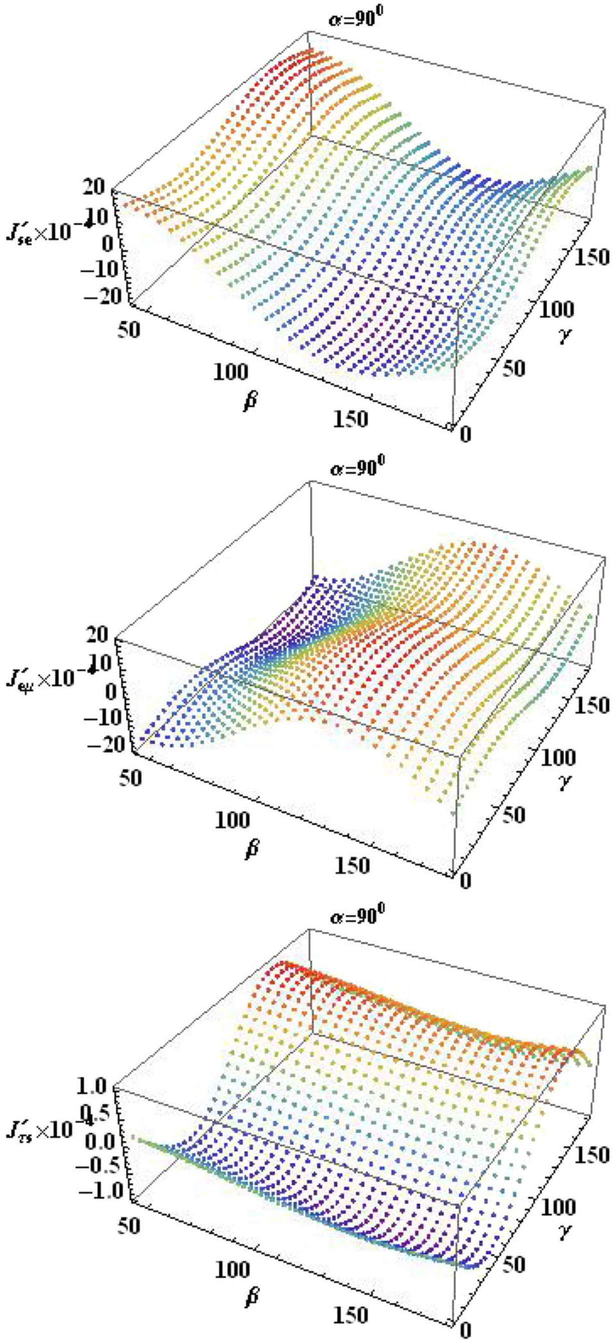

Fig. 3 For $\alpha = 90^0$, the variation of J'_{se} , $J'_{\tau s}$ *and* $J'_{e\mu}$ with β *and* γ. Input values are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\Delta_{41} = 1.7 eV^2$ and mixing angles $\theta_{13} = 10^o, \theta_{23} = 45^o, \theta_{12} = 34^o$,$\theta_{34} = 18.5^o, \theta_{24} = 4^o, \theta_{14} = 3.6^o$

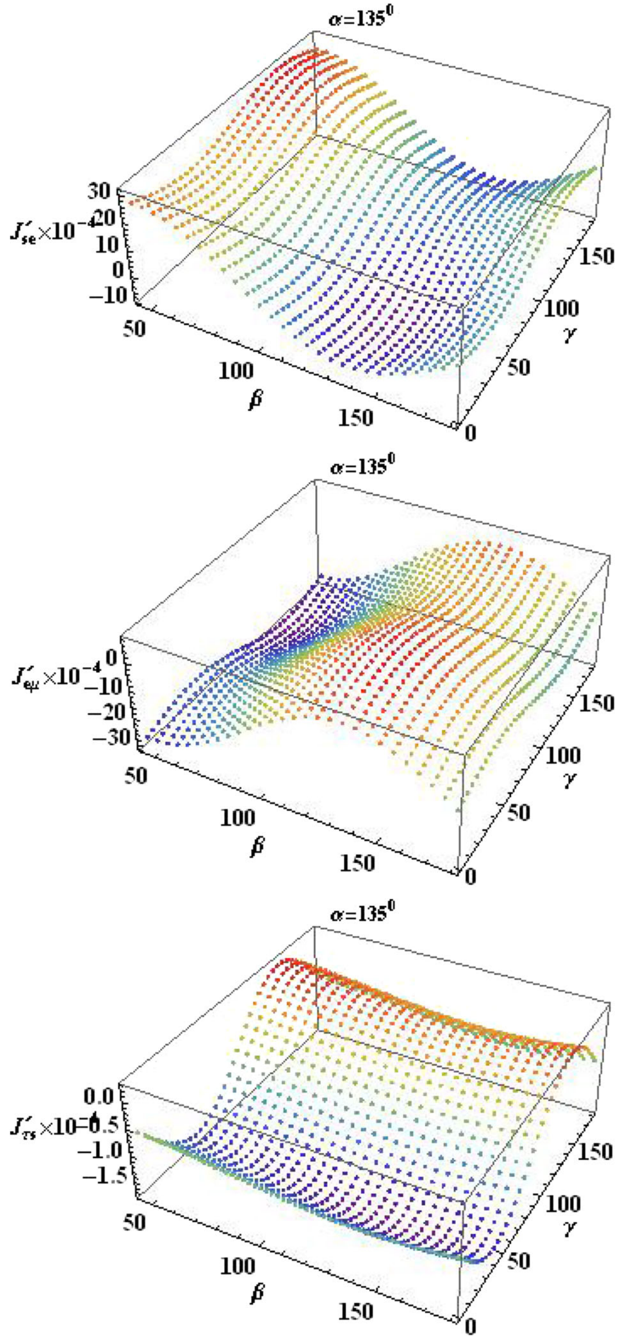

Fig. 4 For $\alpha = 135^0$, the variation of J'_{se}, $J'_{\tau s}$ *and* $J'_{e\mu}$ with β *and* γ. Input values are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\Delta_{41} = 1.7 eV^2$ and mixing angles $\theta_{13} = 10^o, \theta_{23} = 45^o, \theta_{12} = 34^o$,$\theta_{34} = 18.5^o, \theta_{24} = 4^o, \theta_{14} = 3.6^o$

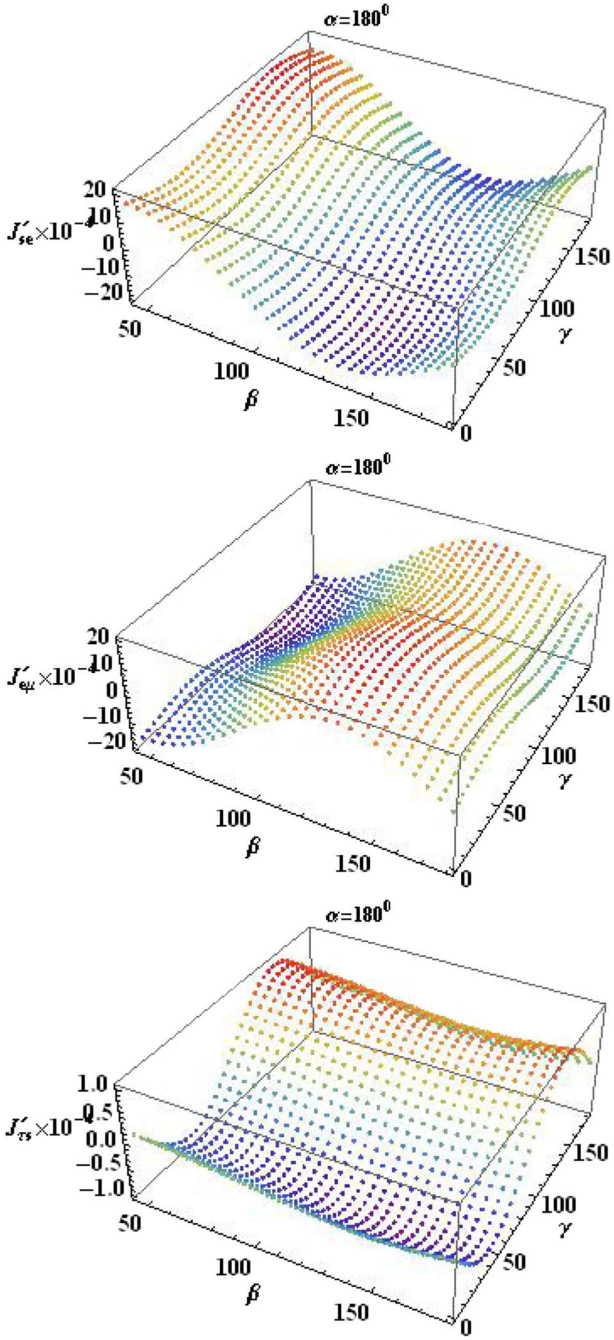

Fig. 5 For $\alpha = 180^0$, the variation of J'_{se}, $J'_{\tau s}$ *and* $J'_{e\mu}$ with β *and* γ. Input values are $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\Delta_{41} = 1.7 eV^2$ and mixing angles $\theta_{13} = 10^o, \theta_{23} = 45^o, \theta_{12} = 34^o$,$\theta_{34} = 18.5^o, \theta_{24} = 4^o, \theta_{14} = 3.6^o$

We know that the neutrino are almost degenerate just above the electroweak breaking scale. Therefore, we take a common neutrino mass to be 2 eV, which is upper limit of tritium beta decay spectrum [25]. The contribution of the term in beyond the GUT scale, $\epsilon = 2(M_i Re(m_{ii}) - M_j Re(m_{jj})$, can be additive or subtractive depending on the values of the phase α, β,γ and phase f_i. We have taken $\Delta_{31} = 0.002eV^2$ [26, 27], $\Delta_{21} = 0.00008eV^2$ [28] and $\Delta_{41} = 1.7eV^2$ [29] . For simplicity, we have set the charge lepton phases $f_1 = f_2 = f_3 = f_4 = 0$and $\delta_{ij} = 0$. The active sterile neutrino mixing angle are $\theta_{14},\ \theta_{24}\ and\ \theta_{34}$. In this work, we consider following value for sterile neutrino mixing angles [30], $\theta_{14} = 3.6^o$, $\theta_{24} = 4^o$, $\theta_{34} = 18.5^o$. In Table 1, we list combine value of Jarlskog Determinant above GUT scale for various value of Majorana phases α, β and γ. In Figs. 1, 2, 3, 4 and 5, we plot the Jarlskog Determinant with β and γ for different α values. As you can see from figures, the trend of graph is nearly same for different values of α but with different magnitudes of Jarlskog determinant which is in vertical axis.

6 Conclusions

The understanding of Jarlskog determinant is important for flavor physics. In both the quark and lepton sectors, this Jarlskog invariant provides the ability to control the magnitude of CP violation. We studied the Jarlskog determinant in four flavor framework above the GUT scale. We calculated the Jarlskog determinant by new MNS matrix in the form of 4x4 matrices, (19), by using four flavor mixing angle above the GUT scale, (23–28) with current neutrino mixing angles. The values of Jarlskog determinant shows some phase inversion for some combinations of Majorana phases α, $\beta\ and\ \gamma$.. Since, the magnitude of Jarlskog determinant above the GUT scale is of the order of $\sim 10^{-23}$ .Hence, we tabulated the Jarlskog determinant which has much larger magnitude for some particular combination of Majorana phases α, $\beta\ and\ \gamma$.. In the lepton sector above the GUT scale, we can constraint the Jarlskog determinant for normal mass order which is given as $-31.3015 \times 10^{-4} \leq J'_{se} \leq 31.3015 \times 10^{-4}$,$-31.6378 \times 10^{-4} \leq J'_{se} \leq 31.6378 \times 10^{-4}$ and $-1.7739 \times 10^{-4} \leq J'_{se} \leq 1.7739 \times 10^{-4}$.

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